CSE-571
Probabilistic Robotics

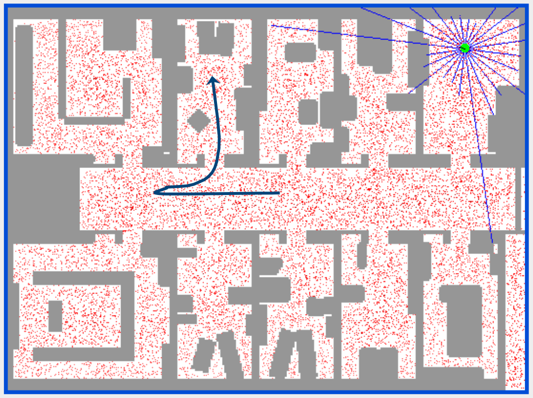
Bayes Filter Implementations

Particle filters

Motivation

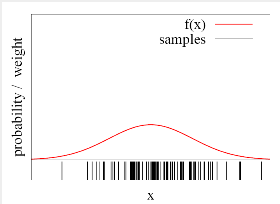
- So far, we discussed the
  - Kalman filter: Gaussian, linearization problems
- Particle filters are a way to **efficiently** represent **non-Gaussian distributions**
- Basic principle
  - Set of state hypotheses (“particles”)
  - Survival-of-the-fittest

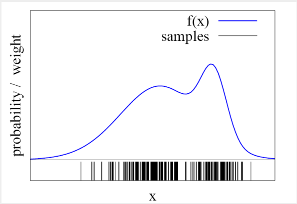
Sample-based Localization (sonar)



Function Approximation

- Particle sets can be used to approximate densities

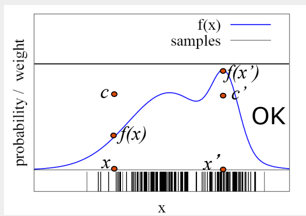




- The more particles fall into an interval, the higher the probability of that interval
- How to draw samples form a function/distribution?

Rejection Sampling

- Let us assume that $f(x) <= 1$ for all x
- Sample x from a uniform distribution
- Sample c from [0,1]
- if $f(x) > c$ keep the sample
  otherwise reject the sampe



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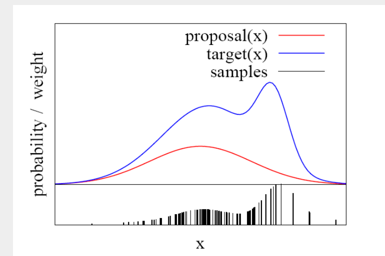
Importance Sampling Principle

- We can even use a different distribution g to generate samples from f
- By introducing an importance weight w, we can account for the "differences between g and f"
- $w = f / g$
- f is often called target
- g is often called proposal



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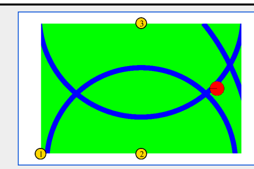
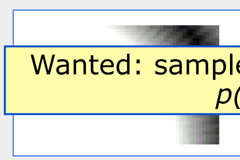
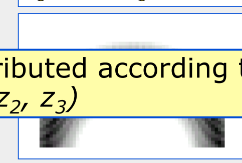
Importance Sampling with Resampling: Landmark Detection Example

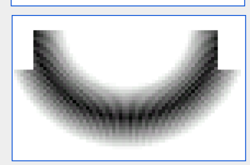
Distributions





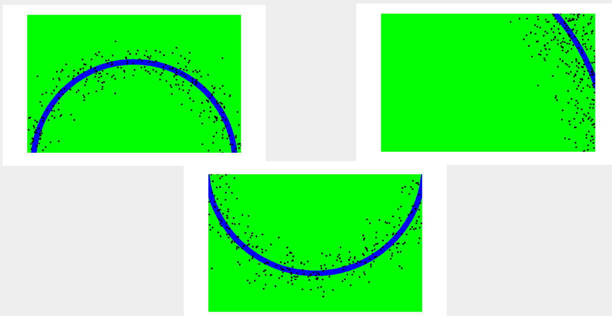


Wanted: samples distributed according to $p(x| z_1, z_2, z_3)$



This is Easy!

We can draw samples from $p(x|z_l)$ by adding noise to the detection parameters.



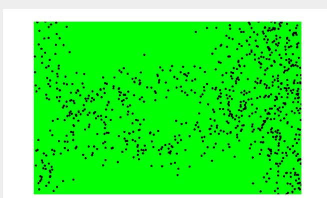
Importance Sampling with Resampling

$$\text{Target distribution f}: p(x|z_1,z_2,\ldots,z_n) = \frac{\prod_k p(z_k|x) \quad p(x)}{p(z_1,z_2,\ldots,z_n)}$$

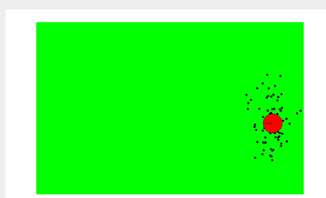
$$\text{Sampling distribution g}: p(x|z_l) = \frac{p(z_l|x)p(x)}{p(z_l)}$$

$$\text{Importance weights w}: \frac{f}{g} = \frac{p(x|z_1,z_2,\ldots,z_n)}{p(x|z_l)} = \frac{p(z_l)\prod_{k\neq l} p(z_k|x)}{p(z_1,z_2,\ldots,z_n)}$$

Importance Sampling with Resampling



Weighted samples

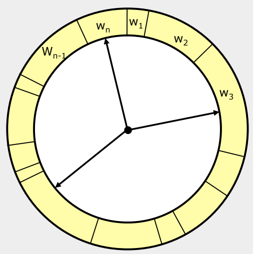


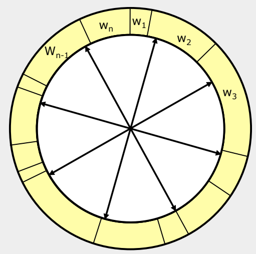
After resampling

Resampling

- **Given**: Set S of weighted samples.
- **Wanted** : Random sample, where the probability of drawing x_i is given by w_i.
- Typically done n times with replacement to generate new sample set S'.

Resampling





- Roulette wheel
- Binary search, n log n

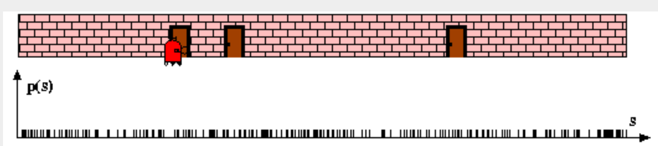
- Stochastic universal sampling
- Systematic resampling
- Linear time complexity
- Easy to implement, low variance

Resampling Algorithm

1. Algorithm **systematic_resampling**(*S*,*n*):
2. $S' = \emptyset, c_1 = w^1$
3. **For** $i = 2 \dots n$ — ***Generate cdf***
4. $\quad c_i = c_{i-1} + w^i$
5. $u_1 \sim U[0, n^{-1}], i = 1$ — ***Initialize threshold***
6. **For** $j = 1 \dots n$ — ***Draw samples …***
7. $\quad$ **While** $(u_j > c_i)$ — ***Skip until next threshold reached***
8. $\qquad i = i + 1$
9. $\quad S' = S' \cup \{< x^i, n^{-1} >\}$ — ***Insert***
10. $\quad u_j = u_j + n^{-1}$ — ***Increment threshold***
11. **Return** S'

Also called **stochastic universal sampling**

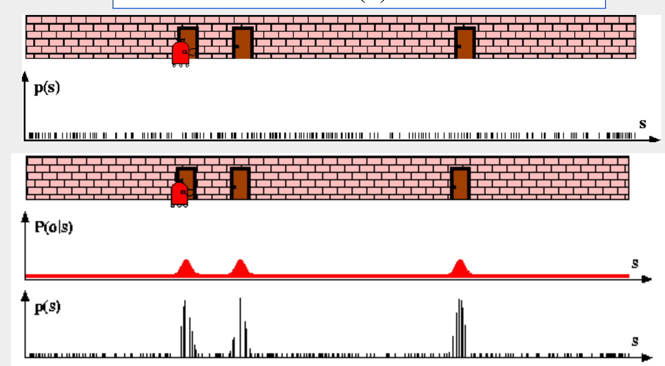
Particle Filters



Sensor Information: Importance Sampling

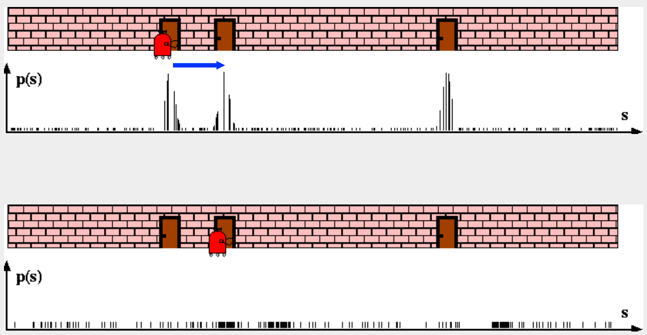
$$Bel(x) \leftarrow \alpha\, p(z \mid x)\, Bel^-(x)$$

$$w \leftarrow \frac{\alpha\, p(z \mid x)\, Bel^-(x)}{Bel^-(x)} = \alpha\, p(z \mid x)$$



Robot Motion

$$Bel^-(x) \leftarrow \int p(x \mid u, x')\, Bel(x')\, \mathrm{d}x'$$



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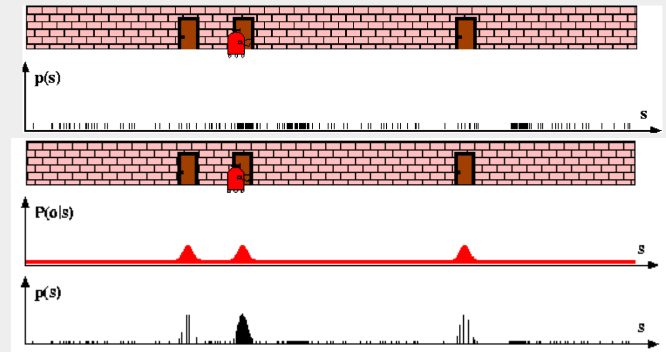

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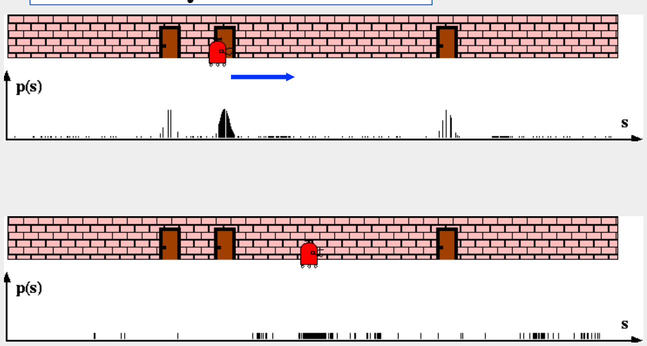


Particle Filter Algorithm

1. Algorithm **particle_filter**(S_{t-1}, u_{t-1} z_t):
2. $S_t = \emptyset, \quad \eta = 0$
3. **For** $i = 1 \ldots n$ — ***Generate new samples***
4. Sample index *j(i)* from the discrete distribution given by w_{t-1}
5. Sample x_t^i from $p(x_t \mid x_{t-1}, u_{t-1})$ using $x_{t-1}^{j(i)}$ and u_{t-1}
6. $w_t^i = p(z_t \mid x_t^i)$ — ***Compute importance weight***
7. $\eta = \eta + w_t^i$ — ***Update normalization factor***
8. $S_t = S_t \cup \{< x_t^i, w_t^i >\}$ — ***Insert***
9. **For** $i = 1 \ldots n$
10. $w_t^i = w_t^i / \eta$ — ***Normalize weights***

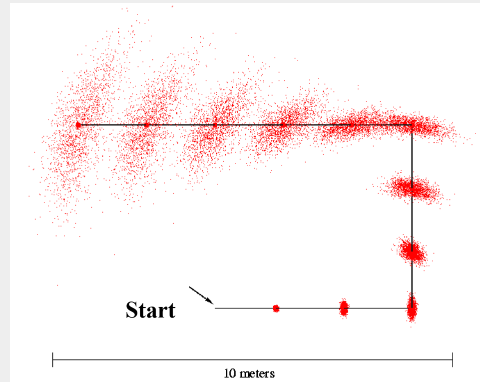
Particle Filter Algorithm

$$Bel(x_t) = \eta \, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_{t-1}) \, Bel(x_{t-1}) \, dx_{t-1}$$

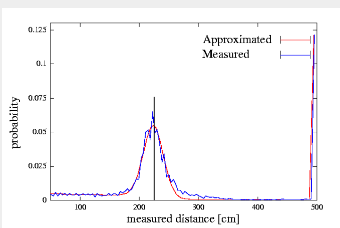
- draw x^i_{t-1} from $Bel(x_{t-1})$
- draw x^i_t from $p(x_t \mid x^i_{t-1}, u_{t-1})$
- Importance factor for x^i_t:

$$w^i_t = \frac{\text{target distribution}}{\text{proposal distribution}} = \frac{\eta \, p(z_t \mid x_t) \, p(x_t \mid x_{t-1}, u_{t-1}) \, Bel(x_{t-1})}{p(x_t \mid x_{t-1}, u_{t-1}) \, Bel(x_{t-1})} \propto p(z_t \mid x_t)$$

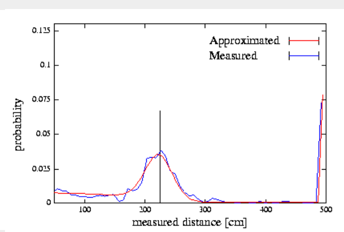
Motion Model Reminder



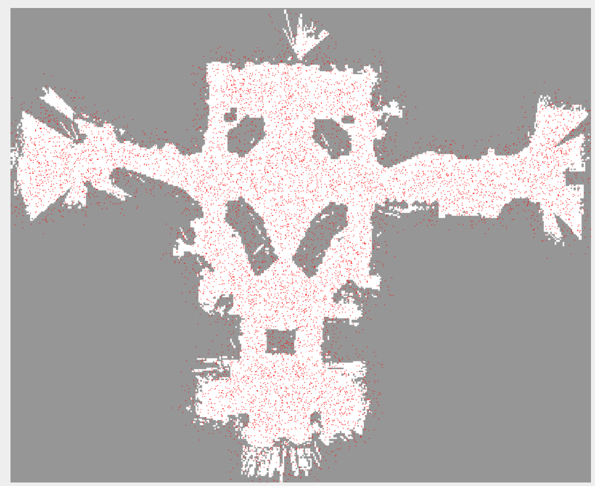
Proximity Sensor Model Reminder



Laser sensor



Sonar sensor



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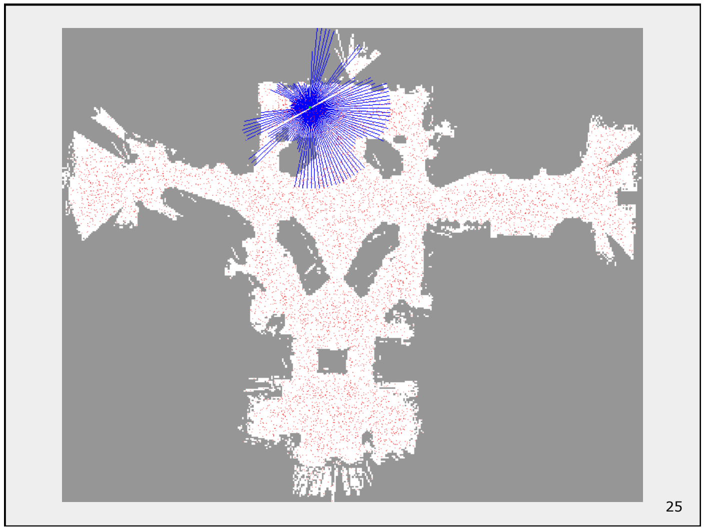

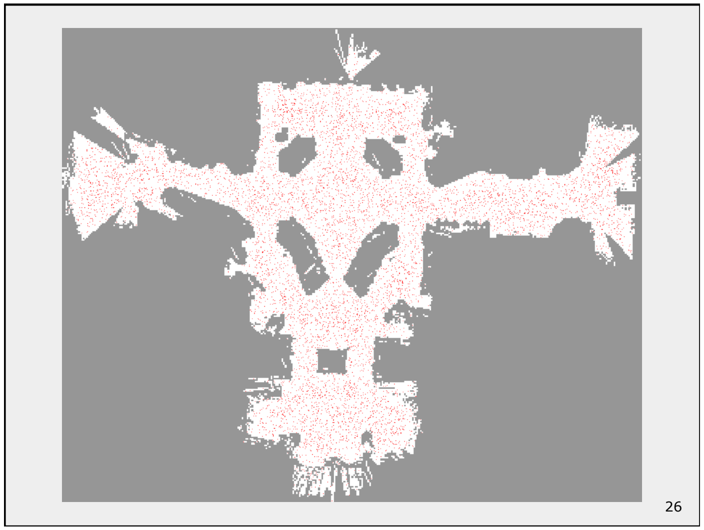

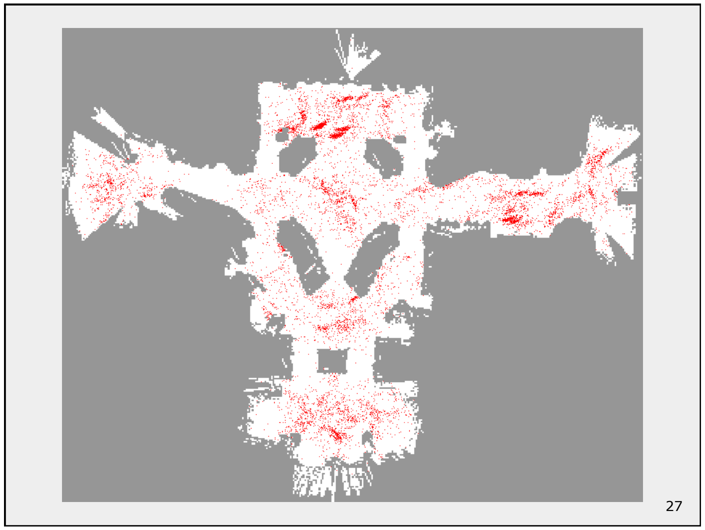

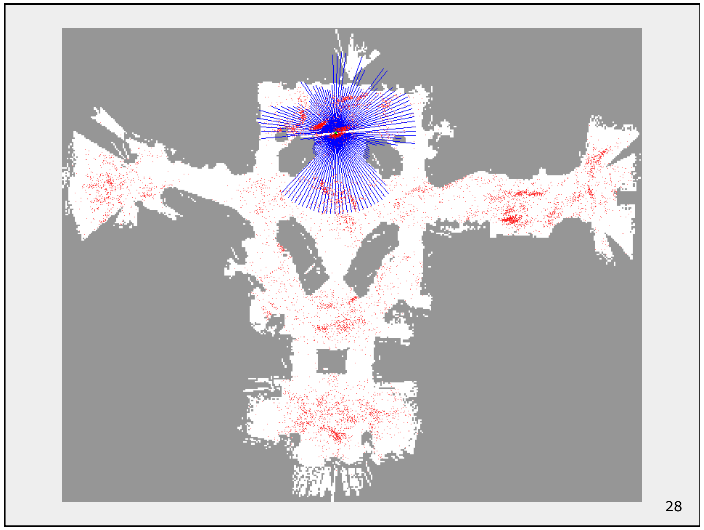

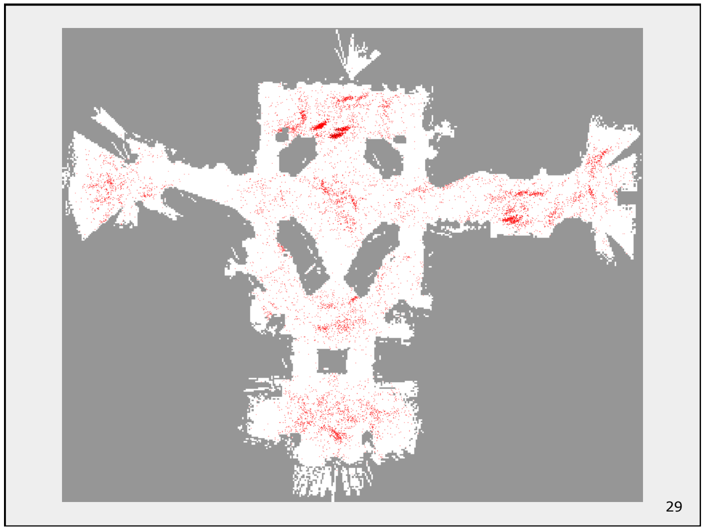
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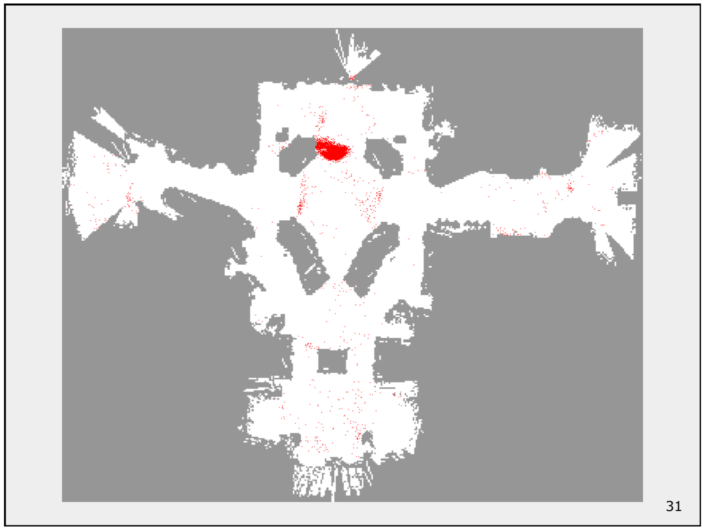
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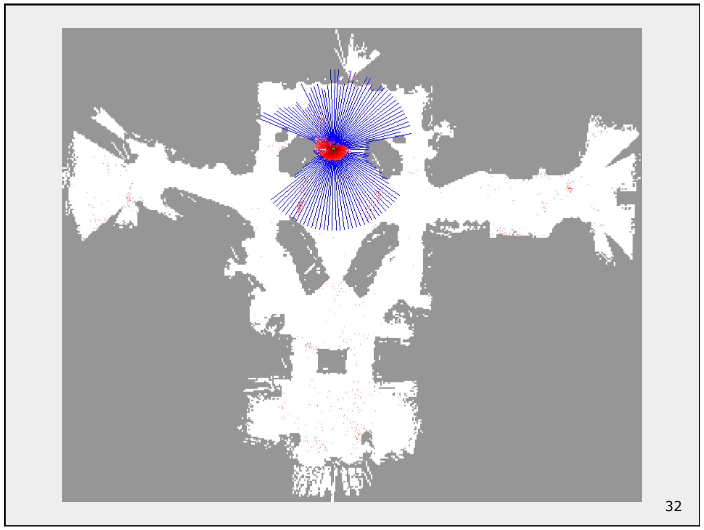
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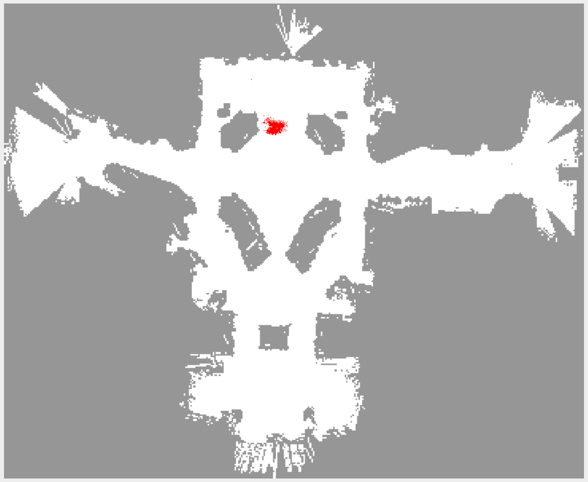

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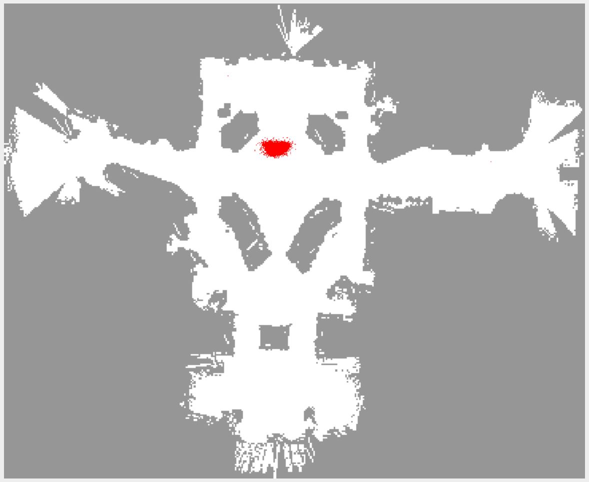

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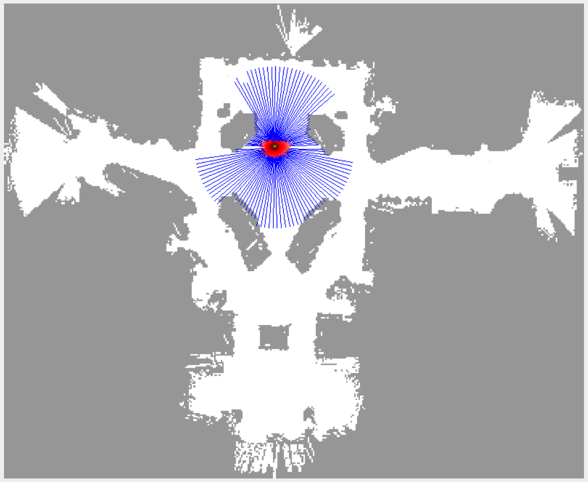

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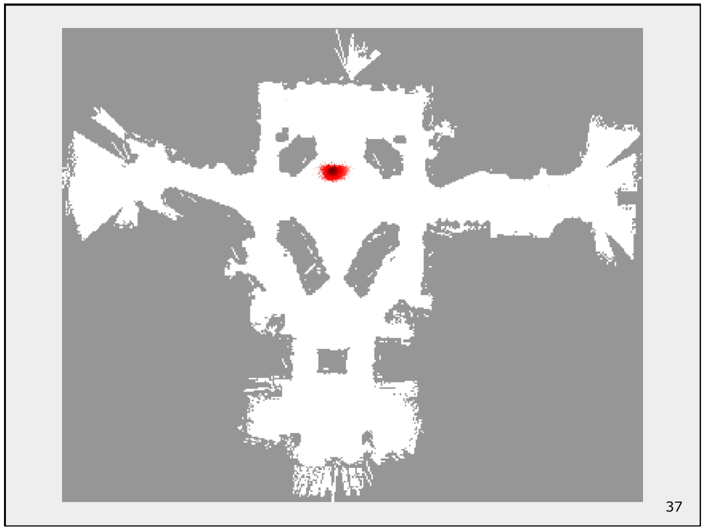
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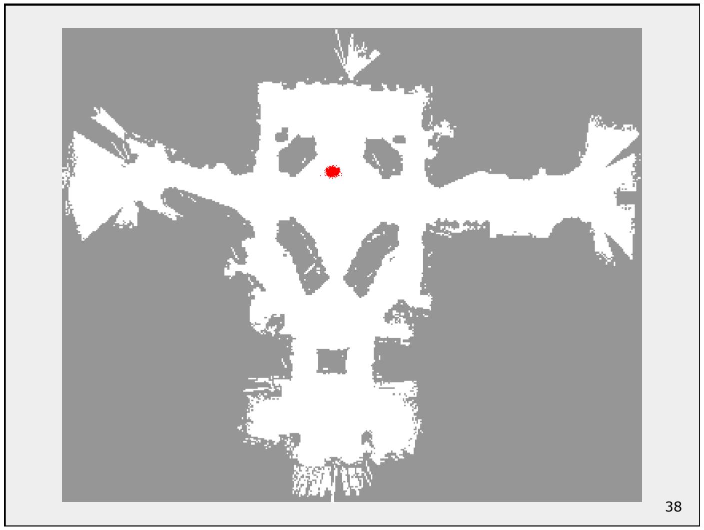
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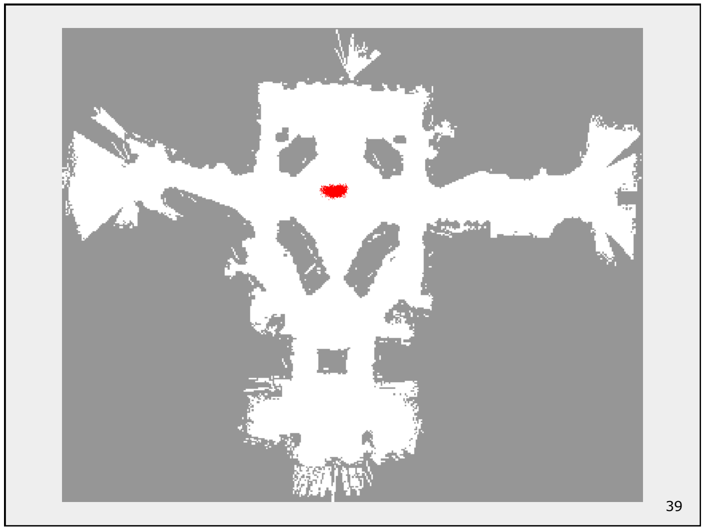
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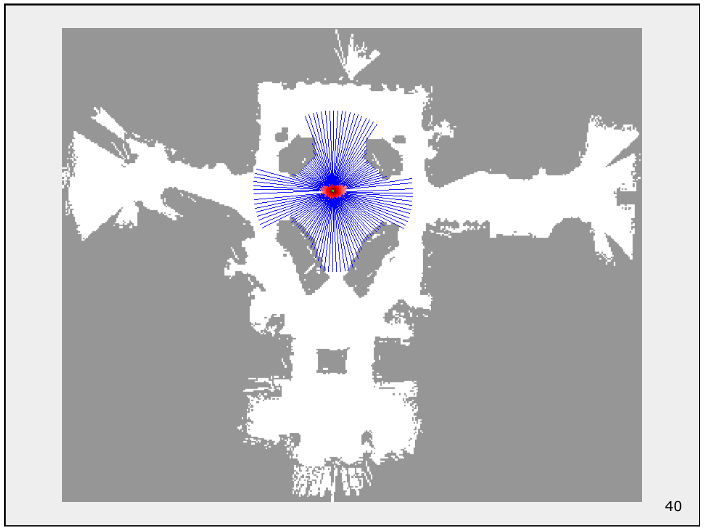
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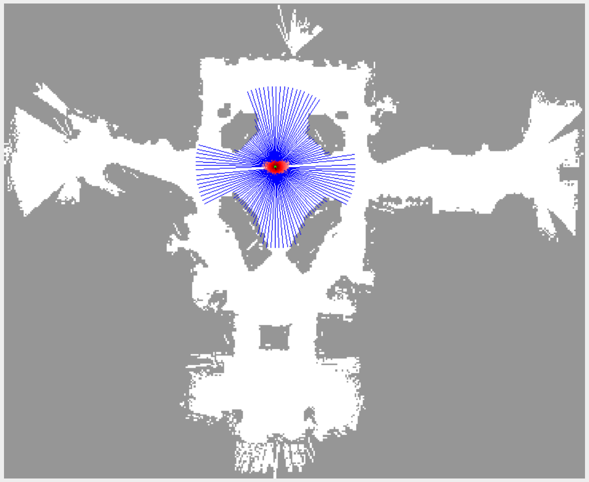

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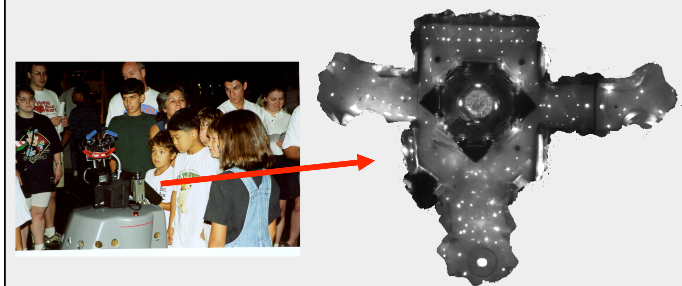
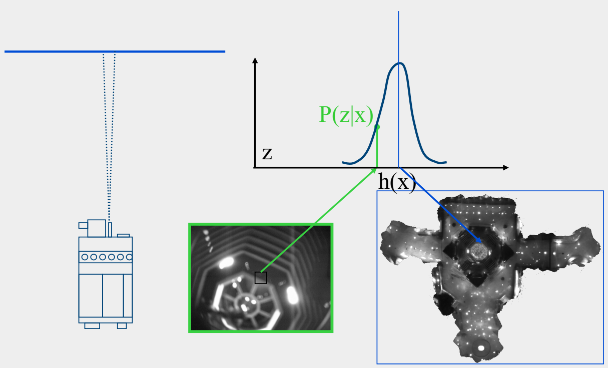
Using Ceiling Maps for Localization



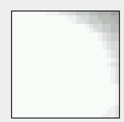
[Dellaert et al. 99]

Vision-based Localization

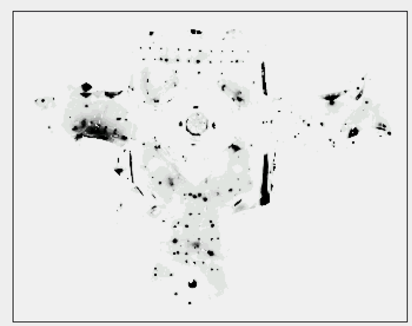


Under a Light

Measurement z:

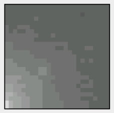


***P(z|x)*:**

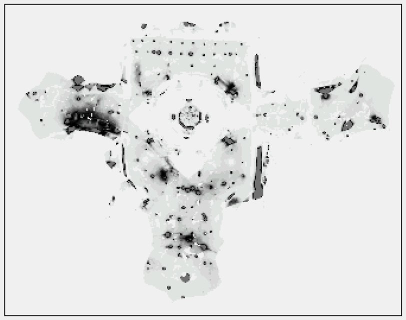


Next to a Light

Measurement z:



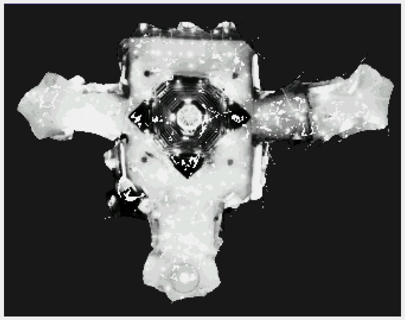
***P(z|x)*:**



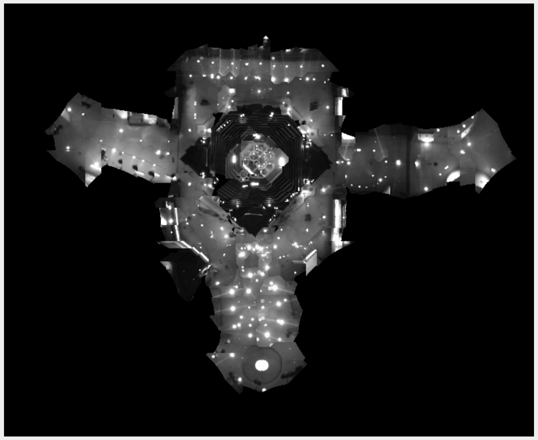
Elsewhere

Measurement z:

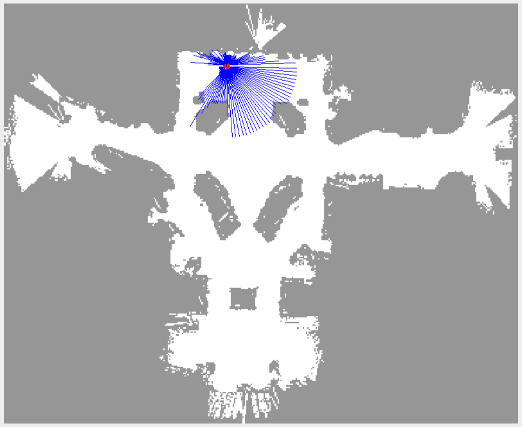
***P(z|x)*:**



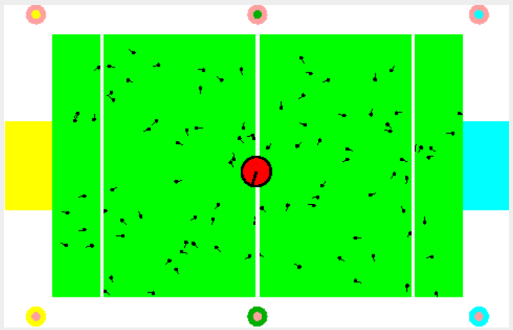
Global Localization Using Vision



Recovery from Failure

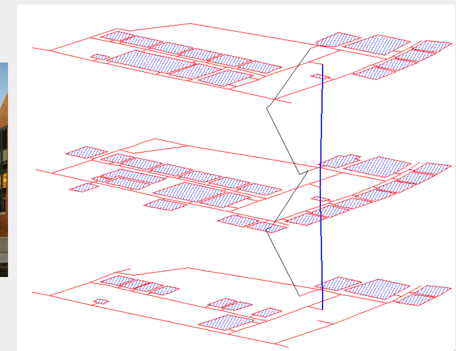


Localization for AIBO robots

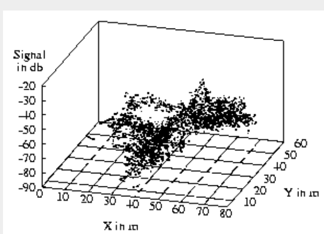


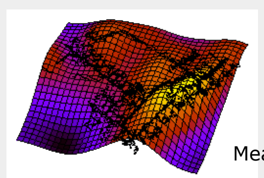
[Ferris-Haehnel-Fox: RSS-06]

Hybrid Model for People Tracking

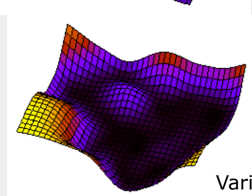


WiFi Sensor Model



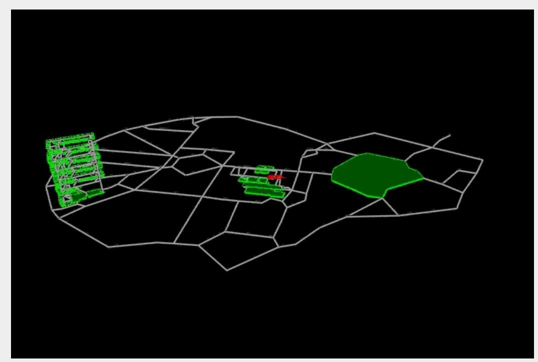


Mean

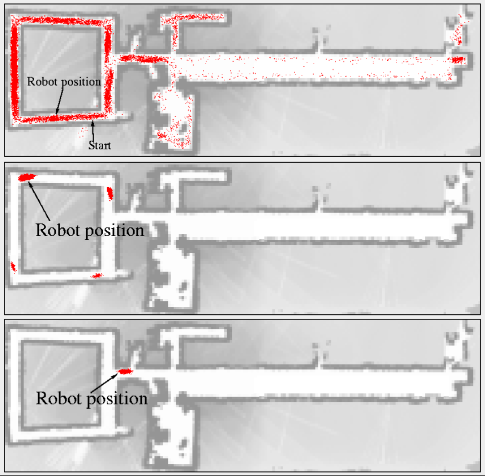


Variance

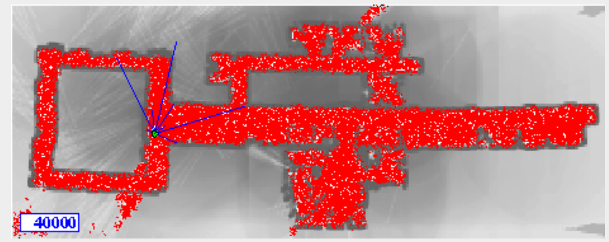
Tracking Example



Adaptive Sampling

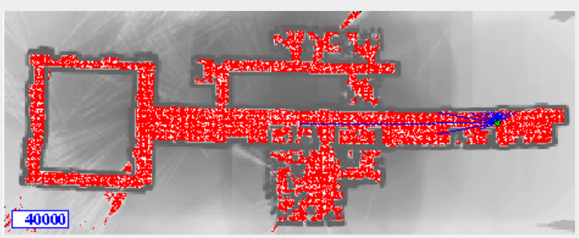


KLD-Sampling Sonar

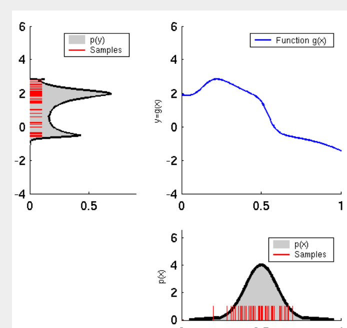


Adapt number of particles on the fly based on statistical approximation measure

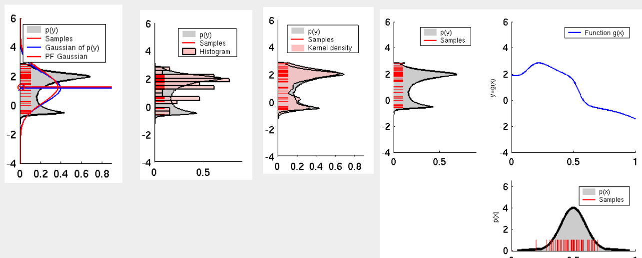
KLD-Sampling Laser



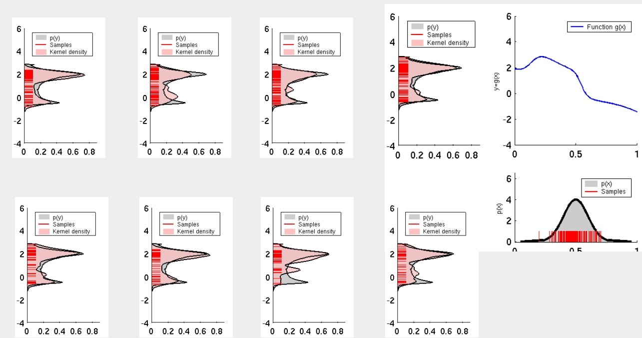
Particle Filter Projection



Density Extraction



Sampling Variance



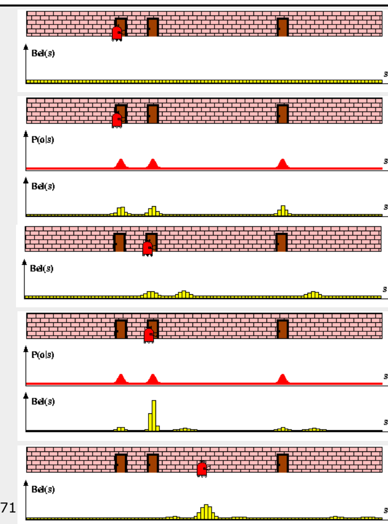
CSE-571
Probabilistic Robotics

Bayes Filter Implementations

Discrete filters

SA-1

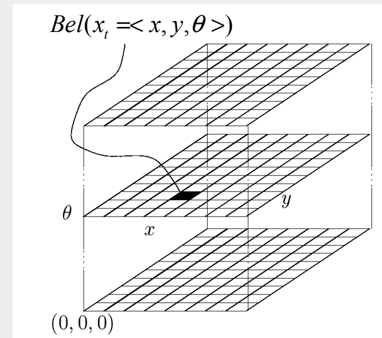
Piecewise Constant



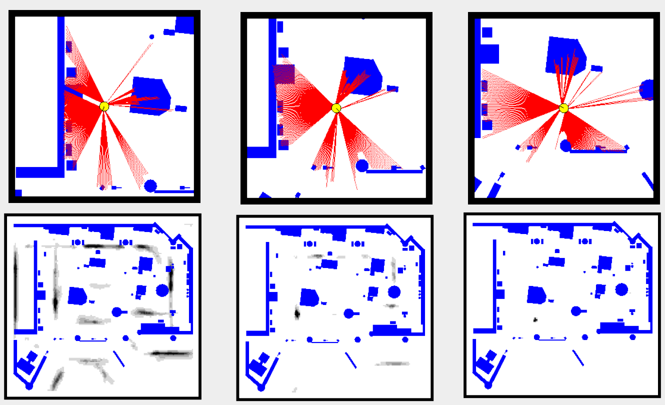
Discrete Bayes Filter Algorithm

Algorithm **Discrete_Bayes_filter**(*Bel(x),d*):
h=0
If *d* is a perceptual data item *z* then
  For all *x* do
    $Bel'(x) = P(z \mid x) Bel(x)$
    $\eta = \eta + Bel'(x)$
  For all *x* do
    $Bel'(x) = \eta^{-1} Bel'(x)$
Else if *d* is an action data item *u* then
  For all *x* do
    $Bel'(x) = \sum_{x'} P(x \mid u, x')\, Bel(x')$
Return *Bel'(x)*

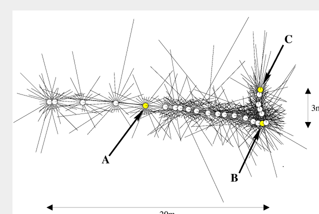
Piecewise Constant Representation



Grid-based Localization

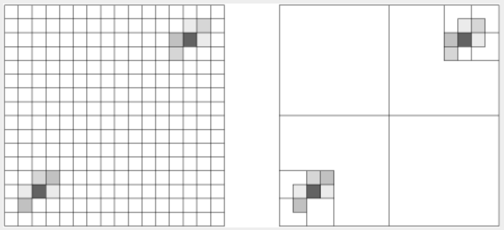


Sonars and Occupancy Grid Map



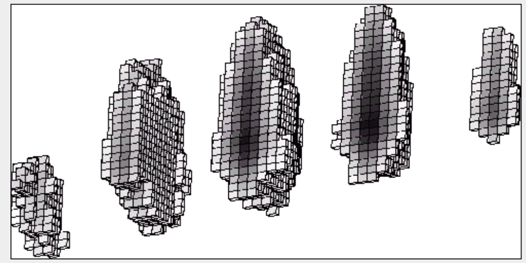
Tree-based Representation

Idea: Represent density using a variant of Octrees

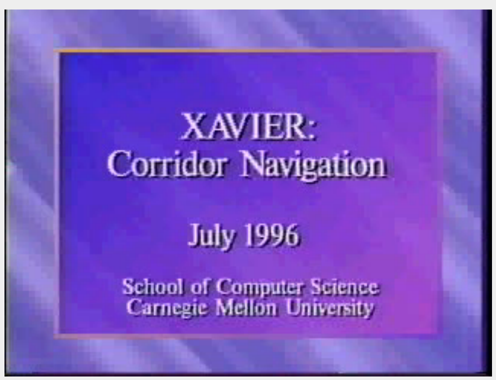


Tree-based Representations

- Efficient in space and time
- Multi-resolution



Topological Localization



Localization Algorithms - Comparison

	Kalman filter	Multi-hypothesis tracking	Topological maps	Grid-based (fixed/variable)	Particle filter
Sensors	Gaussian	Gaussian	Features	Non-Gaussian	Non-Gaussian
Posterior	Gaussian	Multi-modal	Piecewise constant	Piecewise constant	Samples
Efficiency (memory)	++	++	++	-/o	+/++
Efficiency (time)	++	++	++	o/+	+/++
Implementation	+	o	+	+/o	++
Accuracy	++	++	-	+/++	++
Robustness	-	+	+	++	+/++
Global localization	No	Yes	Yes	Yes	Yes