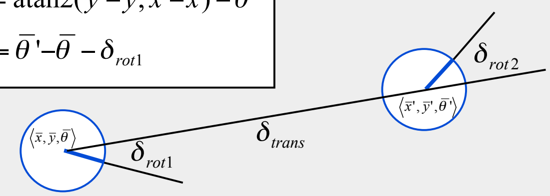
CSE-571
Probabilistic Robotics

Probabilistic Motion Models

Probabilistic Kinematics

- Robot moves from $\langle \bar{x}, \bar{y}, \bar{\theta} \rangle$ to $\langle \bar{x}', \bar{y}', \bar{\theta}' \rangle$.
- Odometry information $u = \langle \delta_{rot1}, \delta_{rot2}, \delta_{trans} \rangle$.

$$\delta_{trans} = \sqrt{(\bar{x}' - \bar{x})^2 + (\bar{y}' - \bar{y})^2}$$
$$\delta_{rot1} = \mathrm{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$$
$$\delta_{rot2} = \bar{\theta}' - \bar{\theta} - \delta_{rot1}$$



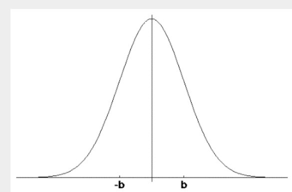
Noise Model for Motion

- The measured motion is given by the true motion corrupted with noise.

$$\hat{\delta}_{rot1} = \delta_{rot1} + \varepsilon_{\alpha_1|\delta_{rot1}| + \alpha_2|\delta_{trans}|}$$
$$\hat{\delta}_{trans} = \delta_{trans} + \varepsilon_{\alpha_3|\delta_{trans}| + \alpha_4|\delta_{rot1} + \delta_{rot2}|}$$
$$\hat{\delta}_{rot2} = \delta_{rot2} + \varepsilon_{\alpha_1|\delta_{rot2}| + \alpha_2|\delta_{trans}|}$$

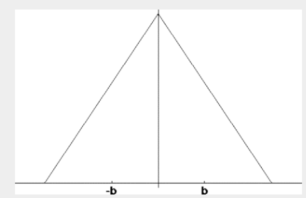
Noise Models

Normal distribution



$$\varepsilon_{\sigma^2}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}$$

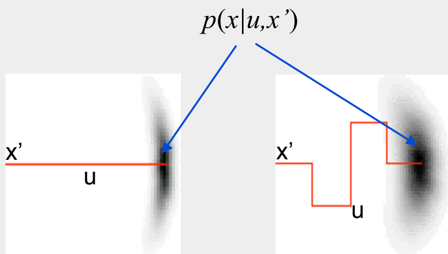
Triangular distribution



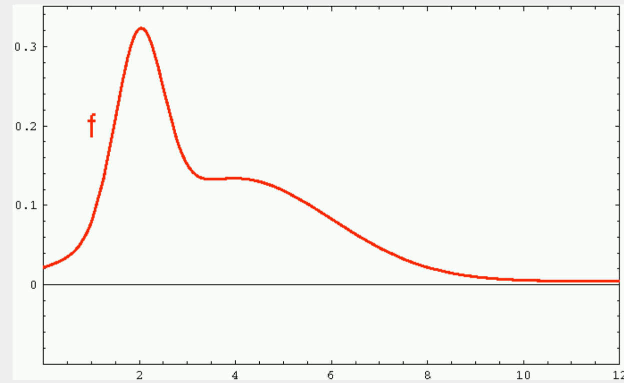
$$\varepsilon_{\sigma^2}(x) = \begin{cases} 0 \text{ if } |x| > \sqrt{6\sigma^2} \\ \frac{\sqrt{6\sigma^2} - |x|}{6\sigma^2} \end{cases}$$

Probabilistic Kinematics

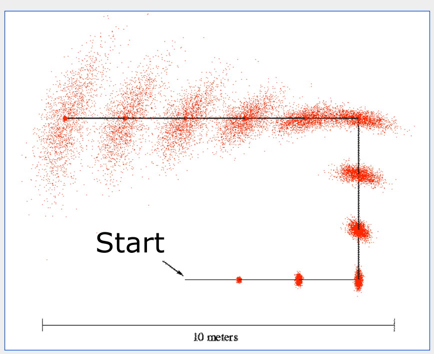
- Odometry information is inherently noisy.
- How can we model this uncertainty?



Sample-based Density Representation



Sample-based Motion



Sample Odometry Motion Model

1. Algorithm **sample_motion_model**(u, x):

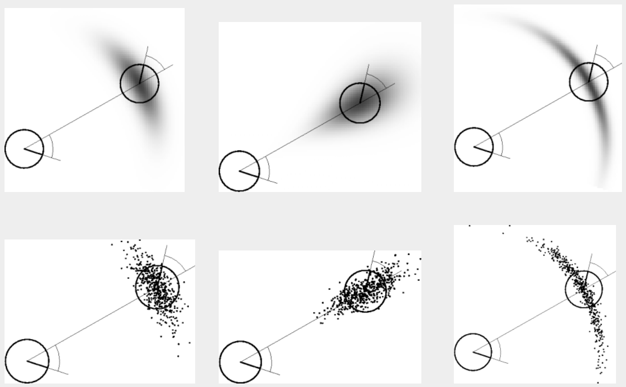
   $u = \langle \delta_{rot1}, \delta_{rot2}, \delta_{trans} \rangle, x = \langle x, y, \theta \rangle$

1. $\hat{\delta}_{rot1} = \delta_{rot1} + \text{sample}(\alpha_1 \, | \delta_{rot1} | + \alpha_2 \, \delta_{trans})$
2. $\hat{\delta}_{trans} = \delta_{trans} + \text{sample}(\alpha_3 \, \delta_{trans} + \alpha_4 \, (| \delta_{rot1} | + | \delta_{rot2} |))$
3. $\hat{\delta}_{rot2} = \delta_{rot2} + \text{sample}(\alpha_1 \, | \delta_{rot2} | + \alpha_2 \, \delta_{trans})$

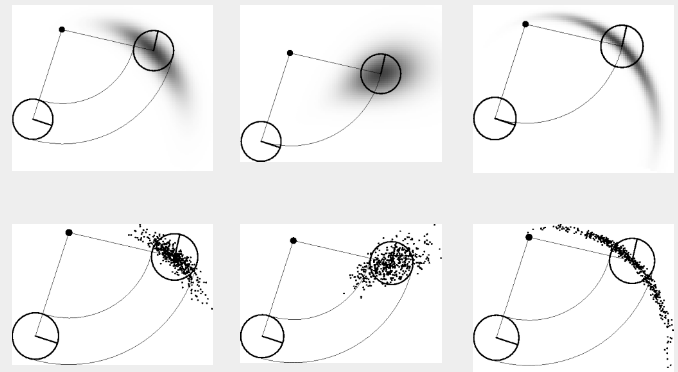
4. $x' = x + \hat{\delta}_{trans} \cos(\theta + \hat{\delta}_{rot1})$
5. $y' = y + \hat{\delta}_{trans} \sin(\theta + \hat{\delta}_{rot1})$
6. $\theta' = \theta + \hat{\delta}_{rot1} + \hat{\delta}_{rot2}$

7. Return $\langle x', y', \theta' \rangle$

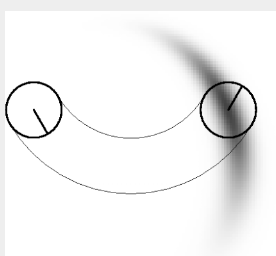
Examples (odometry based)



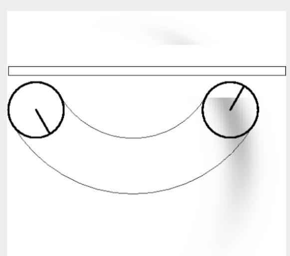
Examples (velocity based)



Motion Model with Map



$P(x \mid u, x')$



$P(x \mid u, x', m) = \eta \, P(x \mid m) \, P(x \mid u, x')$

- When does this approximation fail?