# CSE-571
# Probabilistic Robotics

## Probabilistic Sensor Models

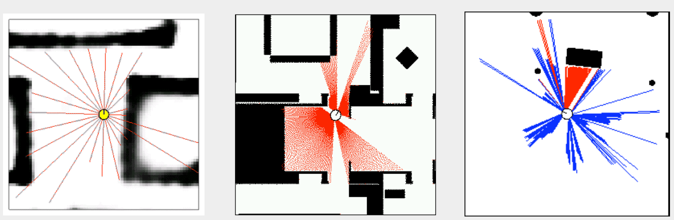
Beam-based
Scan-based
Landmarks

## Sensors for Mobile Robots

- Contact sensors: Bumpers
- Internal sensors
  - Accelerometers (spring-mounted masses)
  - Gyroscopes (spinning mass, laser light)
  - Compasses, inclinometers (earth magnetic field, gravity)
- Proximity sensors
  - Sonar (time of flight)
  - Radar (phase and frequency)
  - Laser range-finders (triangulation, tof, phase)
  - Infrared (intensity)
- Visual sensors: Cameras
- Satellite-based sensors: GPS



## Proximity Sensors



- The central task is to determine *P(z|x)*, i.e. the probability of a measurement *z* given that the robot is at position *x*.
- **Question**: Where do the probabilities come from?
- **Approach**: Let's try to explain a measurement.



## Beam-based Sensor Model

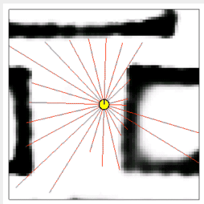
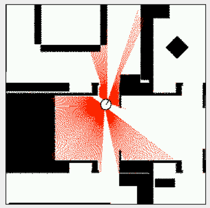
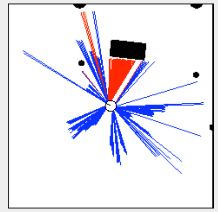
- Scan *z* consists of *K* measurements.

$$z = \{z_1, z_2, \ldots, z_K\}$$

- Individual measurements are independent given the robot position.

$$P(z \mid x, m) = \prod_{k=1}^{K} P(z_k \mid x, m)$$

# Beam-based Sensor Model

$$P(z \mid x, m) = \prod_{k=1}^{K} P(z_k \mid x, m)$$



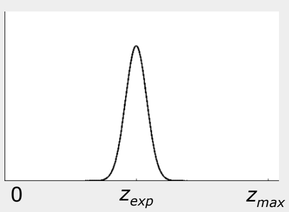
# Proximity Measurement

- Measurement can be caused by …
  - a known obstacle.
  - cross-talk.
  - an unexpected obstacle (people, furniture, …).
  - missing all obstacles (total reflection, glass, …).
- Noise is due to uncertainty …
  - in measuring distance to known obstacle.
  - in position of known obstacles.
  - in position of additional obstacles.
  - whether obstacle is missed.



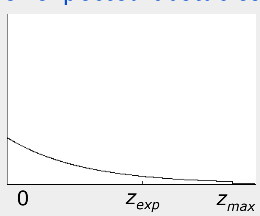
# Beam-based Proximity Model

Measurement noise




$$P_{hit}(z \mid x, m) = \eta \frac{1}{\sqrt{2\pi b}} e^{-\frac{1}{2}\frac{(z - z_{exp})^2}{b}}$$

Unexpected obstacles

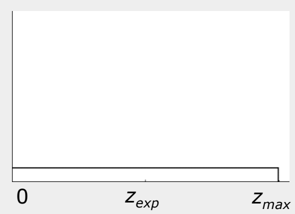



$$P_{unexp}(z \mid x, m) = \eta \; \lambda \; e^{-\lambda z}$$



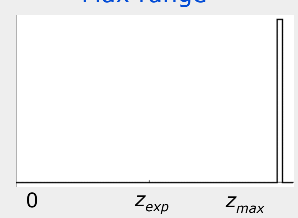
# Beam-based Proximity Model

Random measurement



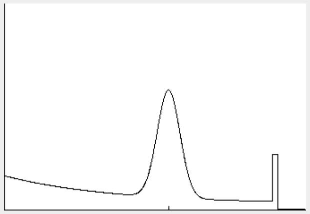

$$P_{rand}(z \mid x, m) = \eta \frac{1}{z_{\max}}$$

Max range




$$P_{\max}(z \mid x, m) = \eta \frac{1}{z_{small}}$$

# Mixture Density



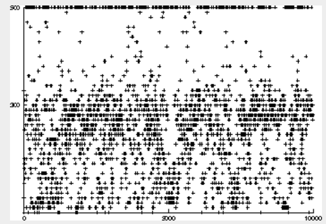
$$P(z \mid x, m) = \begin{pmatrix} \alpha_{\text{hit}} \\ \alpha_{\text{unexp}} \\ \alpha_{\text{max}} \\ \alpha_{\text{rand}} \end{pmatrix}^T \cdot \begin{pmatrix} P_{\text{hit}}(z \mid x, m) \\ P_{\text{unexp}}(z \mid x, m) \\ P_{\text{max}}(z \mid x, m) \\ P_{\text{rand}}(z \mid x, m) \end{pmatrix}$$

How can we determine the model parameters?

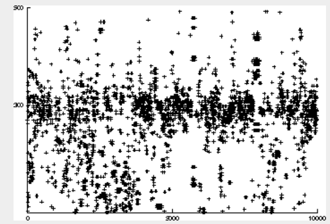


# Raw Sensor Data

Measured distances for expected distance of 300 cm.



Sonar



Laser



# Approximation

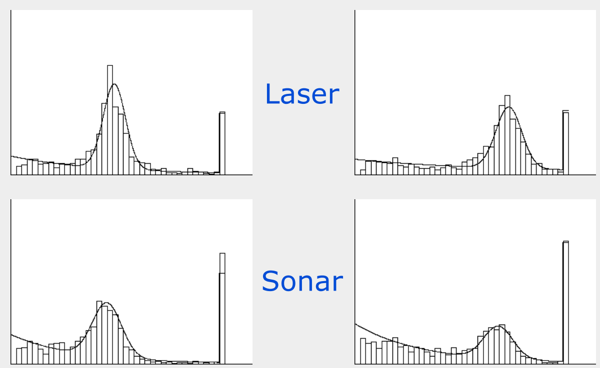
- Maximize log likelihood of the data

  $$P(z \mid z_{\text{exp}})$$

- Search parameter space.
- EM to find mixture parameters $\alpha$?
  - Assign measurements to densities.
  - Estimate densities using assignments.
  - Reassign measurements.



# Approximation Results




300cm 400cm

# Example

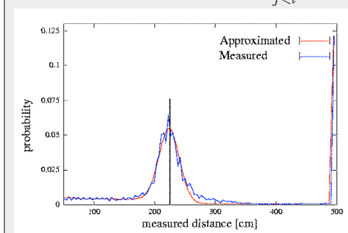
*z* *P(z|x,m)*



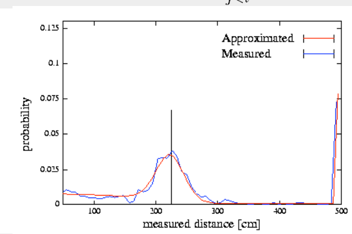
# Discrete Model of Proximity Sensors

- Instead of densities, consider discrete steps along the sensor beam.
- Consider dependencies between different cases.

$$P(d_i \mid l) \;=\; 1 - (1 - (1 - \sum_{j<i} P_u(d_j))\, c_d\, P_m(d_i \mid l))) \cdot (1 - (1 - \sum_{j<i} P(d_j))\, c_r)$$



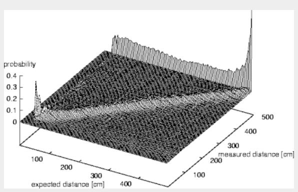

**Laser sensor**



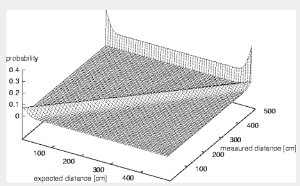

**Sonar sensor**

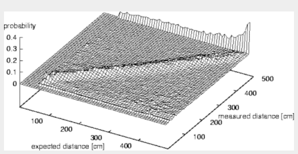


# Approximation Results

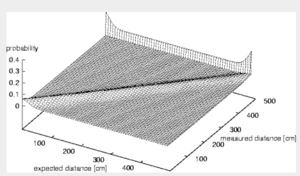


Laser



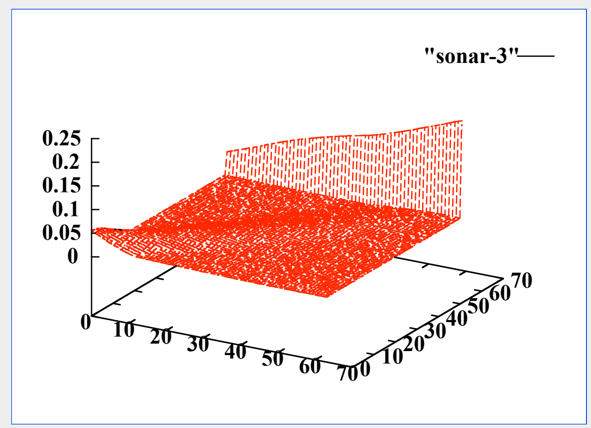


Sonar





# Influence of Angle to Obstacle



## Summary Beam-based Model

- Assumes independence between beams.
  - Justification?
  - Overconfident!
- Models physical causes for measurements.
  - Mixture of densities for these causes.
  - Assumes independence between causes. Problem?
- Implementation
  - Learn parameters based on real data.
  - Different models should be learned for different angles at which the sensor beam hits the obstacle.
  - Determine expected distances by ray-tracing.
  - Expected distances can be pre-processed.



## Scan-based Model

- Beam-based model is …
  - not smooth for small obstacles and at edges.
  - not very efficient.

- Idea: Instead of following along the beam, just check the end point.

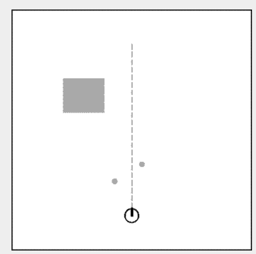


## Scan-based Model

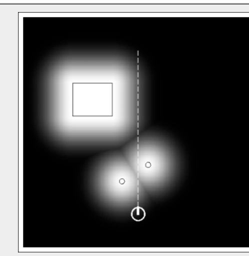
- Probability is a mixture of …
  - a Gaussian distribution with mean at distance to closest obstacle,
  - a uniform distribution for random measurements, and
  - a small uniform distribution for max range measurements.
- Again, independence between different components is assumed.



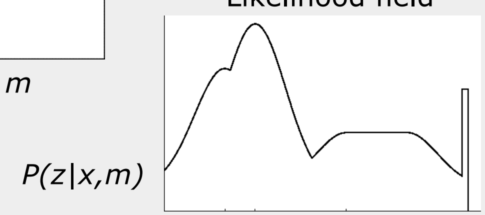
## Example



Map *m*

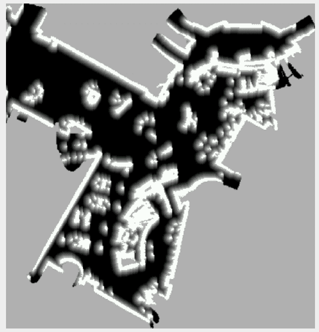


Likelihood field



## San Jose Tech Museum

Occupancy grid map

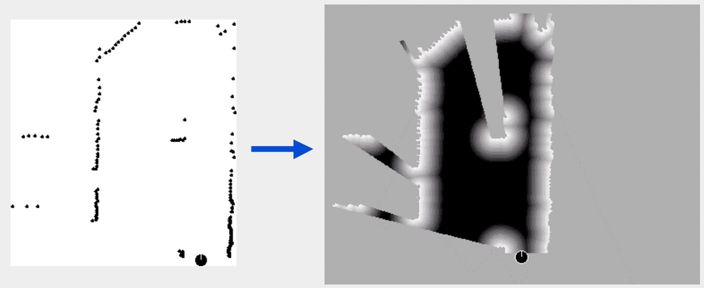


Likelihood field



## Scan Matching

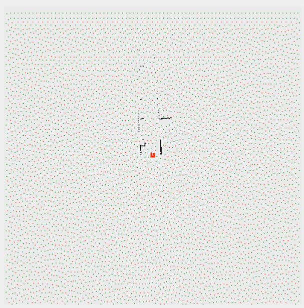
- Extract likelihood field from scan and use it to match different scan.





## Scan Matching

- Extract likelihood field from first scan and use it to match second scan.



~0.01 sec



## Properties of Scan-based Model

- Highly efficient, uses 2D tables only.
- Smooth w.r.t. to small changes in robot position.
- Allows gradient descent, scan matching.
- Ignores physical properties of beams.
- Works for sonars?

## Additional Models of Proximity Sensors

- Map matching (sonar,laser): generate small, local maps from sensor data and match local maps against global model.
- Scan matching (laser): map is represented by scan endpoints, match scan into this map using correlation.
- Features (sonar, laser, vision): Extract features such as doors, hallways from sensor data.



## Landmarks

- Active beacons (*e.g.* radio, GPS)
- Passive (*e.g.* visual, retro-reflective)
- Standard approach is triangulation
- Sensor provides
  - distance, or
  - bearing, or
  - distance and bearing.



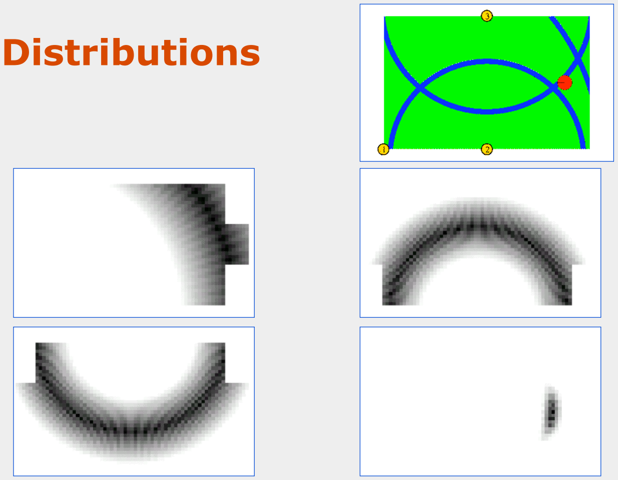
## Distance and Bearing



## Probabilistic Model

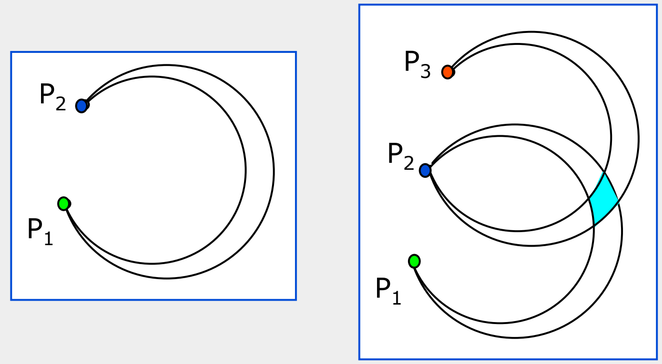
1. Algorithm **landmark_detection_model**(z,x,m):
   $z = \langle i, d, \alpha \rangle, x = \langle x, y, \theta \rangle$
2. $\hat{d} = \sqrt{(m_x(i) - x)^2 + (m_y(i) - y)^2}$
3. $\hat{a} = \text{atan2}(m_y(i) - y, m_x(i) - x) - \theta$
4. $p_{\text{det}} = \text{prob}(\hat{d} - d, \varepsilon_d) \cdot \text{prob}(\hat{\alpha} - \alpha, \varepsilon_\alpha)$
5. Return $z_{\text{det}} p_{\text{det}} + z_{\text{fp}} P_{\text{uniform}}(z \mid x, m)$

## Distributions





## Bearings Only With Uncertainty




Most approaches attempt to find estimation mean.



## Summary of Sensor Models

- Explicitly modeling uncertainty in sensing is key to robustness.
- In many cases, good models can be found by the following approach:
  1. Determine parametric model of noise free measurement.
  2. Analyze sources of noise.
  3. Add adequate noise to parameters (eventually mix in densities for noise).
  4. Learn (and verify) parameters by fitting model to data.
  5. Likelihood of measurement is given by "probabilistically comparing" the actual with the expected measurement.
- This holds for motion models as well.
- It is extremely important to be aware of the underlying assumptions!