

# CSE 562: Mobile Systems & Applications

Quals Course – Systems Area  
Shyam Gollakota

# First Mobile Phone 1973

## SIGMOBILE Outstanding Contribution Award



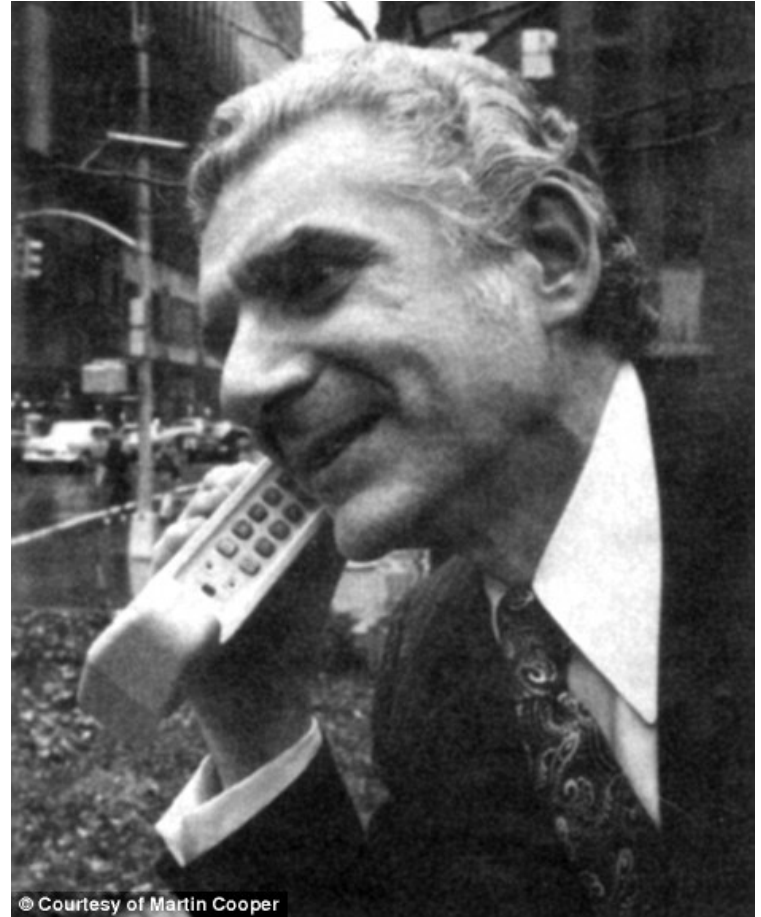
The SIGMOBILE Outstanding Contribution Award is given for significant and lasting contributions to the research on mobile computing and communications and wireless networking.

### 2020 Recipient



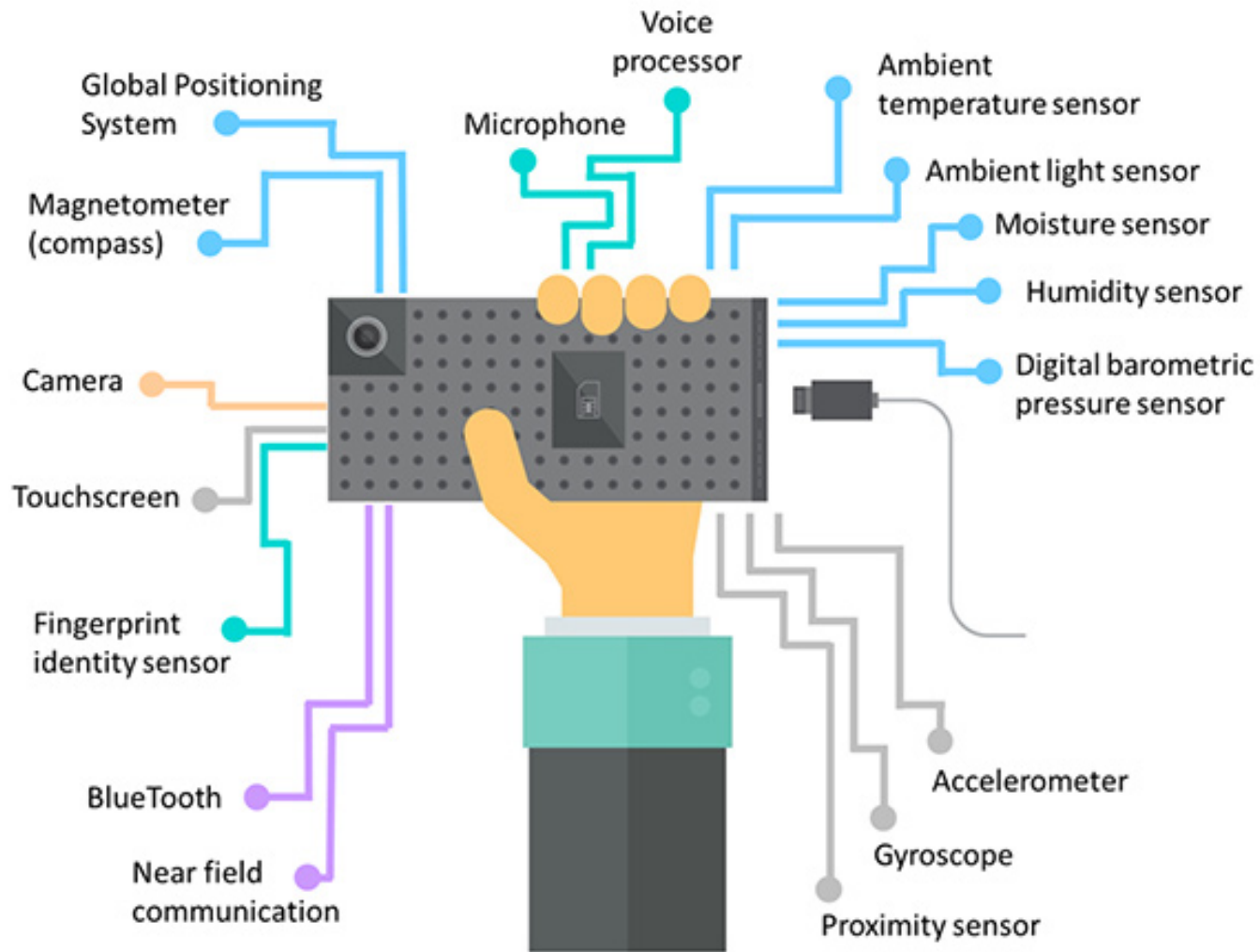
**Marty Cooper** 

For seminal contributions to the conception, practice and adoption of portable telephony.



© Courtesy of Martin Cooper





# Goal of this course

- Have an understanding of state of the art mobile systems research
- Explore applications that are capable with mobile devices

# Course material

1. Signal processing fundamentals
2. Acoustic device and device-free tracking
3. Physiological sensing using phones and speakers
4. IMW tracking and GPS localization
5. Wi-Fi localization and sensing
6. Designing and building IoT device hardware

# Course material

7. Backscatter systems

8. Mobile privacy and security

9. Robotics mobile systems

# Grading

3 hands-on assignments (20+20+20% in all)

- One every two weeks
- Requires programming phones, microcontroller, etc.

Class presentation of one paper (10%)

Final research project (30%)

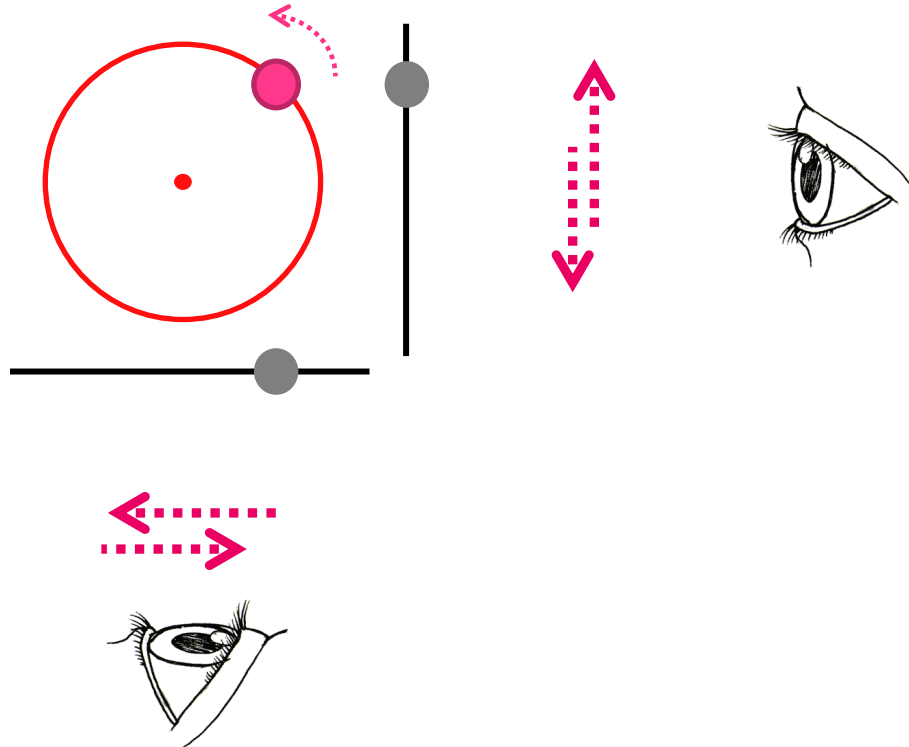
- Proposal due on May 1
- 2-3 person project



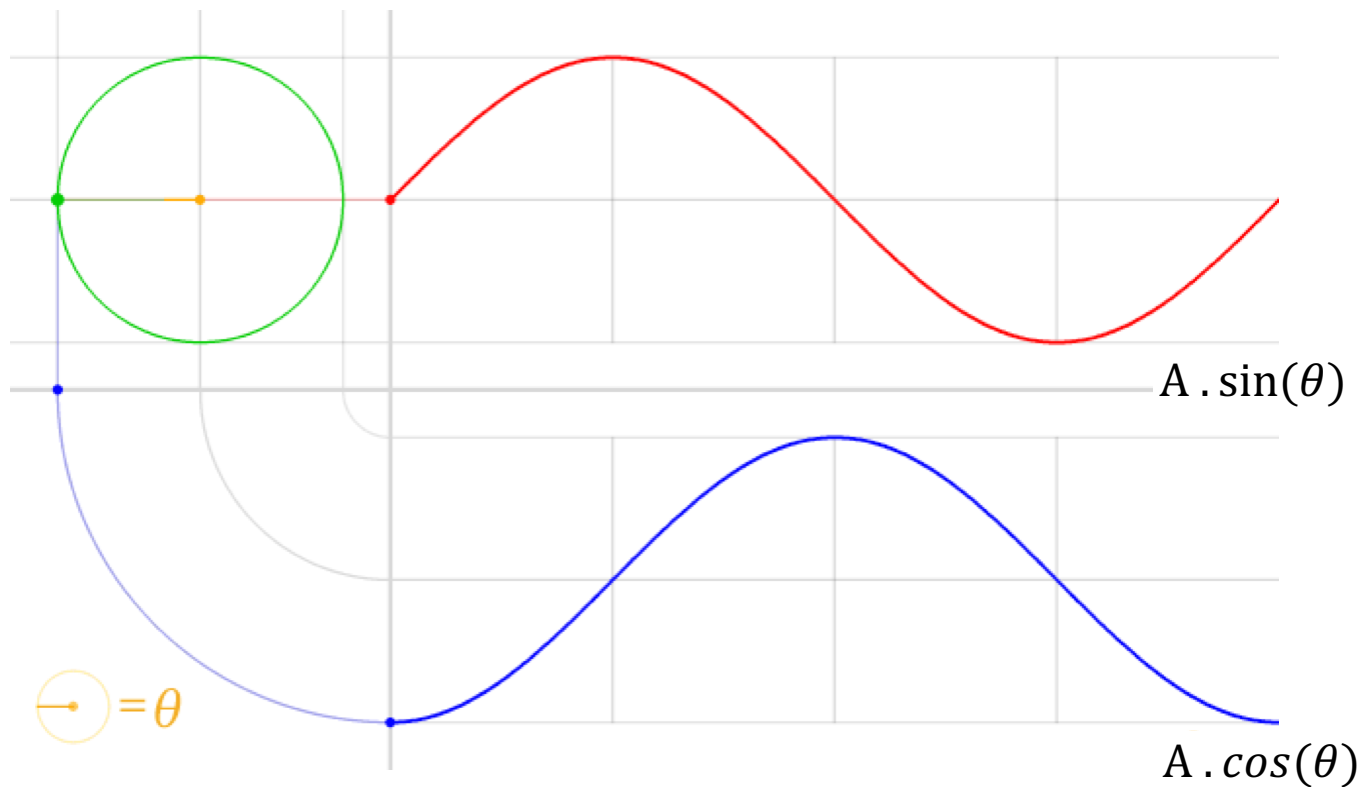
# Signal processing basics

(Slides by Nirupam Roy)

# Model for a signal (frequency, amplitude, and phase)



# Model for a signal (frequency, amplitude, and phase)



# Model for a signal (frequency, amplitude, and phase)



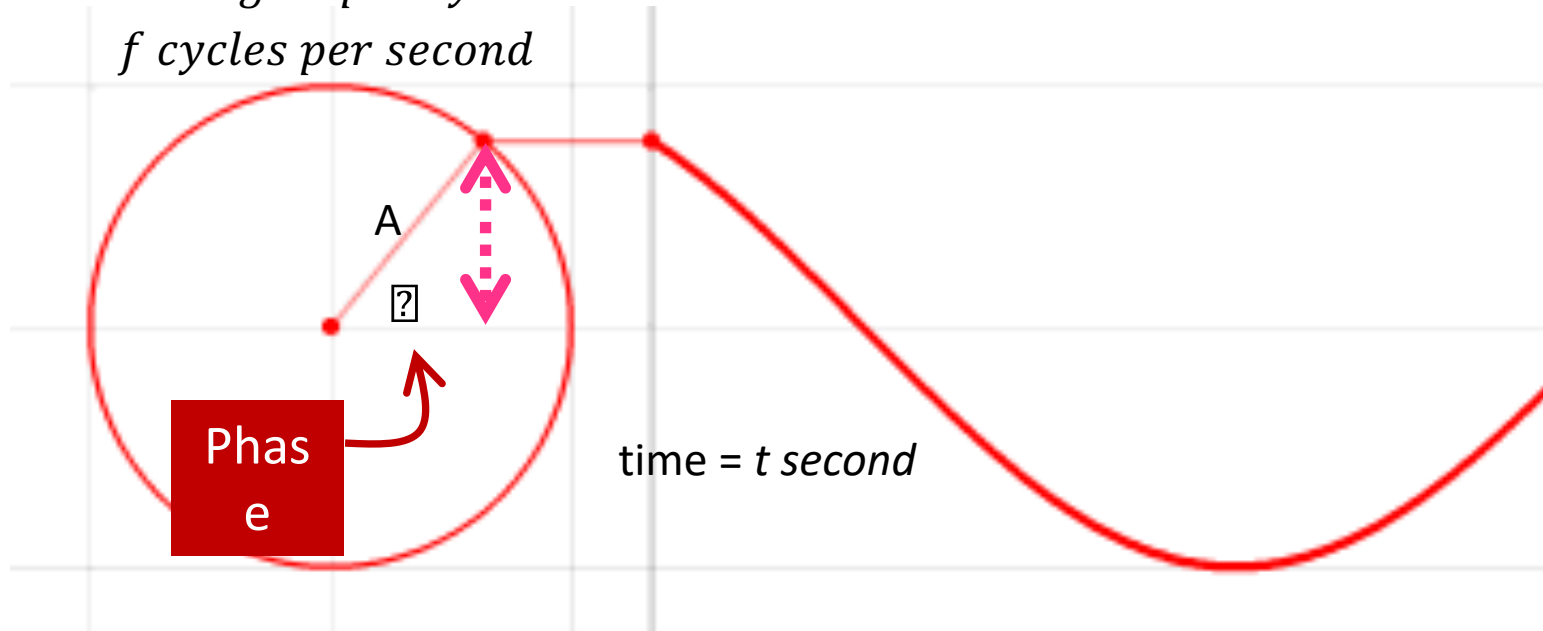
*$f$  cycles per second*

*$2\pi$  angles per cycle*

$$\theta = 2\pi ft$$

# Frequency, Amplitude, and Phase

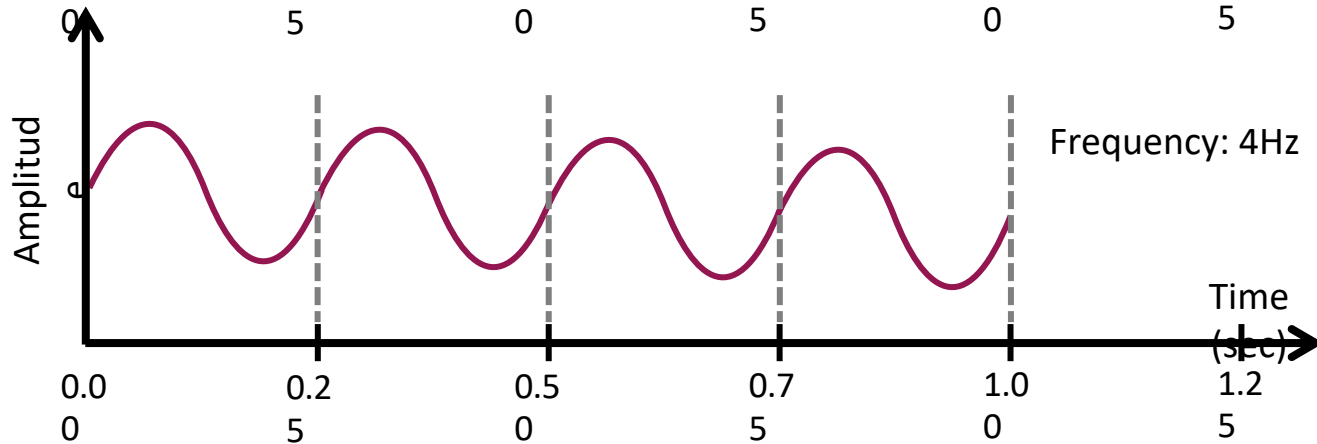
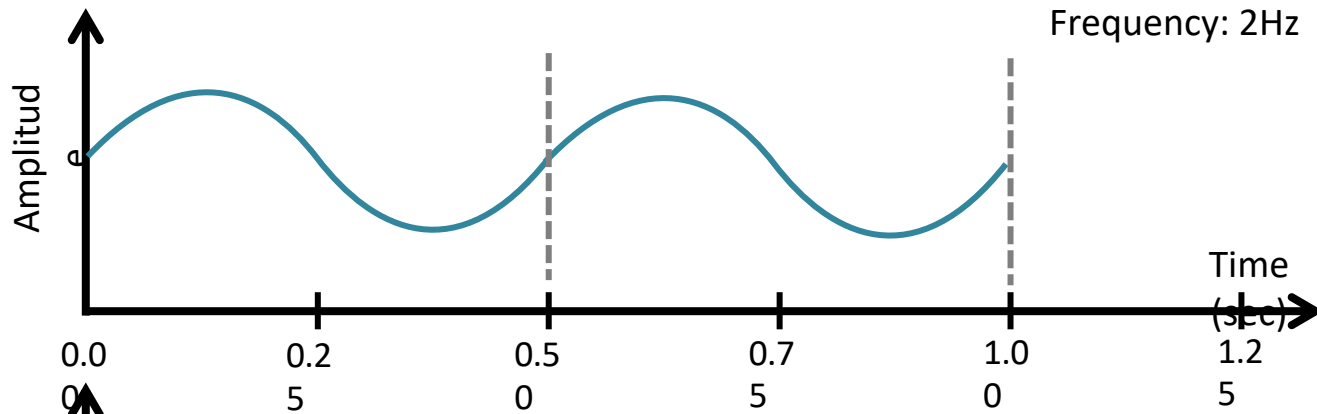
$2\pi$  angles per cycle  
 $f$  cycles per second



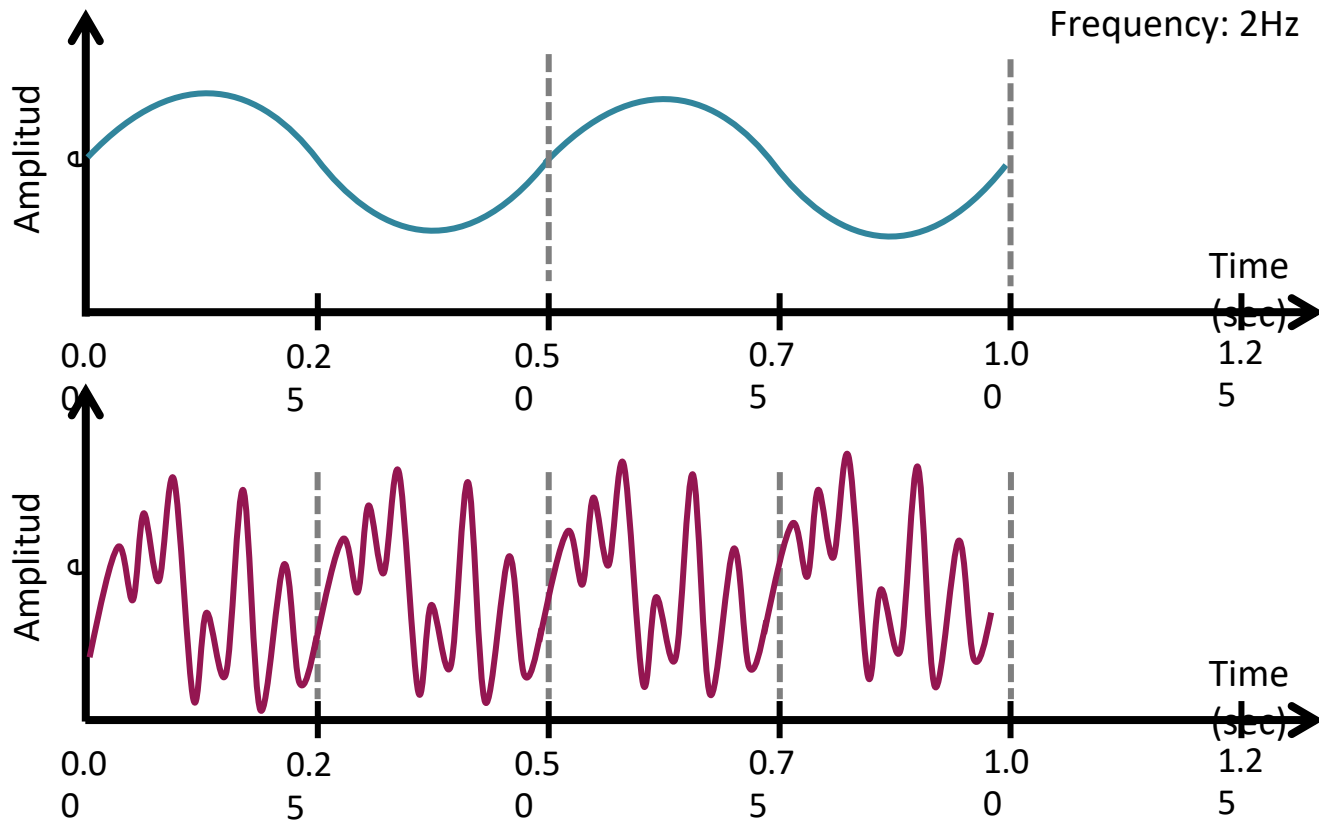
$= A \cdot \sin(\theta) = A \cdot \sin(2\pi ft)$

$A \cdot \sin(2\pi ft + \phi)$  -- with initial/additional phase  $\phi$

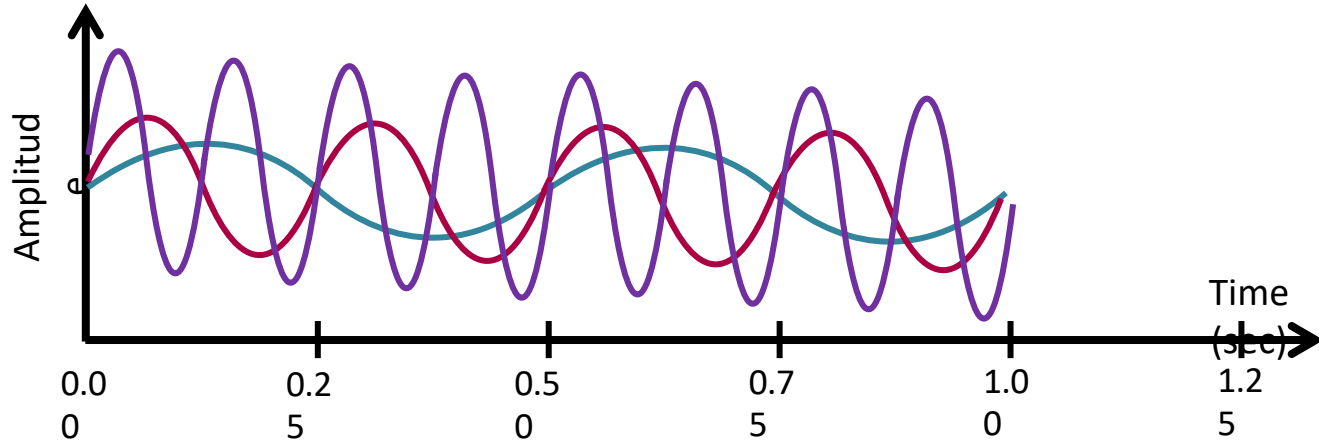
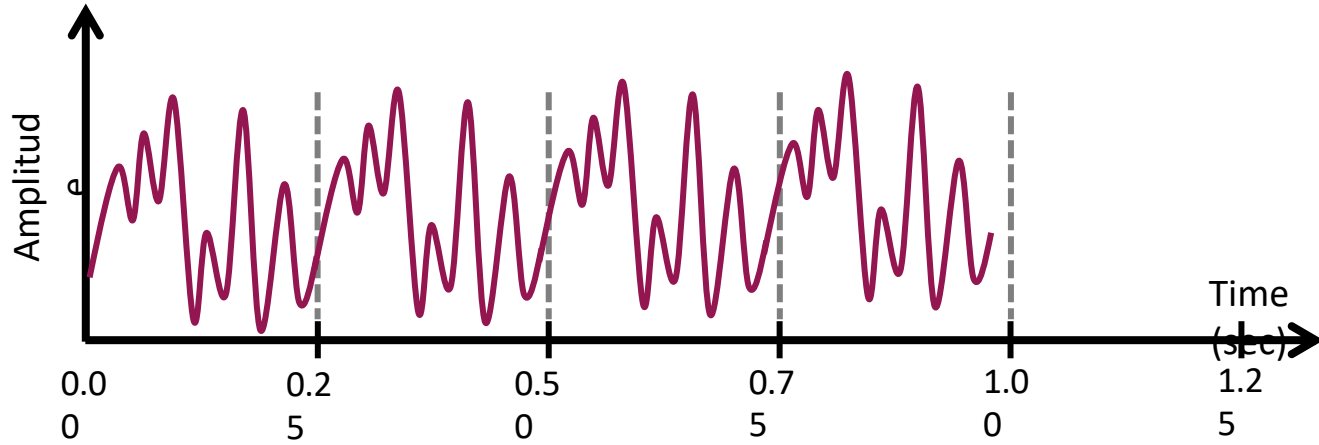
# Frequency, Amplitude, and Phase



# Frequencies of an arbitrary signal

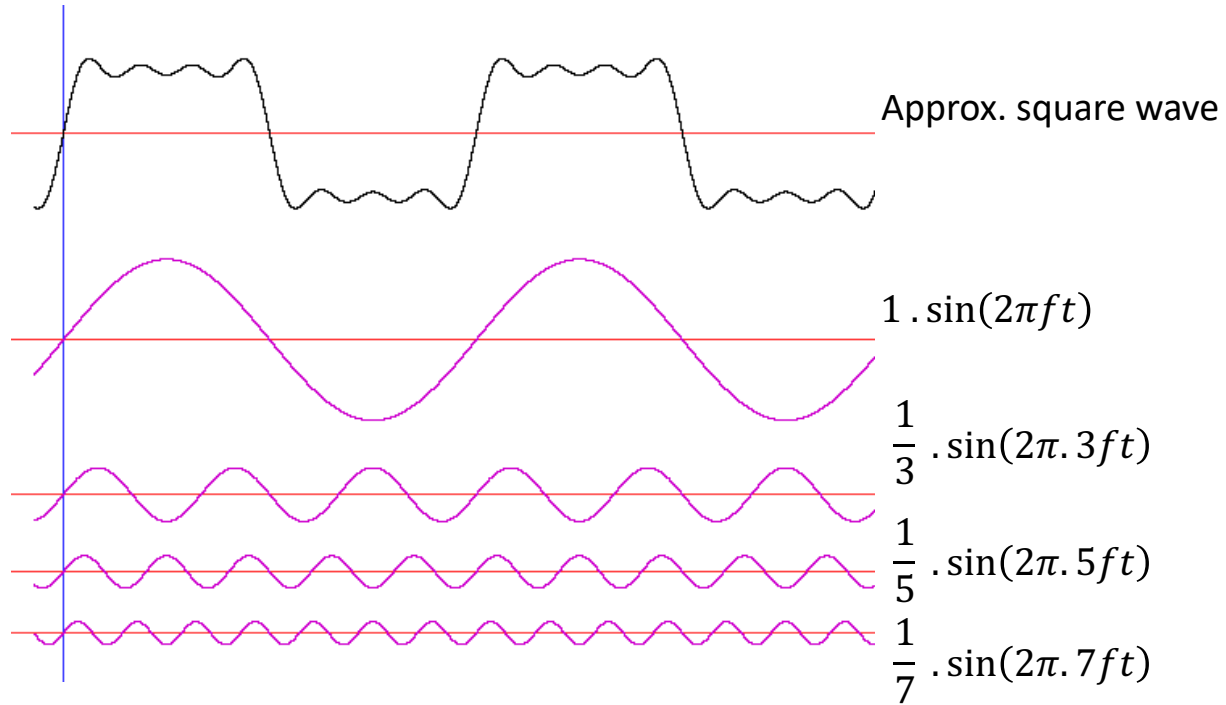


# The concept of the Fourier series

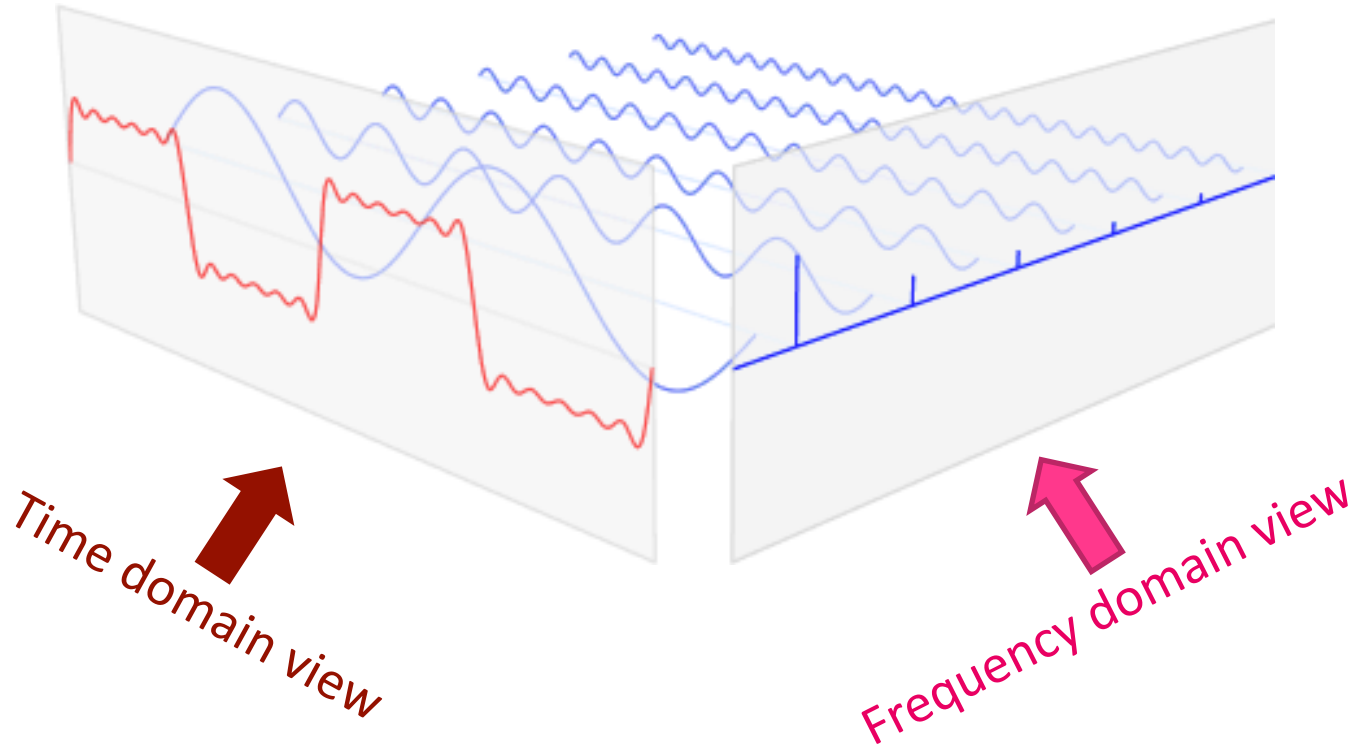




# Time Domain and Frequency Domain



# Time Domain and Frequency Domain



# Analogy: Food coloring chart

Basis for food colors



# Analogy: Food coloring chart

Basis for food colors

|        |     |        |       |      |
|--------|-----|--------|-------|------|
|        | RED | YELLOW | GREEN | BLUE |
| ORANGE | 1   | 2      |       |      |

# Analogy: Food coloring chart

Basis for food colors



|        | RED | YELLOW | GREEN | BLUE |
|--------|-----|--------|-------|------|
| ORANGE | 1   | 2      |       |      |
| PURPLE | 3   |        |       | 1    |

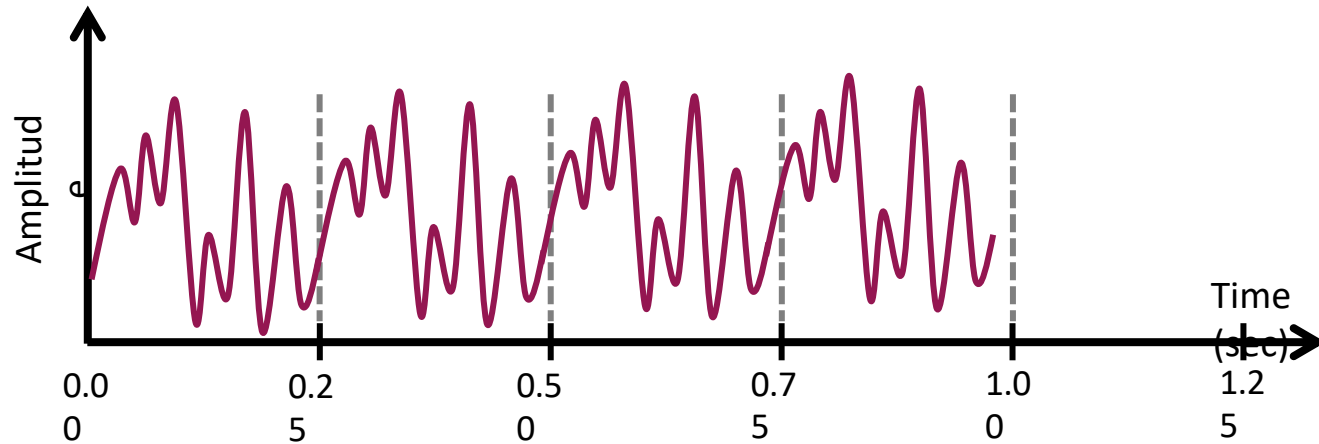
# Analogy: Food coloring chart

Basis for food colors



|            | RED | YELLOW | GREEN | BLUE |
|------------|-----|--------|-------|------|
| ORANGE     | 1   | 2      |       |      |
| PURPLE     | 3   |        |       | 1    |
| DARK GREEN | 1   | 1      |       | 4    |
| LIME GREEN |     | 3      | 1     |      |
| AQUA       |     |        | 2     | 4    |
| FLESH      | 2   | 5      |       |      |
| BROWN      | 6   | 6      |       | 4    |

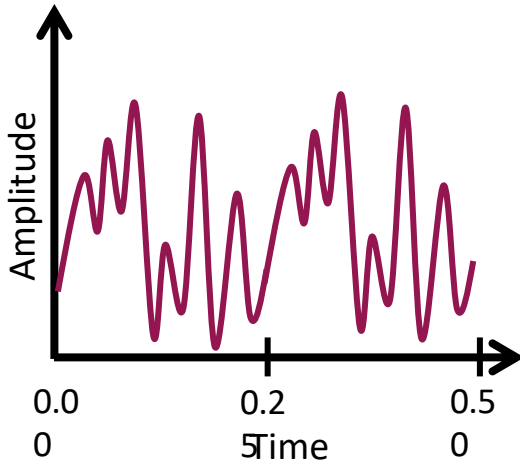
# Time Domain and Frequency Domain



$$\begin{aligned} &= A_1 \cdot \sin(2\pi f_1 t) + B_1 \cdot \cos(2\pi f_1 t) \\ &+ A_2 \cdot \sin(2\pi f_2 t) + B_2 \cdot \cos(2\pi f_2 t) \\ &+ A_3 \cdot \sin(2\pi f_3 t) + B_3 \cdot \cos(2\pi f_3 t) \\ &+ \dots \end{aligned}$$

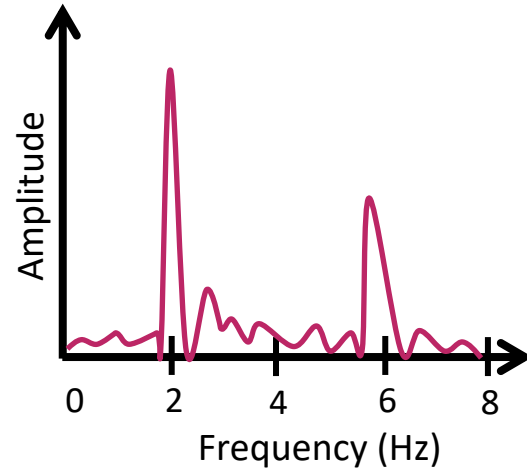
# Time Domain and Frequency Domain

Fourier Transform



Time domain

FFT  
IFFT



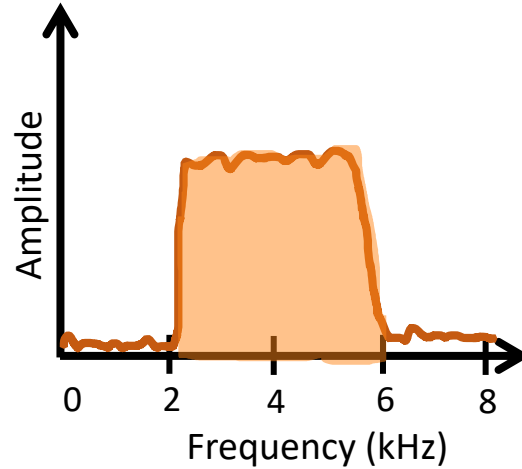
Frequency domain

FFT = Fast Fourier Transform

IFFT = Inverse Fast Fourier Transform



# Frequency band

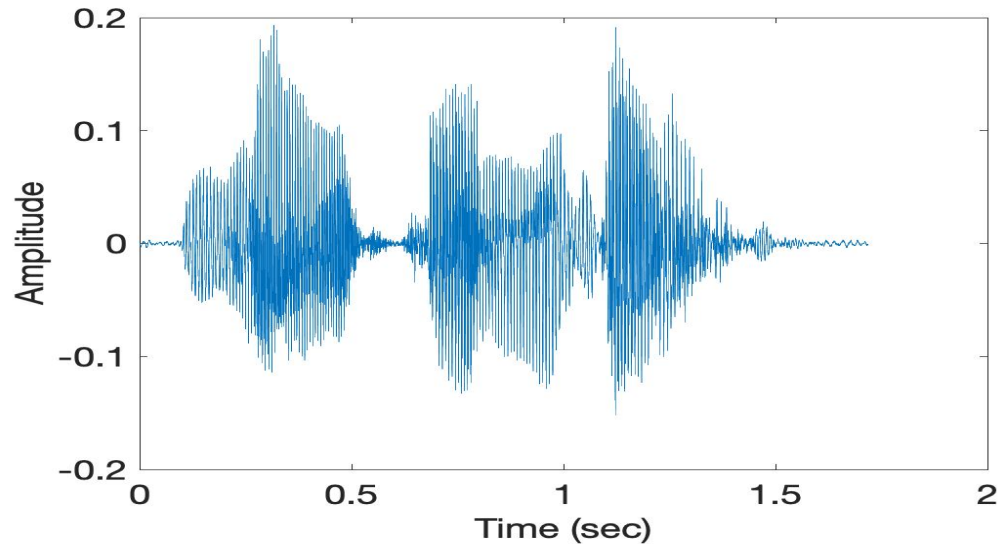


A 4 kHz frequency band starting at 2 kHz

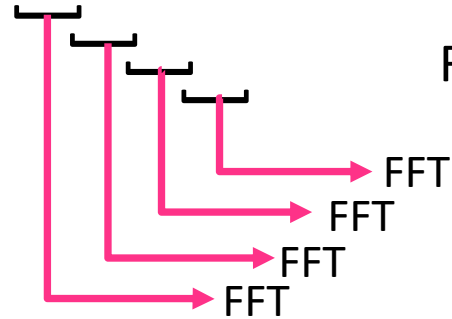
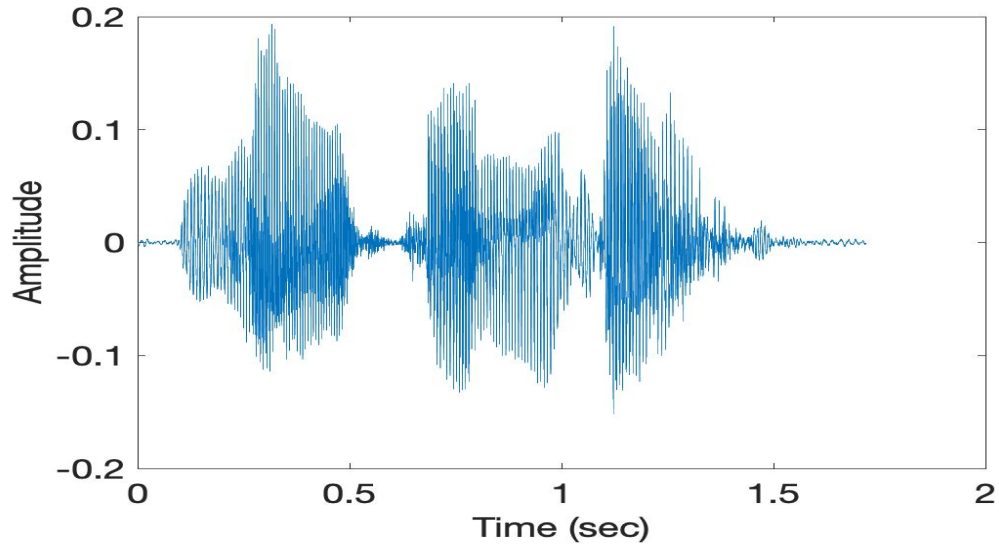
What is bandwidth?

What is center frequency?

# Spectrogram

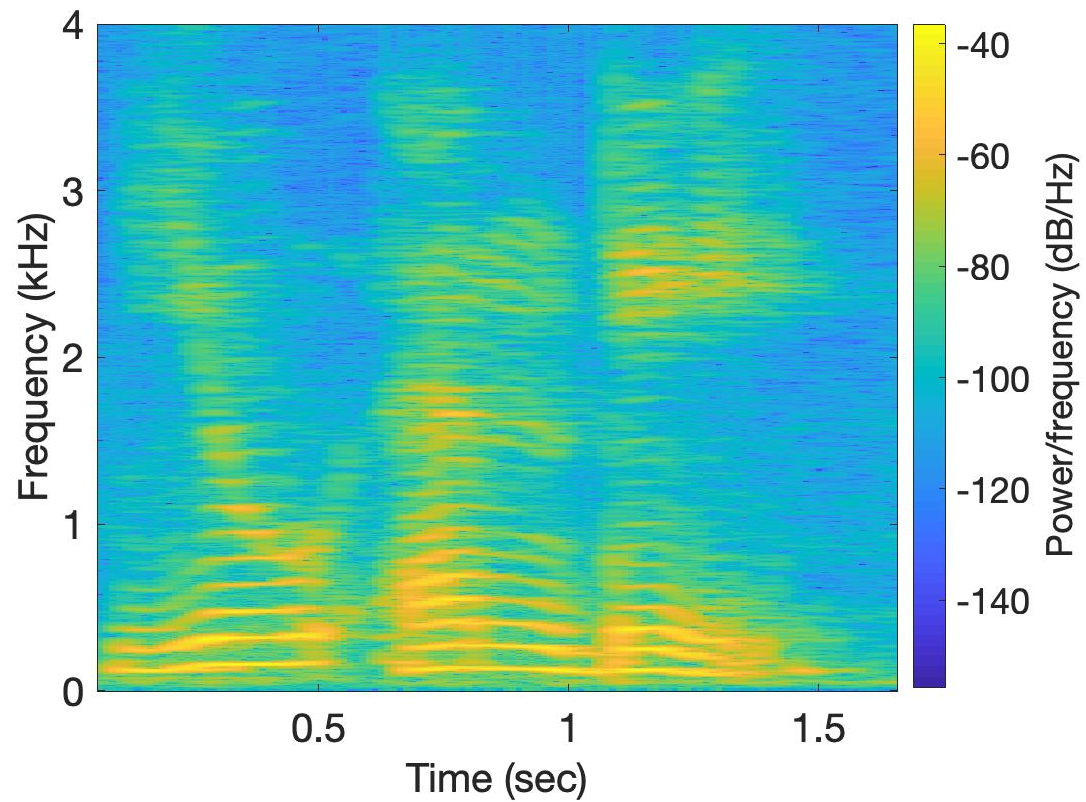


# Spectrogram

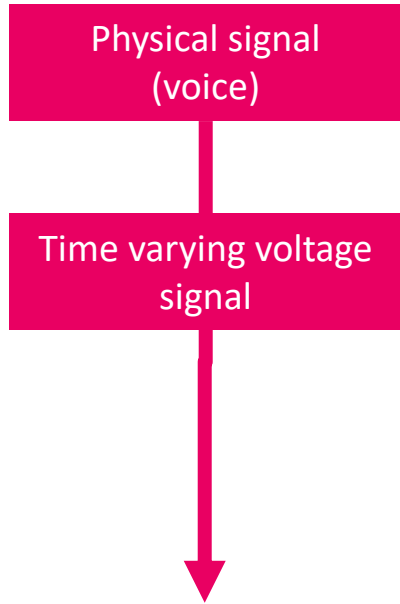


FFT of overlapping windows  
of samples  
(Spectrogram)

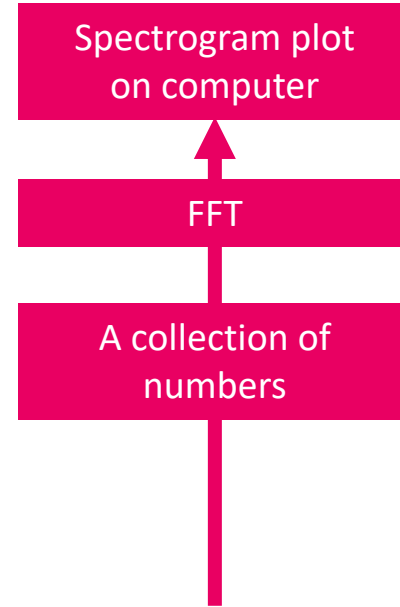
# Spectrogram



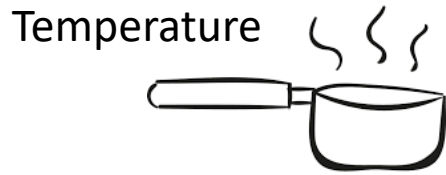
# Analog



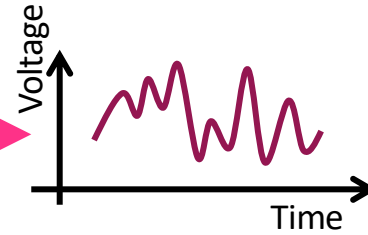
# Digital



# Analog vs Digital World



Thermo-couple



Analog

Digital

# Analog

Physical signal  
(voice)

Time varying voltage  
signal

?

# Digital

Spectrogram plot  
on computer

FFT

A collection of  
numbers

# Analog

Physical signal  
(voice)

Time varying voltage  
signal

ADC

Analog-to-Digital Converter

# Digital

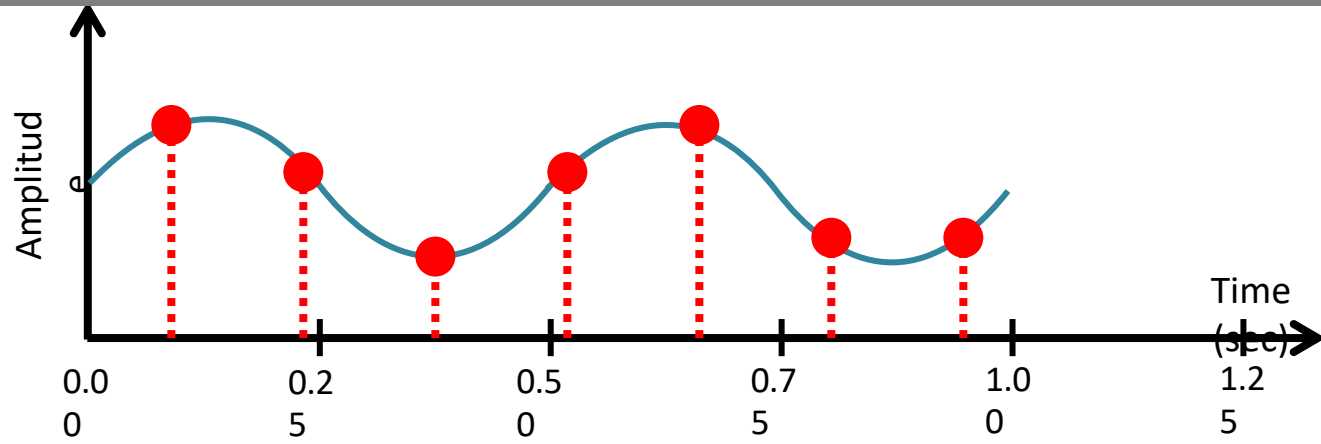
Spectrogram plot  
on computer

FFT

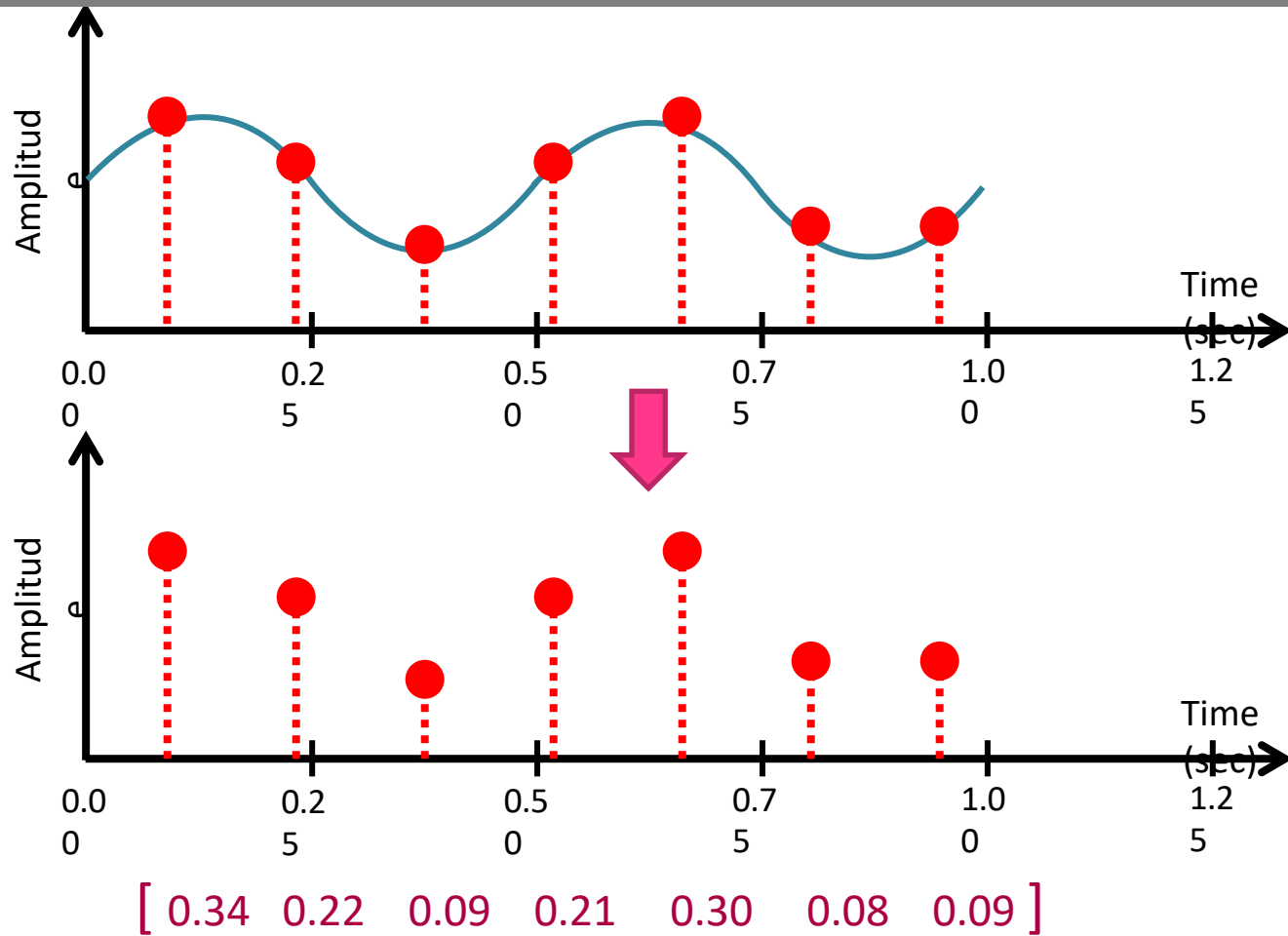
A collection of  
numbers



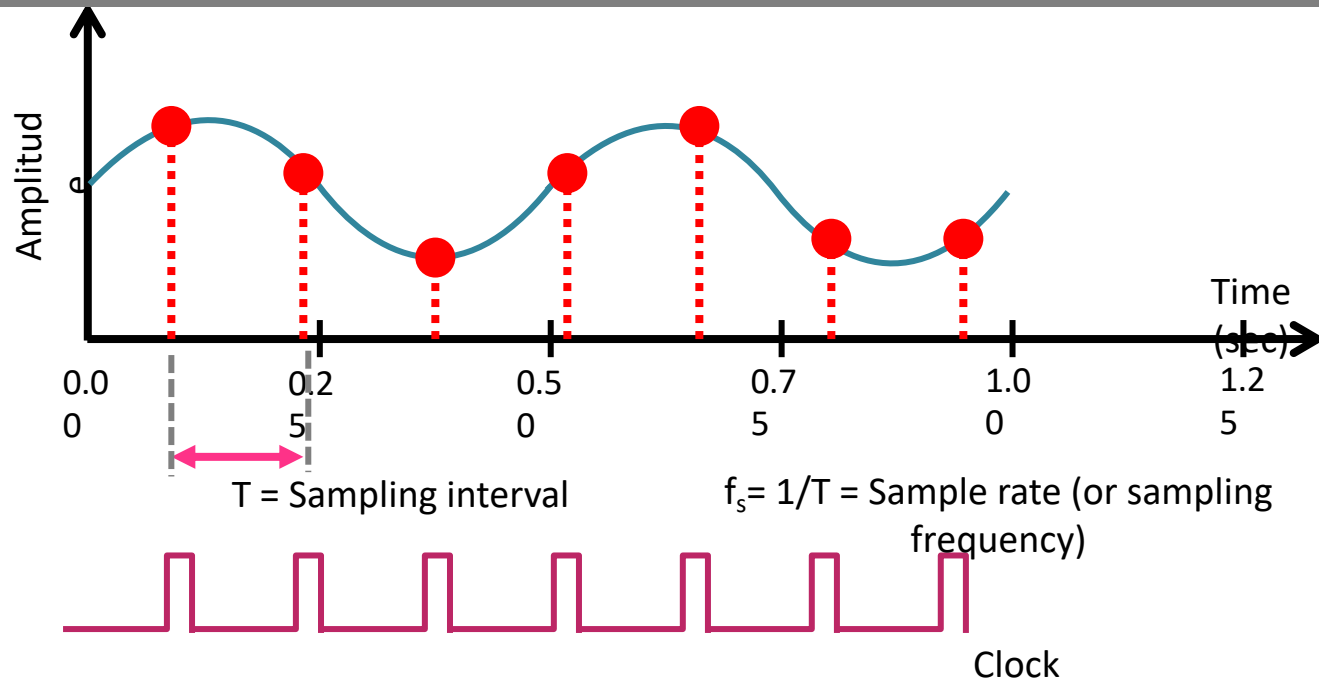
# Sampling theorem

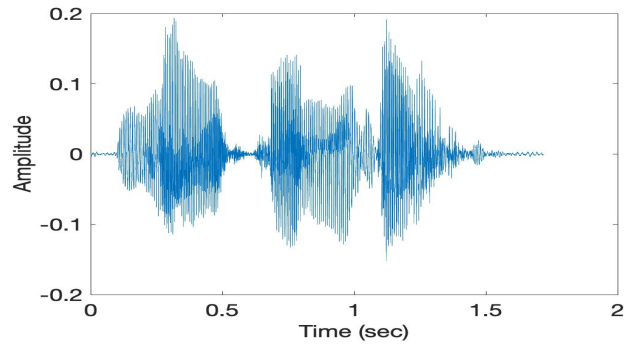


# Sampling theorem

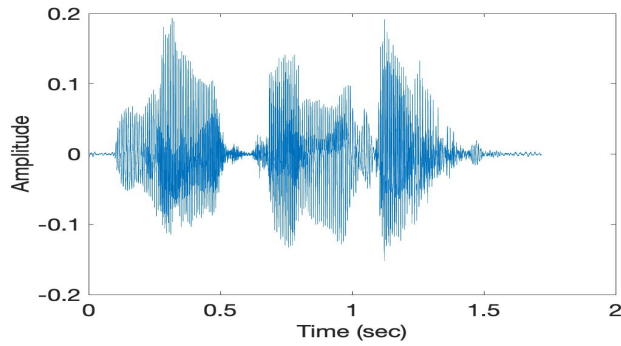


# Sampling theorem

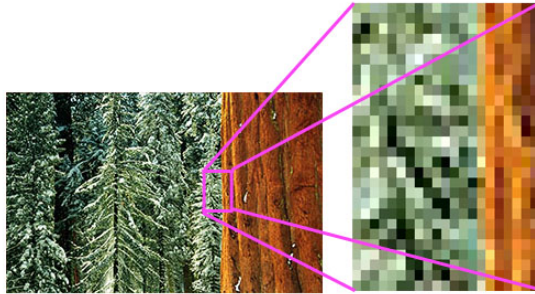




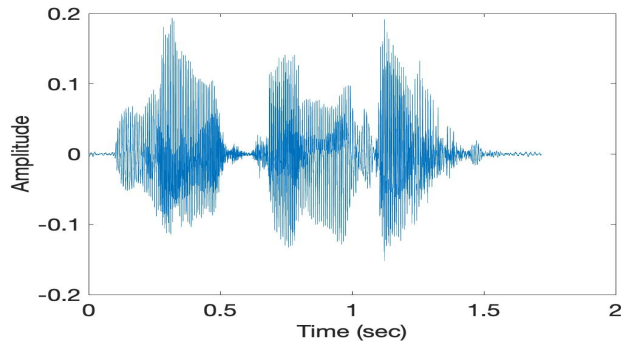
1-dimensional sampling



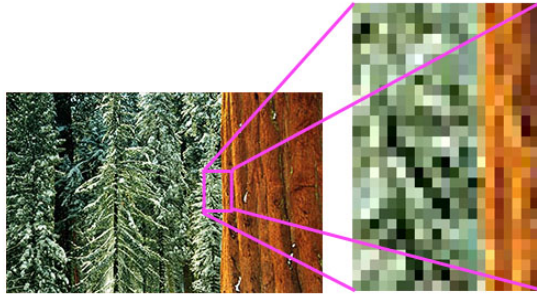
1-dimensional sampling



2-dimensional sampling



1-dimensional sampling

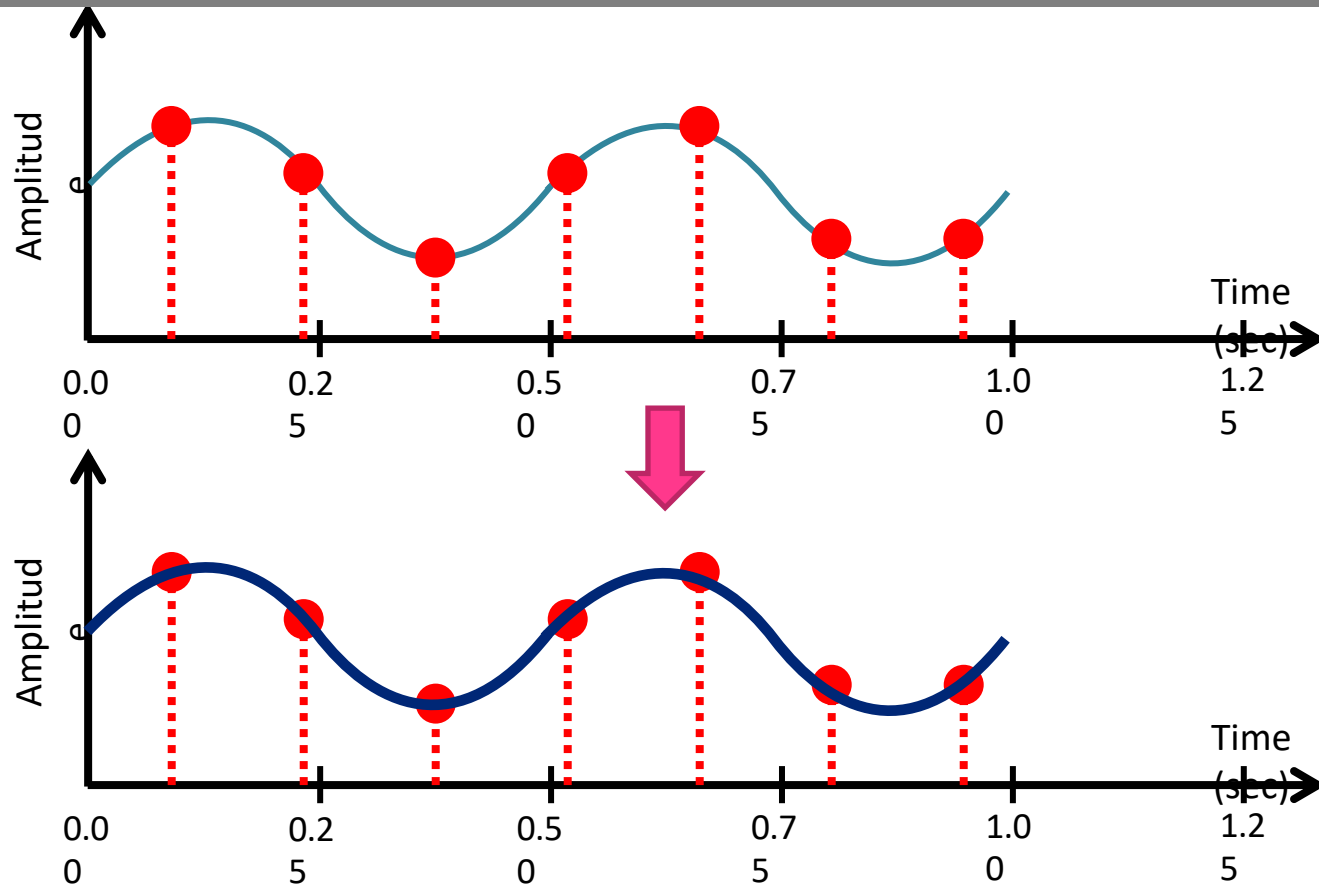


2-dimensional sampling

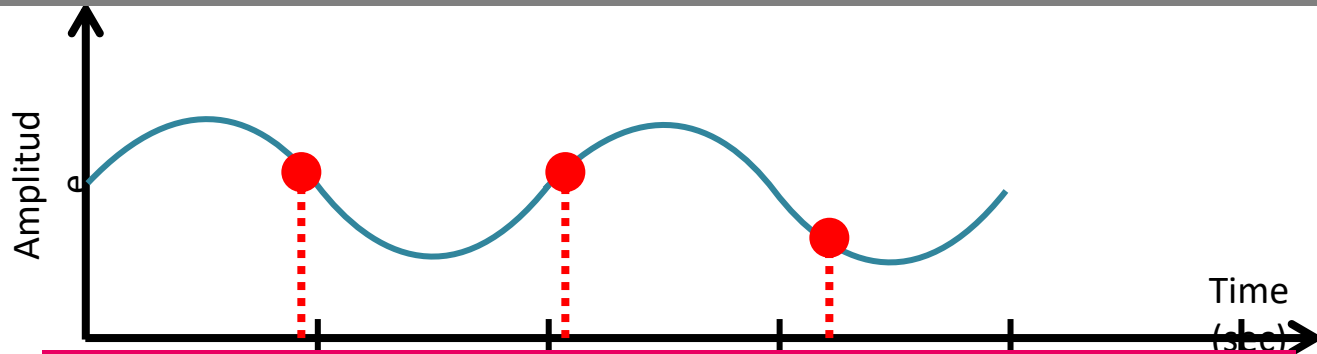


3-dimensional sampling

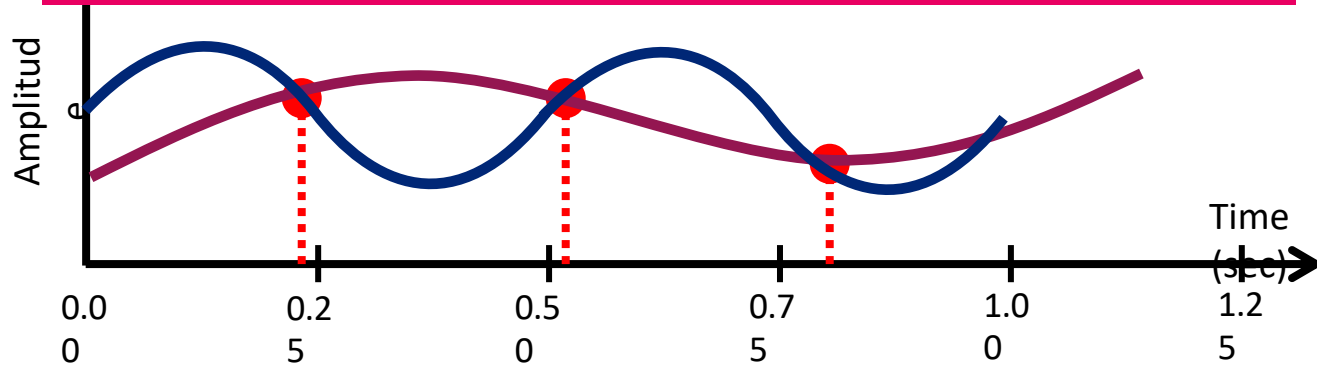
# Sampling theorem



# Sampling theorem

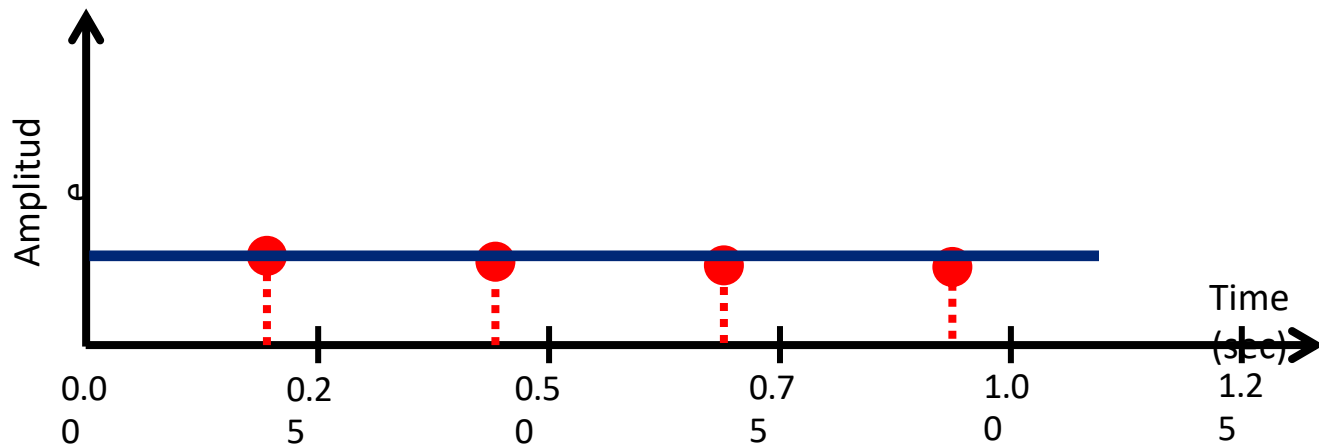
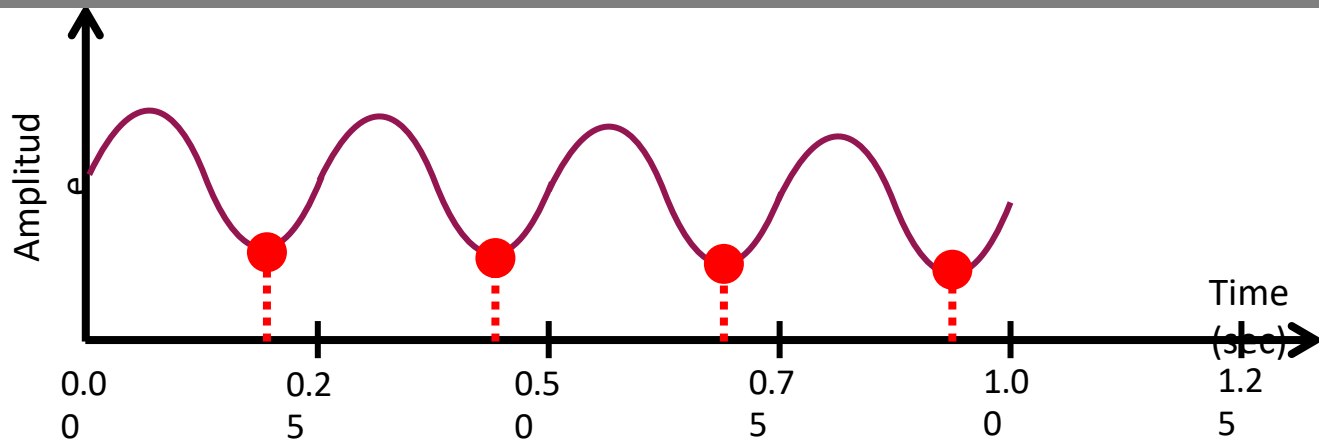


Aliasing: Two signals become indistinguishable after sampling





# Aliasing



# Aliasing



# Aliasing in real life



[https://www.youtube.com/watch?v=QOwzkND\\_ooU](https://www.youtube.com/watch?v=QOwzkND_ooU)

How to find a good sample rate?

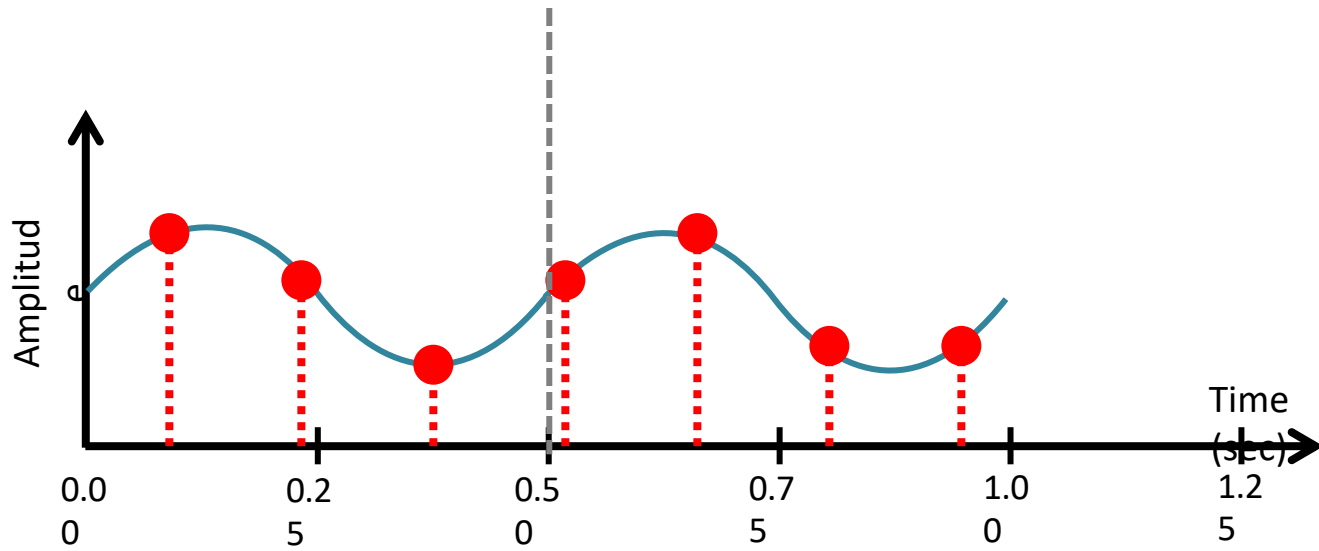
# How to find a good sample rate?

Nyquist sampling theorem:

In order to uniquely represent a signal  $F(t)$  by a set of samples, the sampling rate must be more than twice the highest frequency component present in  $F(t)$ .

If sample rate is  $f_s$  and maximum frequency we want record is  $f_{\max}$ , then

$$f_s > 2f_{\max}$$

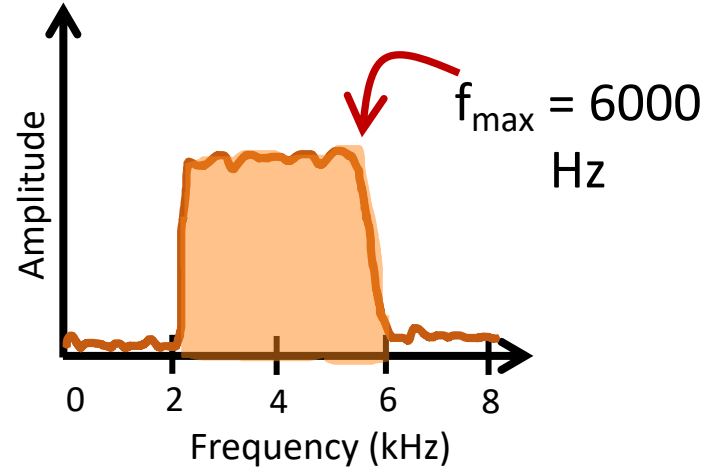
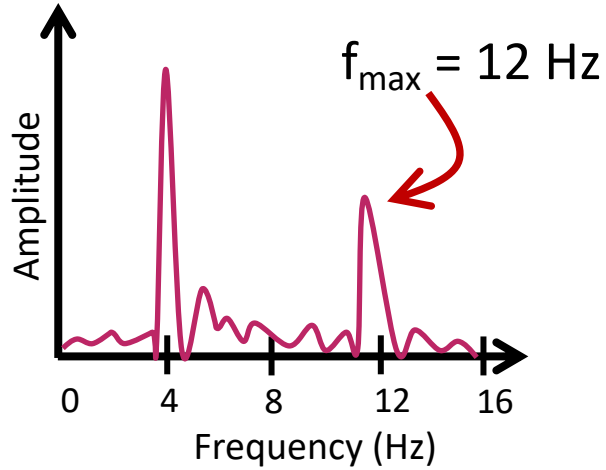


Nyquist frequency = Maximum alias-free frequency for a given sample rate.

Nyquist rate = Lower bound of sample rate for a signal

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT) \cdot \text{sinc} \left( \frac{t - nT}{T} \right),$$

# Nyquist

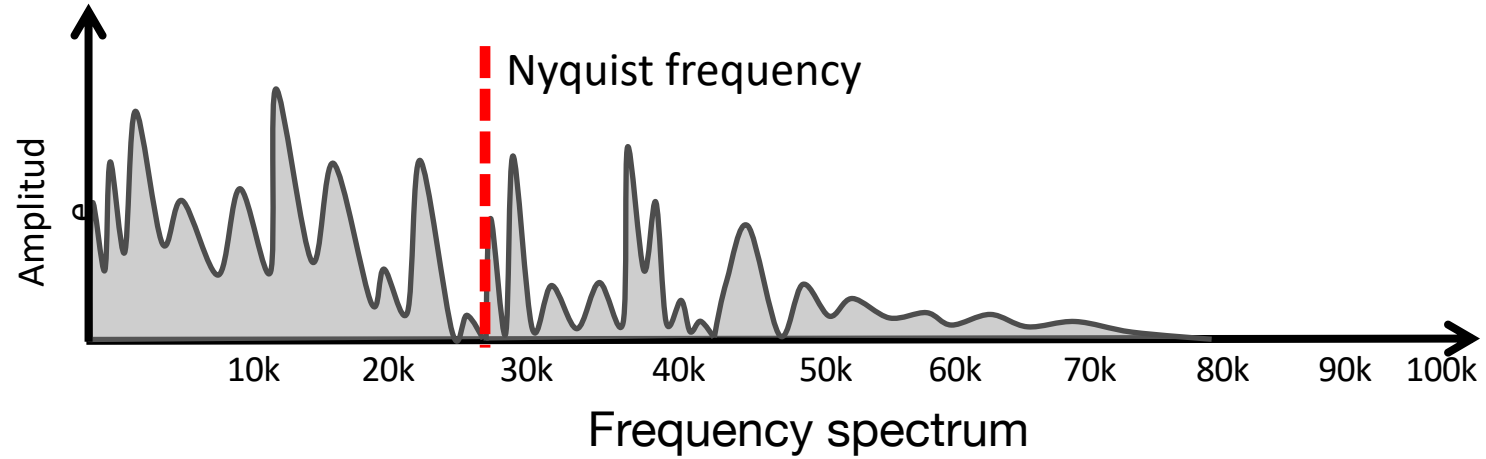


A 4 kHz frequency  
band starting at 2  
kHz

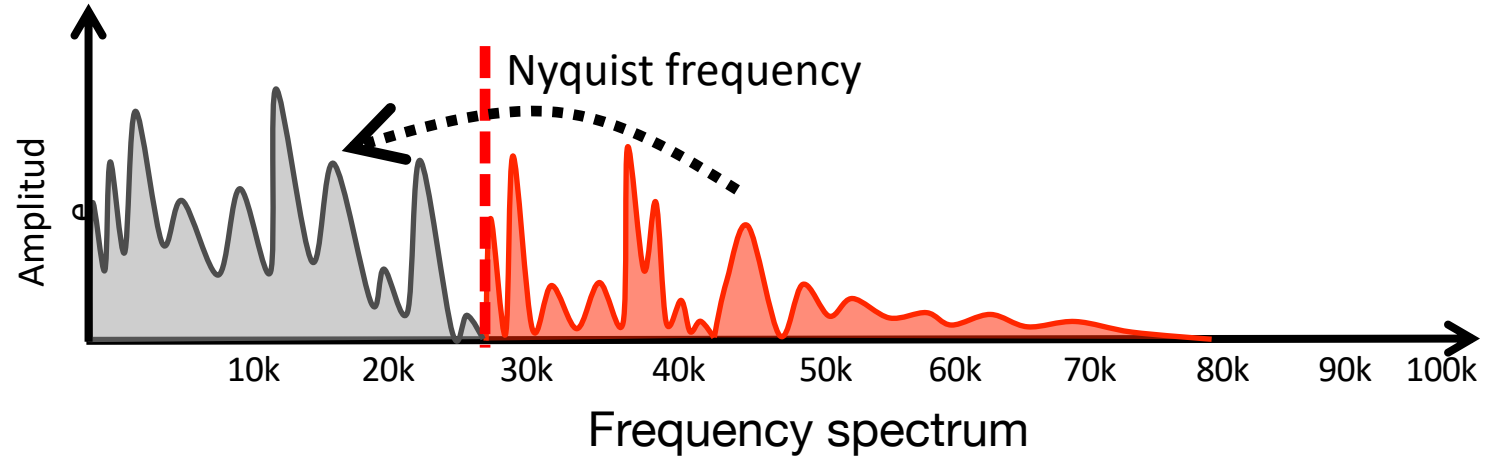


Commonly, the maximum frequency in human voice is 4 kHz, what sample rate will you use in your audio recorder?

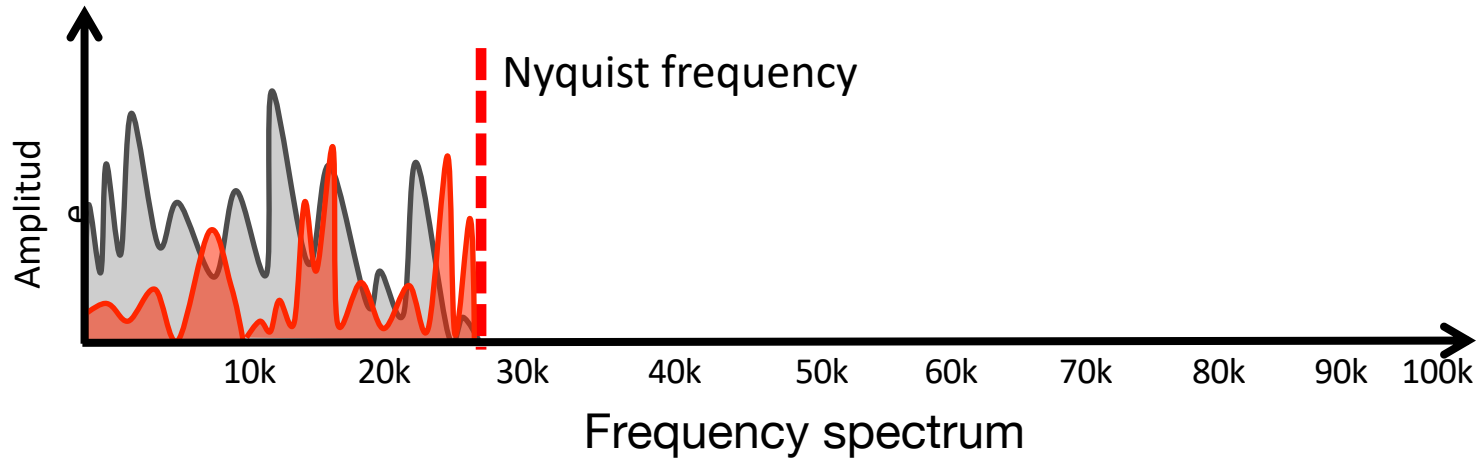
# Aliasing: A real life scenario



# Aliasing: A real life scenario

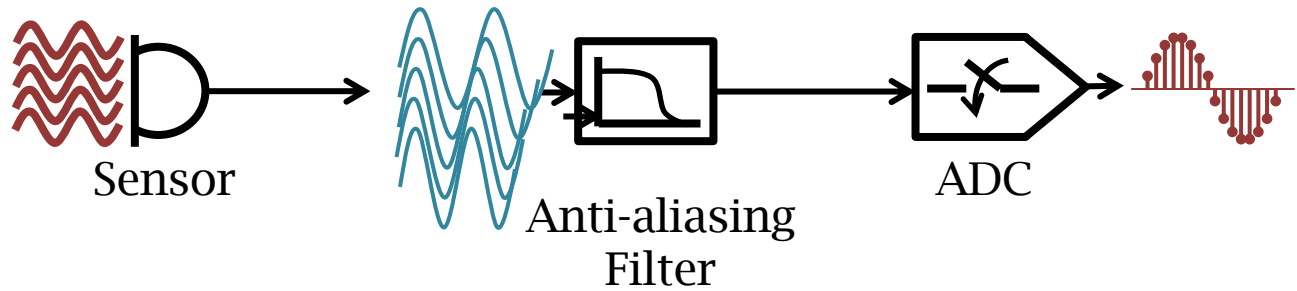
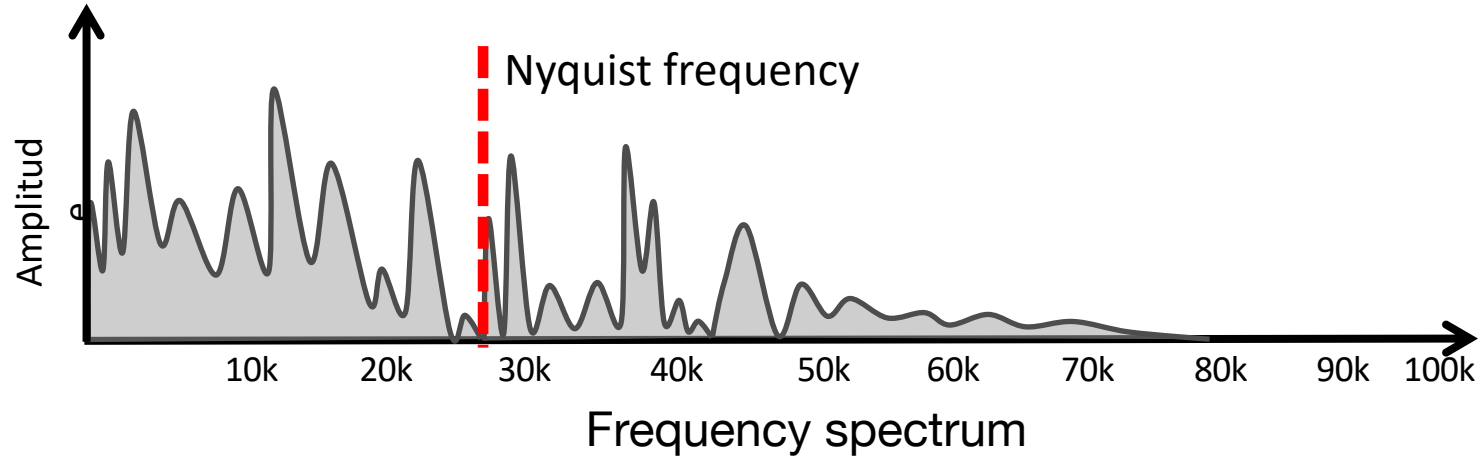


# Aliasing: A real life scenario

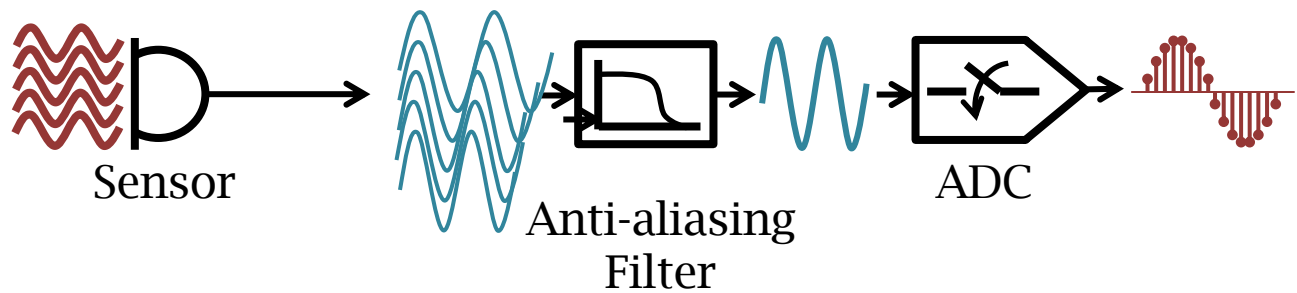
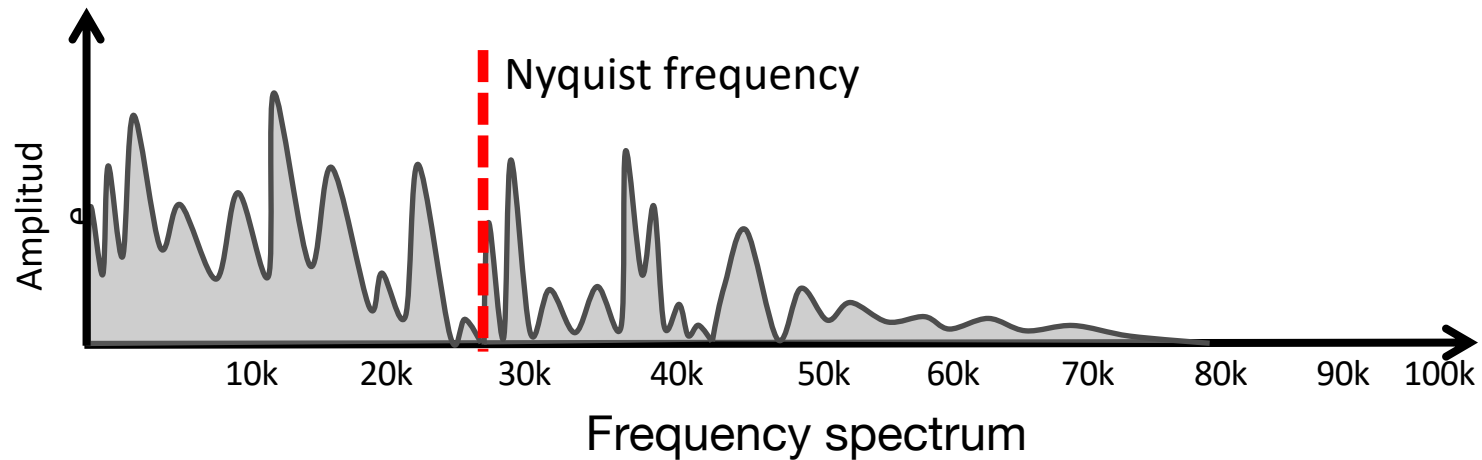


We need a “Low-pass filter”  
to remove unwanted high frequency signals

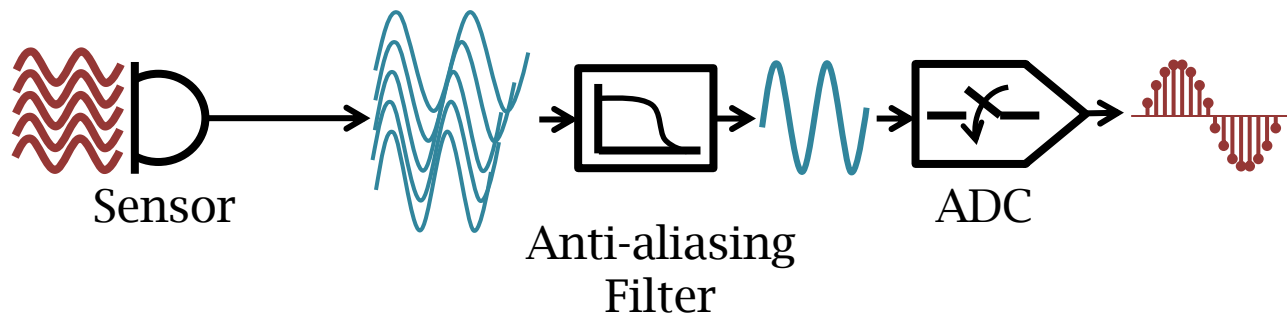
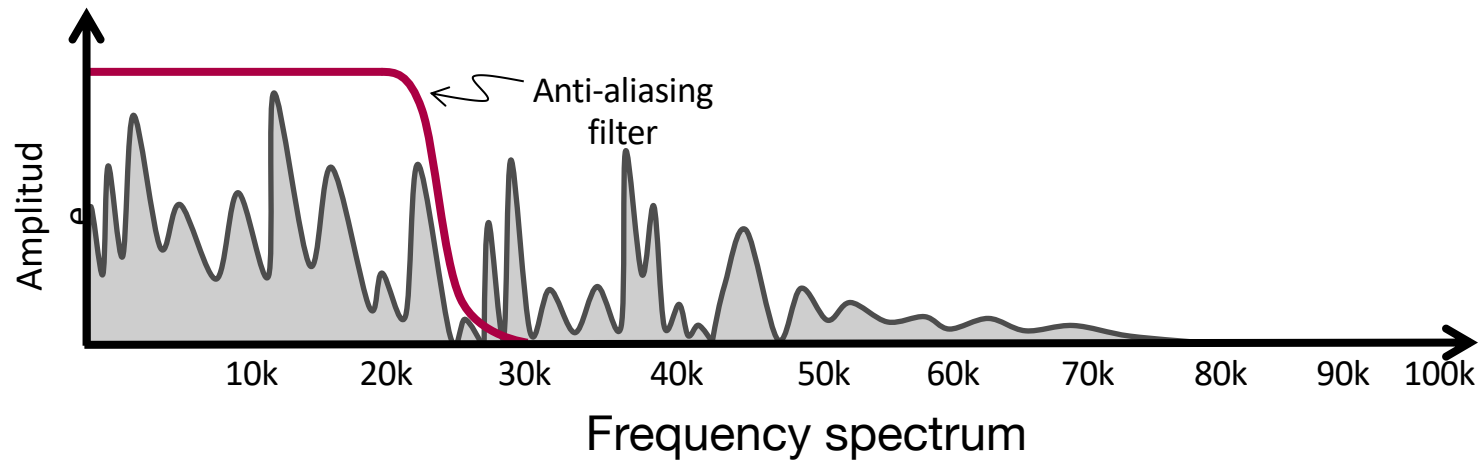
# Anti-aliasing filter



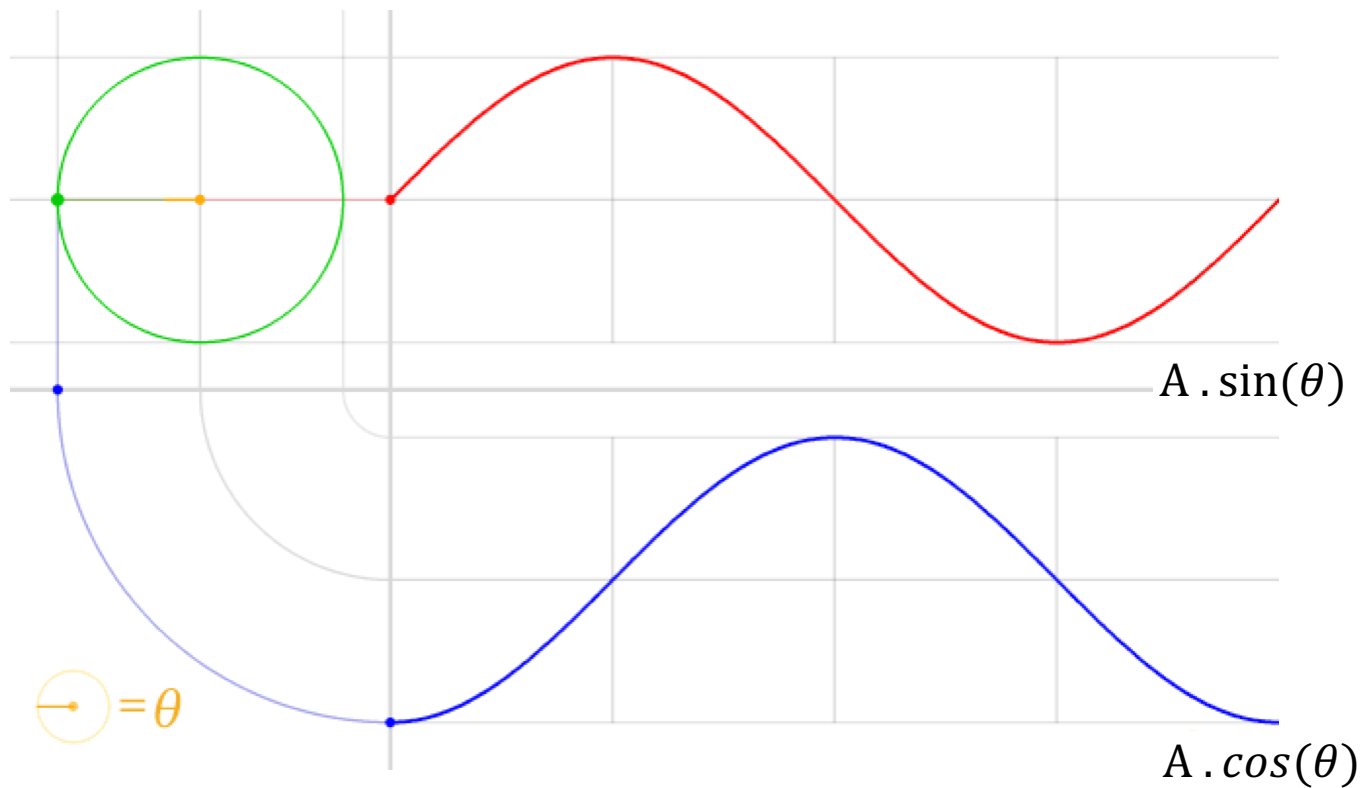
# Anti-aliasing filter



# Anti-aliasing filter

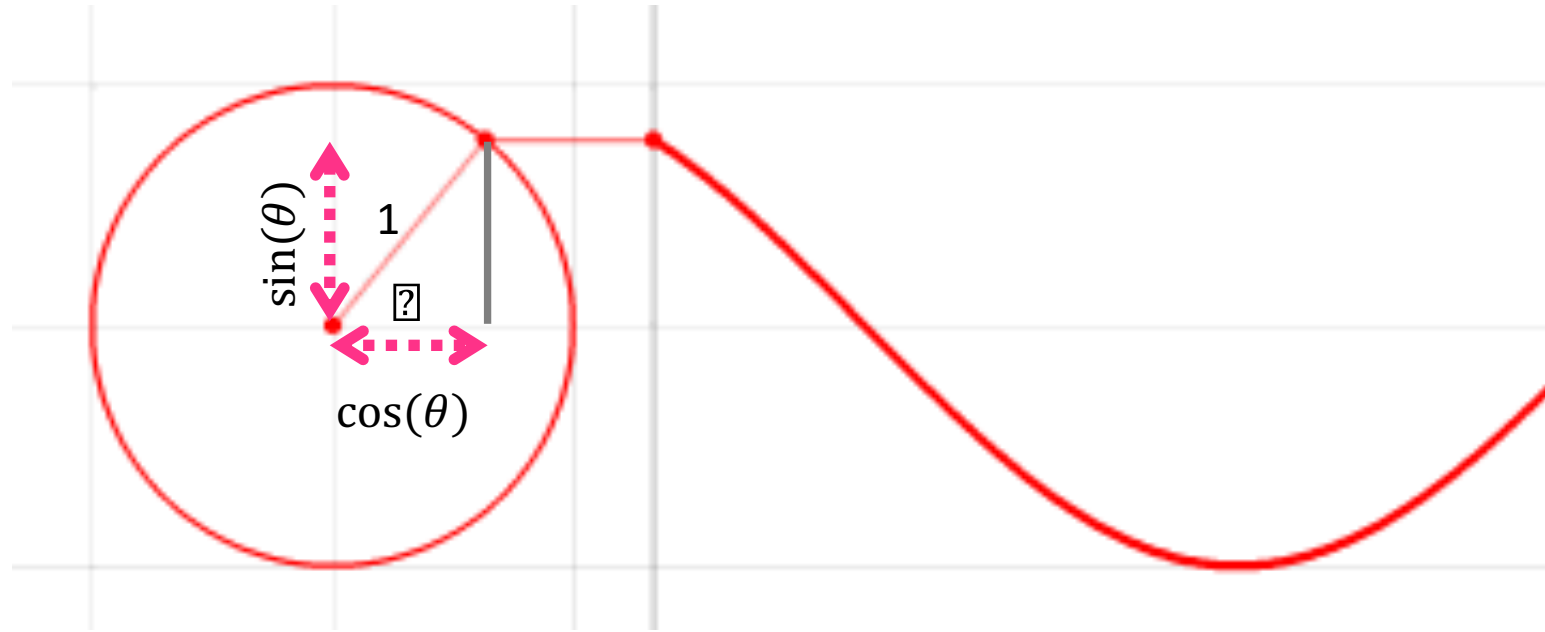


# Model for a signal (frequency, amplitude, and phase)



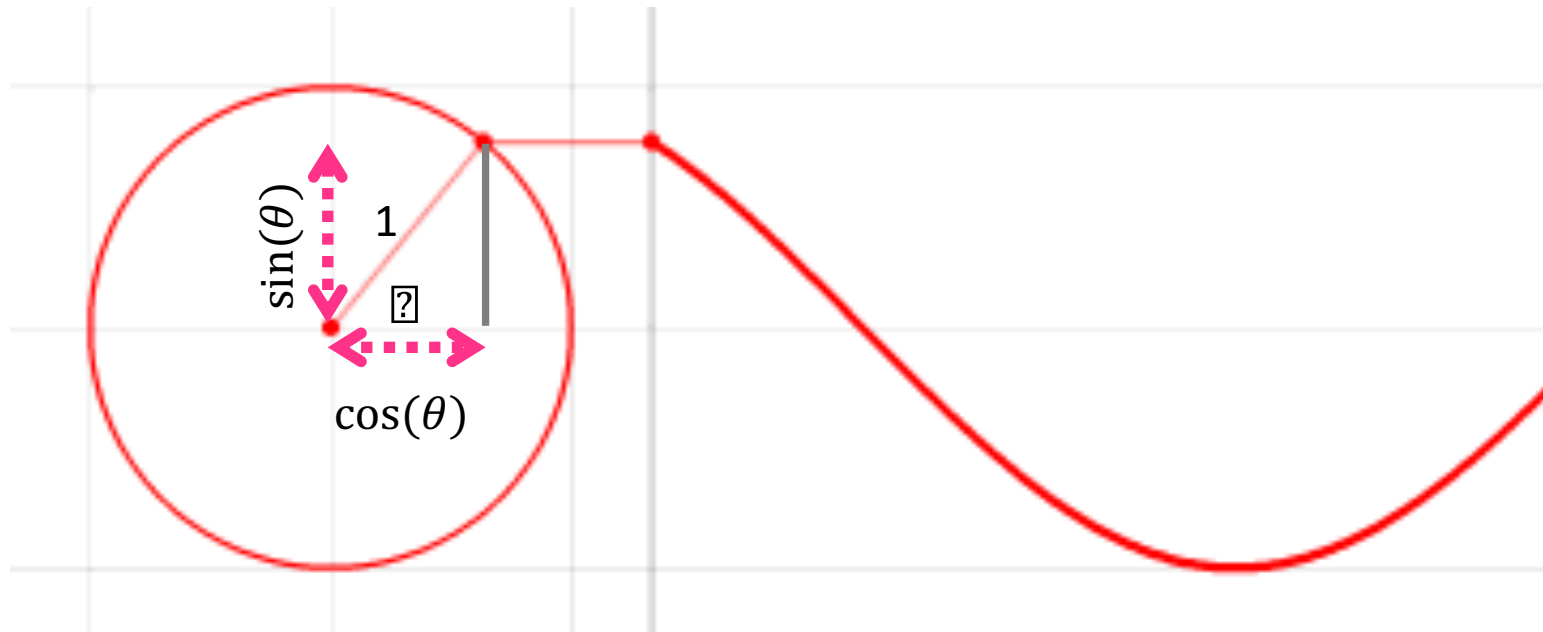


# Model for a signal (frequency, amplitude, and phase)



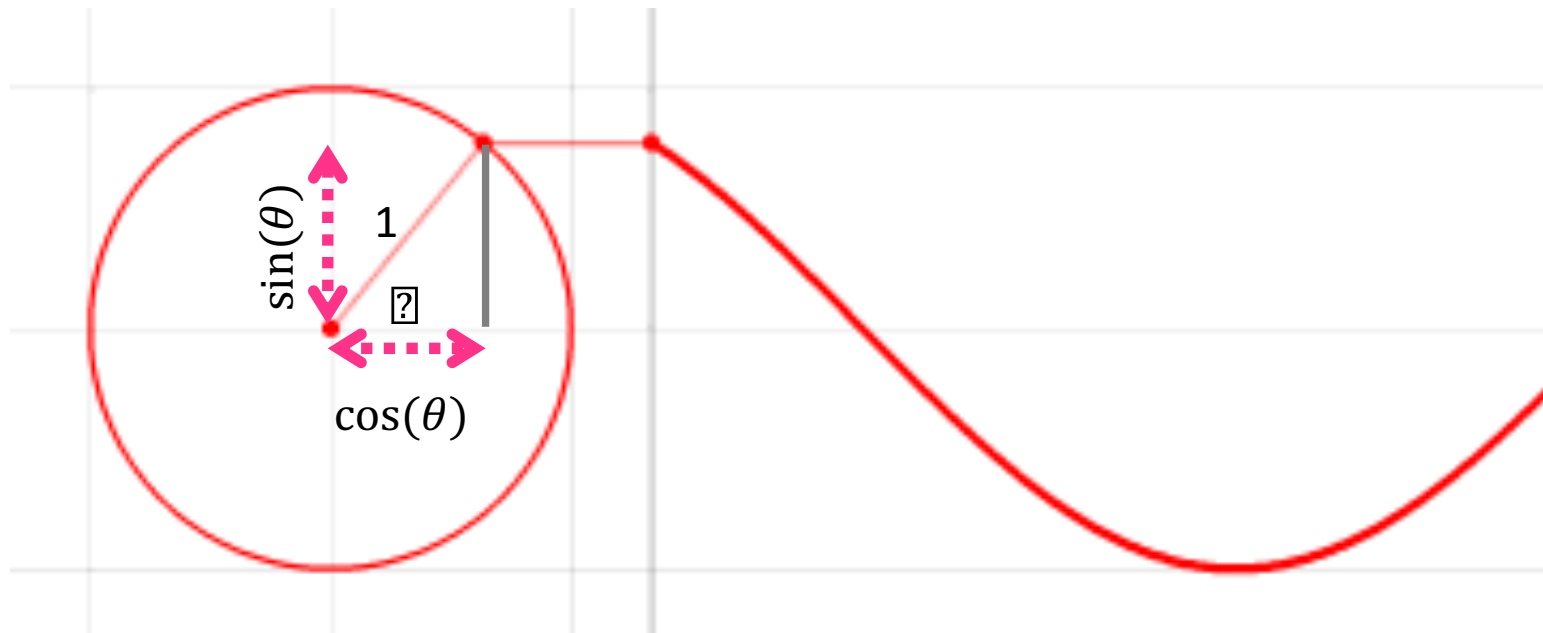
How can we incorporate both Sine and Cosine in the equation?

# Model for a signal (frequency, amplitude, and phase)



1.  $\cos(\theta) + \sin(\theta)$

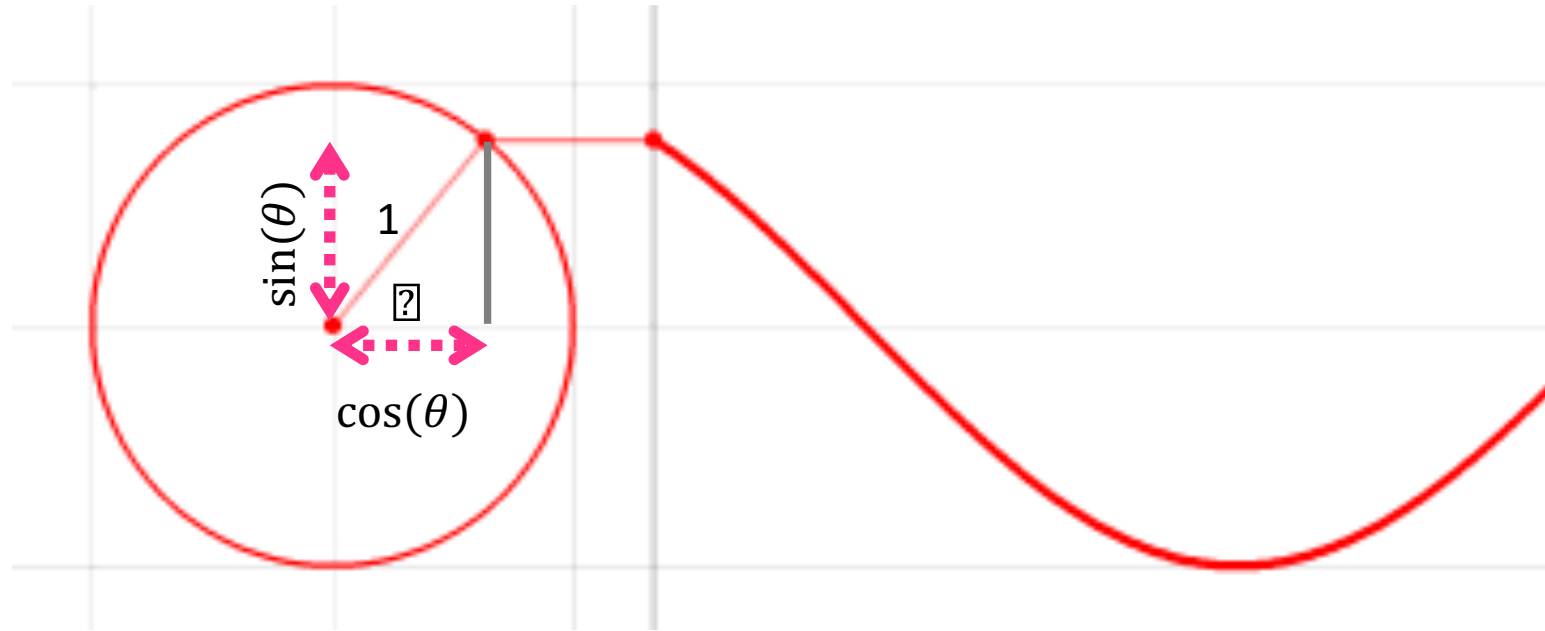
# Model for a signal (frequency, amplitude, and phase)



1.  $\cos(\theta) + \sin(\theta)$

2.  $\langle \cos(\theta), \sin(\theta) \rangle$

# Model for a signal (frequency, amplitude, and phase)




1.  $\cos(\theta) + \sin(\theta)$

2.  $\langle \cos(\theta), \sin(\theta) \rangle$

3.  $\cos(\theta) + j \sin(\theta)$

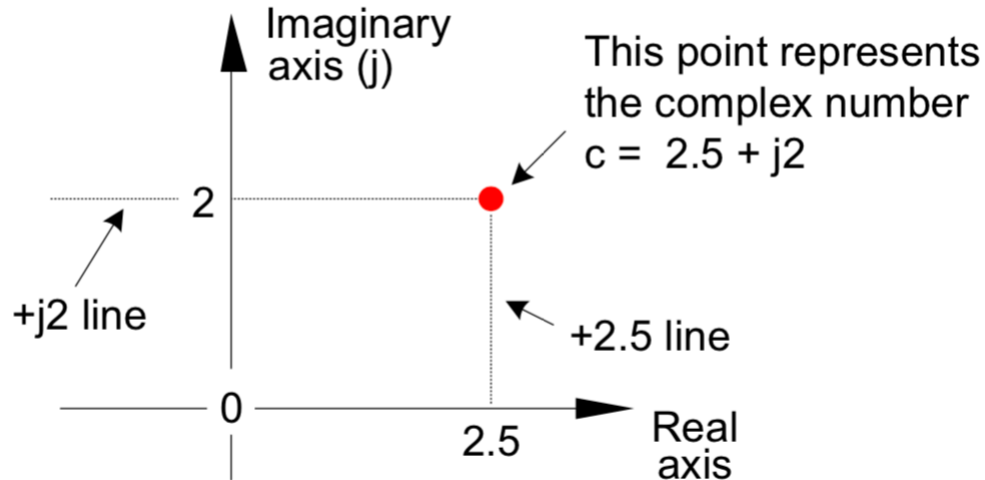
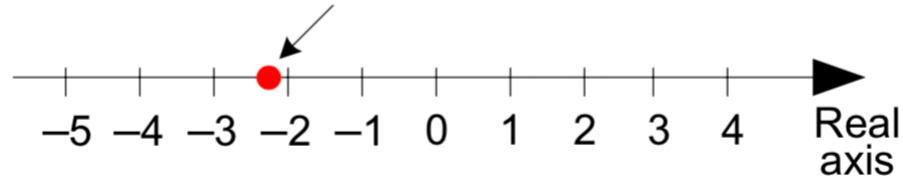
# Complex numbers

Imaginary

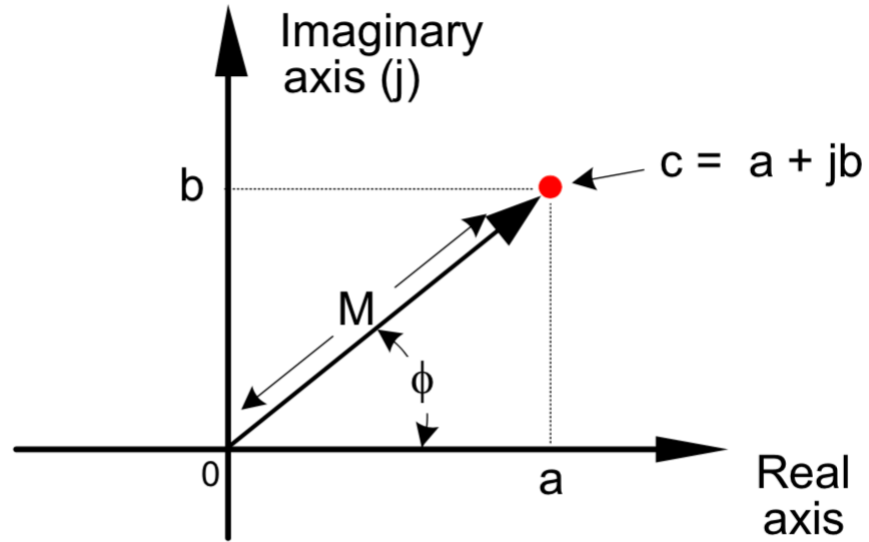

$$j = \sqrt{-1}$$

# Complex numbers

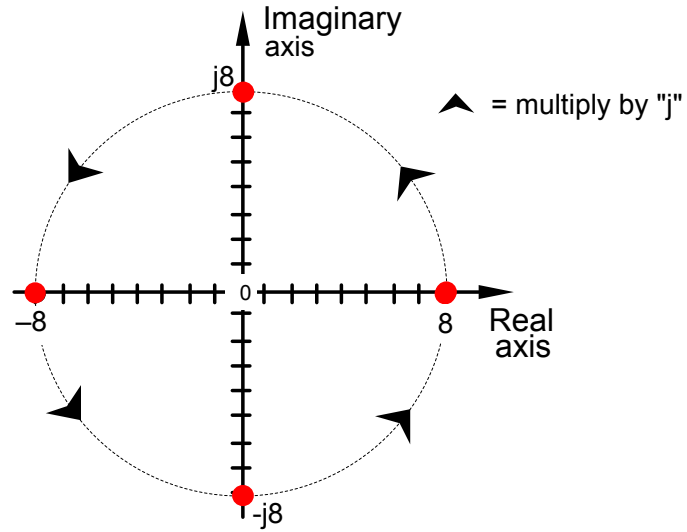
This point represents the  
real number  $a = -2.2$



# Complex numbers



# Complex numbers





# Complex numbers and Natural exponential

$$e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \frac{z^4}{4!} + \frac{z^5}{5!} + \frac{z^6}{6!} + \dots$$

# Complex numbers and Natural exponential

$$e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \frac{z^4}{4!} + \frac{z^5}{5!} + \frac{z^6}{6!} + \dots$$

$$e^{j\phi} = 1 + j\phi + \frac{(j\phi)^2}{2!} + \frac{(j\phi)^3}{3!} + \frac{(j\phi)^4}{4!} + \frac{(j\phi)^5}{5!} + \dots$$

# Complex numbers and Natural exponential

$$e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \frac{z^4}{4!} + \frac{z^5}{5!} + \frac{z^6}{6!} + \dots$$

$$\begin{aligned} e^{j\phi} &= 1 + j\phi + \frac{(j\phi)^2}{2!} + \frac{(j\phi)^3}{3!} + \frac{(j\phi)^4}{4!} + \frac{(j\phi)^5}{5!} + \dots \\ &= 1 + j\phi - \frac{\phi^2}{2!} - j\frac{\phi^3}{3!} + \frac{\phi^4}{4!} + j\frac{\phi^5}{5!} - \frac{\phi^6}{6!} + \dots \end{aligned}$$

# Complex numbers and Natural exponential

$$e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots$$

$$e^{j\phi} = 1 + j\phi + \frac{(j\phi)^2}{2!} + \frac{(j\phi)^3}{3!} + \dots$$

$$= 1 + j\phi - \frac{\phi^2}{2!} - j \frac{\phi^3}{3!} + \dots$$

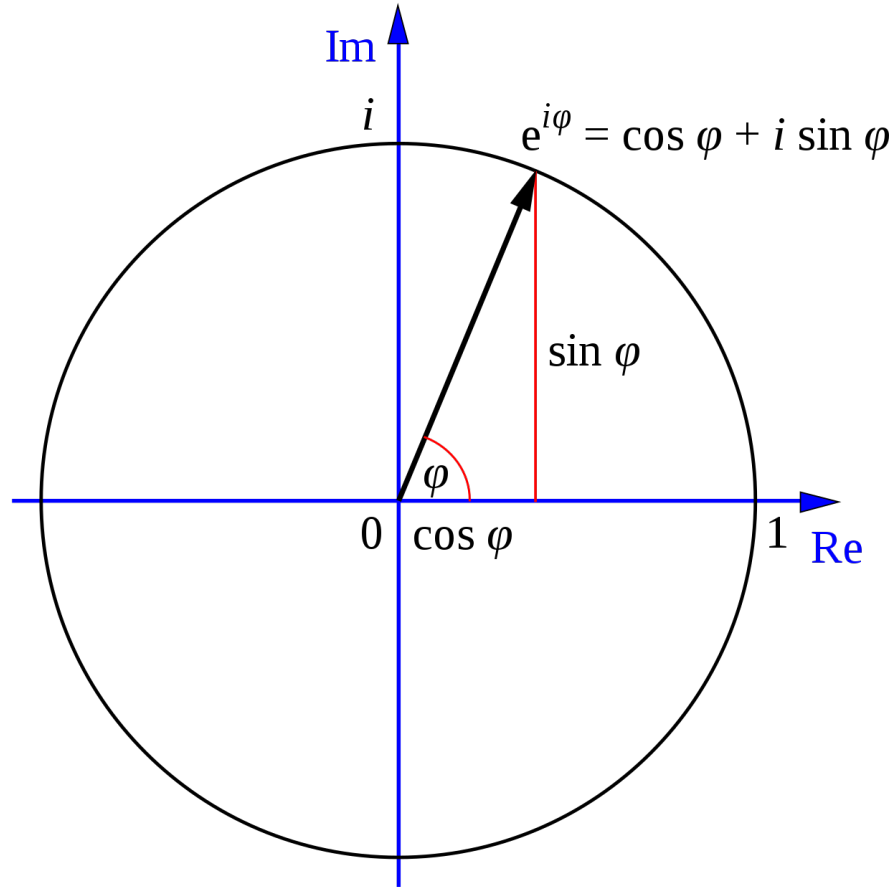
$\cos(\phi)$

$j \sin(\phi)$

$$\frac{\phi^6}{6!} +$$

$$- \frac{\phi^6}{6!}$$

# Complex numbers and Natural exponential



# Model for a signal (frequency, amplitude, and phase)



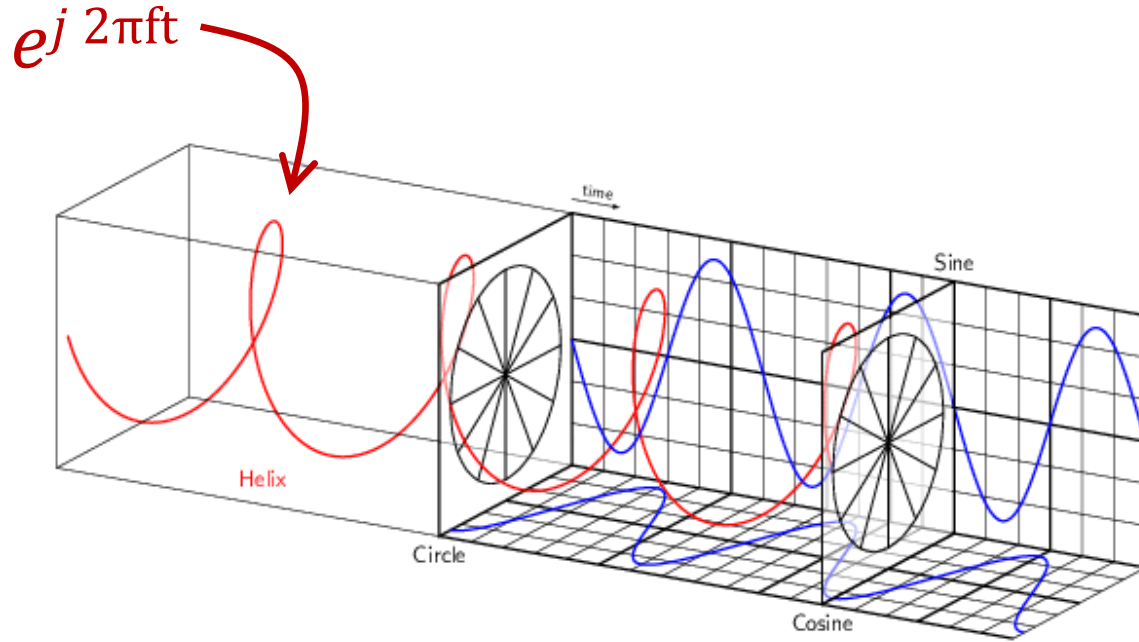
$$\cos(\theta) + j \sin(\theta) = e^{j\theta}$$

# Model for a signal (frequency, amplitude, and phase)



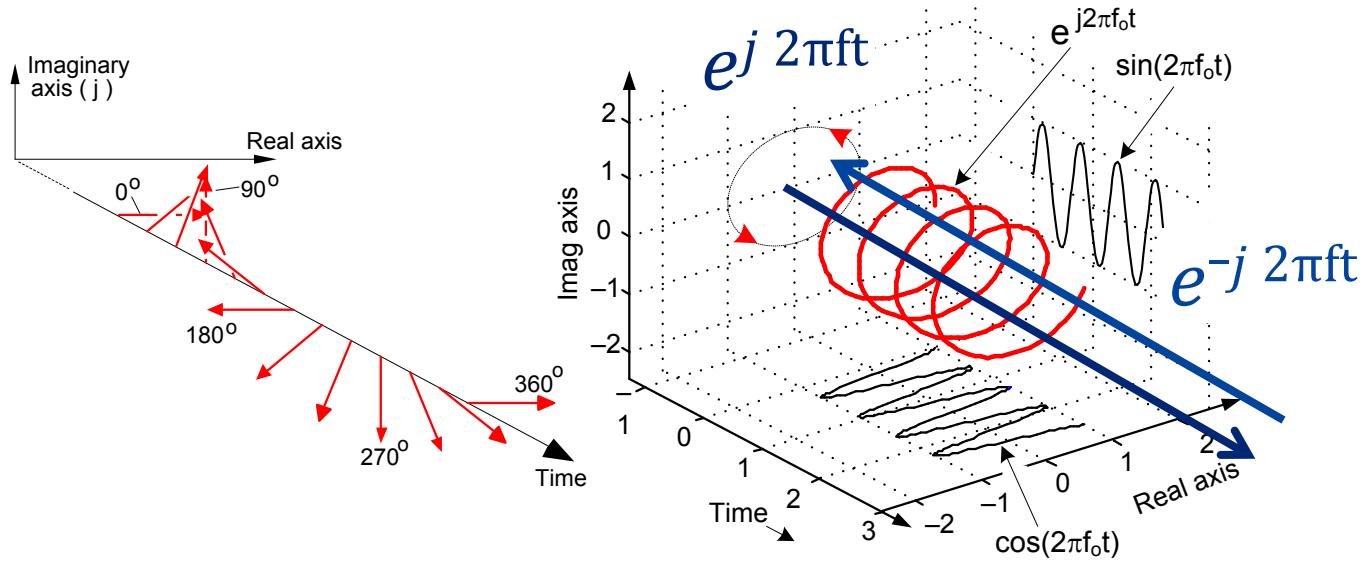
$$\cos(\theta) + j \sin(\theta) = e^{j\theta} = e^{j 2\pi f t}$$

# Model for a signal (frequency, amplitude, and phase)





# Model for a signal (frequency, amplitude, and phase)



## Model for a signal (frequency, amplitude, and phase)

$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

$$e^{-j\theta} = \cos(\theta) - j \sin(\theta)$$

How about real sinusoids?

$$\cos(\theta) = ?$$

$$\sin(\theta) = ?$$

## Presenting real signal with the complex model

$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

$$e^{-j\theta} = \cos(\theta) - j \sin(\theta)$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

## Presenting real signal with the complex model

$$\star e^{j 2\pi ft} = \cos(2\pi ft) + j \sin(2\pi ft)$$

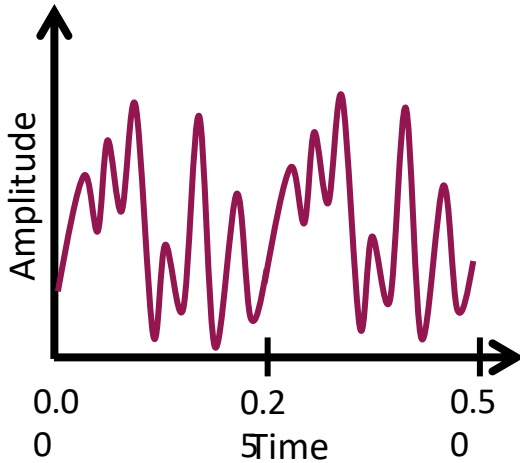
$$\star e^{-j 2\pi ft} = \cos(2\pi ft) - j \sin(2\pi ft)$$

$$\star \cos(2\pi ft) = \frac{e^{j 2\pi ft} + e^{-j 2\pi ft}}{2}$$

$$\star \sin(2\pi ft) = \frac{e^{j 2\pi ft} - e^{-j 2\pi ft}}{2j}$$

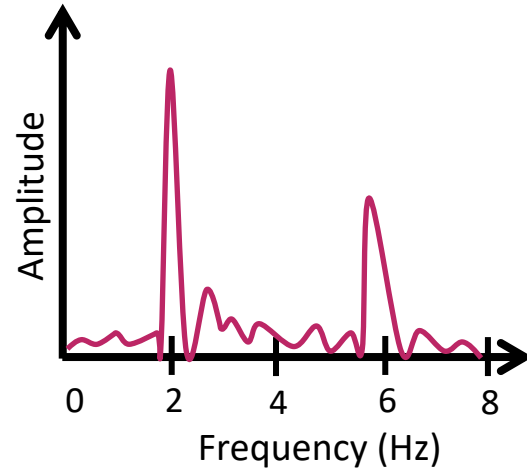
# Time Domain and Frequency Domain

Fourier Transform



Time domain

FFT  
IFFT



Frequency domain

FFT = Fast Fourier Transform

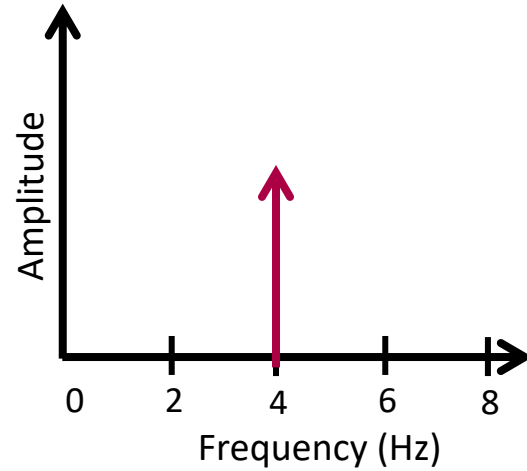
IFFT = Inverse Fast Fourier Transform

# Time Domain and Frequency Domain

Fourier Transform

$$e^{j 2\pi 4t}$$

FFT →



Time domain

Frequency domain

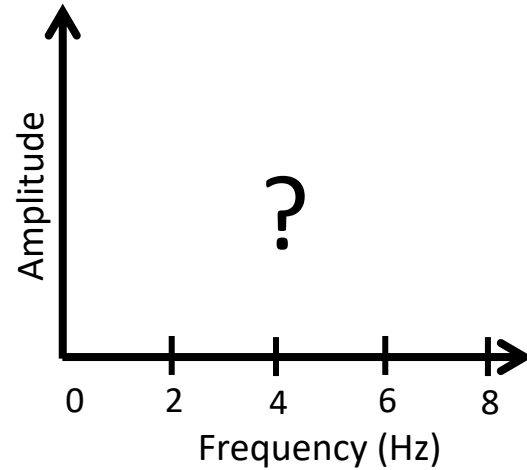
FFT = Fast Fourier Transform

IFFT = Inverse Fast Fourier Transform

# Time Domain and Frequency Domain

Fourier Transform

$\cos(2\pi ft)$



Time domain

Frequency domain

FFT = Fast Fourier Transform

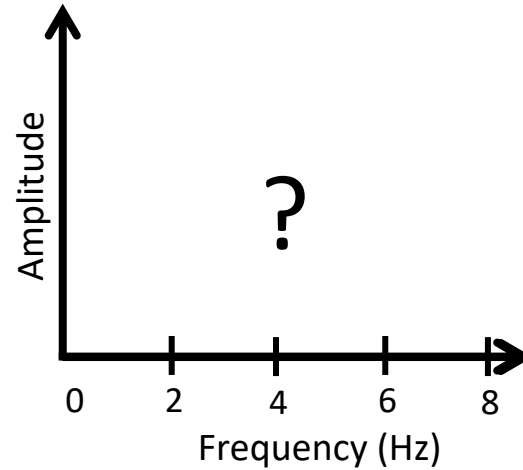
IFFT = Inverse Fast Fourier Transform

# Time Domain and Frequency Domain

Fourier Transform

$$\begin{aligned} & \cos(2\pi ft) \\ = & \frac{e^{j 2\pi ft} + e^{-j 2\pi ft}}{2} \end{aligned}$$

FFT →



Time domain

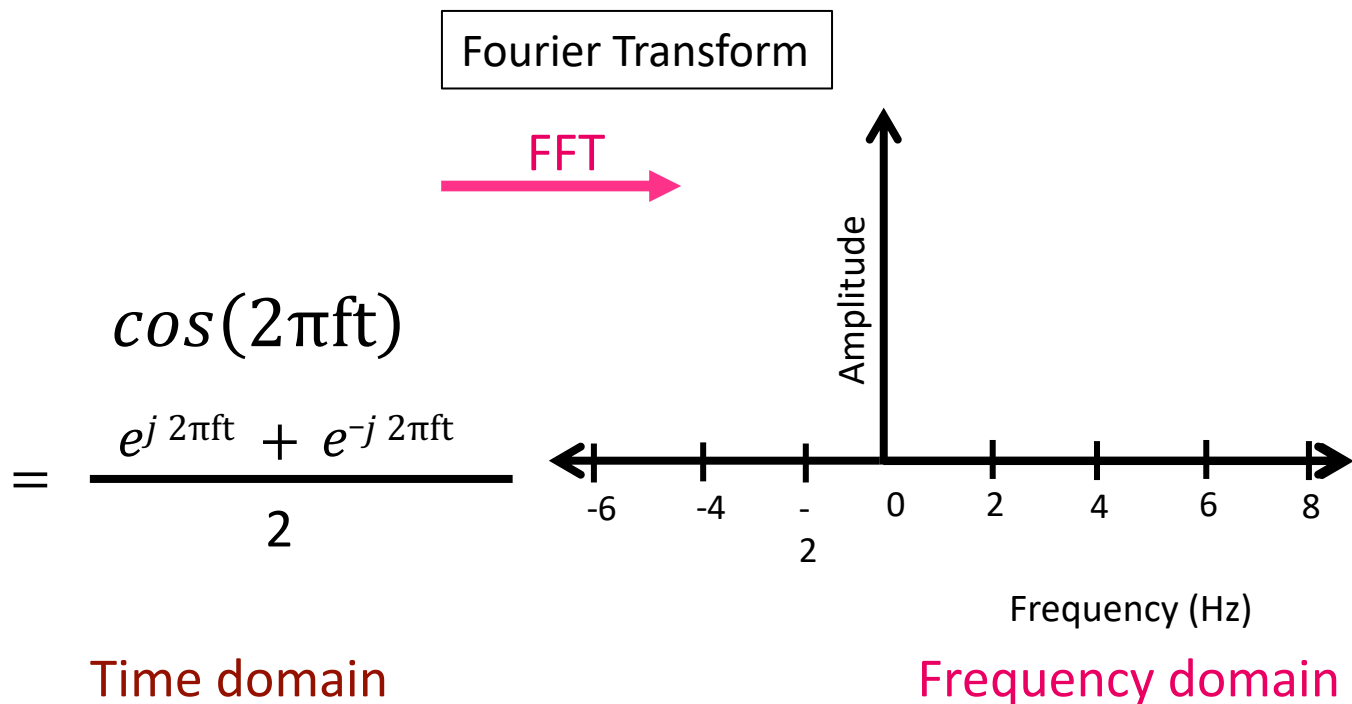
Frequency domain

FFT = Fast Fourier Transform

IFFT = Inverse Fast Fourier Transform



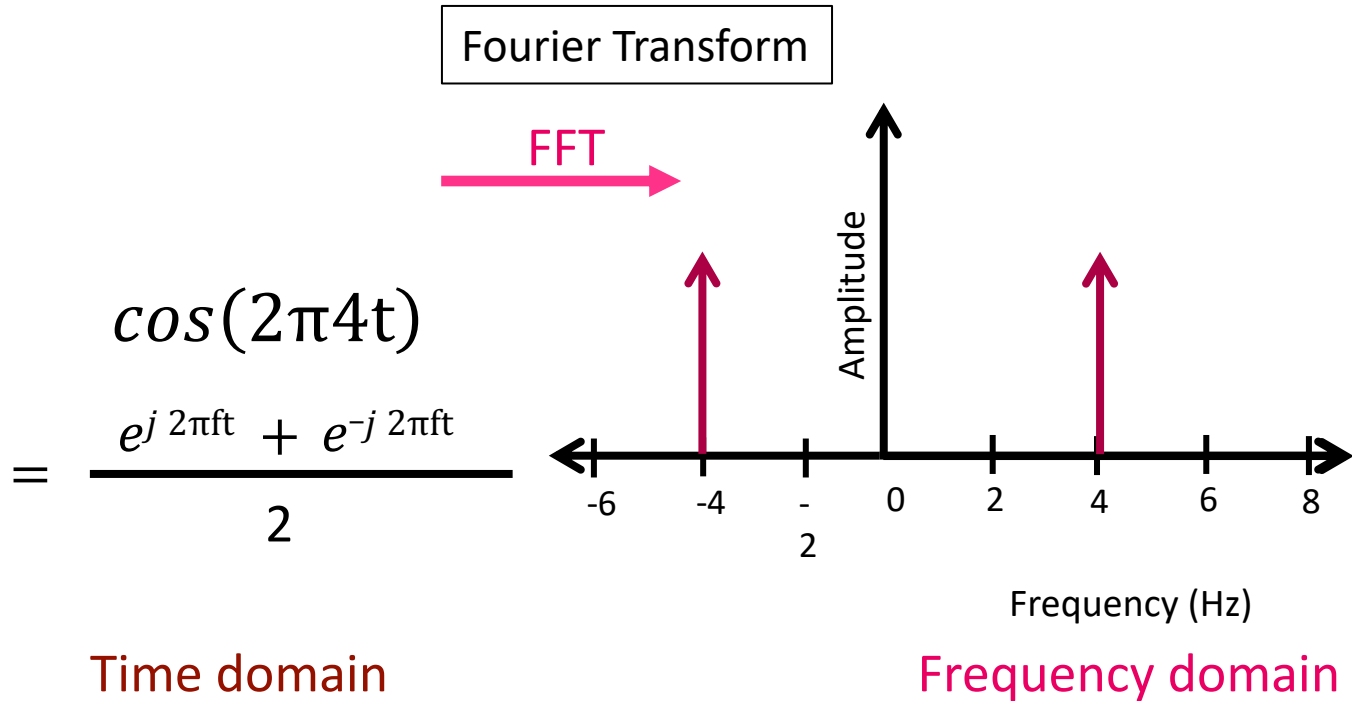
# Time Domain and Frequency Domain



FFT = Fast Fourier Transform

IFFT = Inverse Fast Fourier Transform

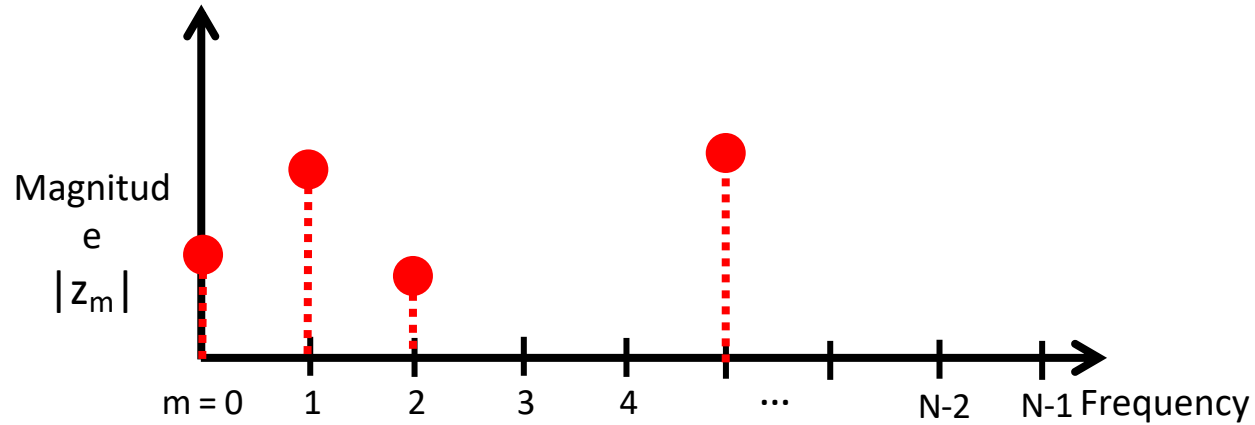
# Time Domain and Frequency Domain



FFT = Fast Fourier Transform

IFFT = Inverse Fast Fourier Transform

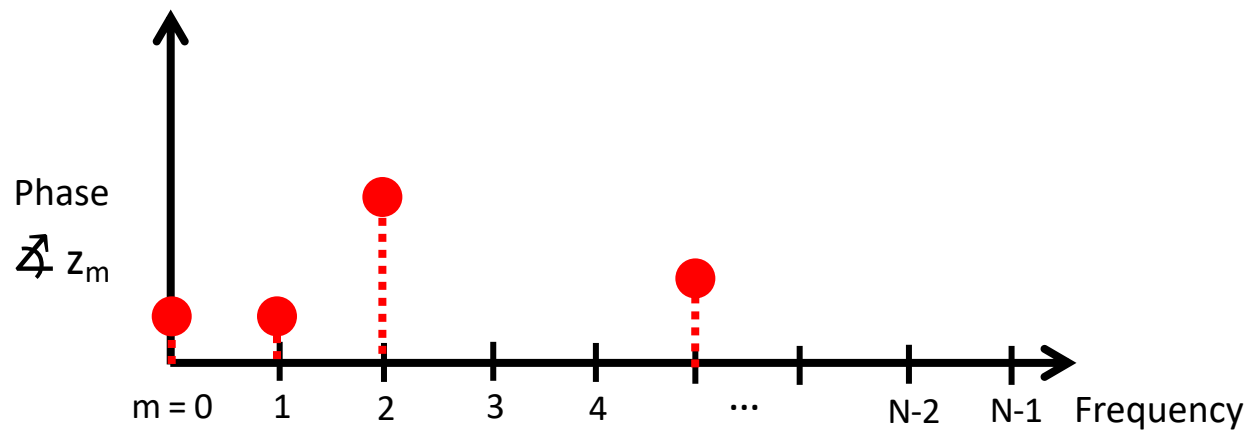
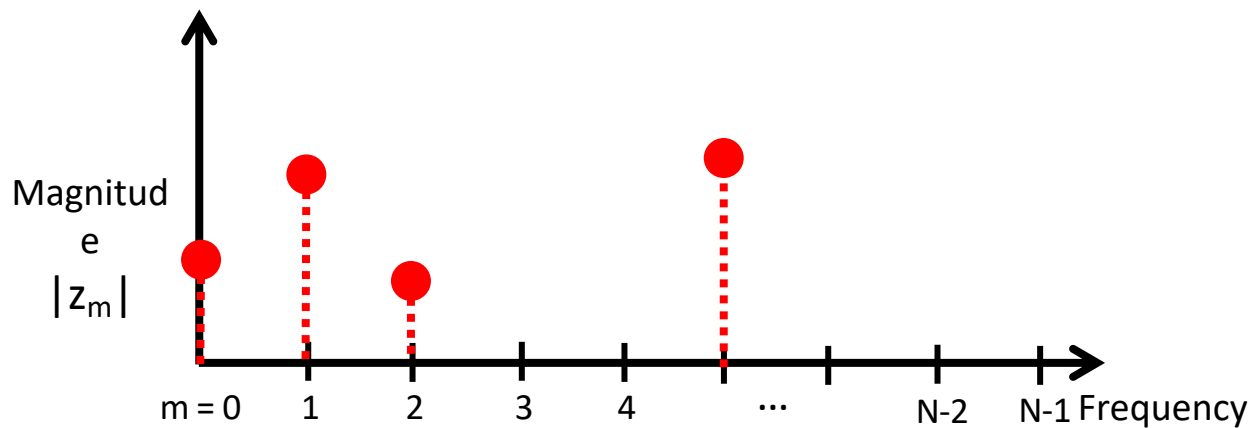
# Plotting the DFT spectrum



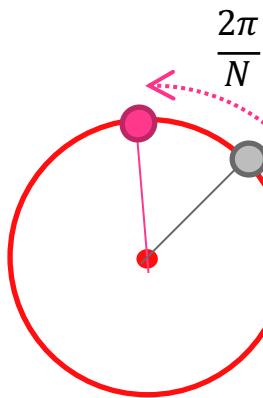
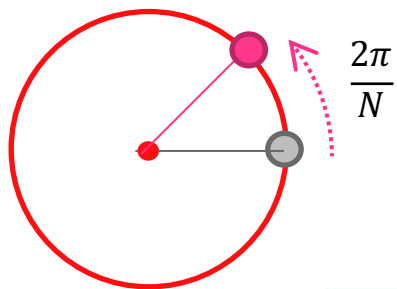
# DFT (Discrete Fourier Transform)

$$\begin{aligned} X_k &= \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{i2\pi}{N}kn} \\ &= \sum_{n=0}^{N-1} x_n \cdot \left[ \cos\left(\frac{2\pi}{N}kn\right) - i \cdot \sin\left(\frac{2\pi}{N}kn\right) \right] \end{aligned}$$

# Plotting the DFT spectrum

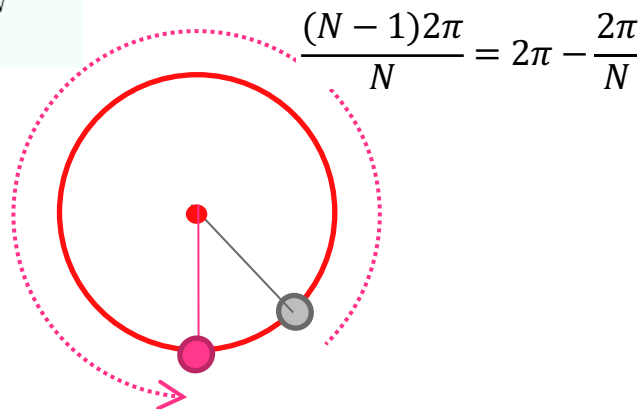
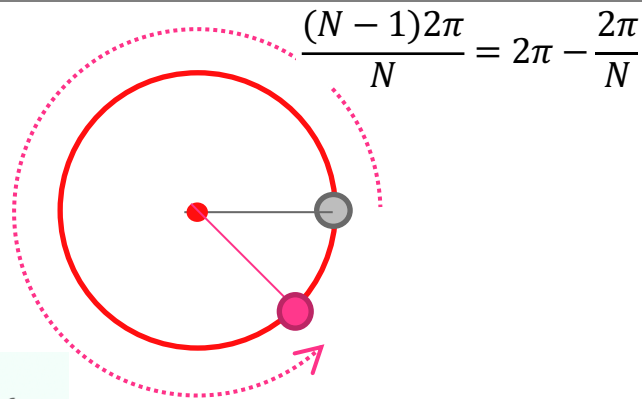


# The Curious Case of “Negative frequency”



Positive rotation with  $\frac{2\pi}{N}$   
radian angle per step

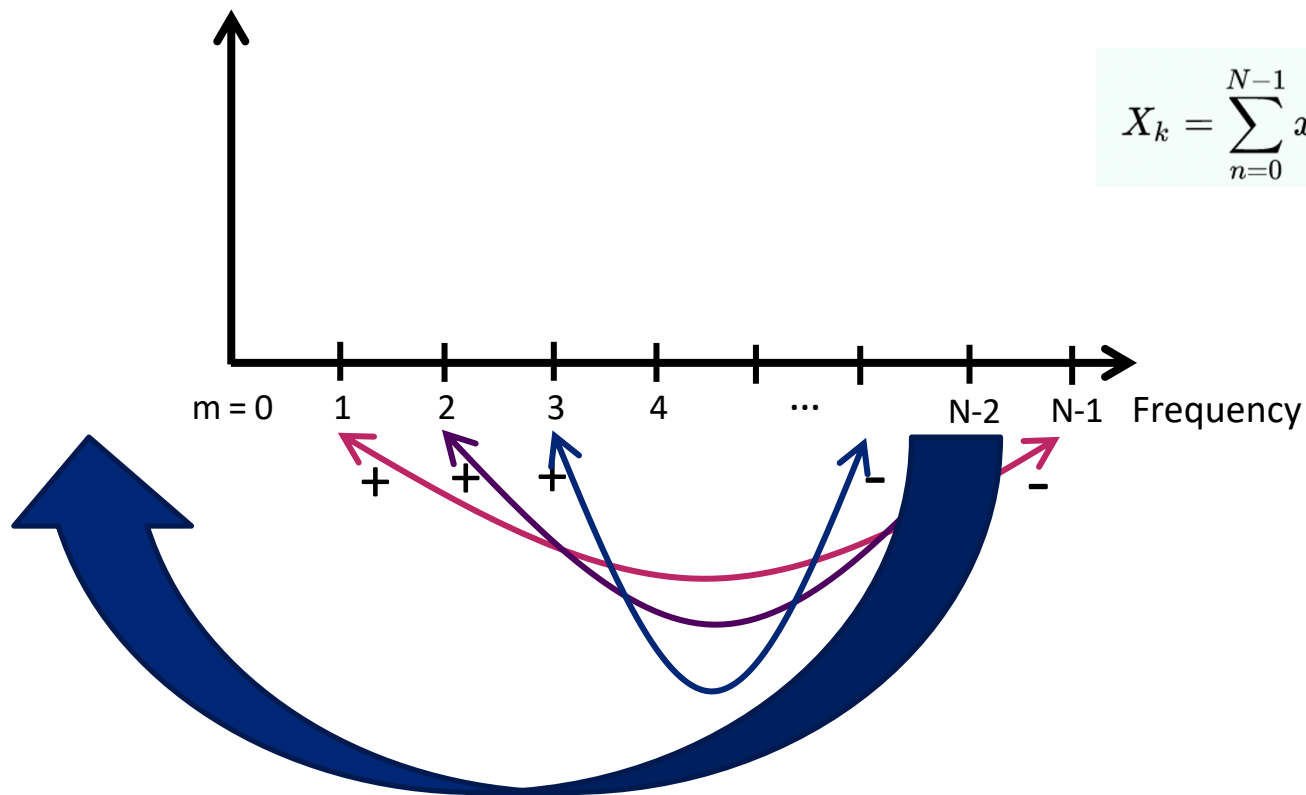
$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{i2\pi}{N}kn}$$



Negative rotation with  $\frac{2\pi}{N}$   
radian angle per step

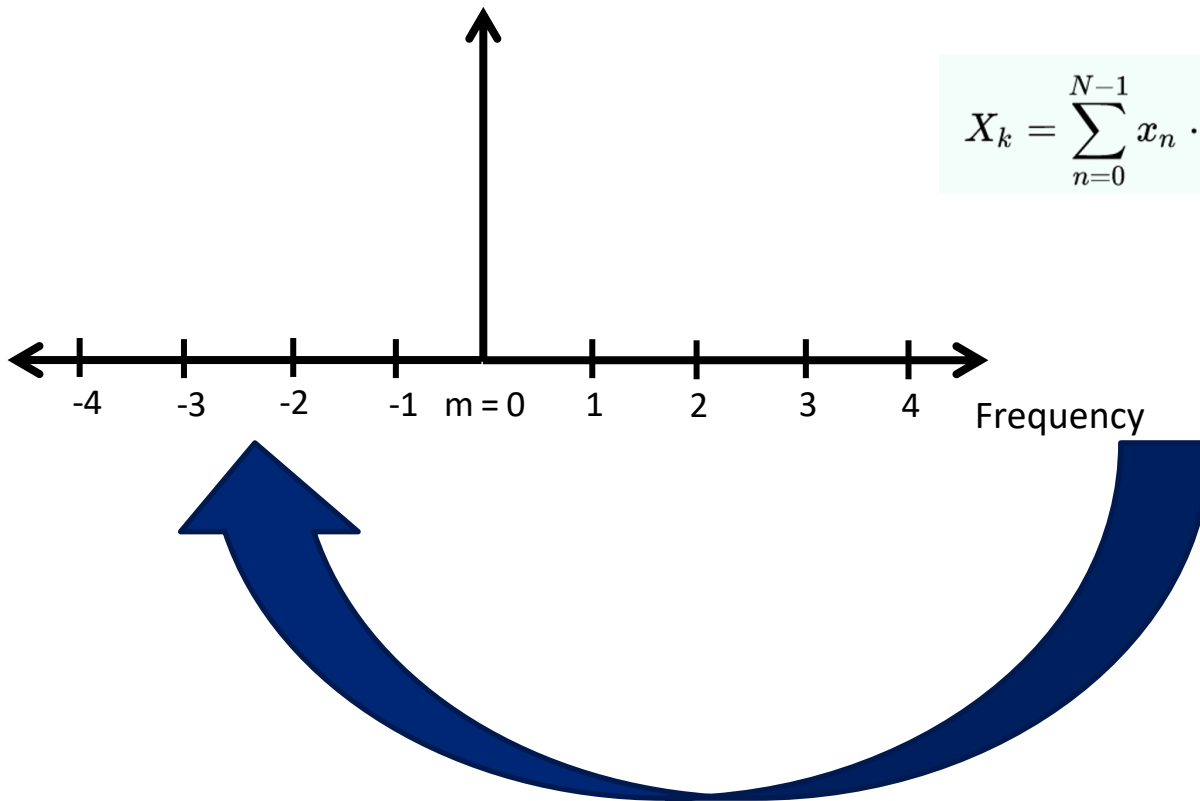
# The Curious Case of “Negative frequency”

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{i2\pi}{N} kn}$$



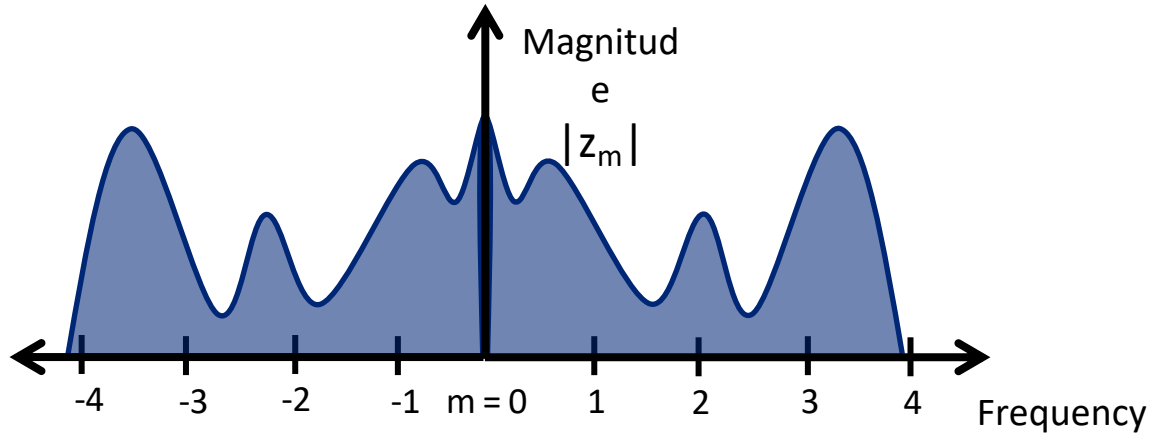
# The Curious Case of “Negative frequency”

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{i2\pi}{N} kn}$$





# The Curious Case of “Negative frequency”

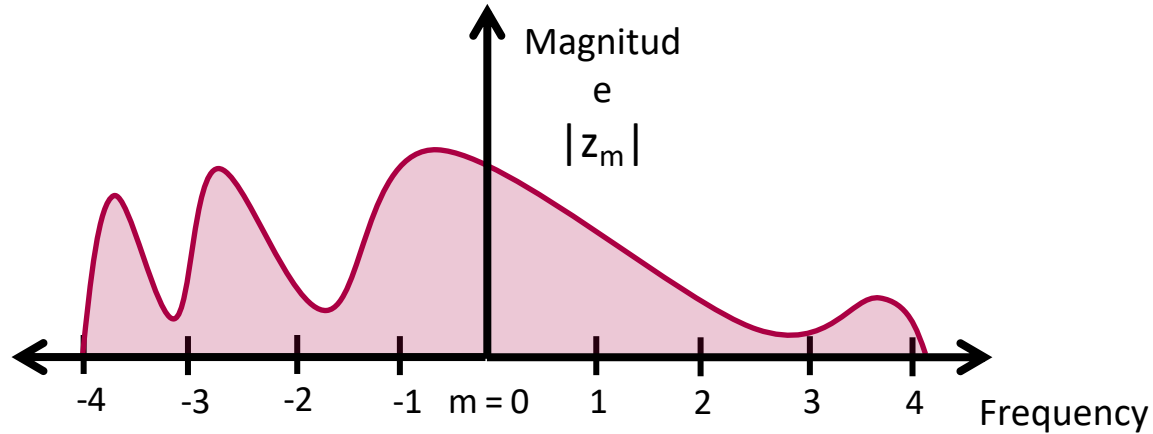


Real signal's magnitude spectrum is symmetric.

Why?

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{i2\pi}{N} kn}$$

# The Curious Case of “Negative frequency”



Complex signal's  
magnitude spectrum  
may or may not be  
symmetric.

Why?

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{i2\pi}{N}kn}$$

# Estimating the real-world frequencies

Sampling frequency =  $f_s$  (i.e.,  $f_s$  samples per second)

Slowest frequency ( $\frac{2\pi}{N}$  radians per step) =  $N$  samples per rotation

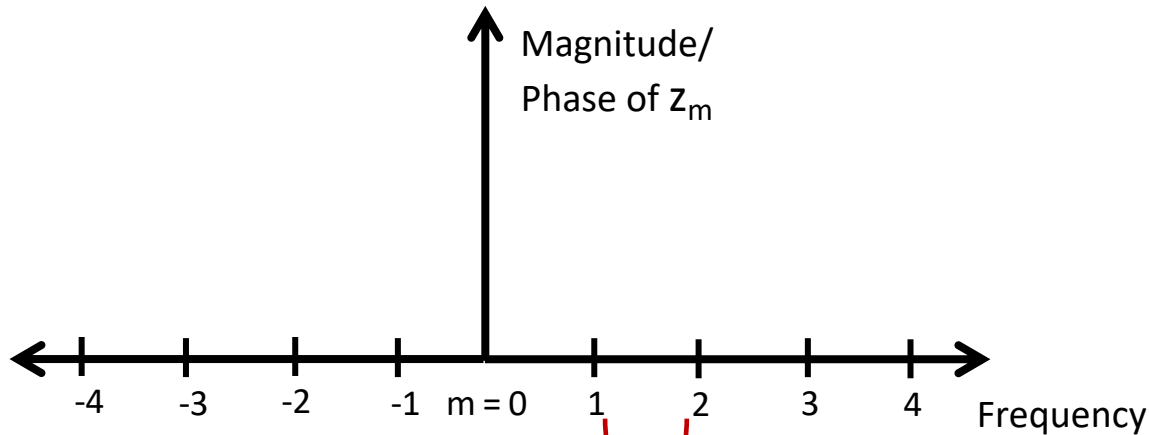
=  $(N/ f_s)$  **seconds** per rotation

Therefore, the slowest frequency =  $(f_s / N)$  Hz

Higher frequencies are integer multiple of  $(f_s / N)$  Hz

$$0, \frac{f_s}{N}, \frac{2f_s}{N}, \frac{3f_s}{N}, \frac{4f_s}{N}, \dots,$$

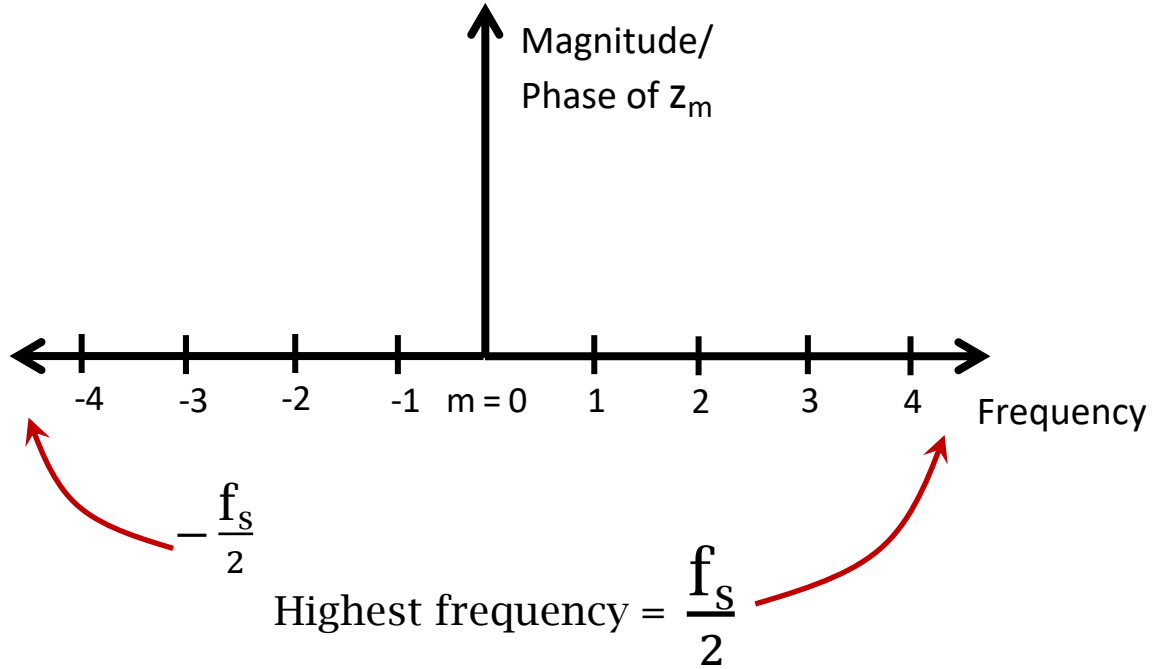
# The resolution and the highest frequency



Resolution  
= minimum observable frequency difference =  $\frac{f_s}{N}$

What if the actual frequency falls in between two frequency bins?

# The resolution and the highest frequency



# The resolution and the highest frequency

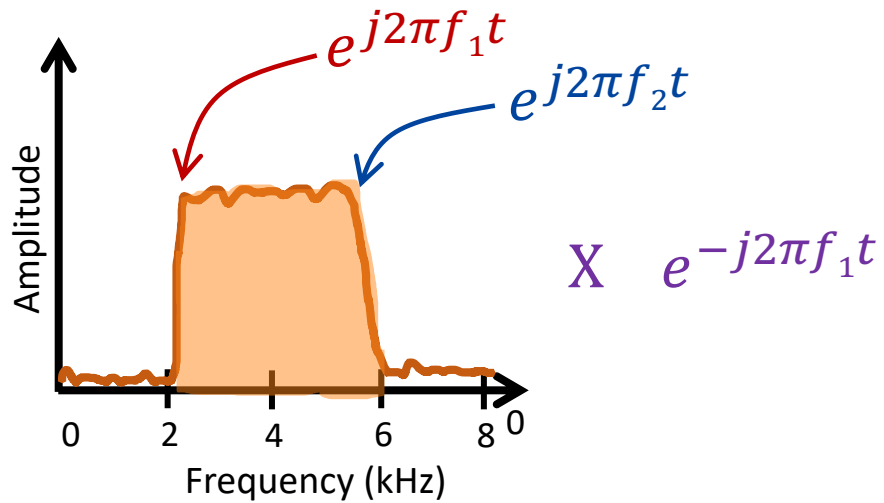
How can we increase the resolution?

$$\frac{f_s}{N} = \frac{\text{sample rate}}{\# \text{ of FFT points}}$$

How can we increase the range of the spectrum?

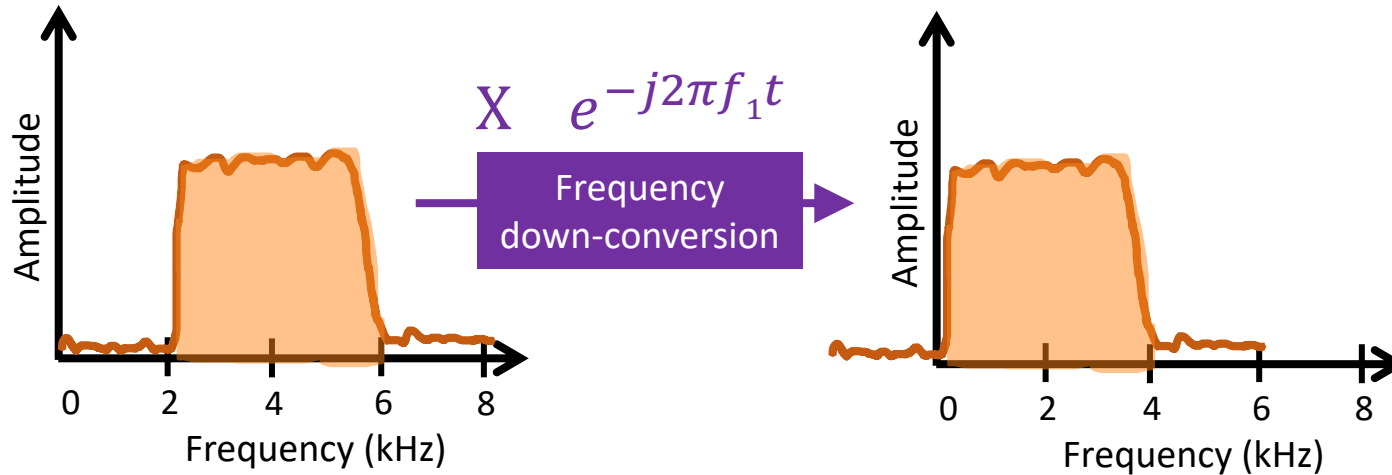
$$\left[ -\frac{f_s}{2}, \frac{f_s}{2} \right]$$

# Downsampling



What should be the sample rate?

# Downsampling



What should be the sample rate?

Generally, bandwidth of the signal determines the sample rate.