CSE-571
AI-based Mobile Robotics

SLAM Mapping

Mapping with Raw Odometry

SLAM:
Simultaneous Localization and Mapping

- Full SLAM:

$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t})$$

- Online SLAM:

$$p(x_t, m \mid z_{1:t}, u_{1:t}) = \iiint p(x_{1:t}, m \mid z_{1:t}, u_{1:t})\, dx_1 dx_2 \ldots dx_{t-1}$$

Integrations typically done one at a time

Techniques for Generating Consistent Maps

- Scan matching
- Probabilistic mapping with a single map and a posterior about poses Mapping + Localization
- EKF SLAM
- Graph-SLAM, SEIF
- Fast-SLAM

Scan Matching

Maximize the likelihood of the i-th pose and map relative to the (i-1)-th pose and map.

$$\hat{x}_t = \underset{x_t}{\operatorname{argmax}} \left\{ p(z_t \mid x_t, \hat{m}^{[t-1]}) \cdot p(x_t \mid u_{t-1}, \hat{x}_{t-1}) \right\}$$

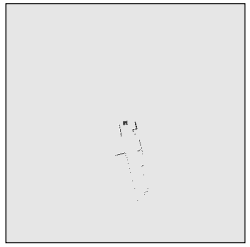
current measurement

map constructed so far

robot motion

Scan Matching Example

Mapping + Localization



Yellow flashes:

artificially distorted map
(30 deg, 50 cm)

[Thrun et al., 00]

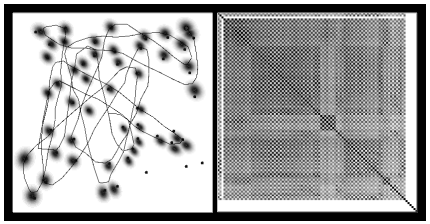
SLAM: Mapping with Kalman Filters

- Map with N landmarks:(2N+3)-dimensional Gaussian

$$Bel(x,m) = \left\langle \begin{pmatrix} l_1 \\ l_2 \\ \vdots \\ l_N \\ x \\ y \\ \theta \end{pmatrix}, \begin{pmatrix} \sigma_{l_1}^2 & \sigma_{l_1 l_2} & \cdots & \sigma_{l_1 l_N} & \sigma_{l_1 x} & \sigma_{l_1 y} & \sigma_{l_1 \theta} \\ \sigma_{l_2 l_1} & \sigma_{l_2}^2 & \cdots & \sigma_{l_2 l_N} & \sigma_{l_2 x} & \sigma_{l_2 y} & \sigma_{l_2 \theta} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \sigma_{l_N l_1} & \sigma_{l_N l_2} & \cdots & \sigma_{l_N}^2 & \sigma_{l_N x} & \sigma_{l_N y} & \sigma_{l_N \theta} \\ \sigma_{l_1 x} & \sigma_{l_2 x} & \cdots & \sigma_{l_N x} & \sigma_x^2 & \sigma_{xy} & \sigma_{x\theta} \\ \sigma_{l_1 y} & \sigma_{l_2 y} & \cdots & \sigma_{l_N y} & \sigma_{xy} & \sigma_y^2 & \sigma_{y\theta} \\ \sigma_{l_1 \theta} & \sigma_{l_2 \theta} & \cdots & \sigma_{l_N \theta} & \sigma_{x\theta} & \sigma_{y\theta} & \sigma_\theta^2 \end{pmatrix} \right\rangle$$

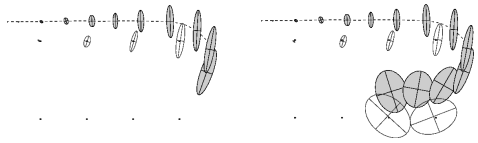
- Can handle hundreds of dimensions

SLAM: Mapping with Kalman Filters

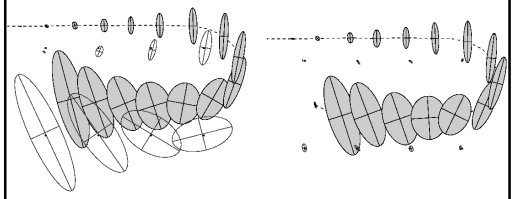


Map Correlation matrix

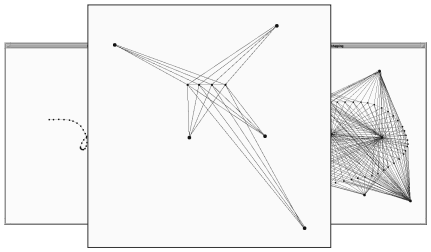
SLAM: Mapping with Kalman Filters



SLAM: Mapping with Kalman Filters



Bearings-only Mapping

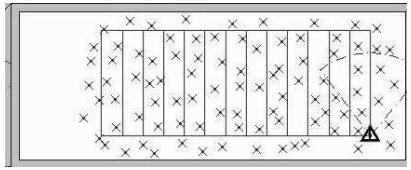


[M. Deans 2002]

Approximations for SLAM

- Local submaps [Leonard et al.99, Bosse et al. 02, Newman et al. 03]
- Sparse links (correlations) [Lu & Milios 97, Guivant & Nebot 01]
- Sparse extended information filters [Frese et al. 01, Thrun et al. 02]
- Thin junction tree filters [Paskin 03]
- Rao-Blackwellisation (FastSLAM) [Murphy 99, Montemerlo et al. 02, Eliazar et al. 03, Haehnel et al. 03]

Sub-maps for EKF SLAM

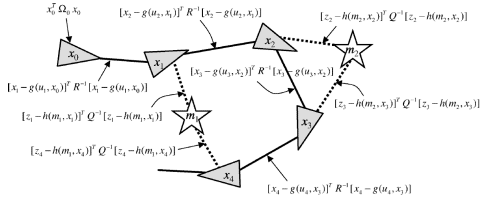


[Leonard et al 1998]

Graph-SLAM

- Full SLAM technique
- Generates probabilistic links
- Computes map only occasionally
- Based on Information Filter form

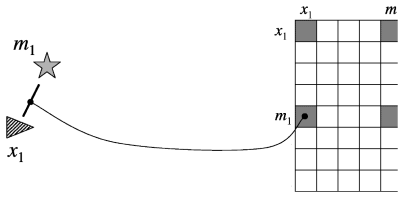
Graph-SLAM Idea



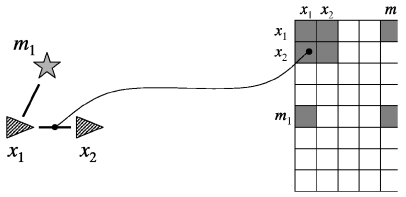
Sum of all constraints:

$$J_{\text{GraphSLAM}} = x_0^T \Omega_0 x_0 + \sum_t [x_t - g(u_t, x_{t-1})]^T R^{-1} [x_t - g(u_t, x_{t-1})] + \sum_t [z_t - h(m_{c_t}, x_t)]^T Q^{-1} [z_t - h(m_{c_t}, x_t)]$$

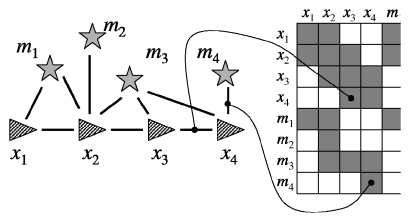
Graph-SLAM Idea (1)



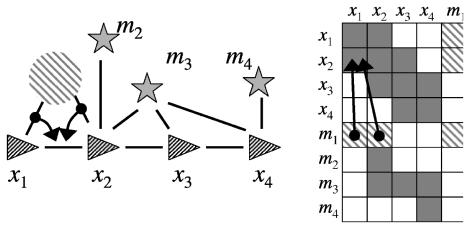
Graph-SLAM Idea (2)



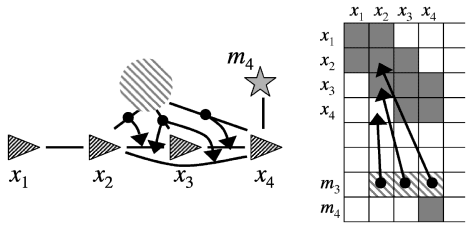
Graph-SLAM Idea (3)



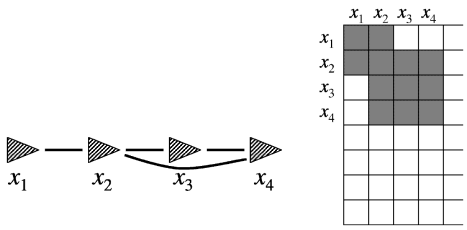
Graph-SLAM Inference (1)



Graph-SLAM Inference (2)



Graph-SLAM Inference (3)

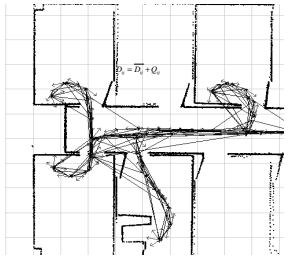


Robot Poses and Scans [Lu and Milios 1997]

- Successive robot poses connected by odometry
- Sensor readings yield constraints between poses
- Constraints represented by Gaussians

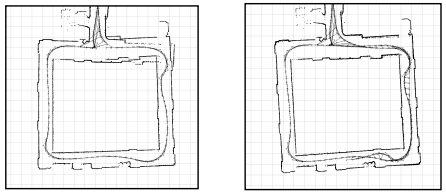
$$D_{ij} = \overline{D_{ij}} + Q_{ij}$$

- Globally optimal estimate $\arg\max_{x} \left[P(D_{ij} \mid \overline{D_{ij}}) \right]$



Loop Closure

- Use scan patches to detect loop closure
- Add new position constraints
- Deform the network based on covariances of matches

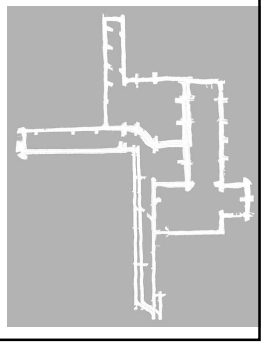


Before loop closure | After loop closure

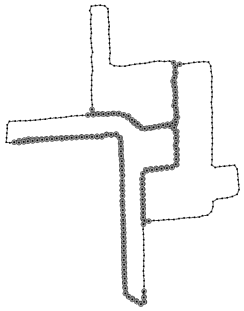
Mapping the Allen Center

Multi-Robot Mapping [Konolige et al.]

Mine Mapping



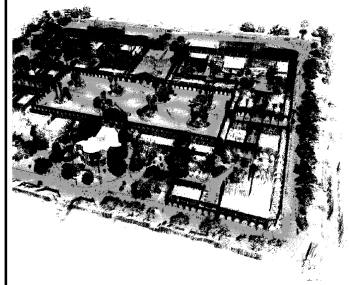
Mine Mapping: Data Associations



Efficient Map Recovery

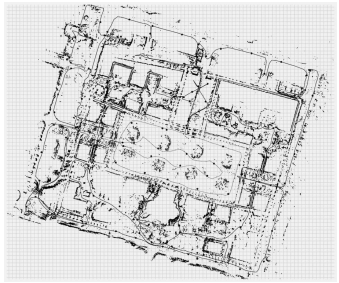
- Information matrix inversion can be avoided if only best map estimate is required
- Minimize constraint function $J_{GraphSLAM}$ using standard optimization techniques (gradient descent, Levenberg Marquardt, conjugate gradient)

3D Outdoor Mapping

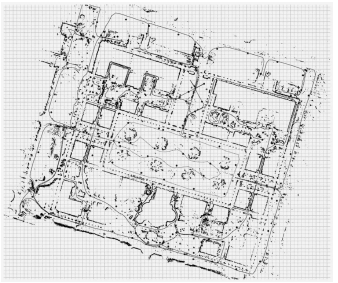


10^8 features, 10^5 poses, only few secs using cg.

Map Before Optimization



Map After Optimization



Graph-SLAM Summary

- Adresses full SLAM problem
- Constructs link graph between poses and poses/landmarks
- Graph is sparse: number of edges linear in number of nodes
- Inference performed by building information matrix and vector (linearized form)
- Map recovered by reduction to robot poses, followed by conversion to moment representation, followed by estimation of landmark positions
- ML estimate by minimization of $J_{GraphSLAM}$
- Data association by iterative greedy search

Information Form

- Represent posterior in canonical form

$\Omega = \Sigma^{-1}$ Information matrix

$\xi = \Sigma^{-1}\mu$ Information vector

- One-to-one transform between canonical and moment representation

$\Sigma = \Omega^{-1}$

$\mu = \Omega^{-1}\xi$

Kalman Filter Algorithm

1. Algorithm **Kalman_filter**(μ_{t-1}, Σ_{t-1}, u_t, z_t):

2. Prediction:
3. $\bar{\mu}_t = A_t\mu_{t-1} + B_t u_t$
4. $\bar{\Sigma}_t = A_t\Sigma_{t-1}A_t^T + R_t$

5. Correction:
6. $K_t = \bar{\Sigma}_t C_t (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$
7. $\mu_t = \bar{\mu}_t + K_t(z_t - C_t\bar{\mu}_t)$
8. $\Sigma_t = (I - K_t C_t)\bar{\Sigma}_t$
9. Return μ_t, Σ_t

Information Filter Algorithm

1. Algorithm **Information_filter**(ξ_{t-1}, Ω_{t-1}, u_t, z_t):

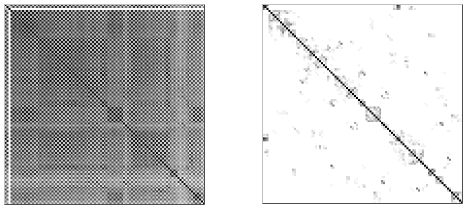
2. Prediction:
3. $\bar{\Omega}_t = (A_t\Omega_{t-1}^{-1}A_t^T + R_t)^{-1}$ $\quad$ $\bar{\mu}_t = A_t\mu_{t-1} + B_t u_t$
4. $\bar{\xi}_t = \bar{\Omega}_t(A_t\Omega_{t-1}^{-1}\xi_{t-1} + B_t u_t)$ $\quad$ $\bar{\Sigma}_t = A_t\Sigma_{t-1}A_t^T + R_t$

5. Correction:
6. $\quad$ $K_t = \bar{\Sigma}_t C_t (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$
7. $\Omega_t = C_t^T Q_t^{-1} C_t + \bar{\Omega}_t$ $\quad$ $\mu_t = \bar{\mu}_t + K_t(z_t - C_t\bar{\mu}_t)$
8. $\xi_t = C_t^T Q_t^{-1} z_t + \bar{\xi}_t$ $\quad$ $\Sigma_t = (I - K_t C_t)\bar{\Sigma}_t$
9. Return ξ_t, Ω_t

Information vs. Moment Form

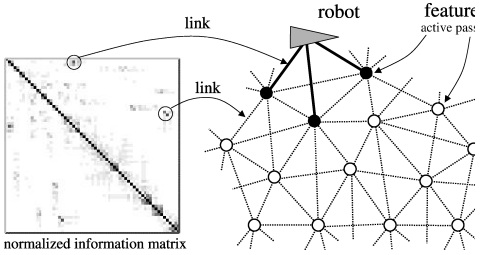
- Complexity of prediction step:
  - KF: additive, at most $O(n^2)$
  - IF: two inversions of *nxn* matrices: $O(n^{2.376})$
- Correction step:
  - KF: inversion of *nxn* matrix: $O(n^{2.376})$
  - IF: additive, at most $O(n^2)$

Information vs. Moment Form



Corelation matrix Information matrix

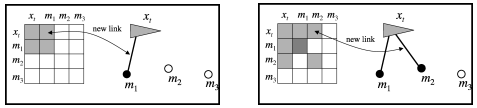
Information Matrix and Links



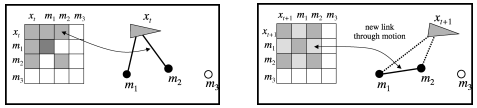
Sparse Extended Information Filters (SEIFs)

- Information form representation
- Online version of Graph-SLAM:
  - Maintains only most recent robot pose
  - Observation update same as Graph-SLAM
  - Motion update integrates previous robot pose out: adds links between landmarks

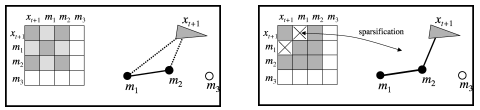
SEIF Illustration (1)



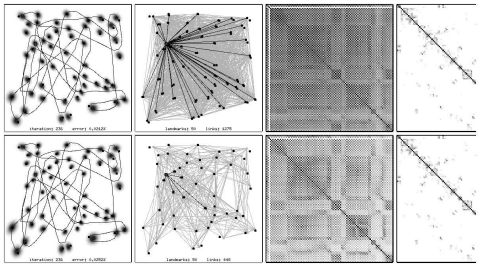
SEIF Illustration (2)



SEIF Illustration (3)



SEIF Illustration: Sparsity



SEIF Summary

- Adresses online SLAM problem
- Adds links between landmarks
- Inference performed by building information matrix and vector
- Maintain sparsity of information matrix by removing weak links (done by shifting information through the matrix)
- Update independent of map size!
- Map estimate is recovered only occasionally
- Data associations can be undone