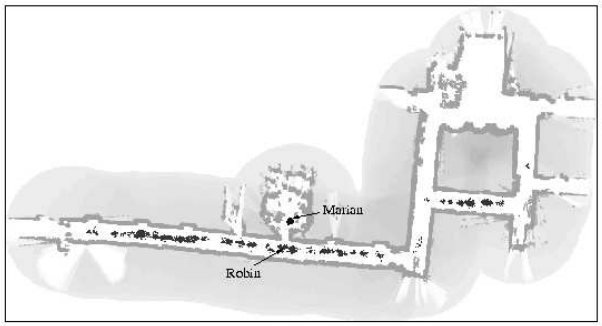
CSE-571
AI-based Mobile Robotics

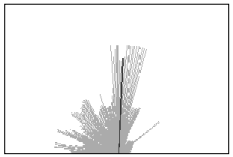
Some more particle filters

Multi-robot Localization: Idea



Robot Detection

Camera image



Laser scan

Multi-robot Localization

- Combined belief state $X = X_1 \otimes X_2 \otimes \ldots \otimes X_N$ has dimension $3N$ ⟶ complexity grows **exponentially** in number of robots
- **Factorial** representation of the belief

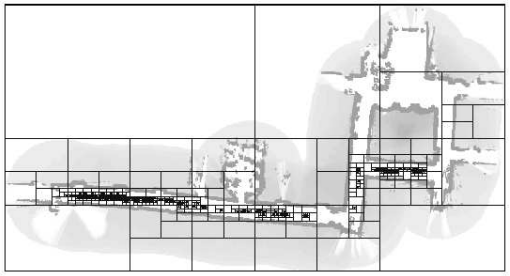
$$P(X_1^{(t)}, \ldots, X_N^{(t)} \mid d) = P(X_1^{(t)} \mid d) \cdot \ldots \cdot P(X_N^{(t)} \mid d)$$

- Perform localization for each robot and use detections to **constrain the beliefs**

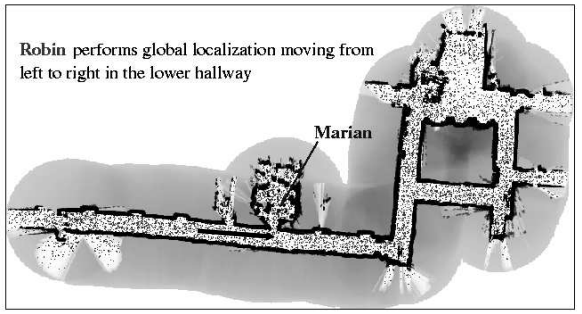
Density Trees

Integration of robot detection requires a **density**

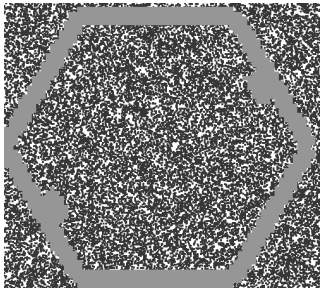
$$Bel(x_m) \leftarrow \int P(x_m \mid x_n, r_n)\, Bel(x_n)\, dx_n \cdot Bel(x_m)$$



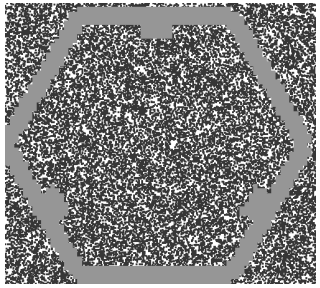
Example Run



Heterogeneous Robots

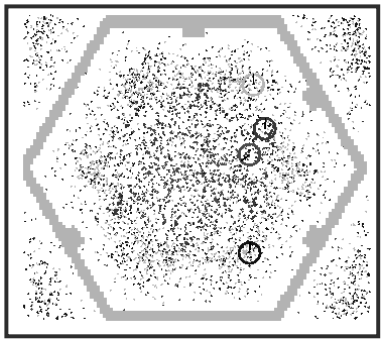


Laser



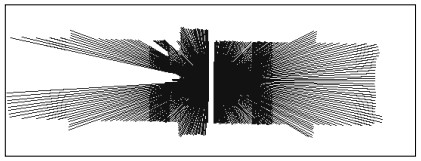
Sonar

Example Run



People Tracking

- **Key questions**
  - How many people are there?
  - Where do they go?
- **Requirements**
  - Real time
  - No model of the environment
  - Robot in motion



[Schulz et al. 2001]

Tracking Multiple Objects

- **Kalman filters**
  - MHT (Multiple hypothesis tracking)

    JPDAF (Joint probabilistic data association filters)
    - Efficient, especially JPDAF
    - Track individual objects using Kalman filters
- **Particle filters**
  - One sample set for multiple objects
    - Efficient
    - Brittle, introduces dependencies
  - Joint state space
    - Exact representation
    - Exponential state space

Multi Object Tracking

- **Idea**: Track each person with an individual particle filter.
- Data association: Which observation corresponds to which object/sample set?
  - Indistinguishable features
  - Missing features
  - False alarms
- Track management: When to add or delete tracks/sample sets?

Joint Probabilistic Data Association

- Solve data association problem
  - Compute assignment probabilities of features to objects
  - Consider missing features by introducing special feature z_0
  - Handle false alarms by Poisson distribution
- Track management outside JPDAF

Assignment Probabilities

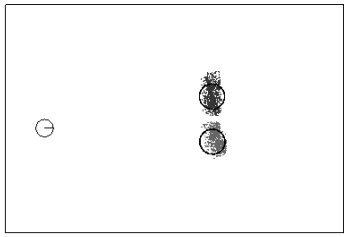
- Consider all possible assignments Θ of features to sample sets.
- Compute marginal probability β_{ji} of feature z_j originating from sample set x_i.

$$\beta_{ji} = \sum_{\theta \in \Theta_{ji}} P(\theta \mid Z^t)$$

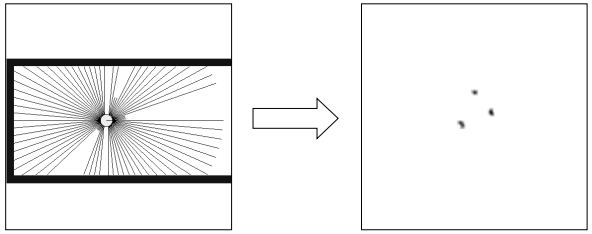
$$P(\theta \mid Z^t) = \eta \, P(\theta \mid X^t) \prod_{(j,i) \in \theta} p(z_j^t \mid x_i^t)$$

Particle Filter + JPDAF

$$\omega_{i,n}^t = \eta \sum_{j=0}^{m_t} \beta_{ji} p(z_j^t \mid x_{i,n}^t)$$

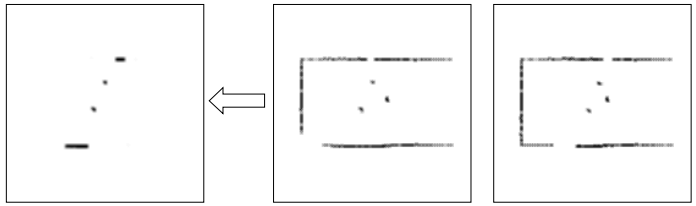


Laser-based People Tracking



- Legs appear as local minima in range scans.
- Transform such features into a position probability grid indicating P(legs$_{x,y}$).

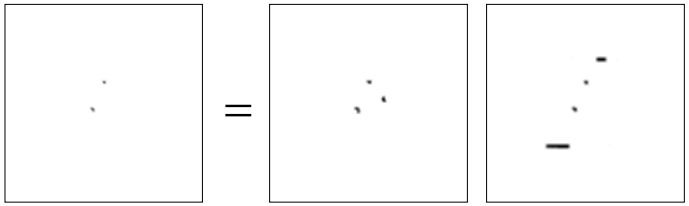
Observing Movements



Difference between consecutive range scans:

$$P(new_{x,y}^t) = P(occ_{x,y}^t) \cdot (1 - P(occ_{x,y}^{t-1}))$$

Integrating Observations



$$P(z_j^t \mid x_{i,n}^t) = P(\text{legs}_{x_{i,n}}^t) \cdot P(\text{new}_{x_{i,n}}^t)$$

Handling Occlusions

Reasons for a missing feature z^t_0:

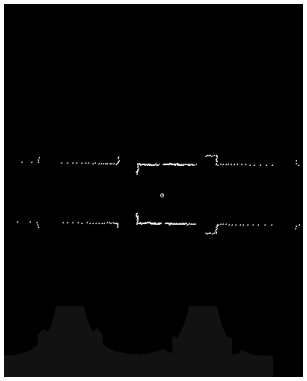
- Feature **not detected** $P(\neg\text{Detect})$
- Feature **occluded** by other object $P(\text{occluded}(x_i^t))$

Idea

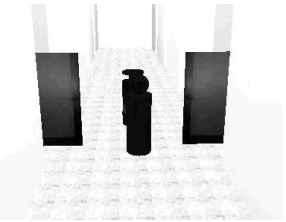
- Compute **occlusion grid**

$$P(z_0^t \mid x_i^t) = P(\text{occluded}(x_i^t) \vee \neg\text{Detect})$$

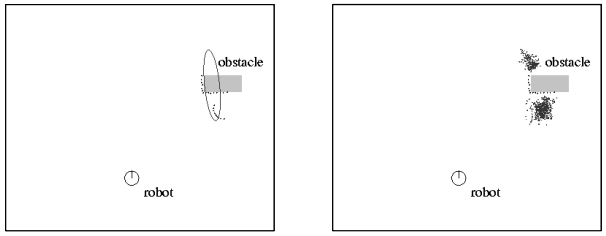
Example Run



Tracking with a Moving Robot

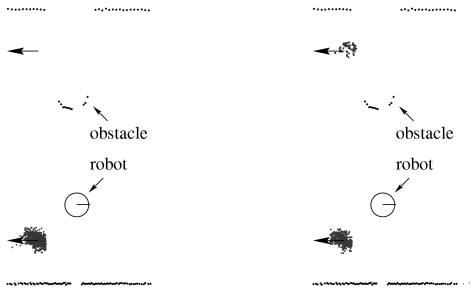


Comparison to Kalman Filters



Kalman Filter Particle Filter

One Particle Filter vs. SJPDAF

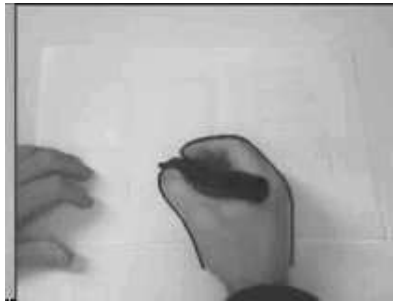


One filter SJPDAF

Condensation

[Isard et al. 96]

Mixed-State Condensation

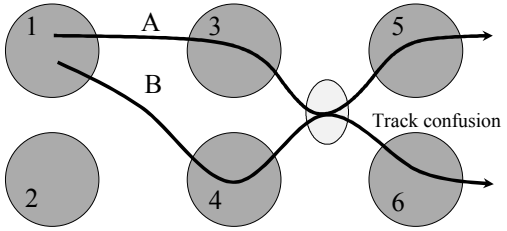


[Isard et al. 96]

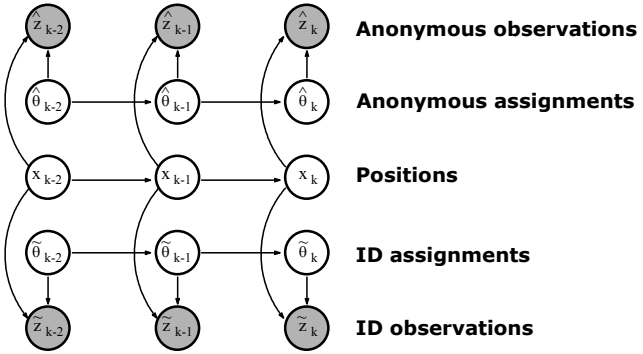
People Tracking with Anonymous and ID Sensors

- Anonymous sensors (laser range-finders, radar) provide accurate location but no identity information.
- ID sensors (infrared, ultrasound badge systems) provide object identity but only coarse location information.

Tracking with Anonymous and ID Sensors



Generative Model



Complexity of Tracking with Unknown Data Associations

- Assignments between tracks and observations are not known
- Need to estimate tracks and assignments
- Anonymous sensors
  - At each iteration *m*! possible assignments between observations and tracks
  - For tracks of length *k* there are $m!^k$ assignments
- ID sensors
  - *m*! possible assignments of id's to tracks

Rao-Blackwellised Particle Filters for People Tracking

- Problem:
  Joint posterior over assignments and tracks is complex
- Observation:
  Tracking is relatively easy once we know the assignments
- Idea:
  Sample assignments and track objects conditioned on each sample using Kalman filters

Rao-Blackwellised Particle Filters for MHT

$$p(x_k, \hat{\theta}_{1:k} \mid \hat{z}_{1:k}) = p(x_k \mid \hat{\theta}_{1:k}, \hat{z}_{1:k}) p(\hat{\theta}_{1:k} \hat{z}_{1:k})$$

$$\begin{aligned} p(\hat{\theta}_{1:k} \mid \hat{z}_{1:k}) &= p(\hat{\theta}_k \mid \hat{\theta}_{1:k-1}, \hat{z}_{1:k}) p(\hat{\theta}_{1:k-1} \mid \hat{z}_{1:k}) \\ &= \frac{p(\hat{z}_k \mid \hat{\theta}_{1:k}, \hat{z}_{1:k-1}) p(\hat{\theta}_k \mid \hat{\theta}_{1:k-1}, \hat{z}_{1:k-1})}{p(\hat{z}_k \mid \hat{\theta}_{1:k-1}, \hat{z}_{1:k-1})} p(\hat{\theta}_{1:k-1} \mid \hat{z}_{1:k}) \\ &= \frac{p(\hat{z}_k \mid \hat{\theta}_k, t_{k-1}) p(\hat{\theta}_k \mid t_{k-1})}{p(\hat{z}_k \mid t_{k-1})} p(\hat{\theta}_{1:k-1} \mid \hat{z}_{1:k}) \end{aligned}$$

$$t_k = \left\langle \mu_k^i, \Sigma_k^i \right\rangle_{i=1,\ldots,m} \Big| \hat{\theta}_{1:k-1}, \hat{z}_{1:k-1}$$

Rao-Blackwellised Particle Filters for MHT

$$p(\hat{\theta}_{1:k} \mid \hat{z}_{1:k}) = \frac{p(\hat{z}_k \mid \hat{\theta}_k, t_{k-1}) p(\hat{\theta}_k \mid t_{k-1})}{p(\hat{z}_k \mid t_{k-1})} p(\hat{\theta}_{1:k-1} \mid \hat{z}_{1:k})$$

Optimal sampling distribution given sample ι

$$p(\hat{\theta}_k \mid \hat{\theta}_{1:k-1}^{(\iota)}, \hat{z}_{1:k}) \propto p(\hat{z}_k \mid \hat{\theta}_k, t_{k-1}^{(\iota)}) p(\hat{\theta}_k \mid t_{k-1}^{(\iota)})$$

Sample from optimal distribution using MCMC

Rao-Blackwellised Particle Filters for MHT

Optimal distribution is NOT target distribution

$$p(\hat{\theta}_{1:k} \mid \hat{z}_{1:k}) = \frac{p(\hat{z}_k \mid \hat{\theta}_k, t_{k-1}) p(\hat{\theta}_k \mid t_{k-1})}{p(\hat{z}_k \mid t_{k-1})} p(\hat{\theta}_{1:k-1} \mid \hat{z}_{1:k})$$

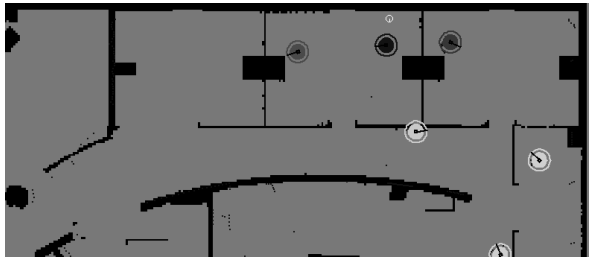
$$p(\hat{\theta}_k \mid \hat{\theta}_{1:k-1}^{(\iota)}, \hat{z}_{1:k}) \propto p(\hat{z}_k \mid \hat{\theta}_k, t_{k-1}^{(\iota)}) p(\hat{\theta}_k \mid t_{k-1}^{(\iota)})$$

$$\hat{w}_{k-1}^{(\iota)} = \frac{p(\hat{\theta}_{1:k-1} \mid \hat{z}_{1:k})}{p(\hat{\theta}_{1:k-1} \mid \hat{z}_{1:k-1})} \propto w_{k-1}^{(\iota)} p(\hat{z}_k \mid t_{k-1}^{(\iota)})$$

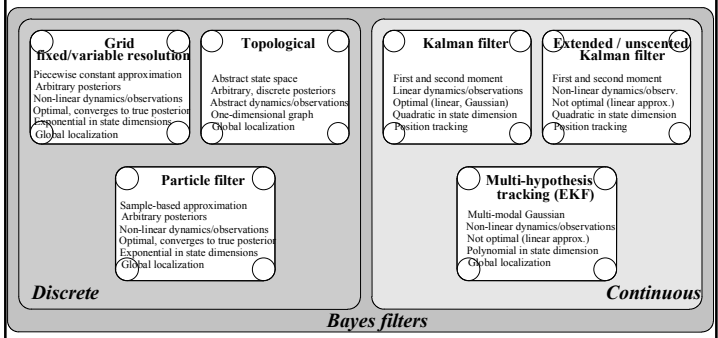
MHT-RBPF Algorithm

1. **Inputs:** $S_{k-1} = \{\langle s_{k-1}^{(\iota)}, w_{k-1}^{(\iota)} \rangle \mid \iota = 1, \ldots, N\}$, observation $\hat{z}_k$
2. $S_k := \emptyset$ *// Initialize*
3. **for** $\iota := 1, \ldots, N$ **do** *// Generate lookahead assignments*
4. Generate M samples $\hat{\theta}_k^{(\iota,m)}$ prop. to $p(\hat{z}_k \mid \hat{\theta}_k^{(\iota,m)}, t_{k-1}^{(\iota)})\, p(\hat{\theta}_k^{(\iota,m)} \mid t_{k-1}^{(\iota)})$ using MCMC.
5. **for** $\iota := 1, \ldots, N$ **do** *// Update importance weights using lookahead assignments*
6. $\widehat{w}_{k-1}^{(\iota)} \propto \frac{w_{k-1}^{(\iota)}}{M} \sum_{m=1\ldots M} p(\hat{z}_k \mid \hat{\theta}_k^{(\iota,m)}, t_{k-1}^{(\iota)})\, p(\hat{\theta}_k^{(\iota,m)} \mid t_{k-1}^{(\iota)})$
7. **for** $\iota := 1, \ldots, N$ **do** *// Sample $s_{k-1}^{(\iota)}$ using updated weights and Markov chain*
8. Sample $s_{k-1}^{(\iota)} = \langle t_{k-1}^{(\iota)}, \hat{\theta}_{1:k-1}^{(\iota)} \rangle$ from S_{k-1} prop. to updated importance weights $\widehat{w}_{k-1}^{(\iota)}$.
9. Draw an assignment $\hat{\theta}_k^{(\iota)}$ from the corresponding Markov chain generated in step 4.
10. Update the position estimates $t_k^{(\iota)}$ using Kalman filter updates with $\hat{z}_k$, $t_k^{(\iota)}$, and $\hat{\theta}_k^{(\iota)}$
11. $s_k^{(\iota)} := \langle t_k^{(\iota)}, \hat{\theta}_{1:k}^{(\iota)} \rangle$; $S_k := S_k \cup \{\langle s_k^{(\iota)}, \frac{1}{N} \rangle\}$
12. **return** S_k

Example: Six People



Adding ID Observations

- Problem:
  In the beginning, assignments of id's to tracks are highly uncertain
- Solution: Two stage process
  - Identity estimation stage
    - Track people using anonymous sensors only
    - Estimate id's of people using id sensors
  - Full Rao-Blackwellisation
    - Sample both anonymous and id assignments

Bayes Filters for Robot Localization