Waves

Slides adapted from Nirupam Roy

Sound

Physical vibrations

Ripples in water

...

Visible light

WiFi signal

Infrared

...

Mechanical Wave

Sound

Physical vibrations

Ripples in water

...

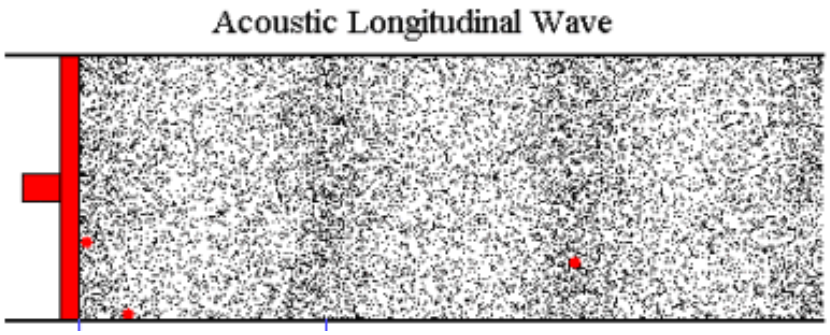
Electromagnetic Wave

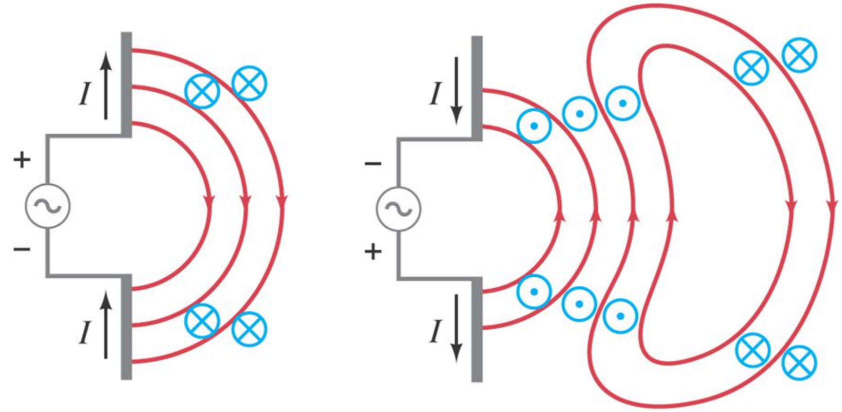
Visible light

WiFi signal

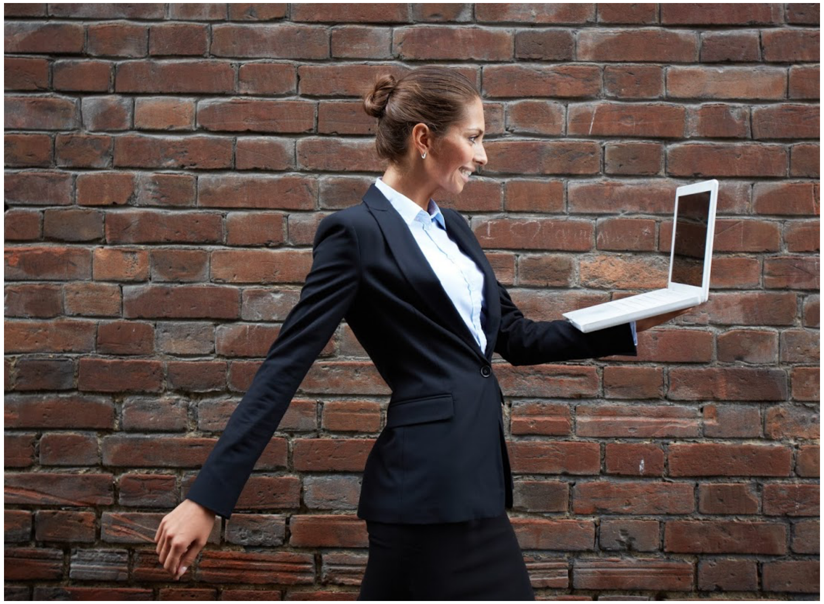
Infrared

...

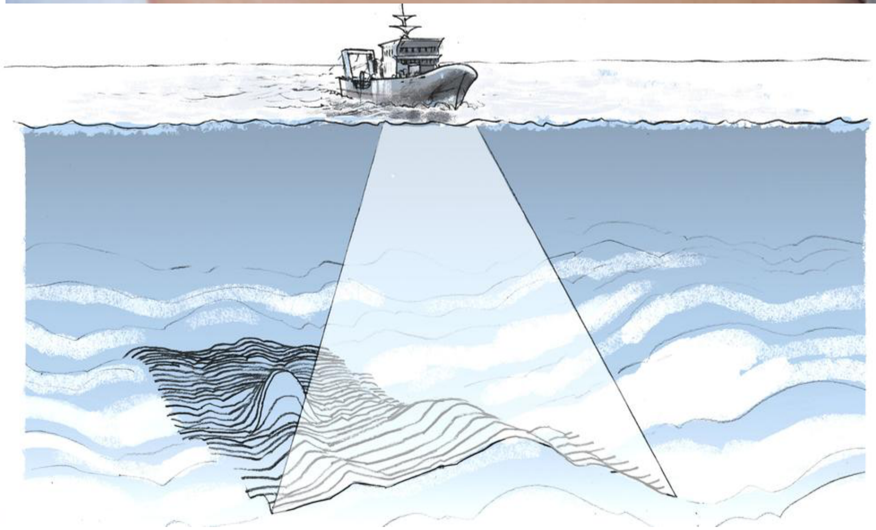




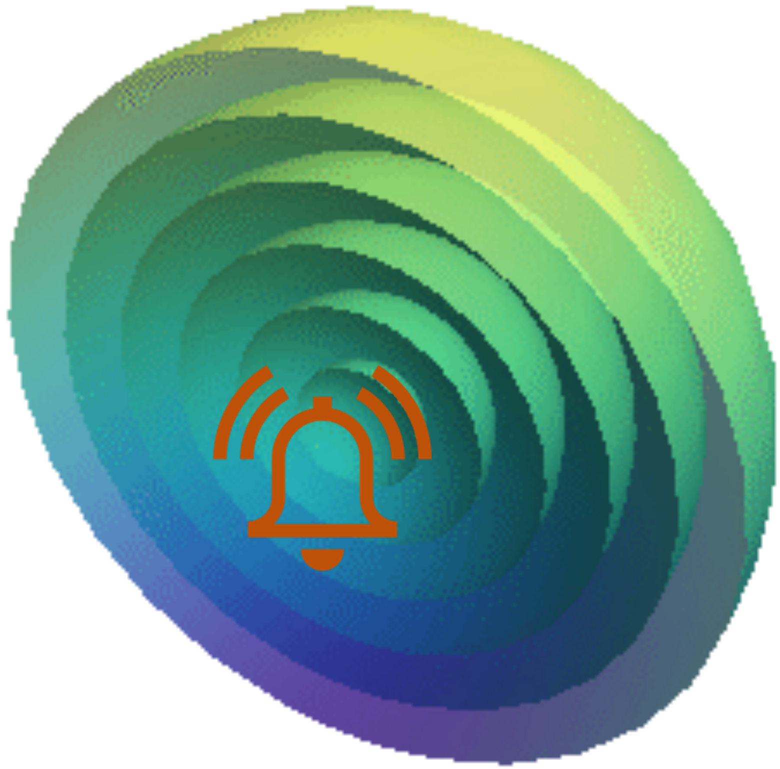
Waves made this possible



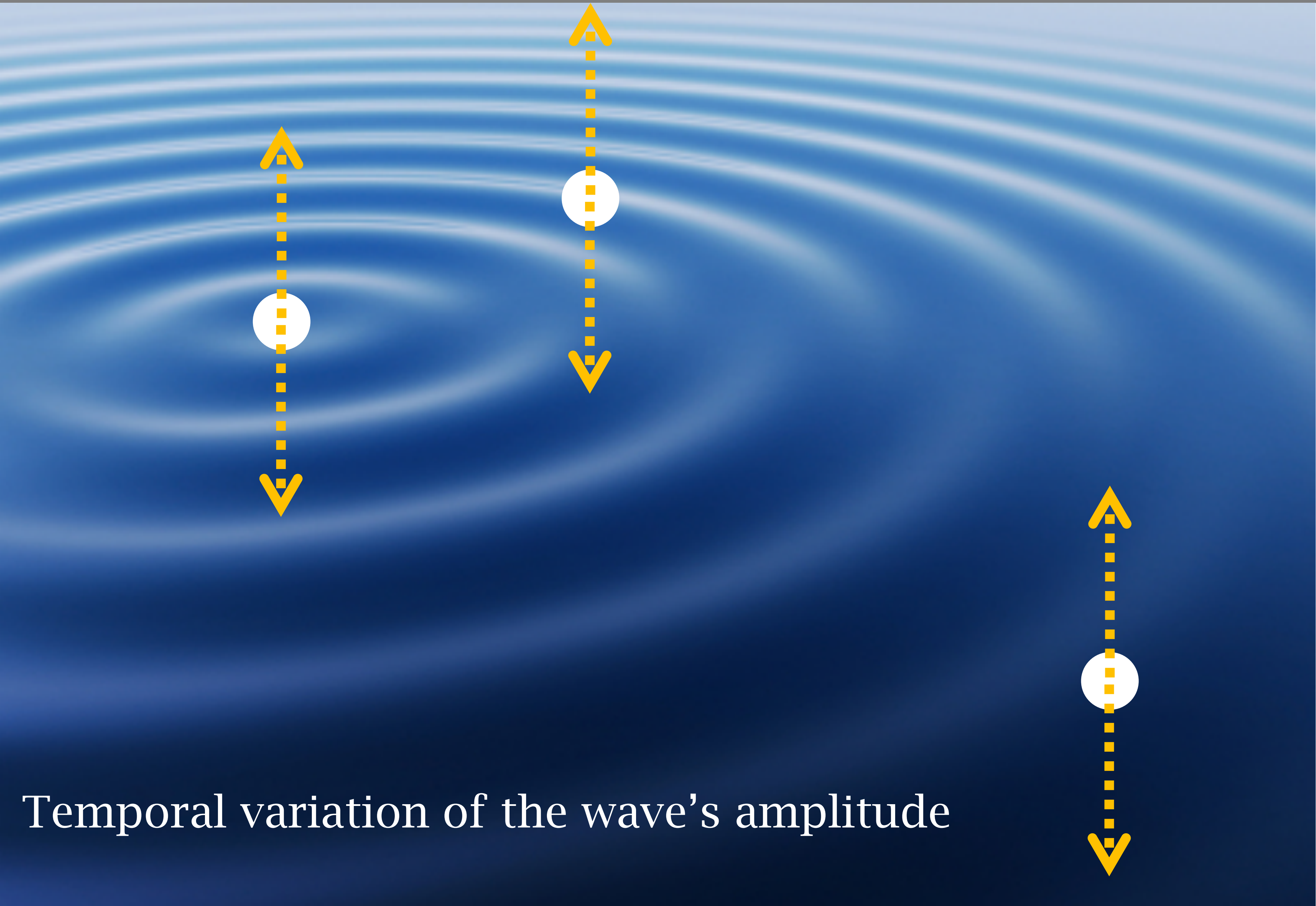
Waves made this possible



Equation of waves in time and space



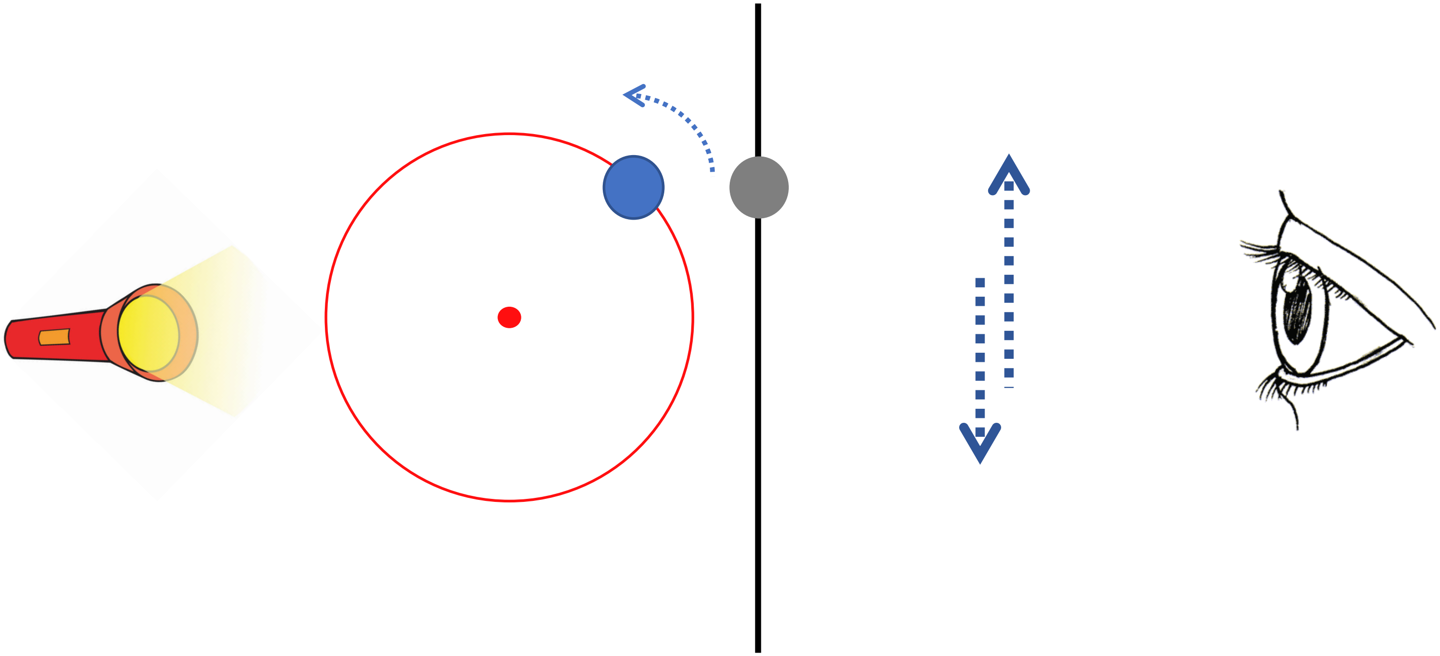
Equation of waves in time and space



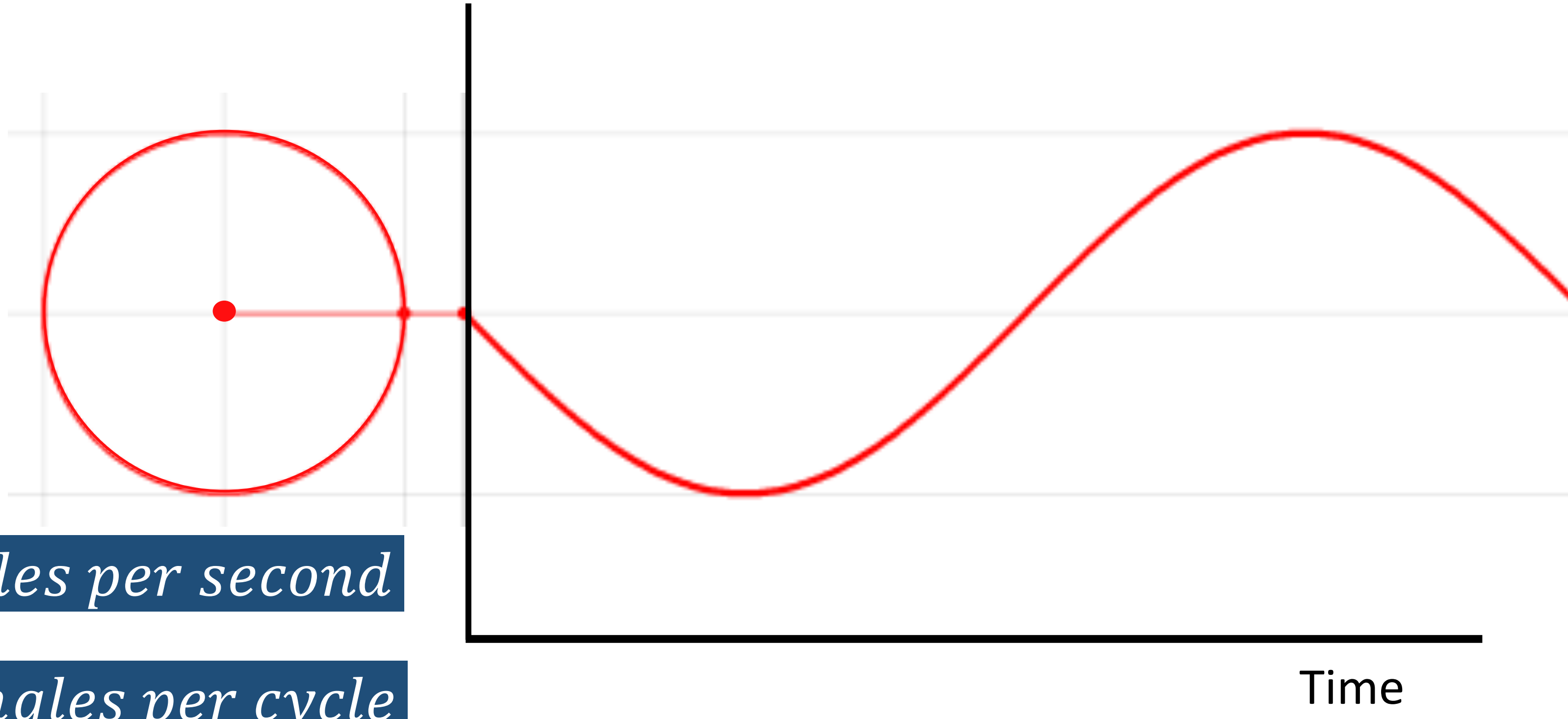
Temporal variation of the wave's amplitude

Equation of waves in time and space

Frequency, Amplitude, and Phase



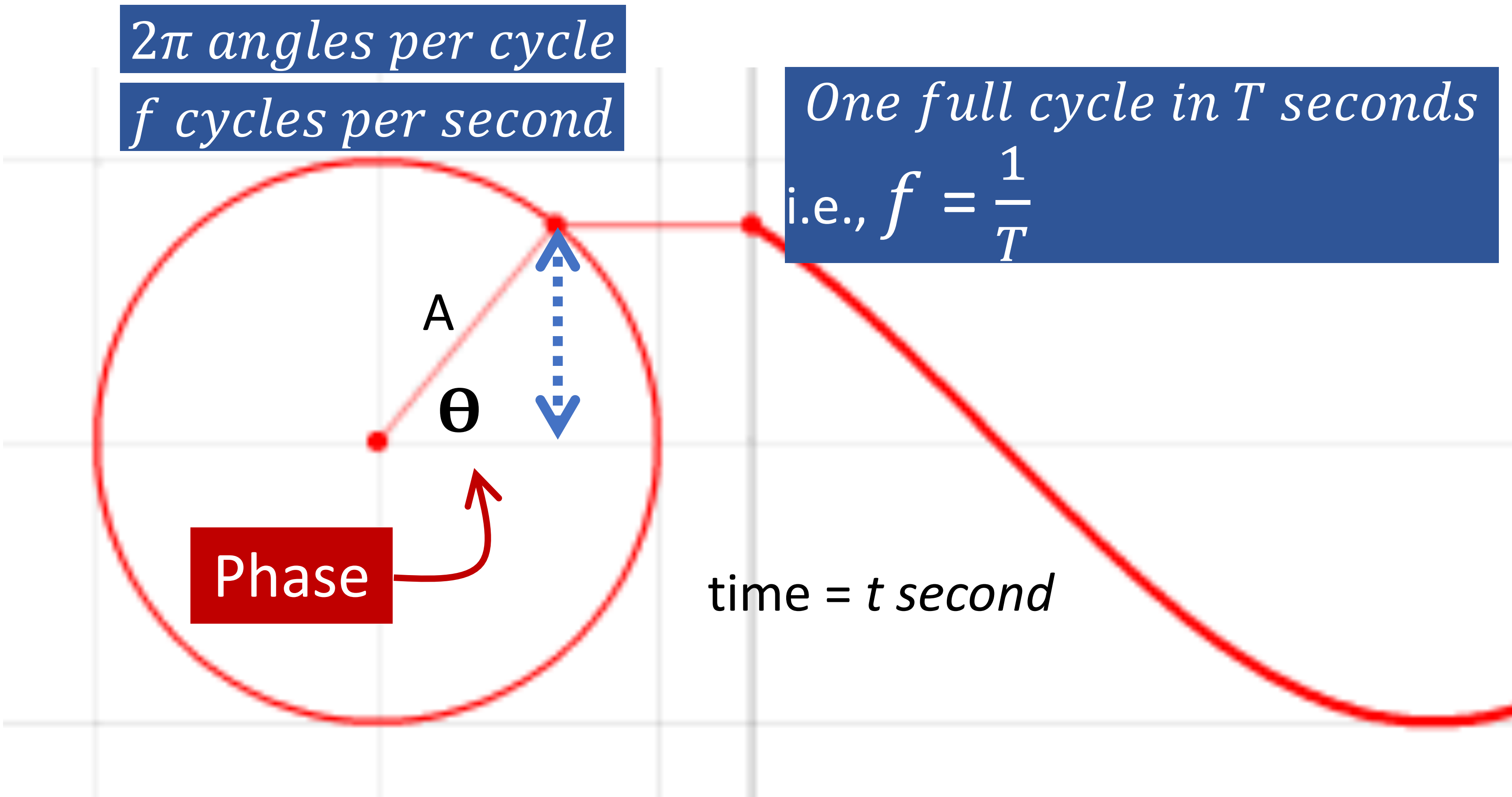
Frequency, Amplitude, and Phase



f cycles per second

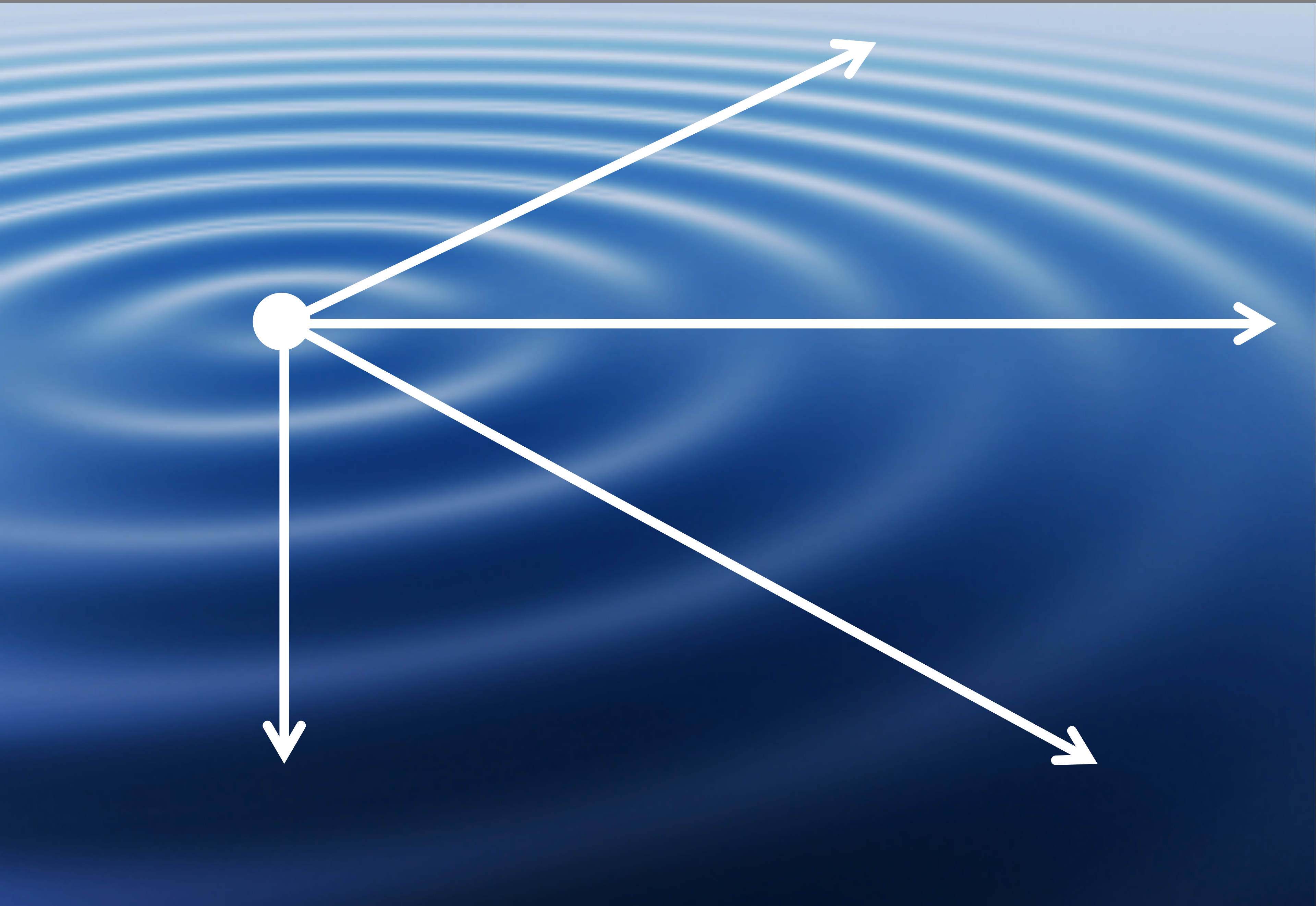
2π angles per cycle

Frequency, Amplitude, and Phase

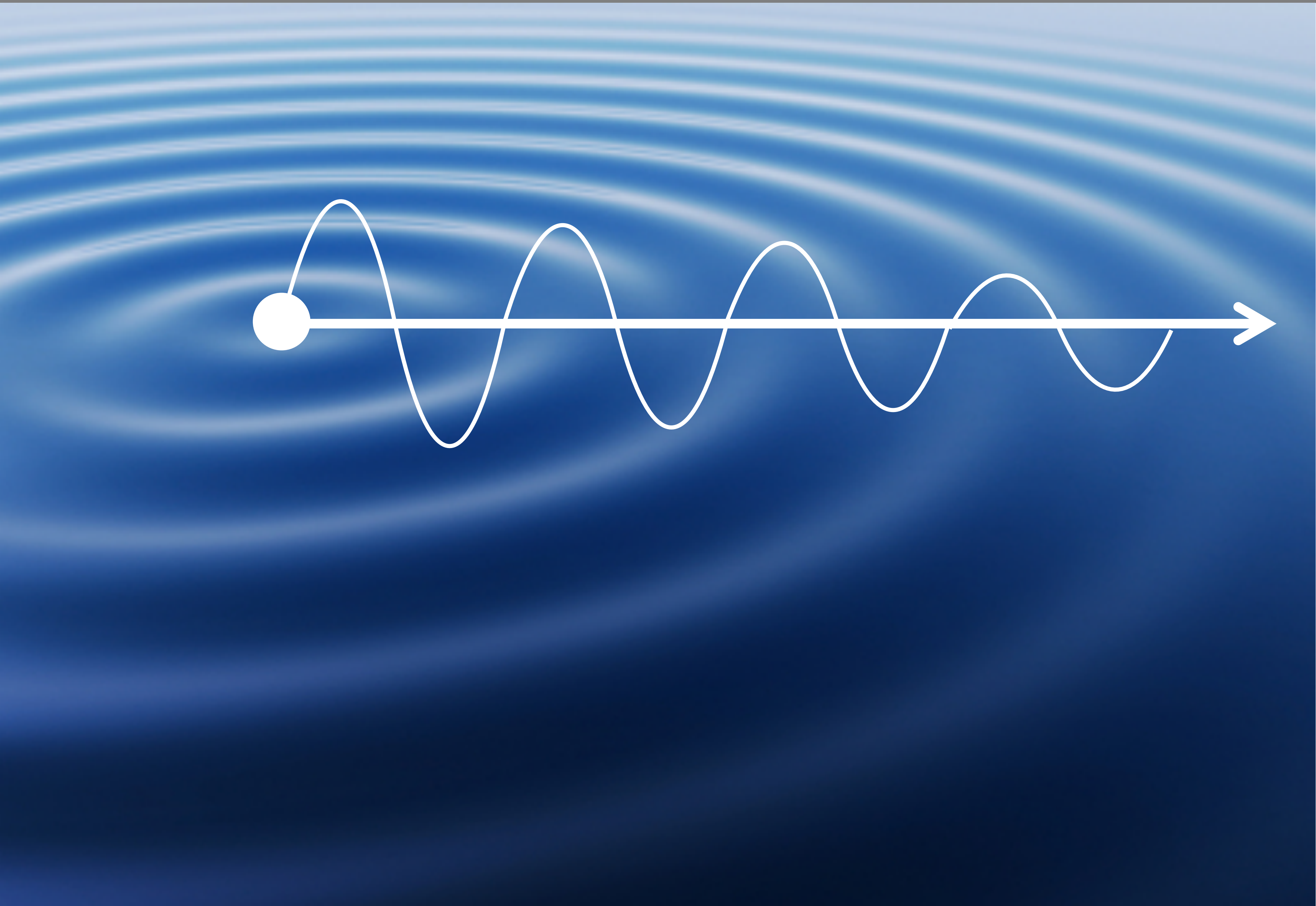


$$\Psi(\mathrm{t}) = \mathrm{A}.\sin(\theta(t))$$
$$= \mathrm{A}.\sin(2\pi f t) = \mathrm{A}.\sin\left(\frac{2\pi}{T}t\right)$$

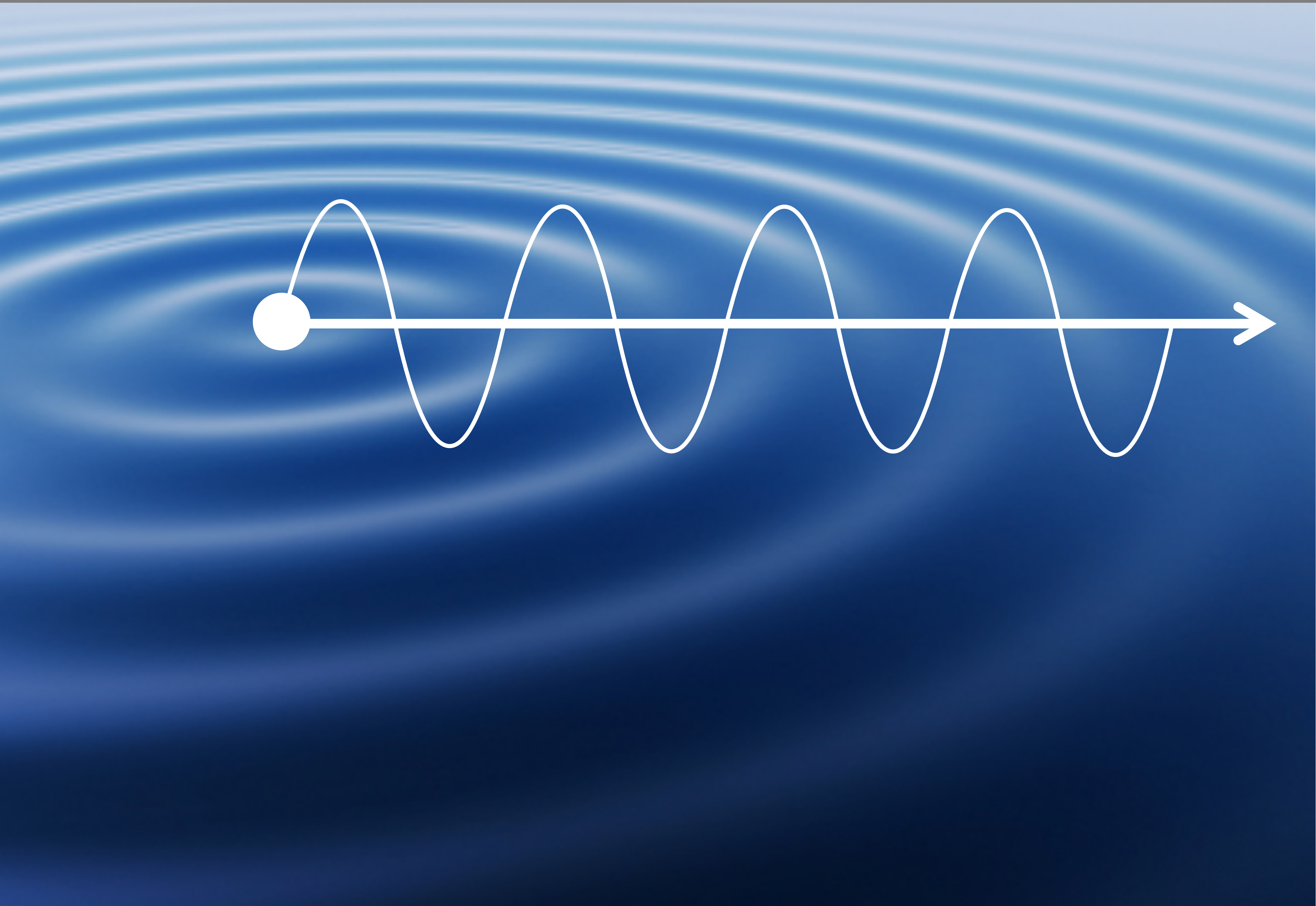
Equation of waves in time and space



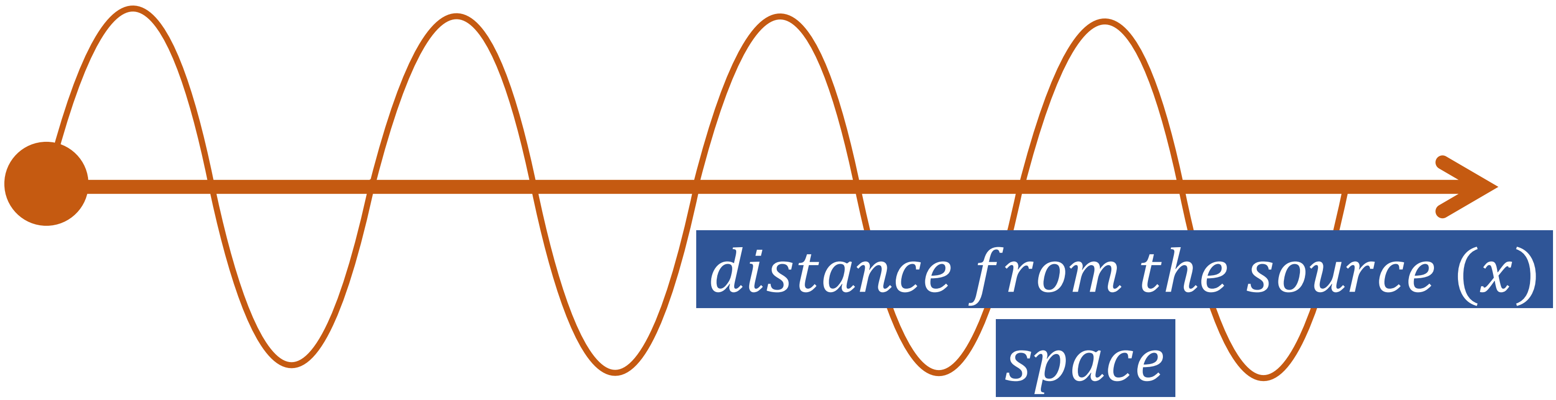
Equation of waves in time and space



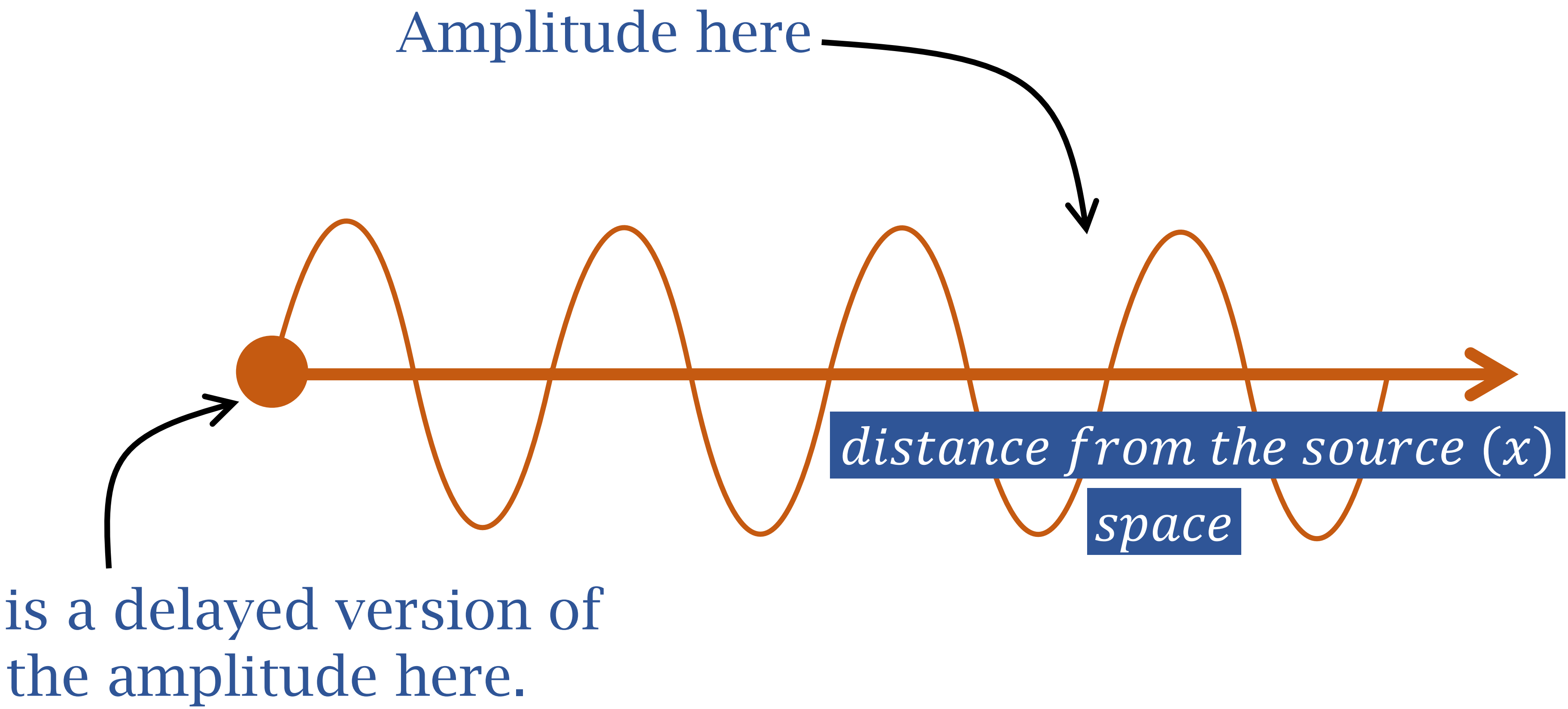
Equation of waves in time and space



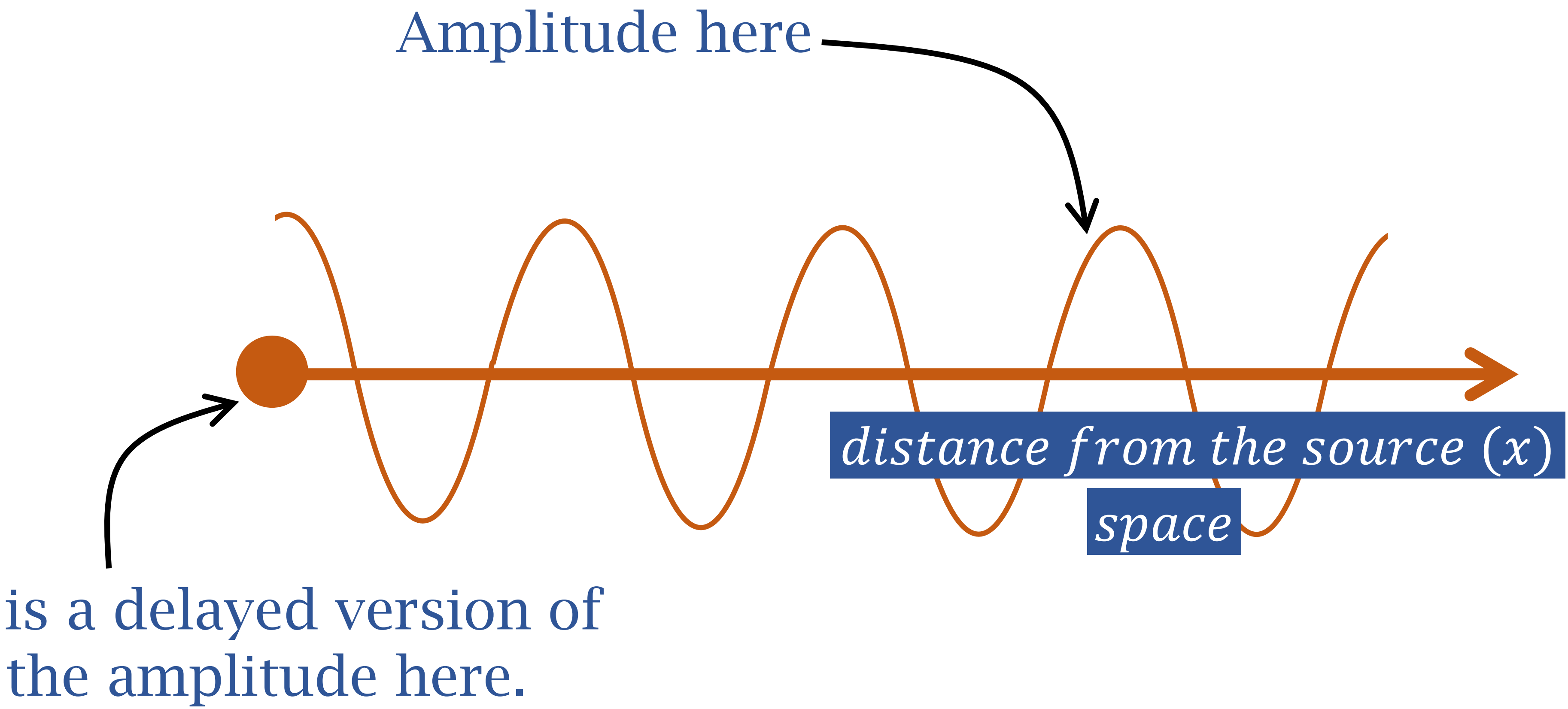
Equation of waves in time and space



Equation of waves in time and space

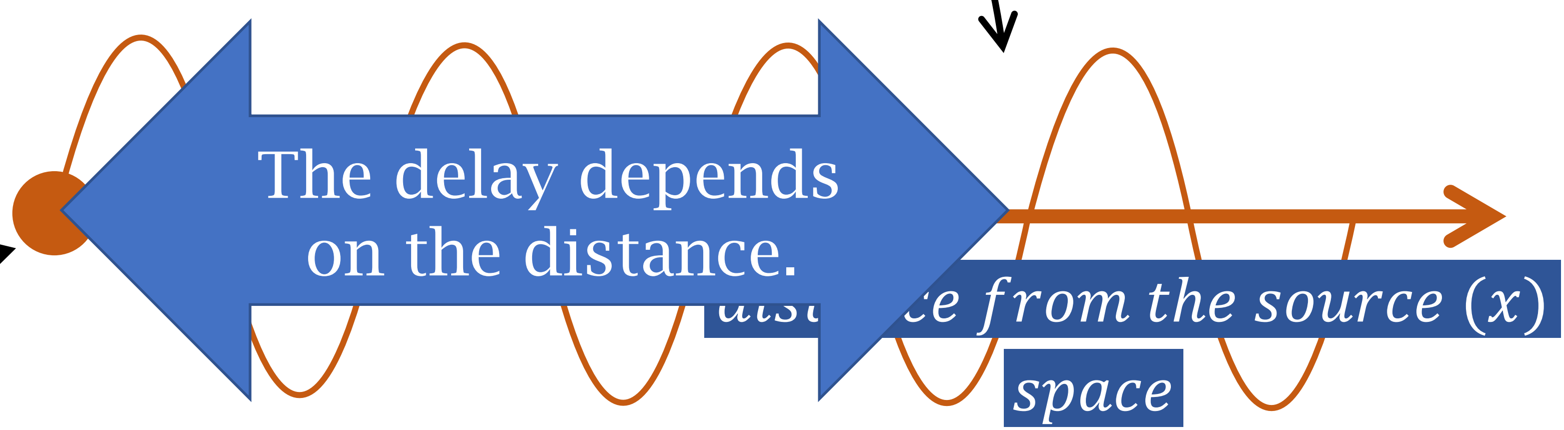


Equation of waves in time and space



Equation of waves in time and space

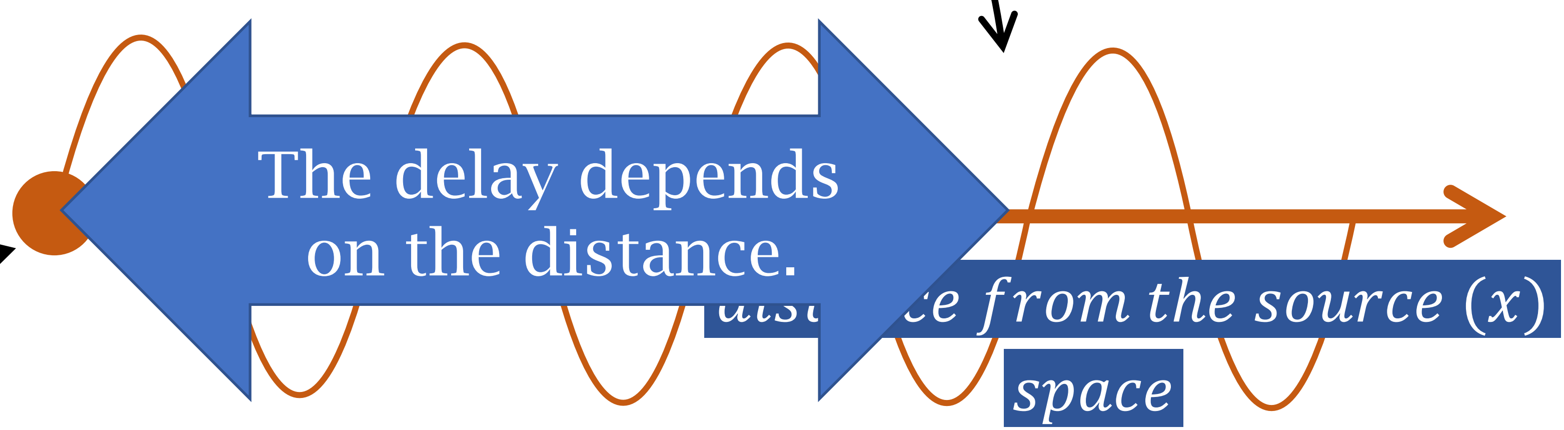
Amplitude here



is a delayed version of
the amplitude here.

Equation of waves in time and space

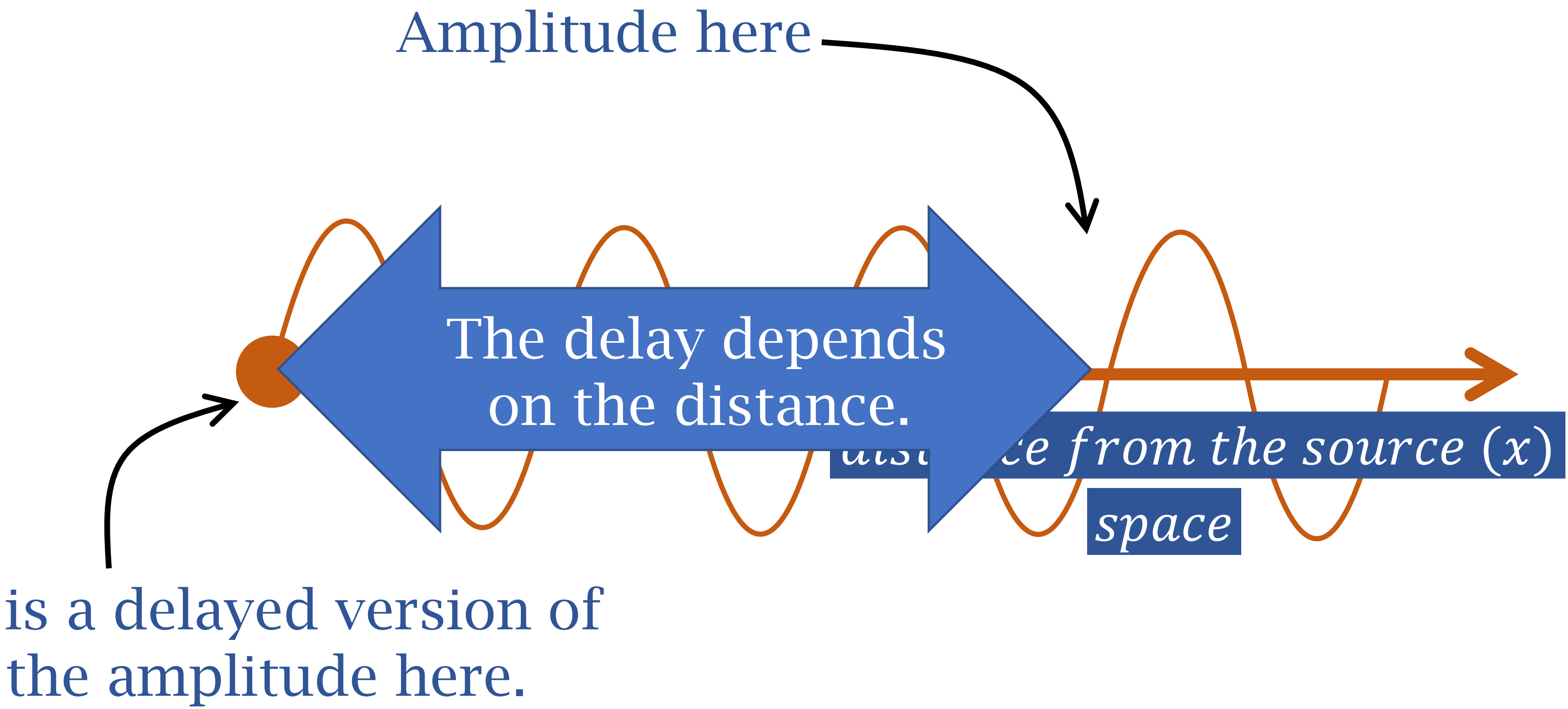
Amplitude here



is a delayed version of
the amplitude here.

$$\Psi(t) = A\,.\sin(2\pi f t) = A\,.\sin(\frac{2\pi}{T}t)$$

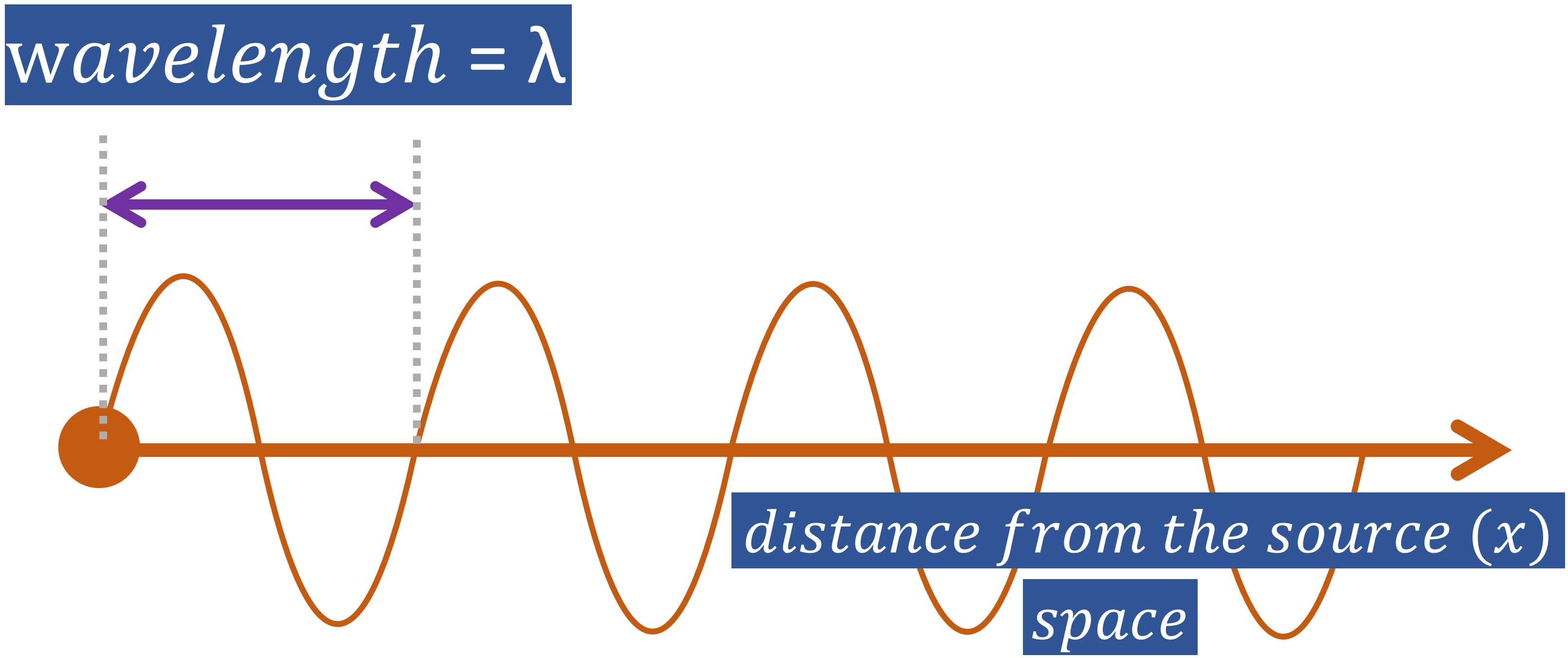
Equation of waves in time and space



$$\Psi(t, x) = A\,.\sin(2\pi f t - \theta(x))$$

Why is this negative?

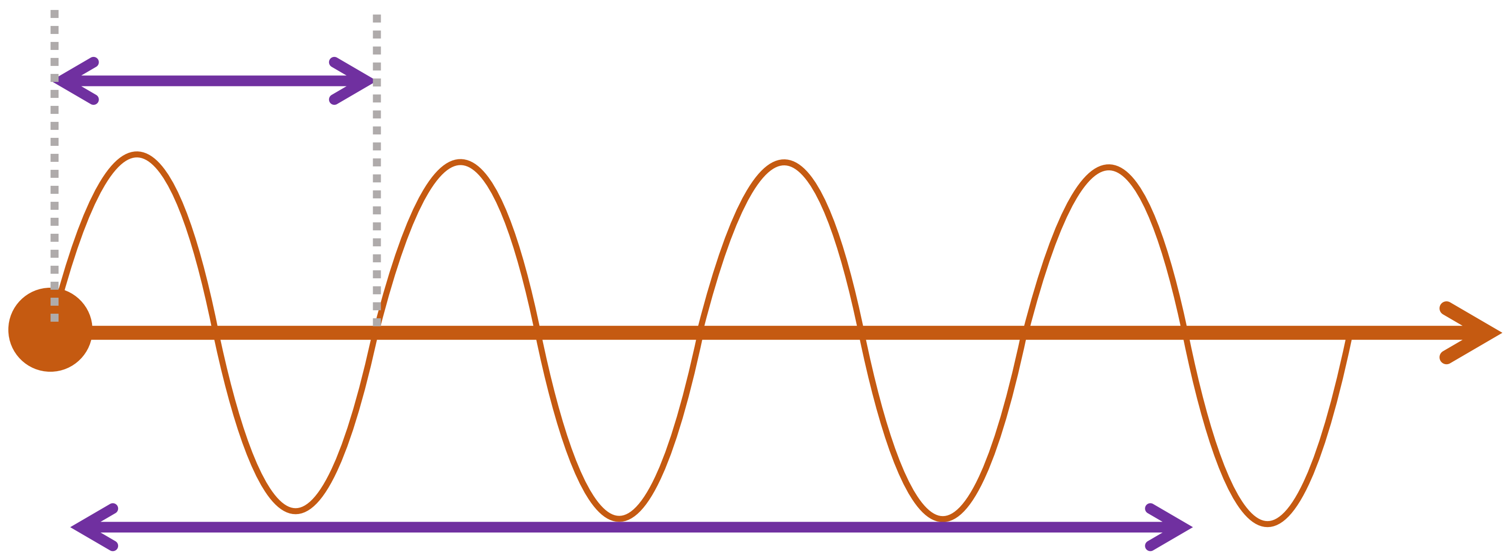
Equation of waves in time and space



$$\Psi(\mathrm{t}, \mathrm{x}) = \mathrm{A} \,.\, \sin(2\pi f t - \theta(x))$$

Equation of waves in time and space

$wavelength = \lambda$



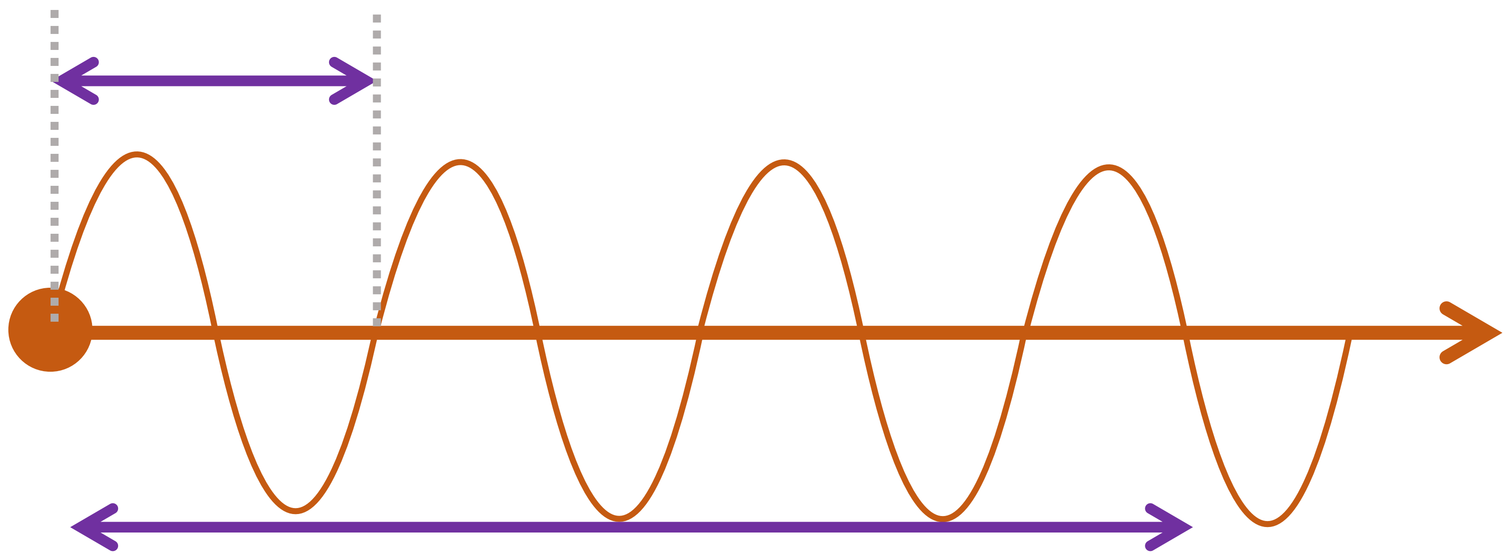
f *number of cycles per second*

C *meters of distance per second (speed of the wave)*

$distance\ per\ cycle = \lambda = C\ /\ f$

Equation of waves in time and space

wavelength = λ



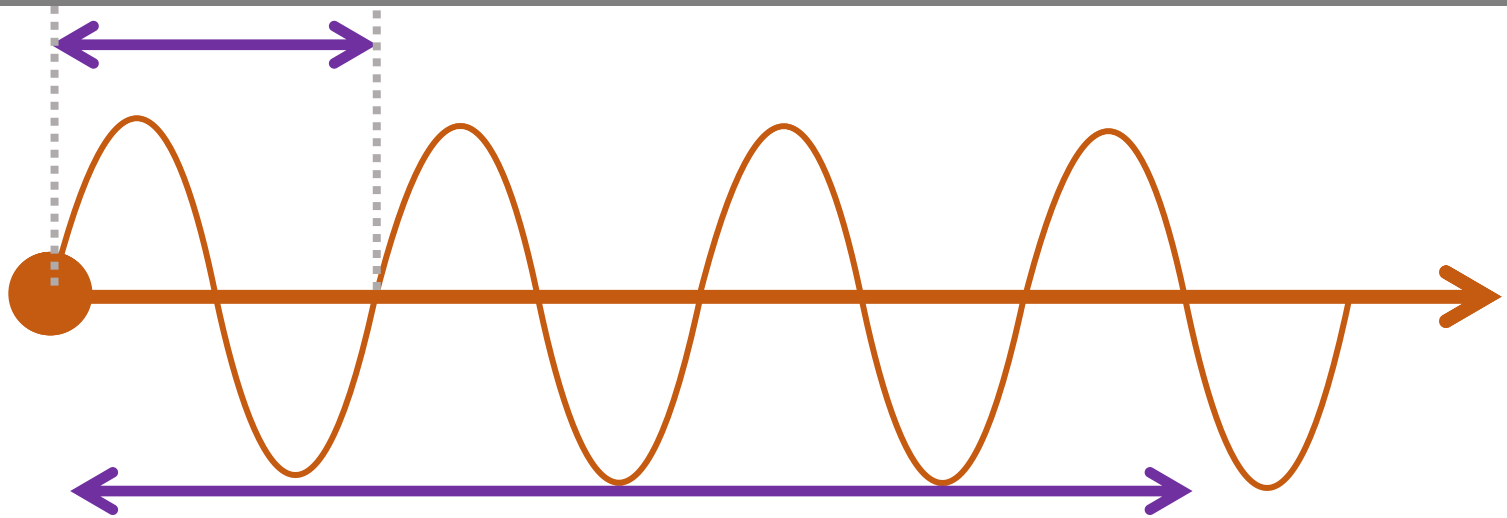
f number of cycles per second

C meters of distance per second (speed of the wave)

distance per cycle = λ = *C / f*

2π radians of angle is covered per cycle

Equation of waves in time and space



f *number of cycles per second*

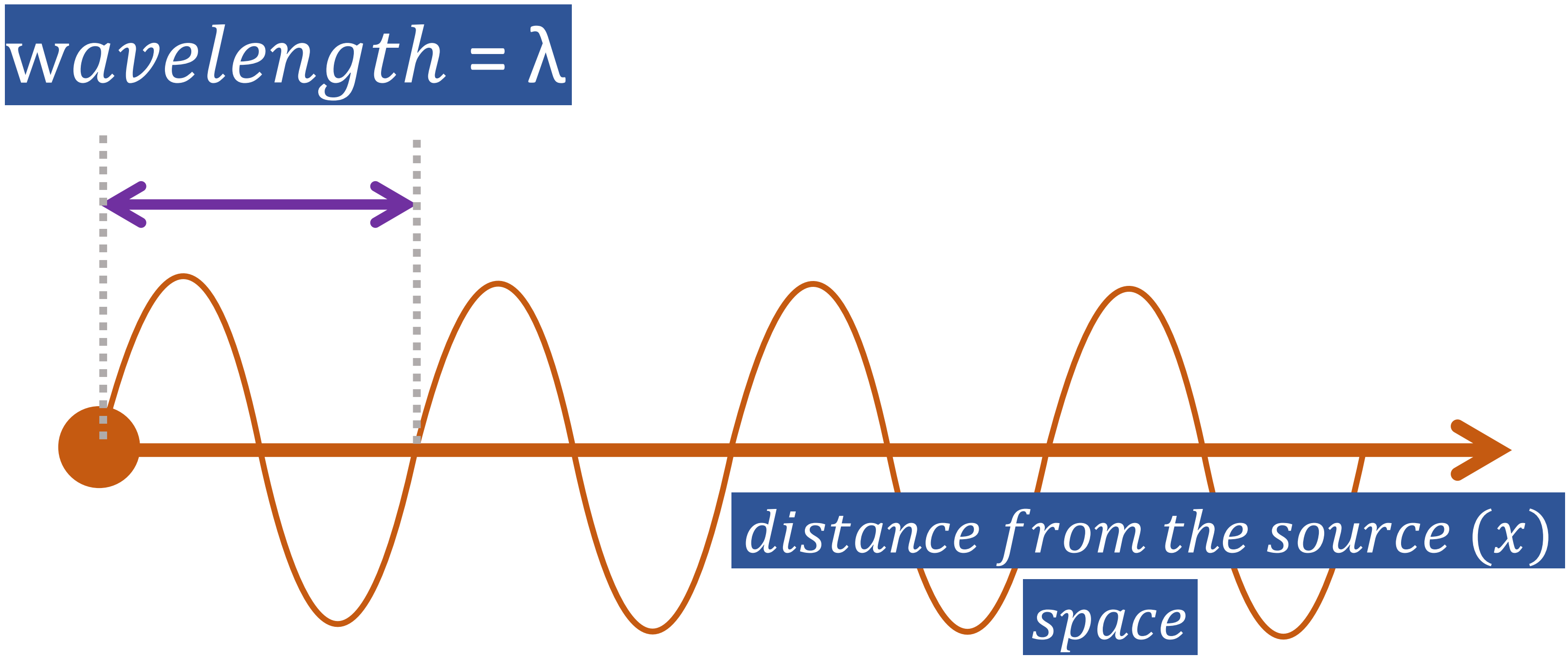
C *meters of distance per second (speed of the wave)*

distance per cycle $= \lambda = C / f$

2π *radians of angle is covered per cycle*

$\frac{2\pi}{\lambda}$ *radians of angle is covered per unit distance*

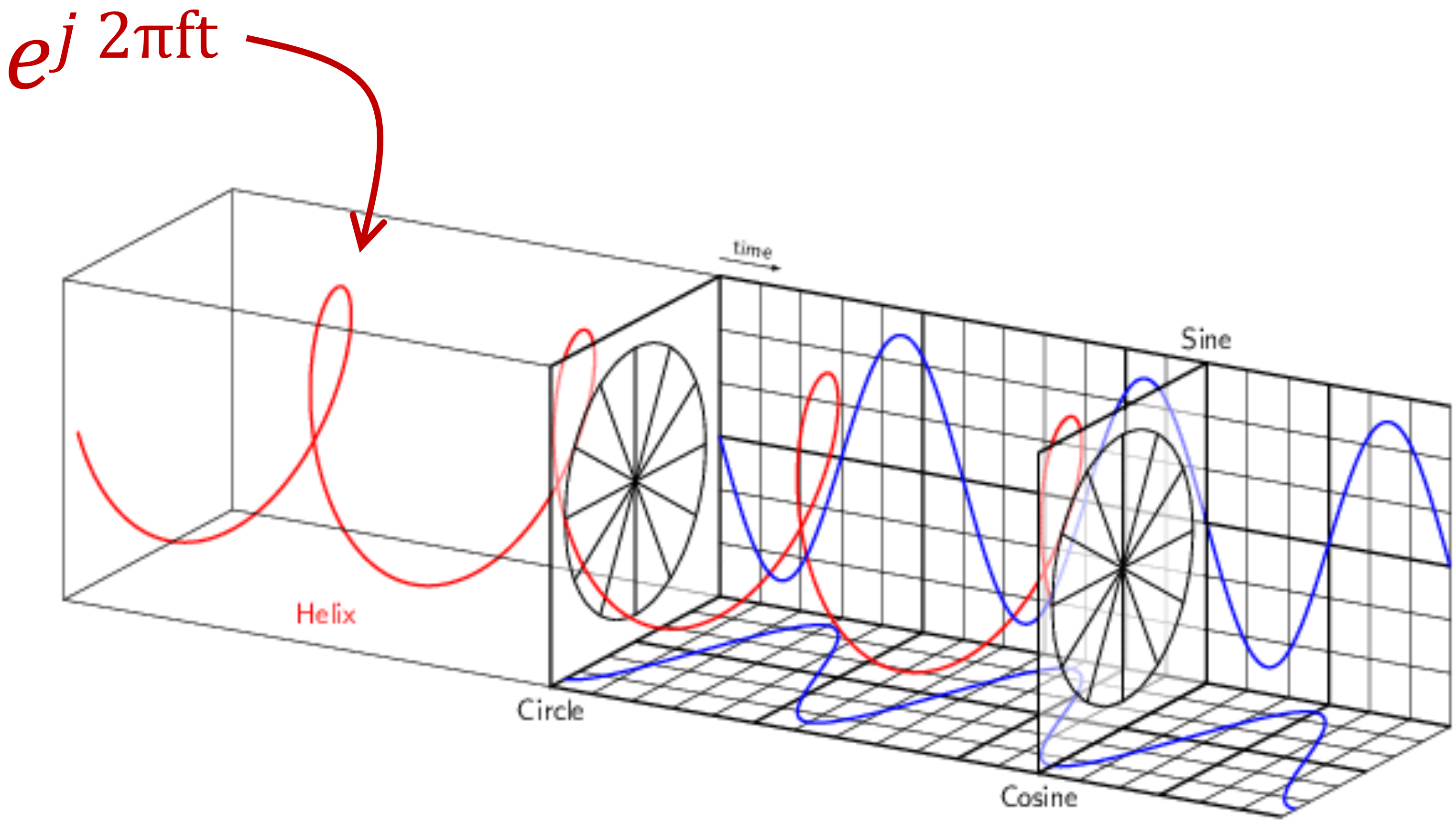
Equation of waves in time and space



$$\Psi(t, x) = A\,.\sin(2\pi f t - \theta(x))$$

$$= A\,.\sin(2\pi f t - \frac{2\pi}{\lambda} x)$$

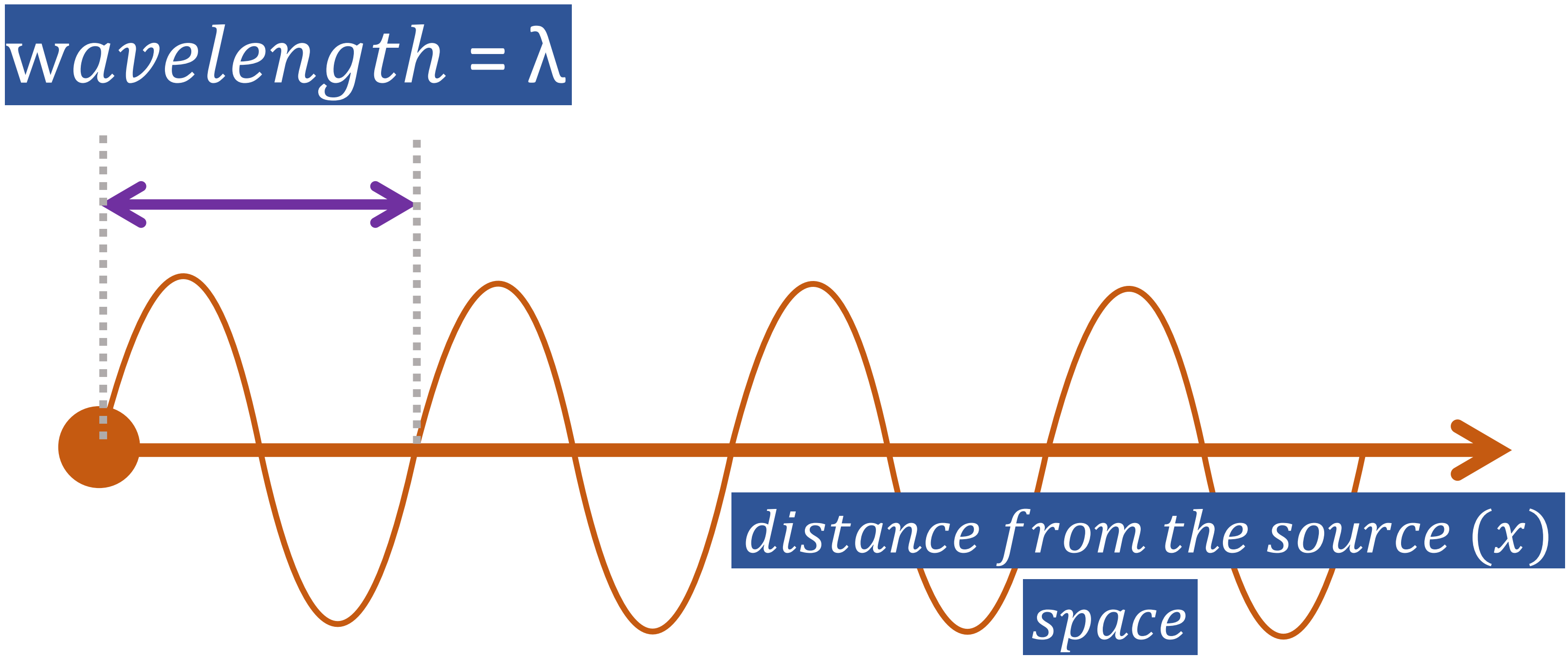
Model for a signal (frequency, amplitude, and phase)



Presenting real signal with the complex model

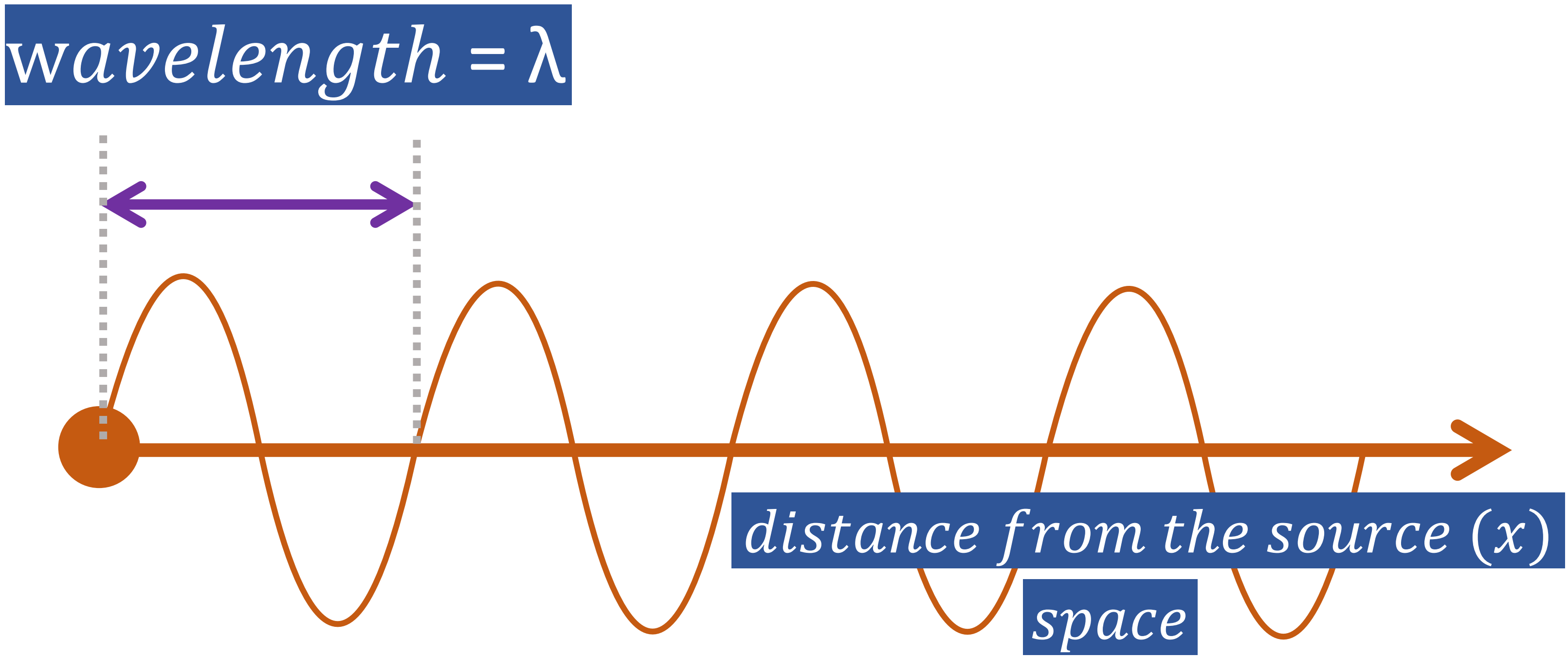
- $$e^{j\,2\pi ft} = cos(2\pi ft) + j\ \sin(2\pi ft)$$
- $$e^{-j\,2\pi ft} = cos(2\pi ft) - j\ \sin(2\pi ft)$$
- $$cos(2\pi ft) = \frac{e^{j\,2\pi ft} + e^{-j\,2\pi ft}}{2}$$
- $$sin(2\pi ft) = \frac{e^{j\,2\pi ft} - e^{-j\,2\pi ft}}{2j}$$

Equation of waves in time and space



$$\Psi(\mathrm{t}, \mathrm{x}) = \mathrm{A}\,.\sin(2\pi f t - \frac{2\pi}{\lambda} x)$$

Equation of waves in time and space



$$\Psi(t, x) = A \cdot e^{j\left(2\pi f t - \frac{2\pi}{\lambda} x\right)}$$

Time and space

Cycles per sec = frequency = f Hz

Distance per cycle = wavelength = λ meters

Distance per second = speed = C meters/sec

$$C = f \,.\, \lambda$$

Speed of sound in air: 343 m/s

Speed of sound in water: 1493 m/s

Speed of sound in iron: 5130 m/s

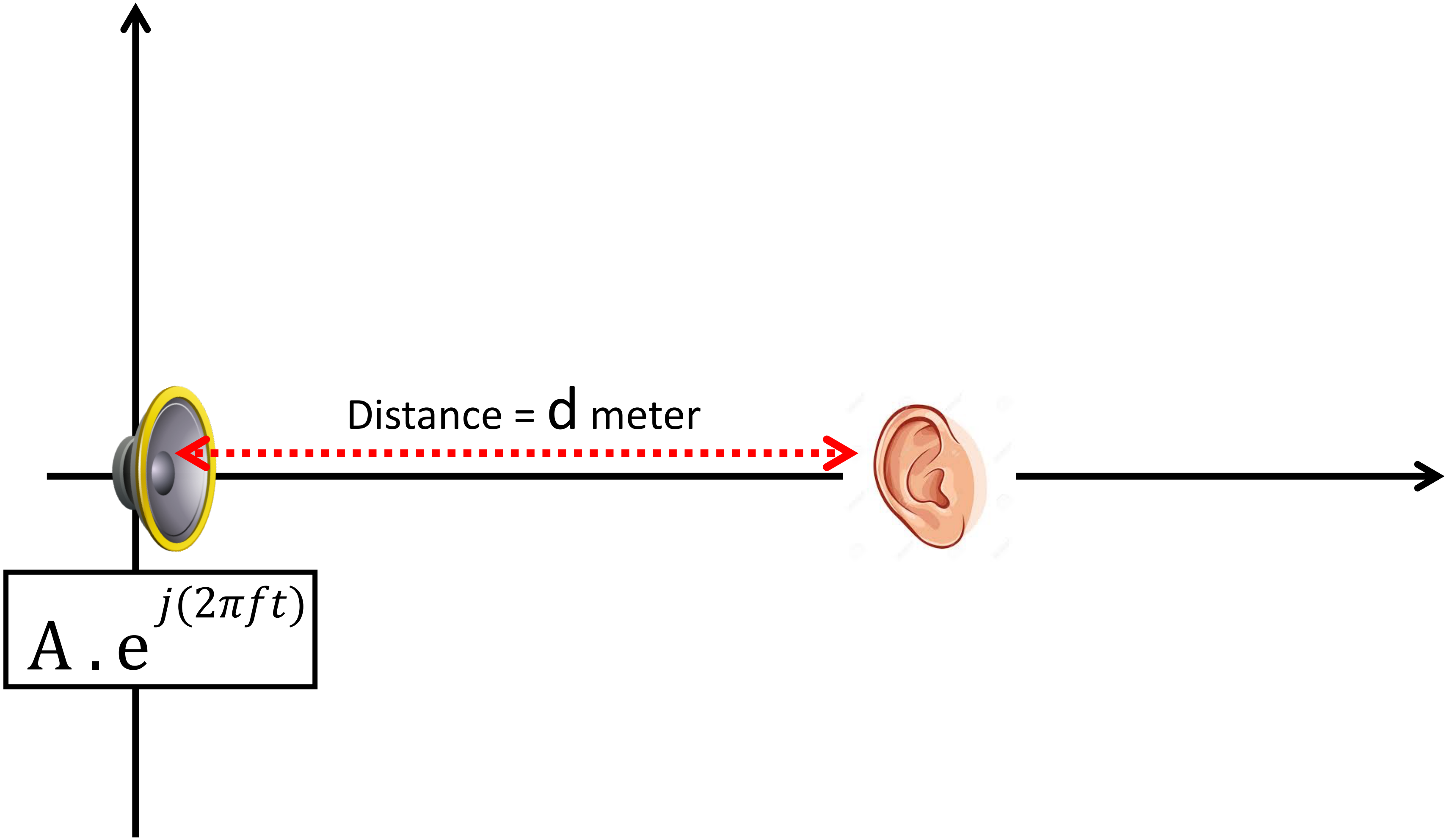
Speed of electromagnetic waves: $3 * 10^8$ m/s
(~ a million times faster than sound)

Time and space

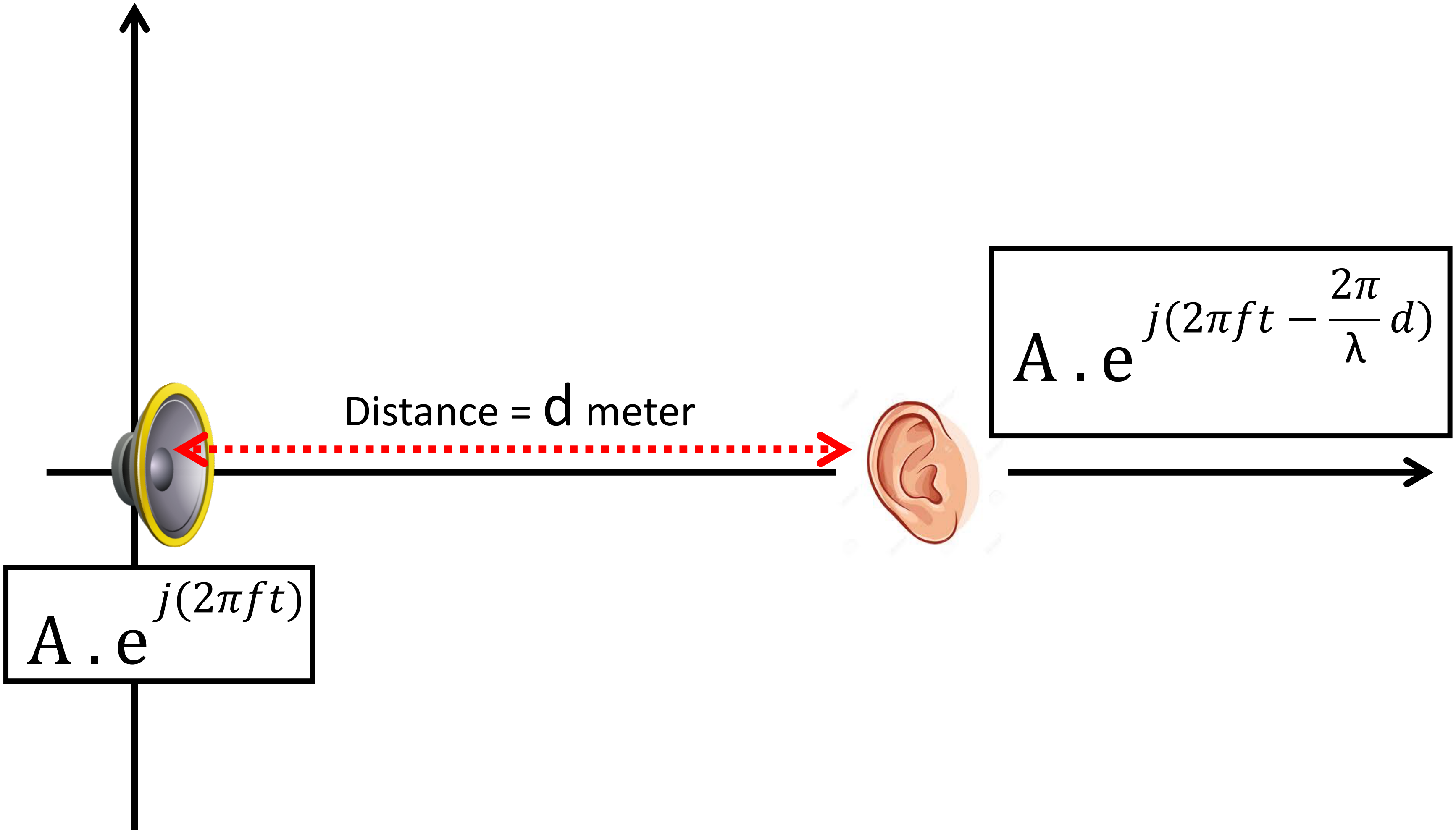
Time and space

$$A \cdot e^{j(2\pi f t)}$$

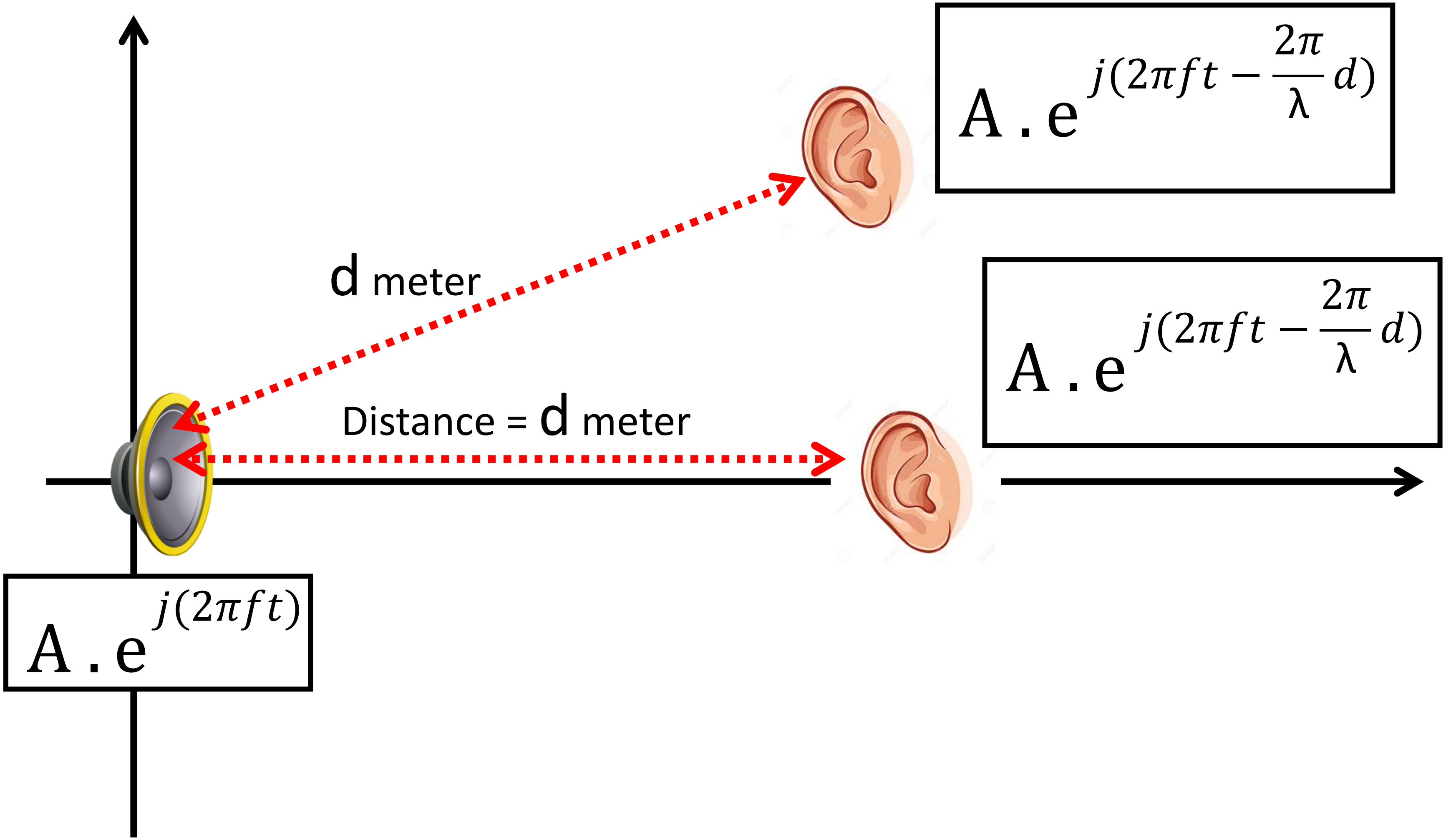
Time and space



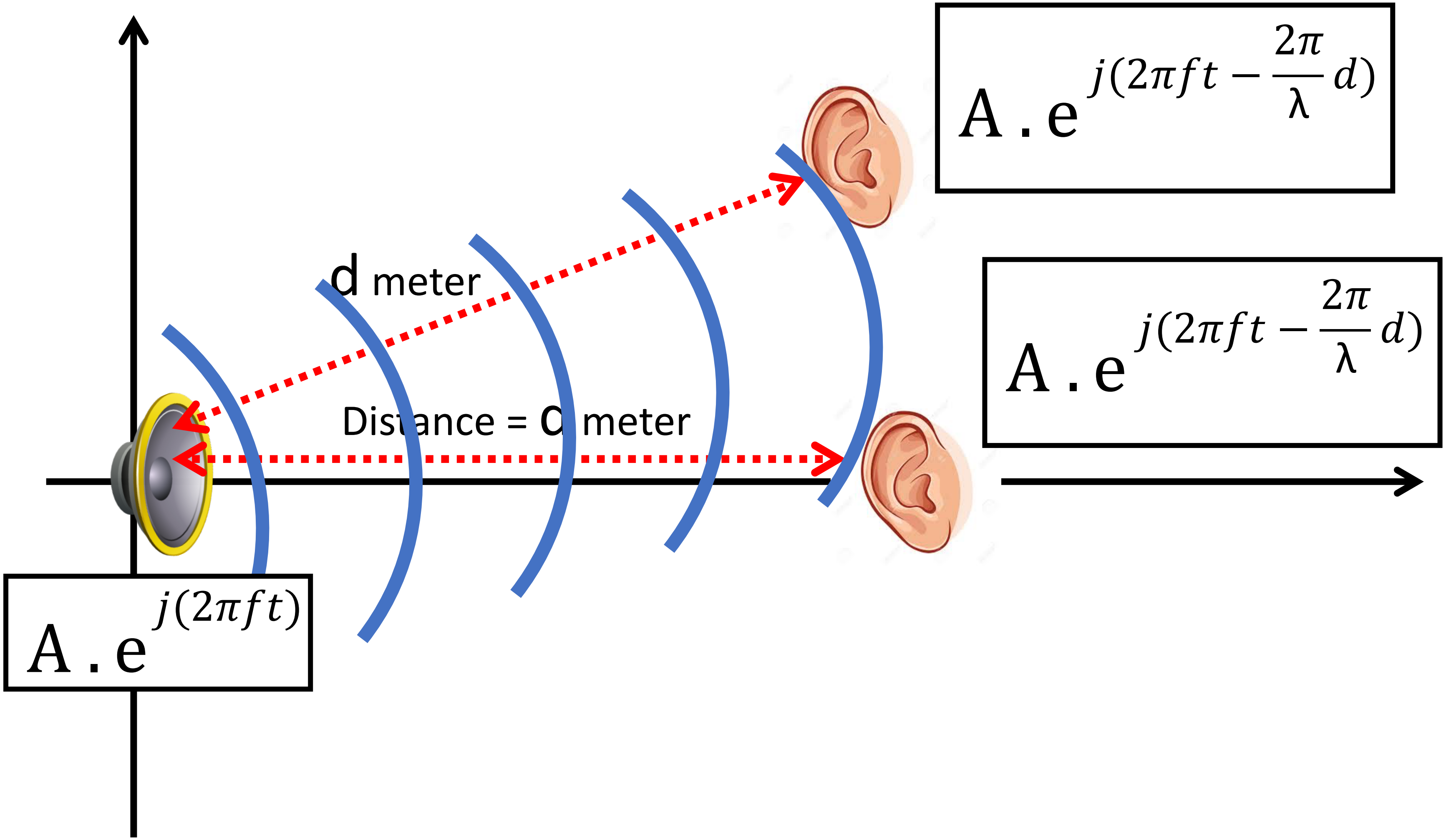
Time and space



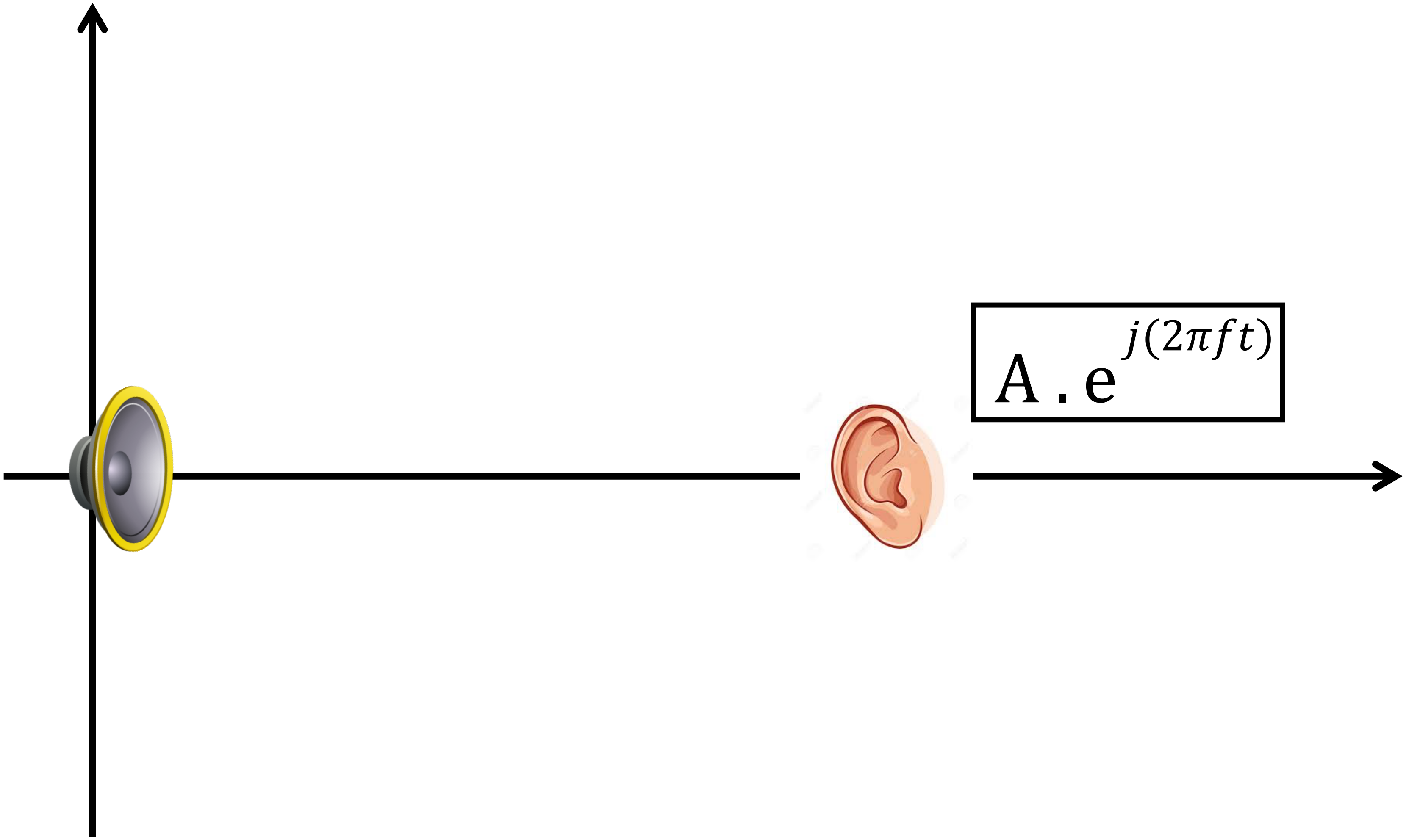
Time and space



Time and space

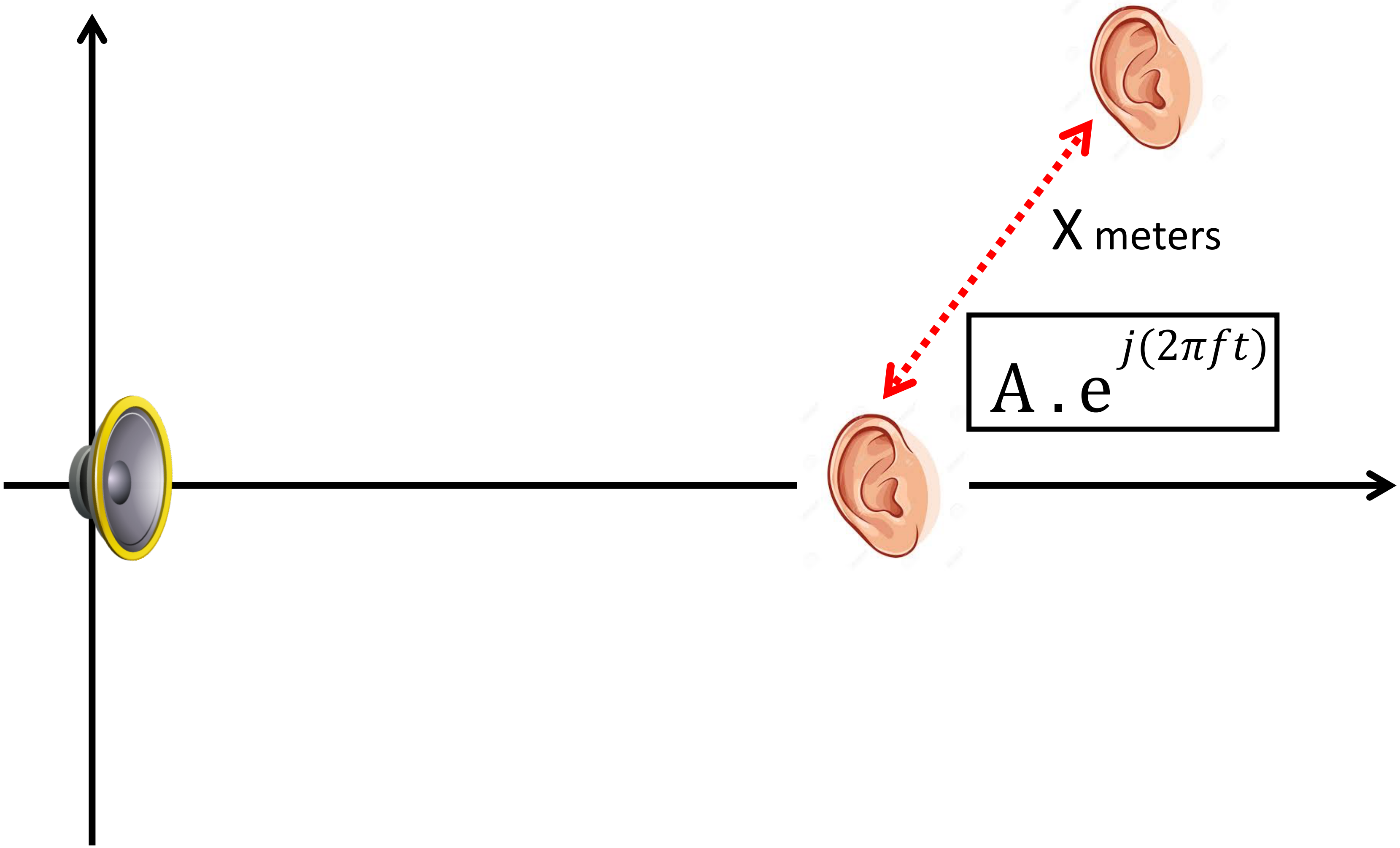


Time and space

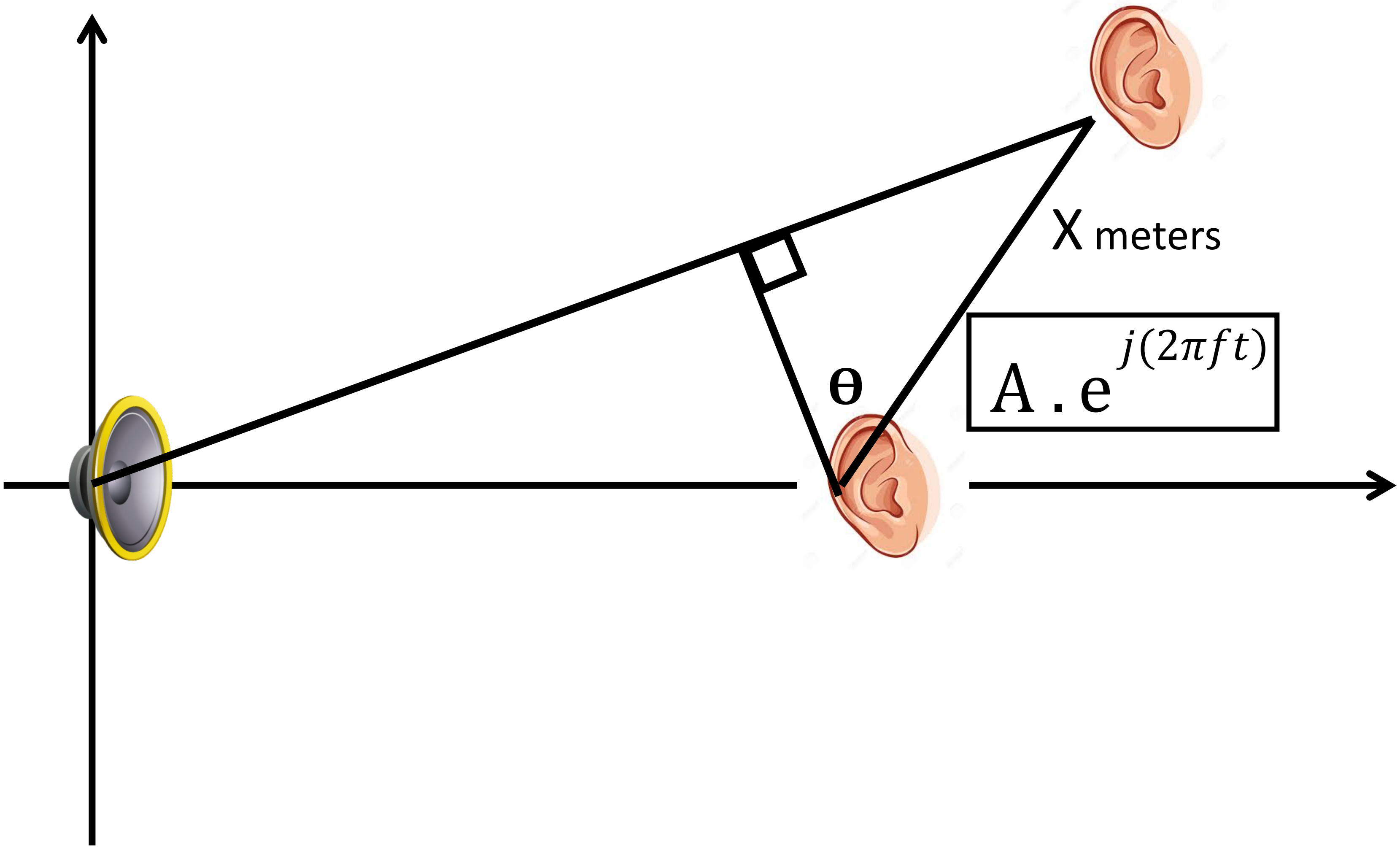


$$A \,.\, e^{j(2\pi f t)}$$

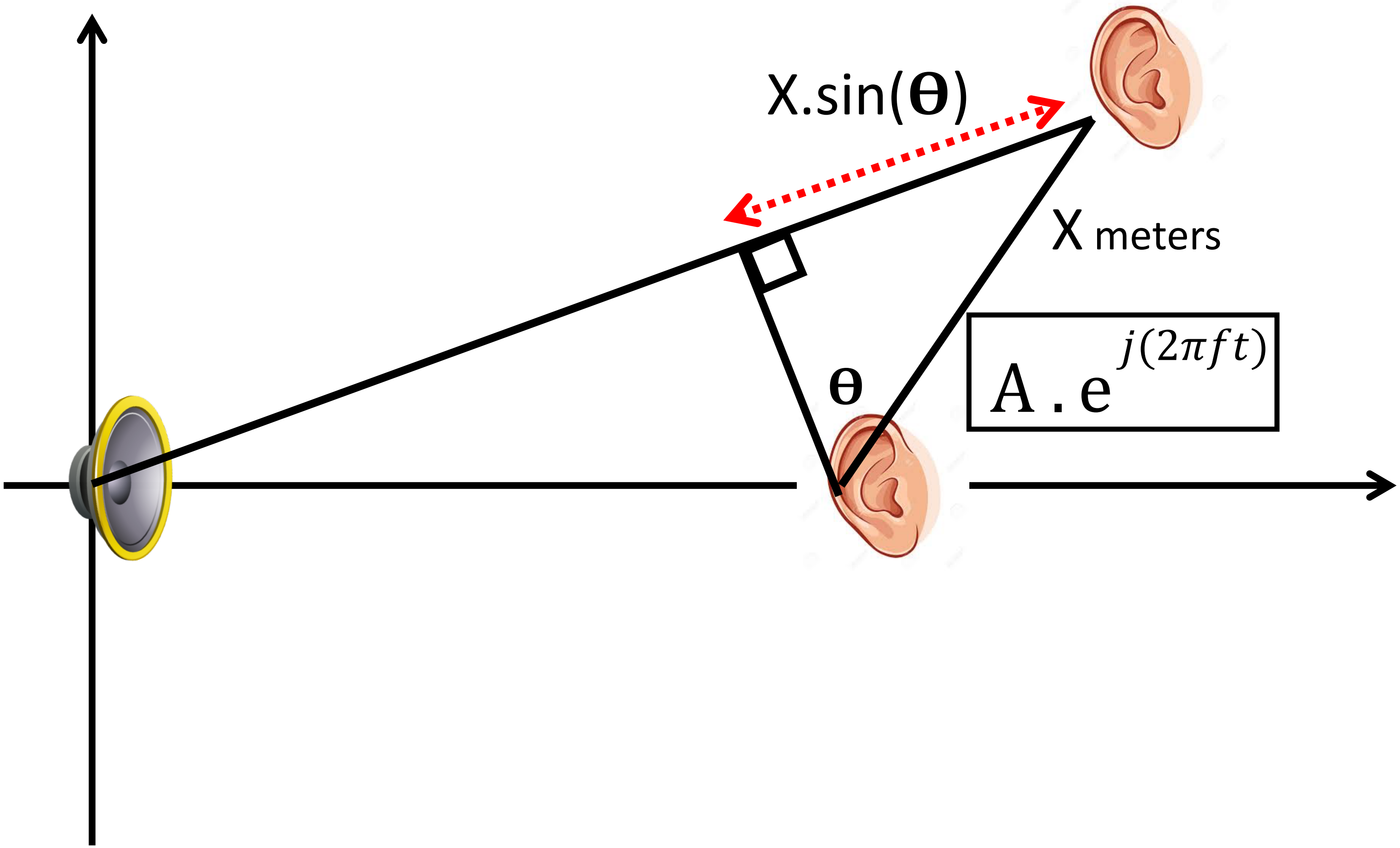
Time and space



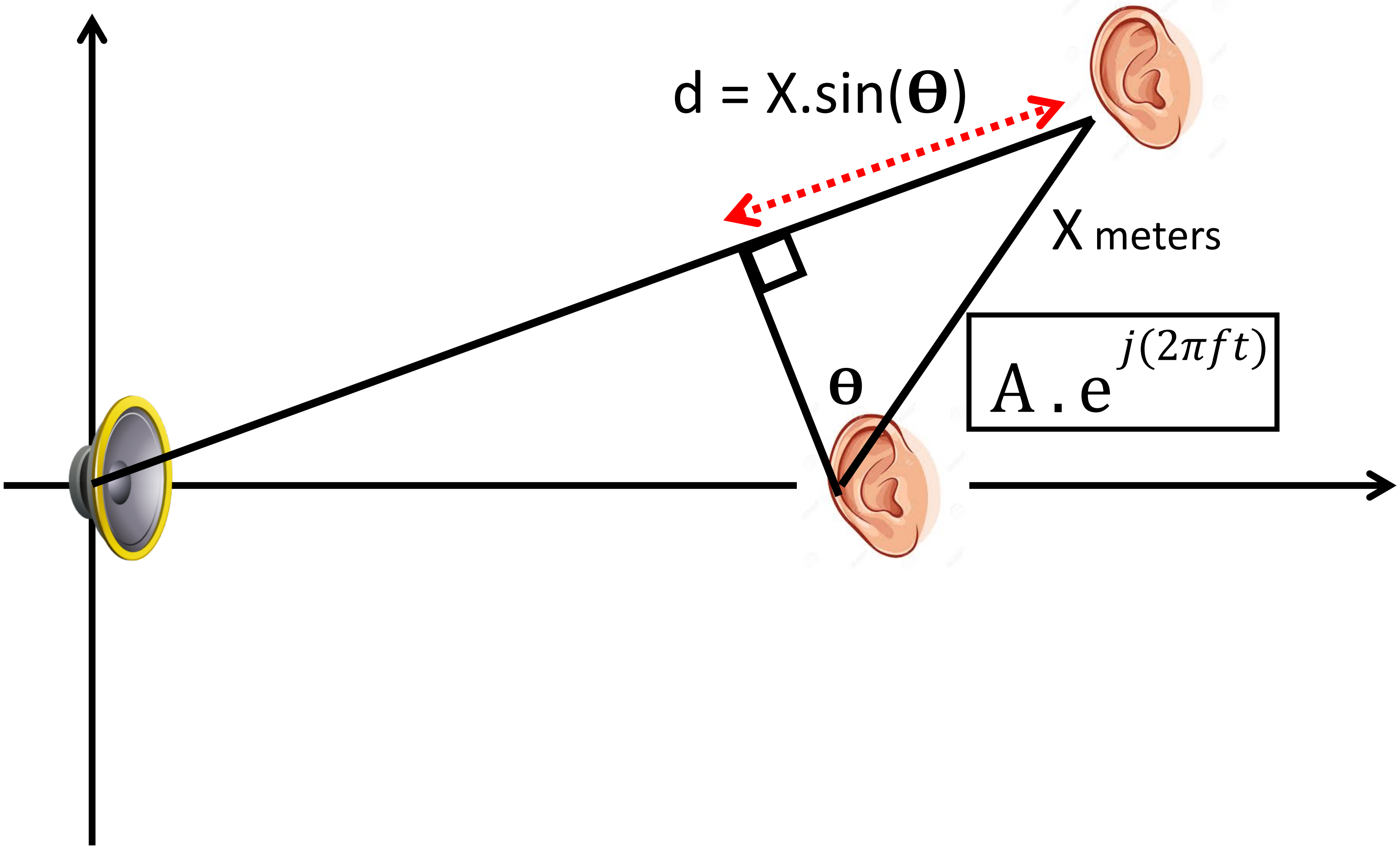
Time and space



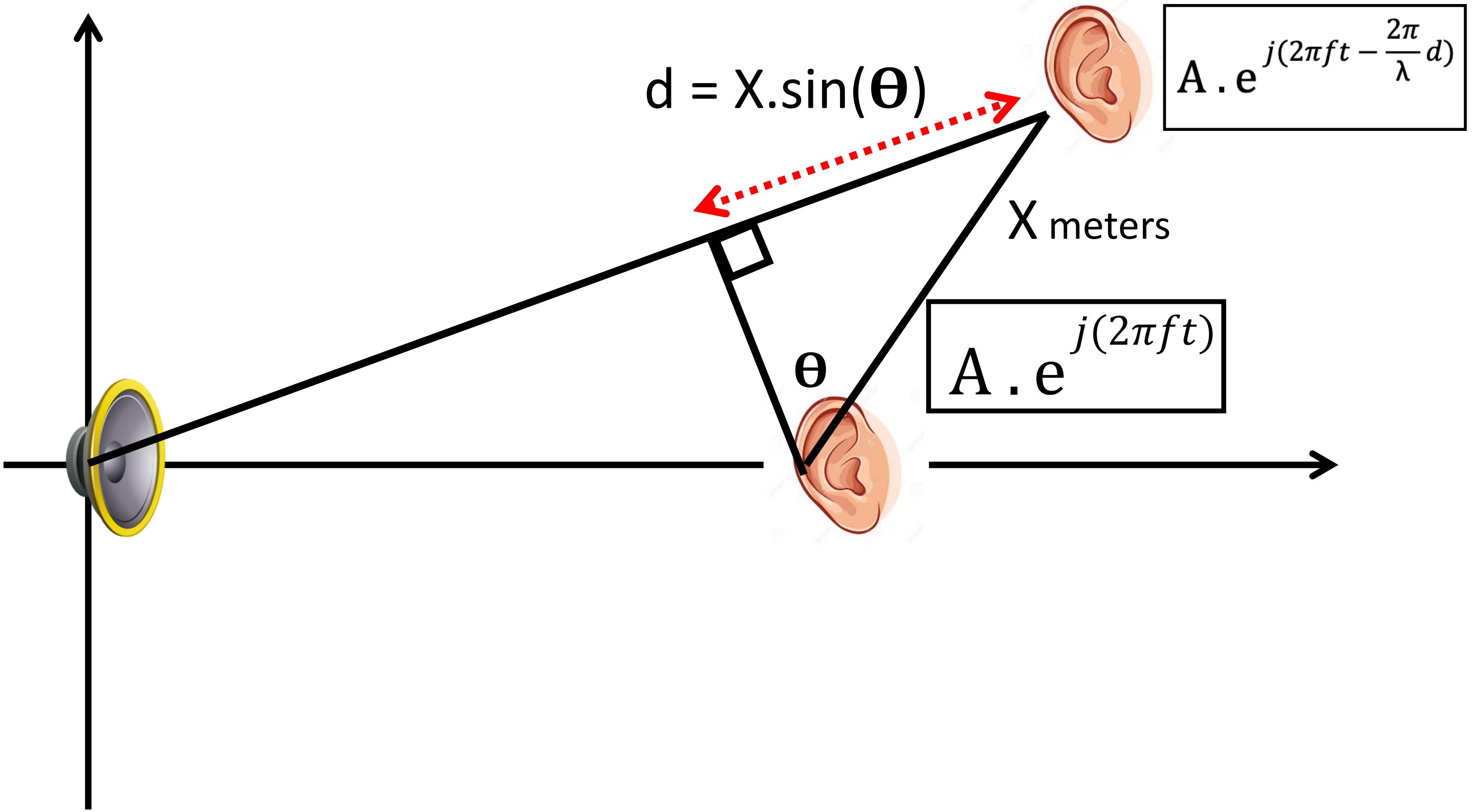
Time and space



Time and space

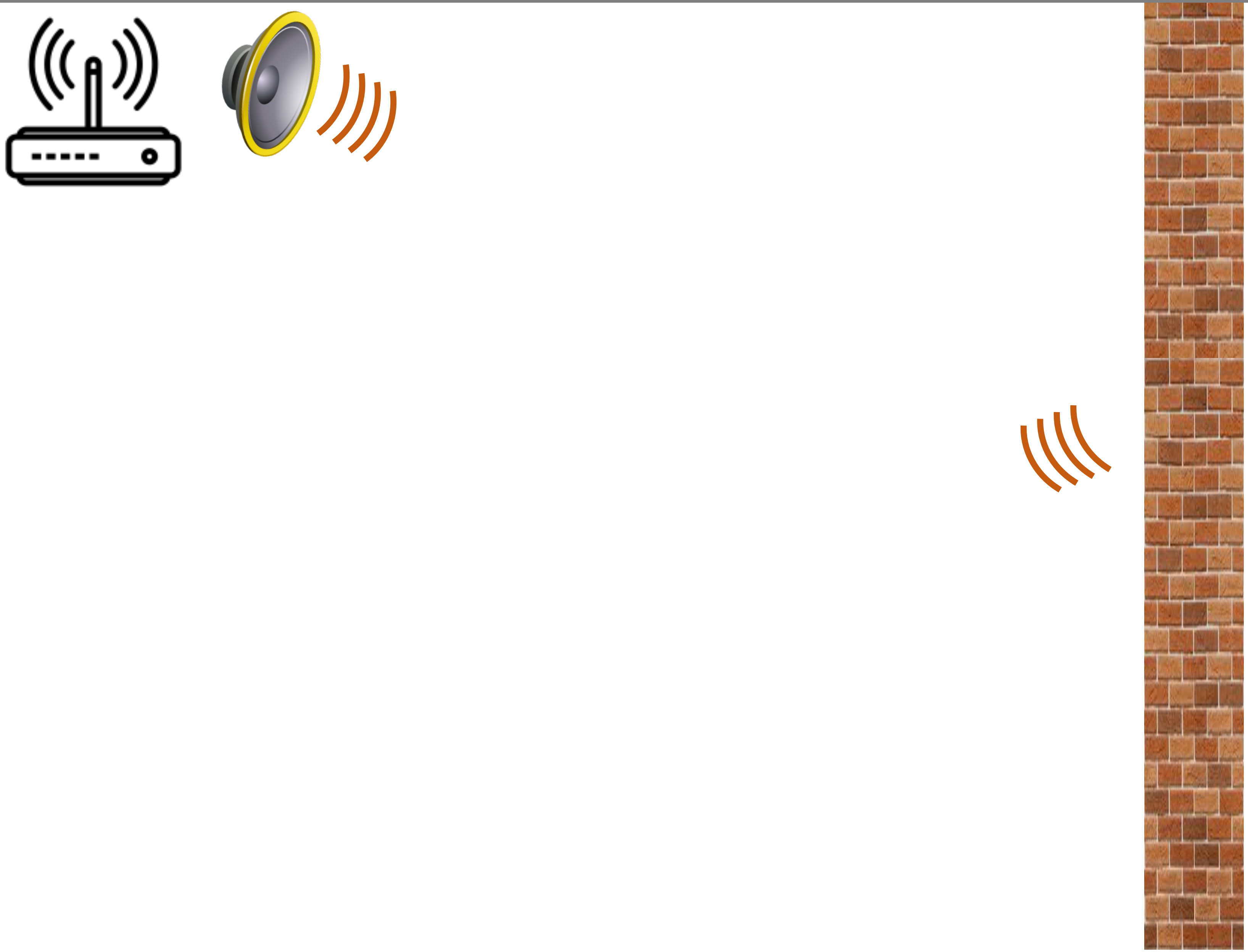


Time and space

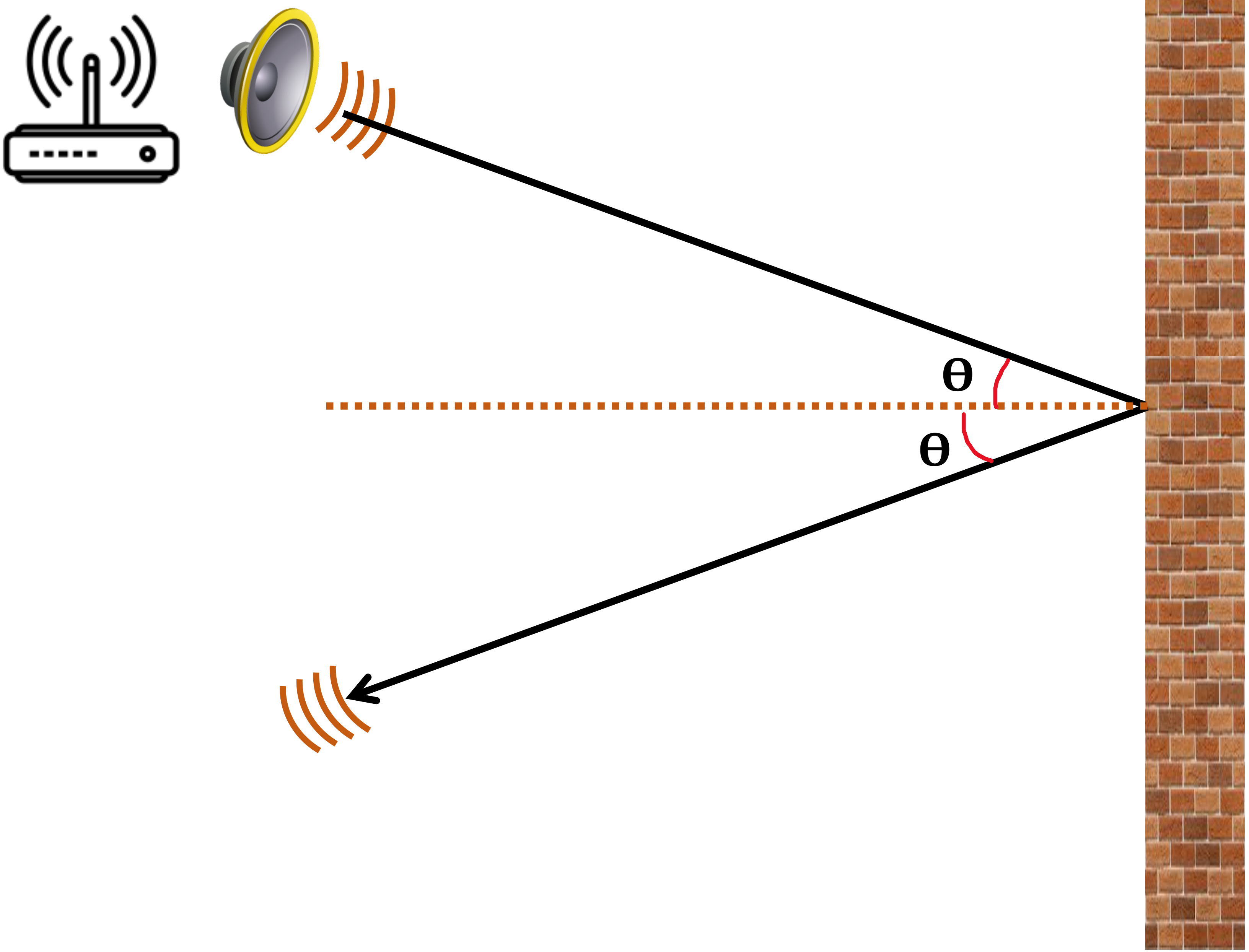


Reflection of waves

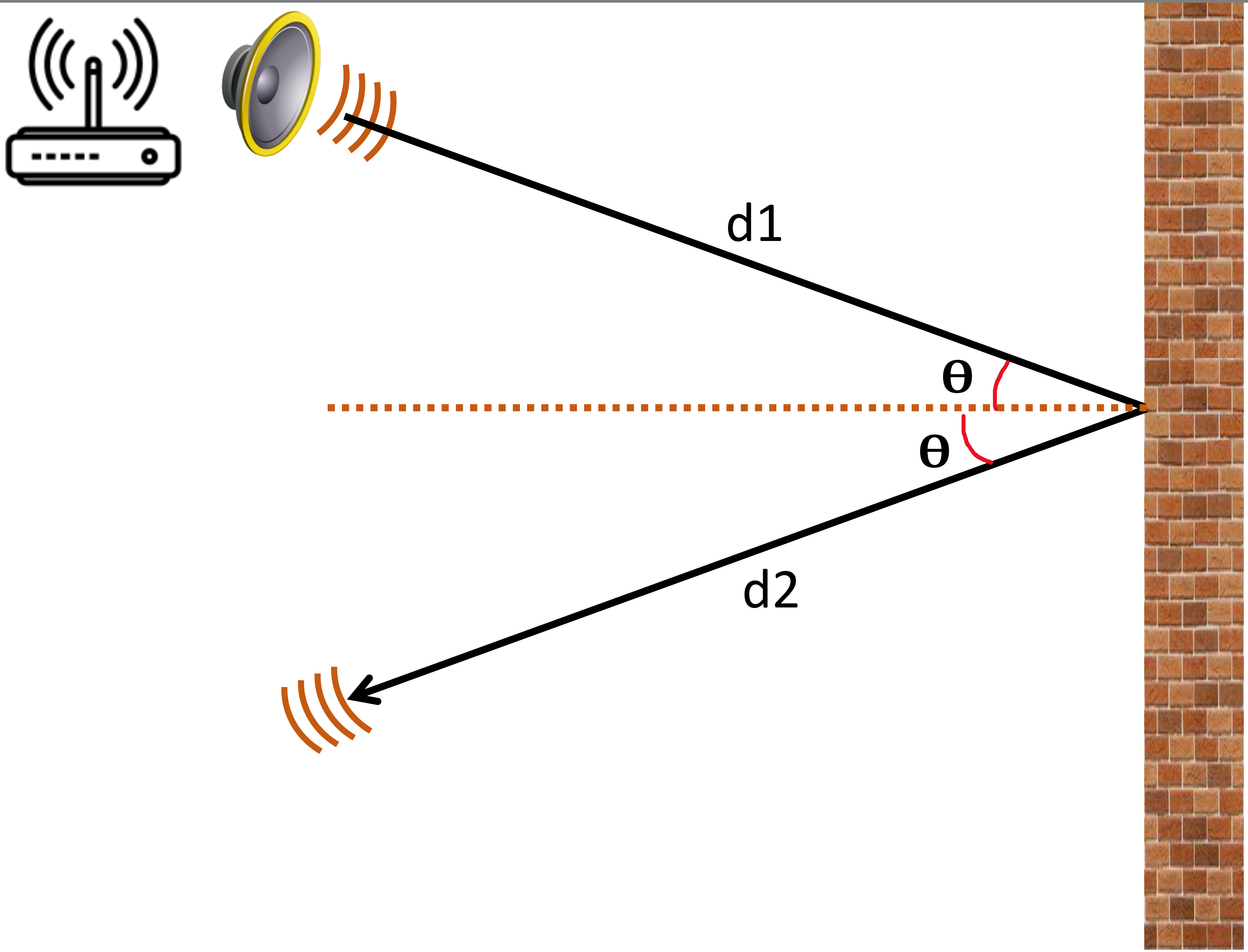
Reflection



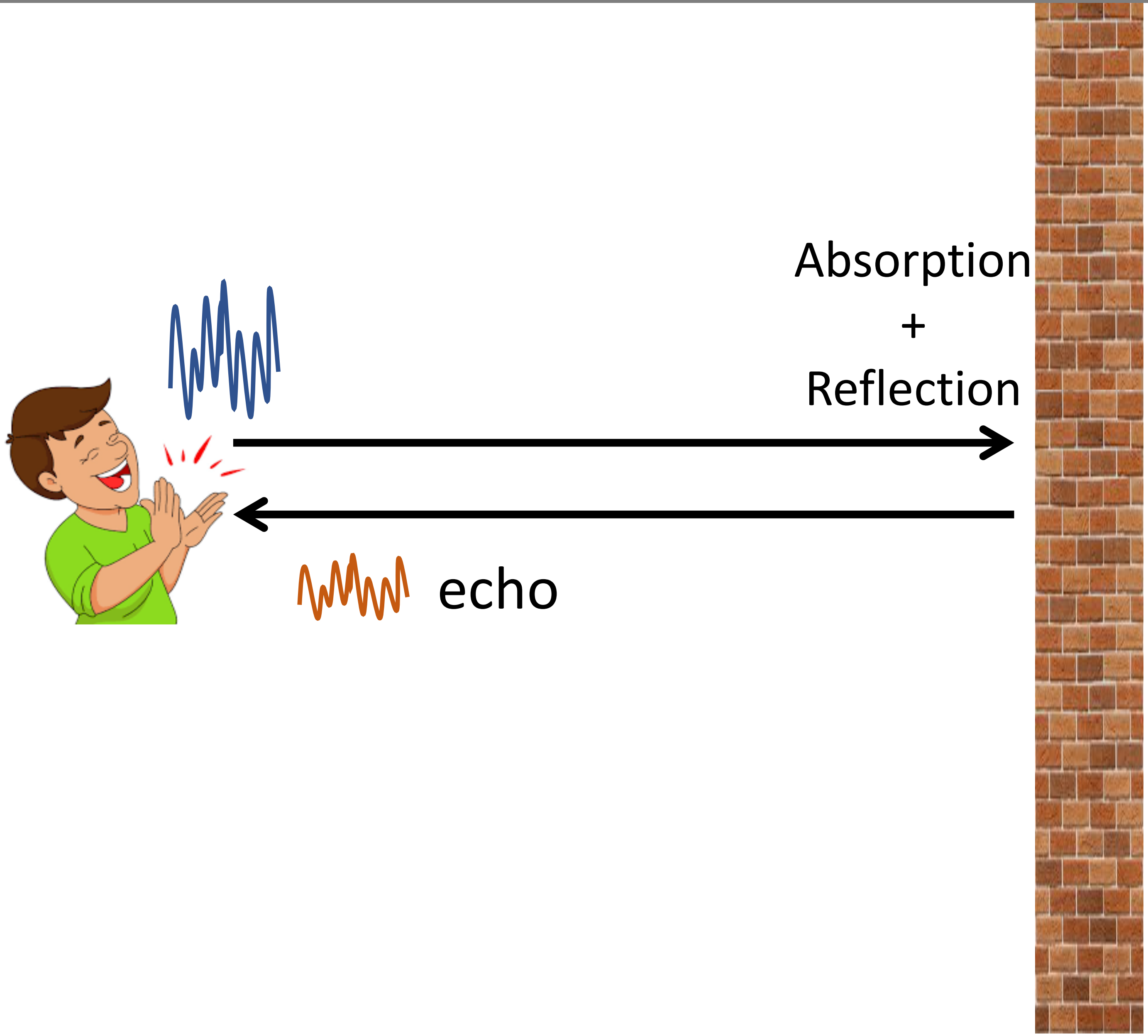
Reflection



Reflection

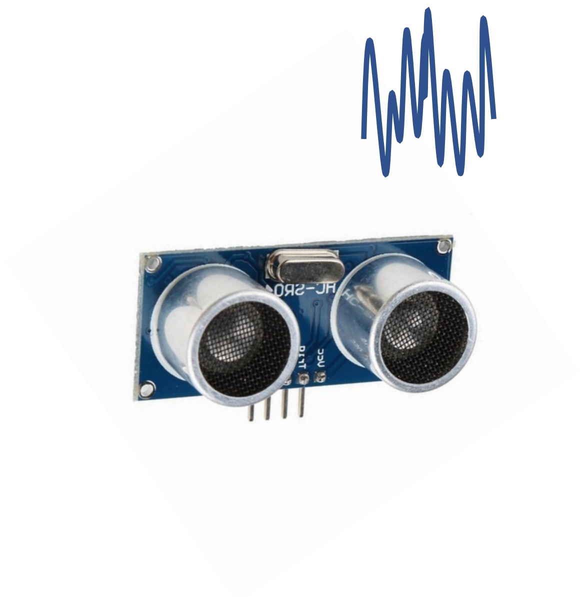


Reflection

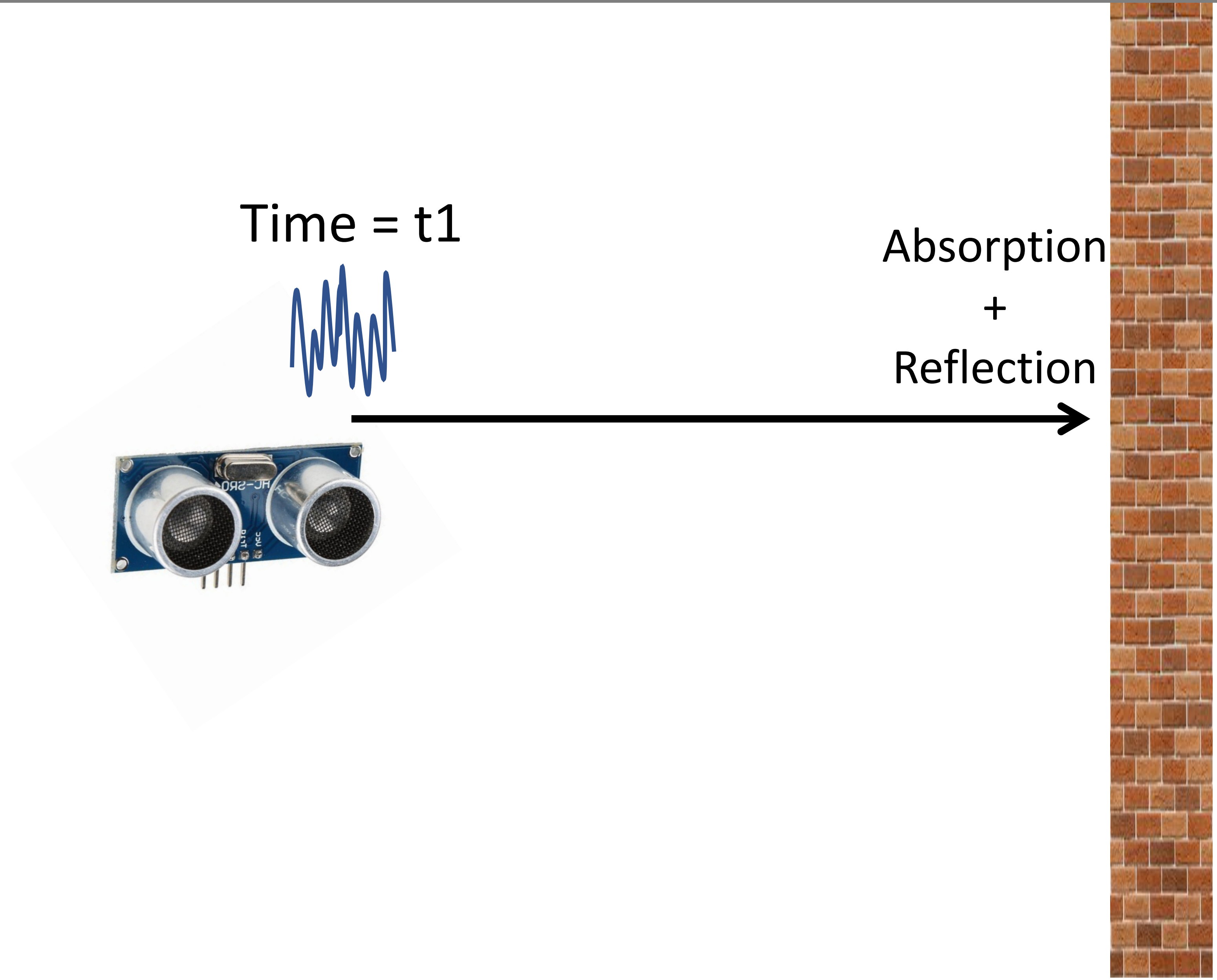


Reflection

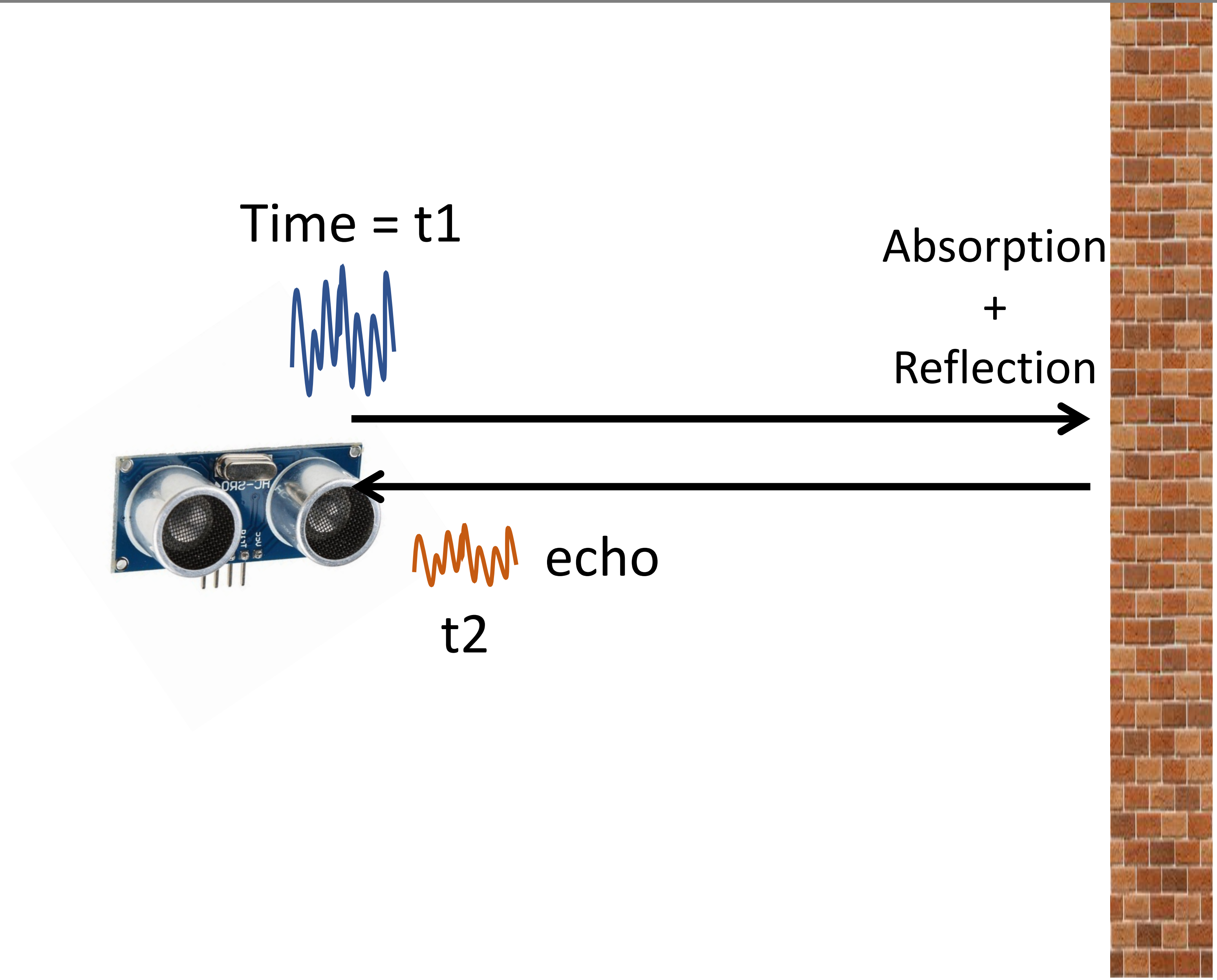
Time = t1



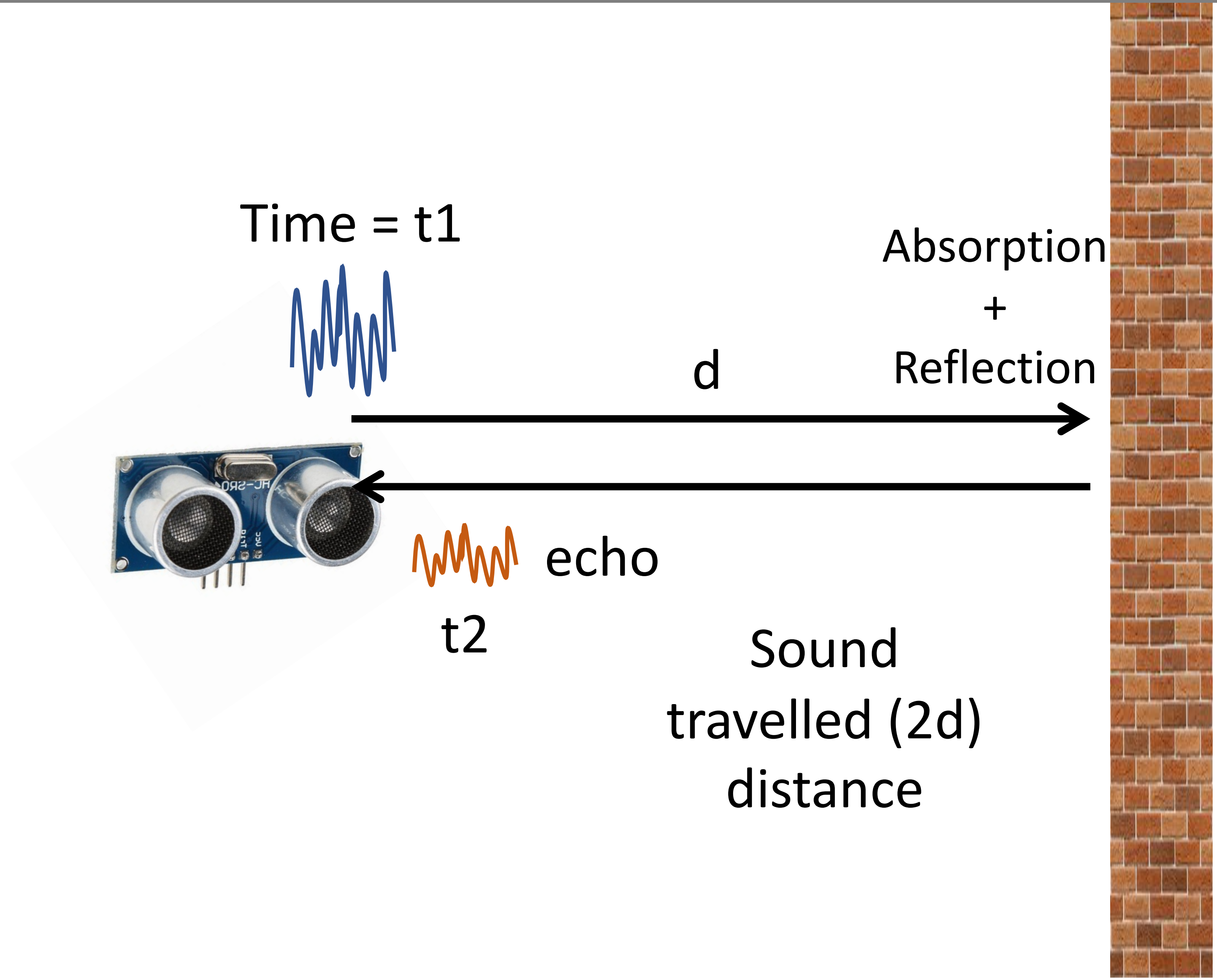
Reflection



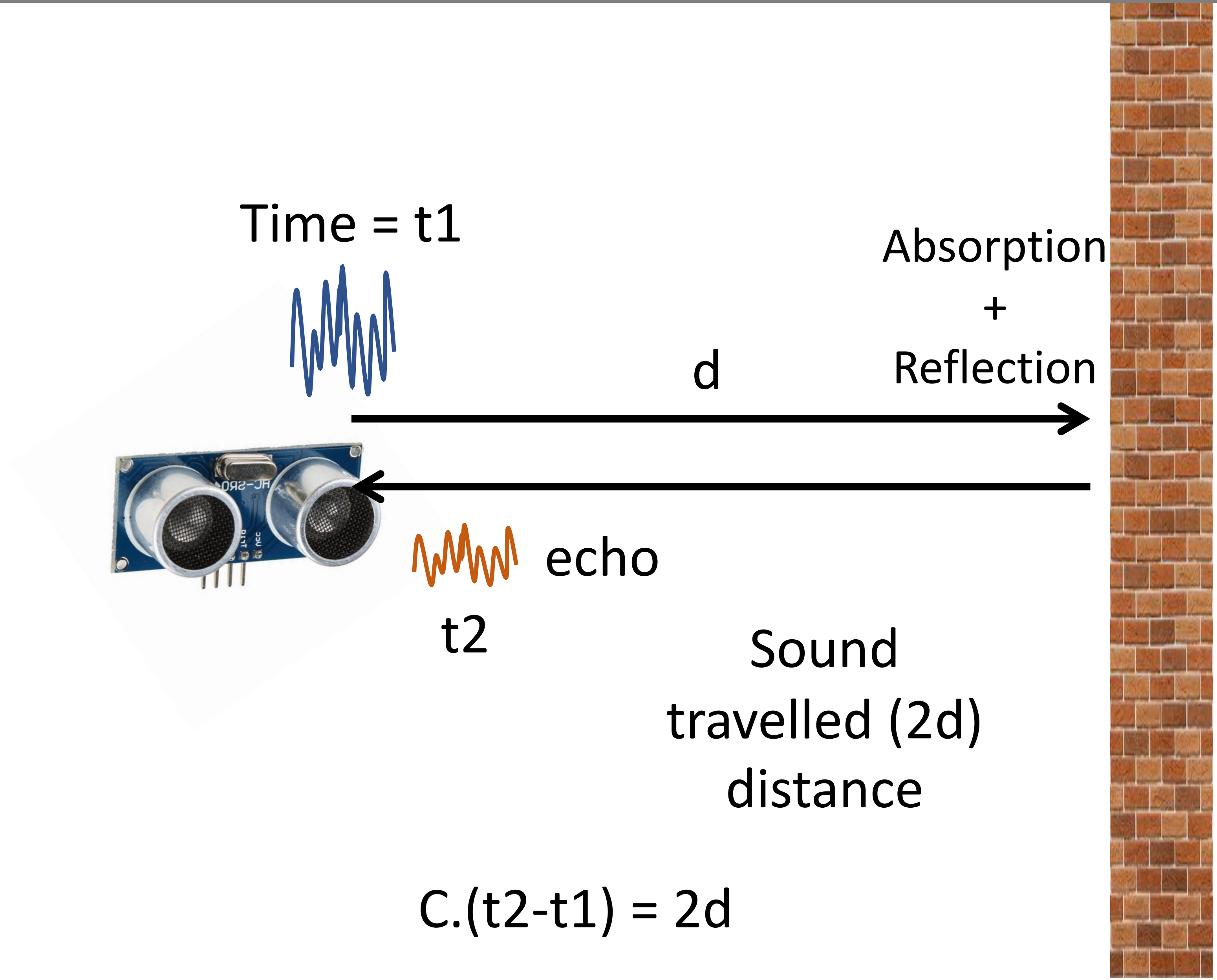
Reflection



Reflection


Time = t1
Absorption
+
Reflection
d
echo
t2
Sound
travelled (2d)
distance

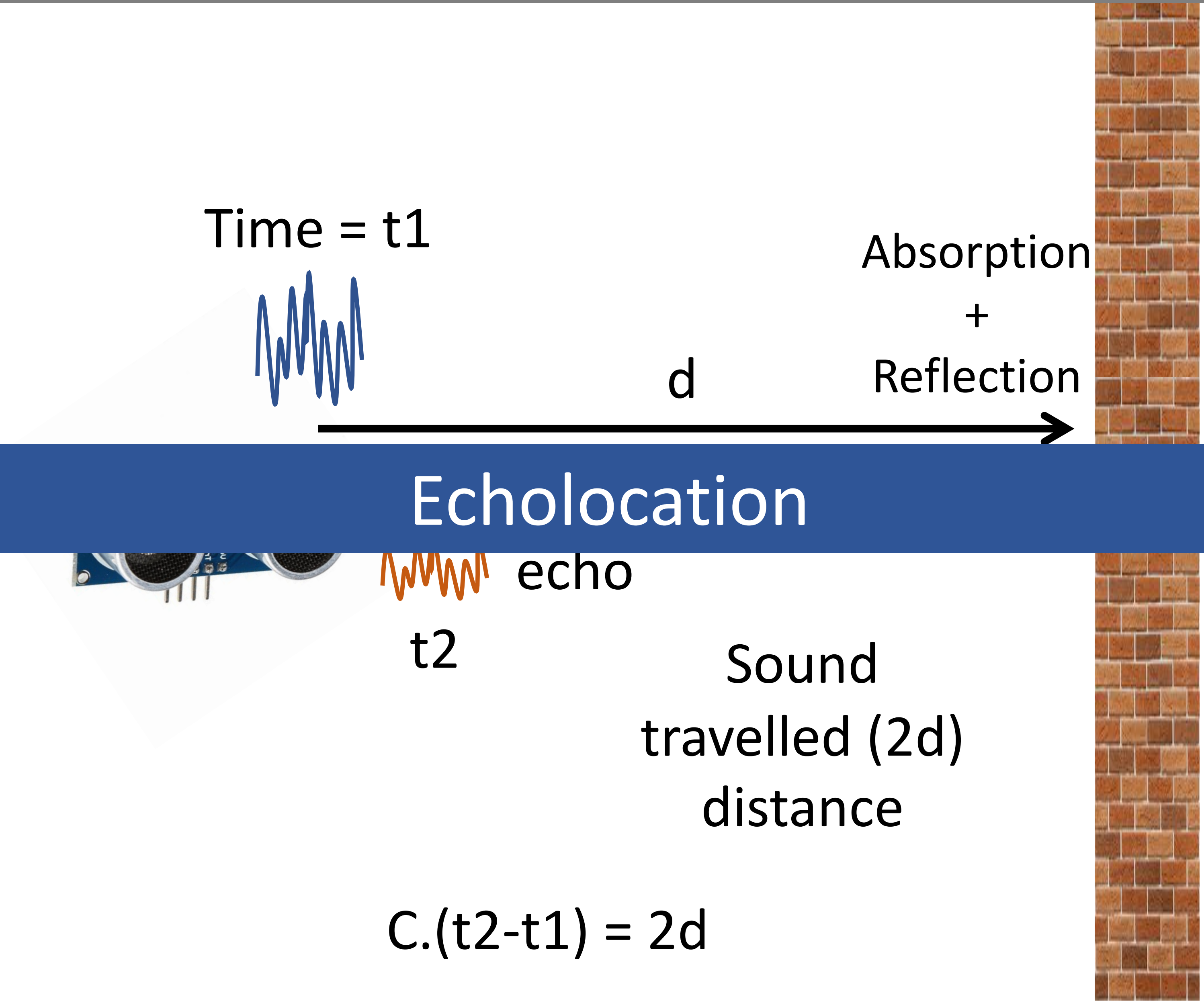
Reflection



C.(t2-t1) = 2d

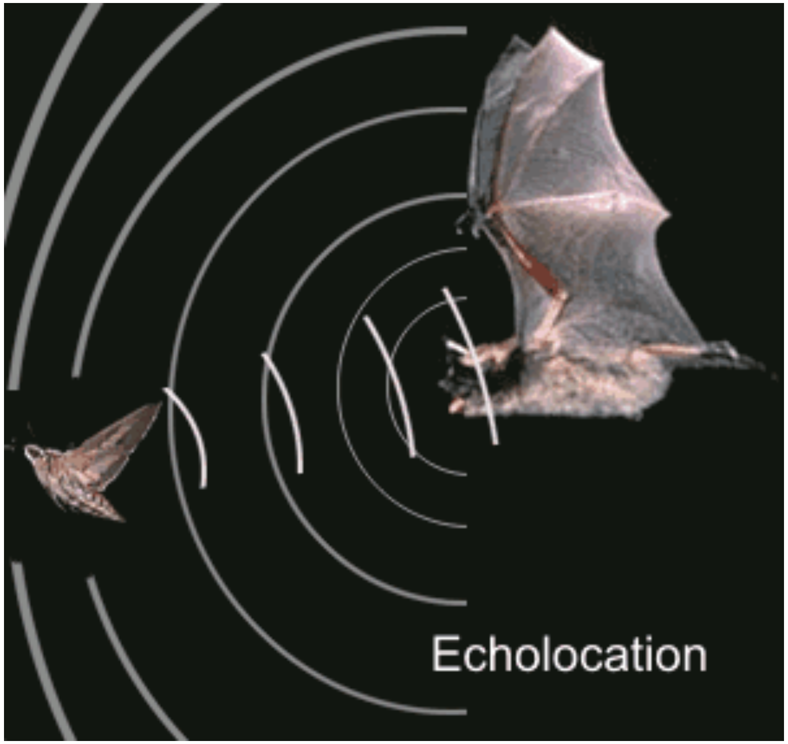
Reflection

Echolocation



$$C.(t2-t1) = 2d$$

Echolocation in nature

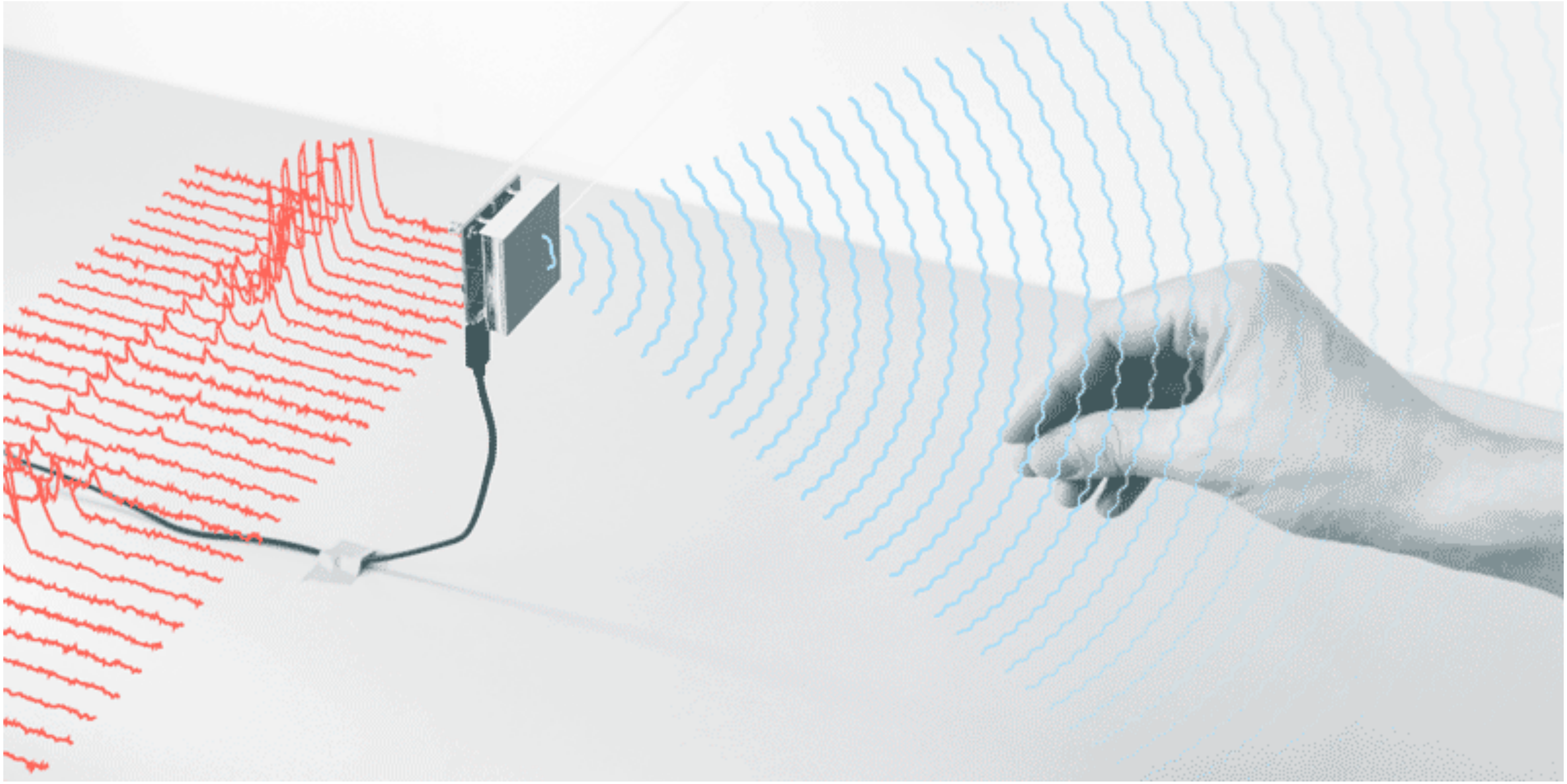


Bat

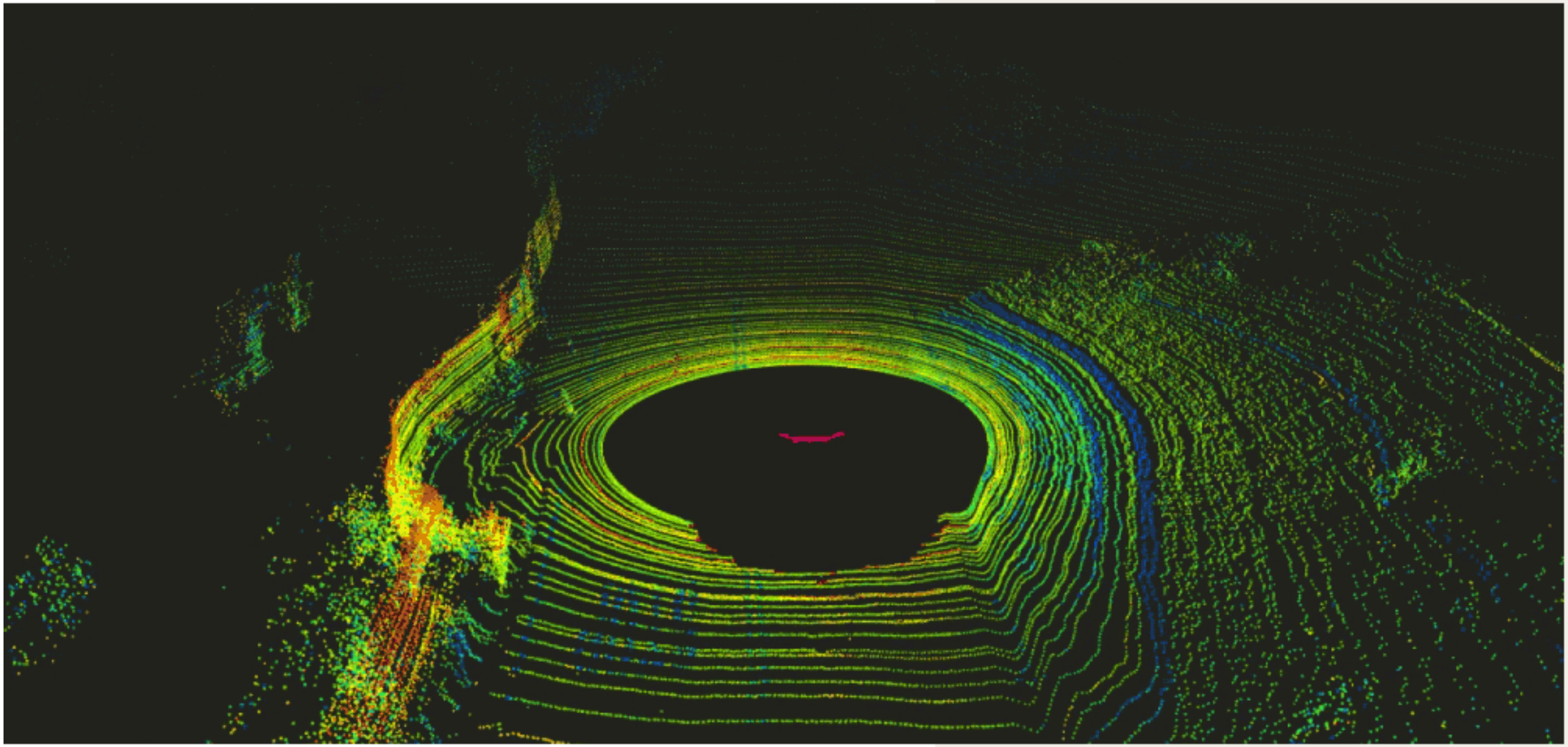
Echolocation in nature

Beluga whale

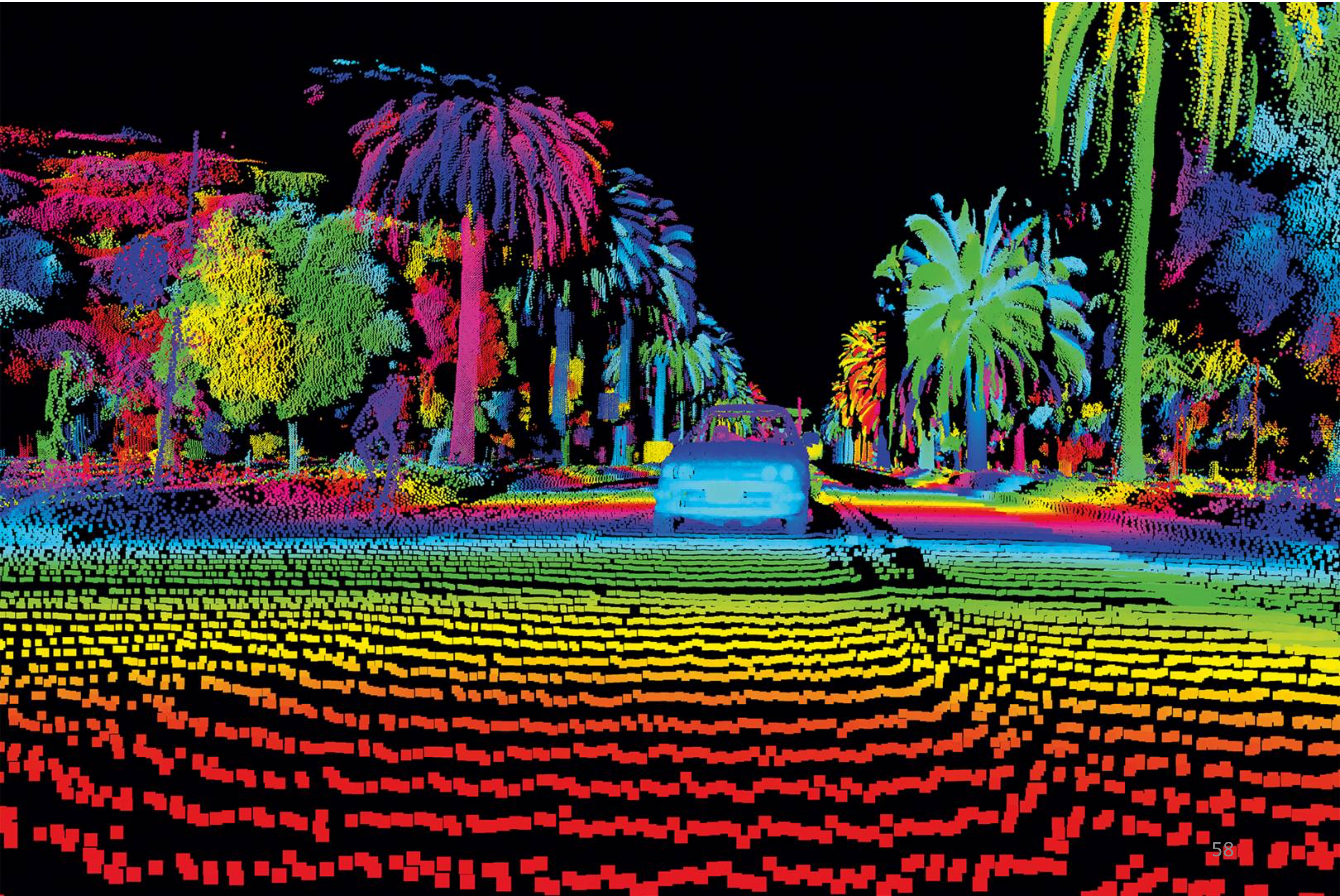
Waves for gesture detection [Project Soli]



LiDAR (Light Detection and Ranging)

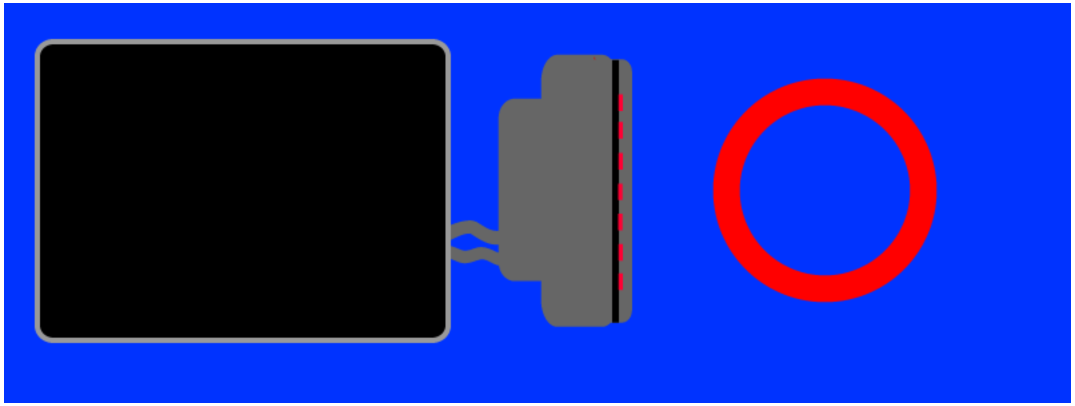


LiDAR (Light Detection and Ranging)



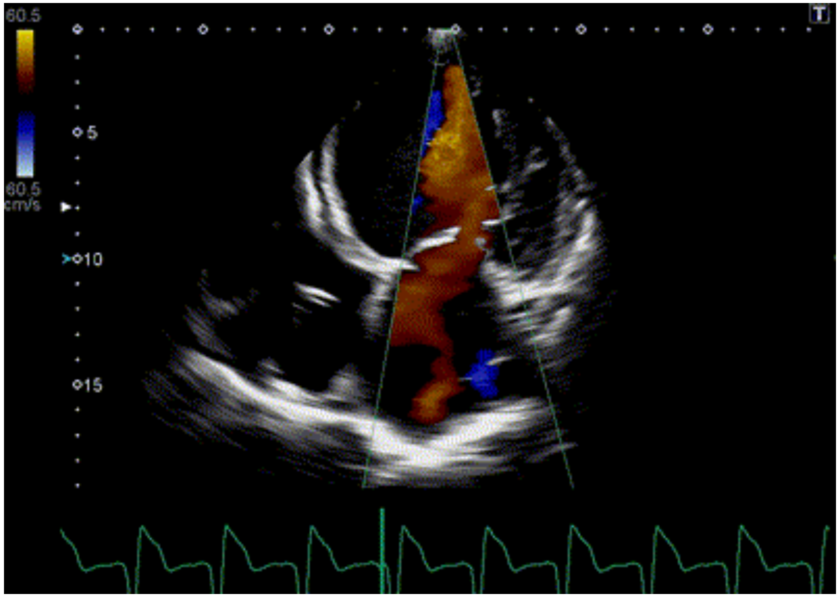
Acoustic imaging

Acoustic imaging



Sonogram

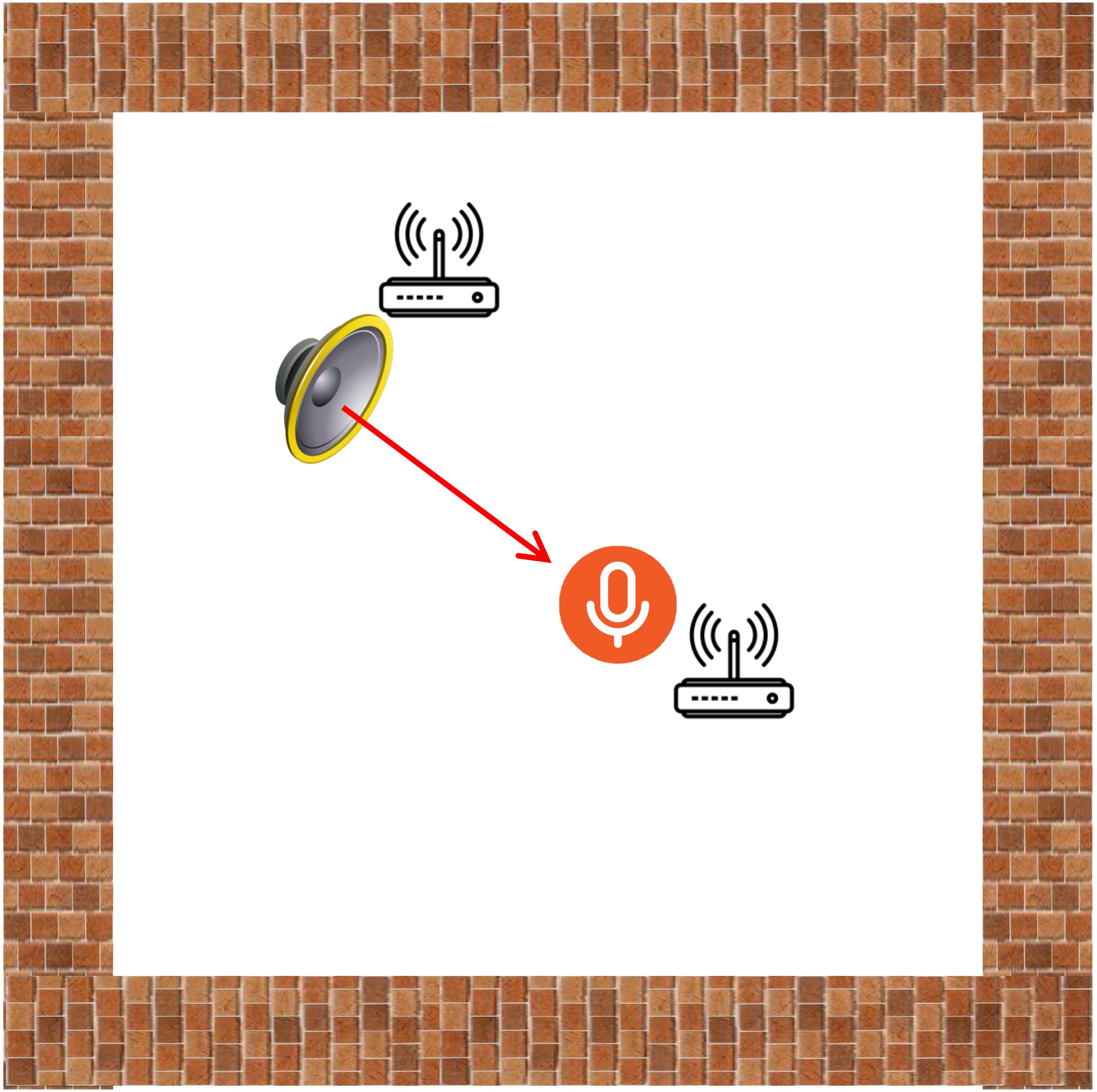
Acoustic imaging



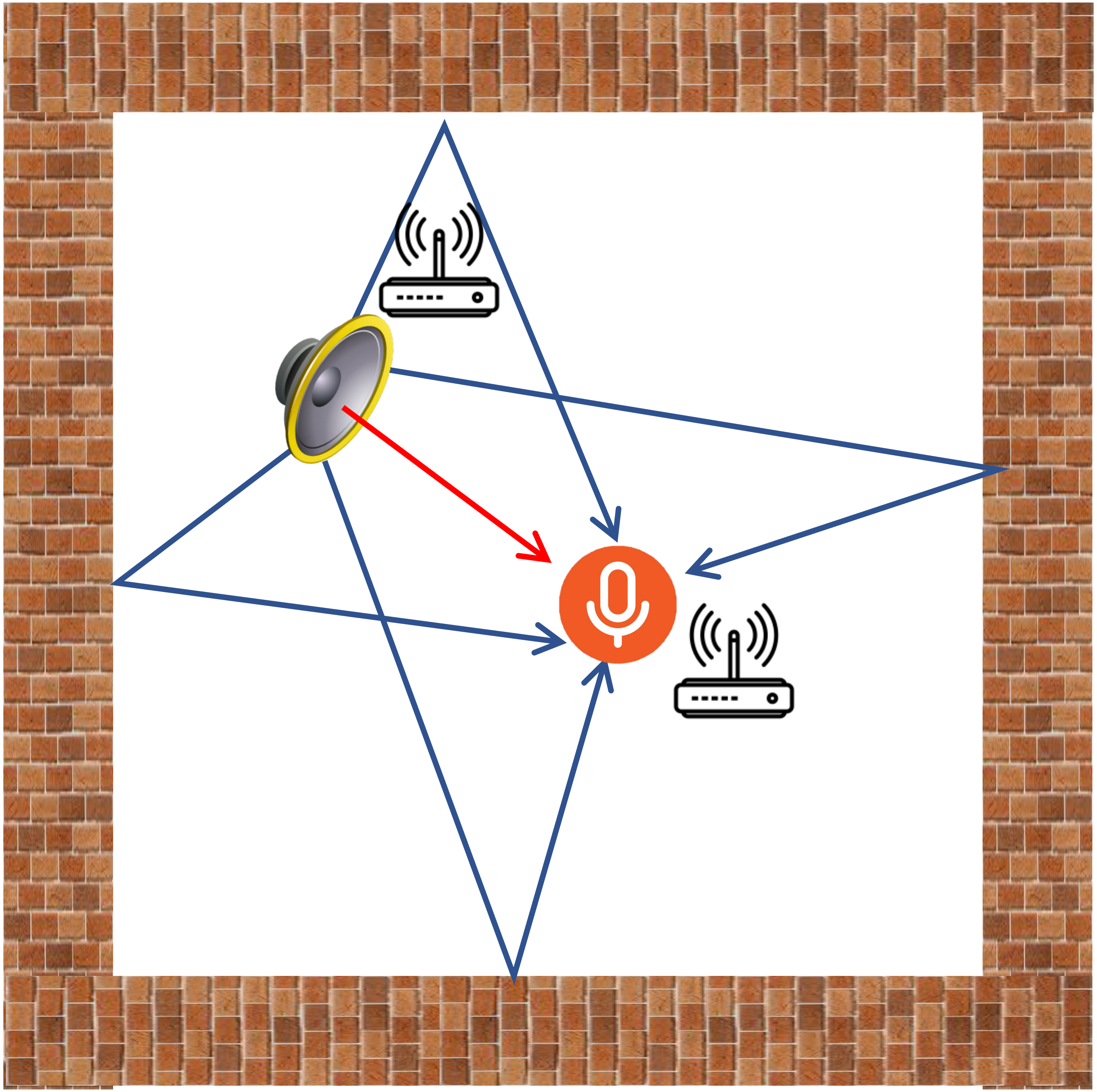
Sonogram

Multipath

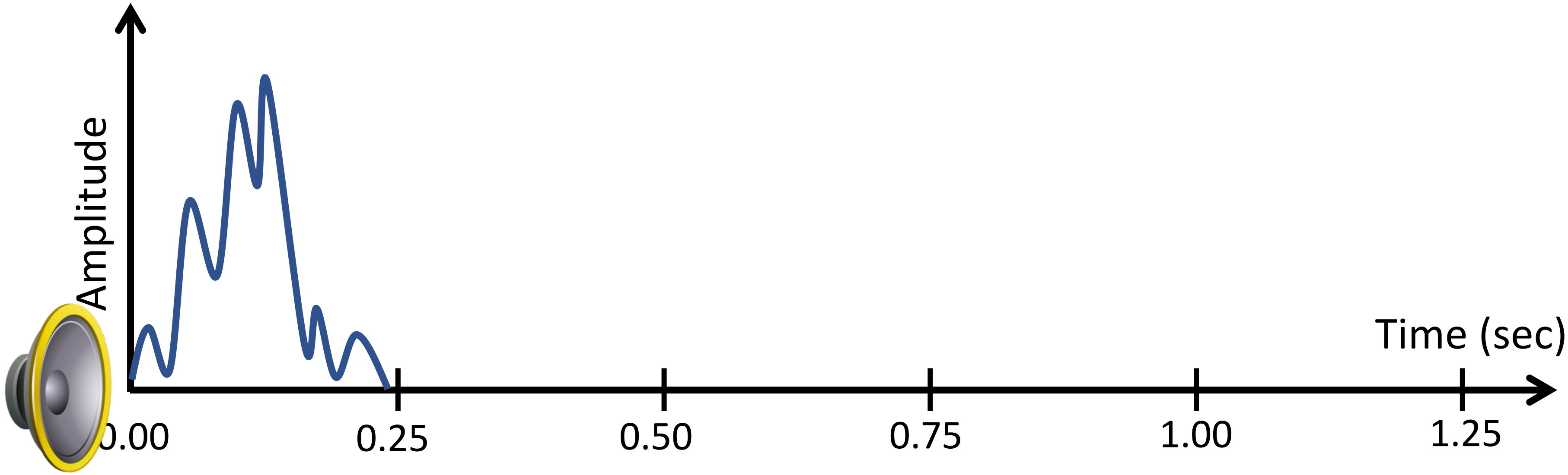
Multipath



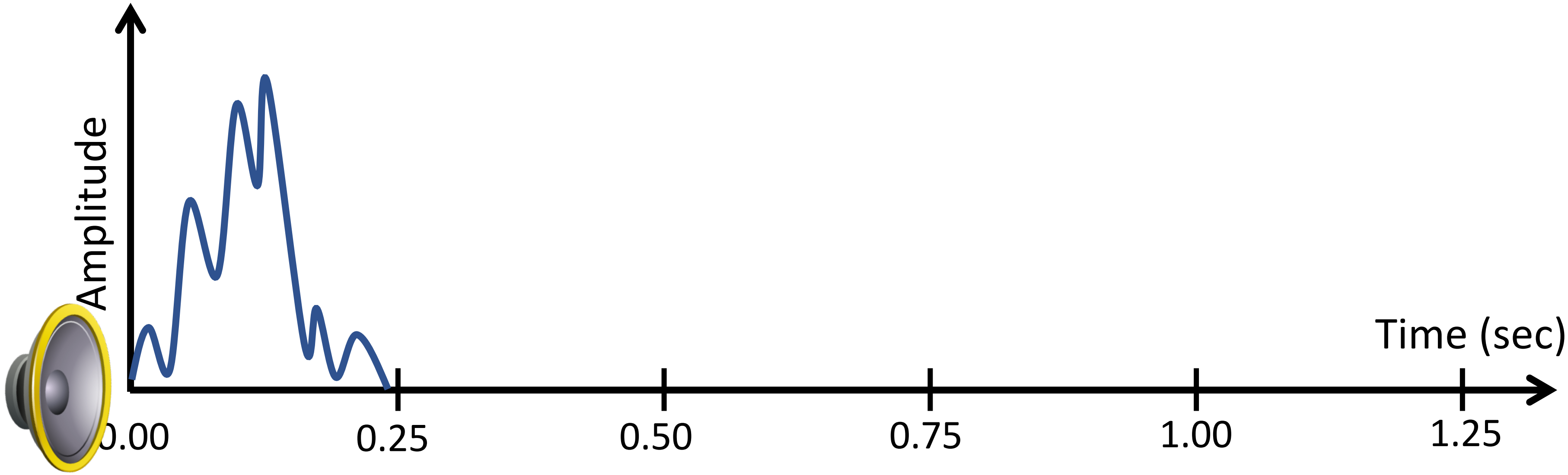
Multipath

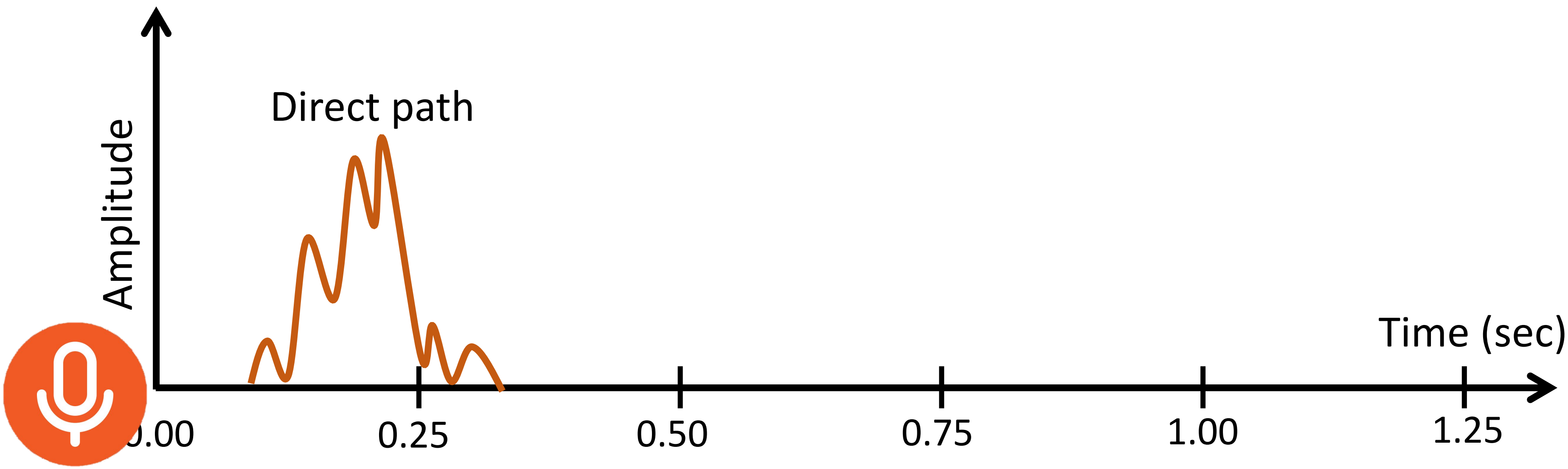


Multipath: Convolution

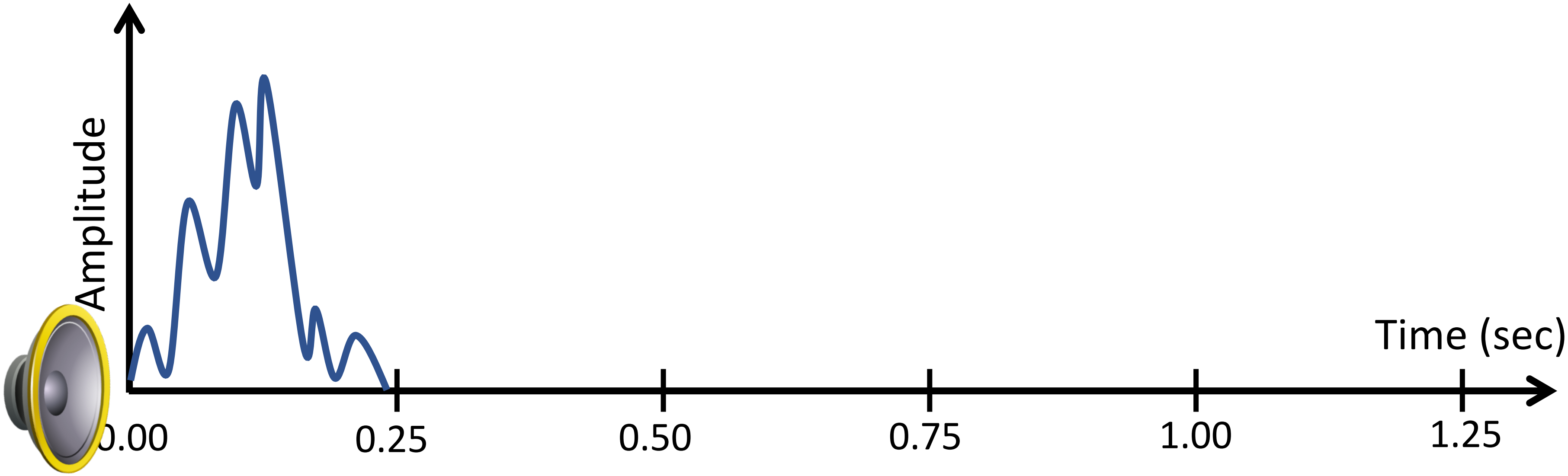


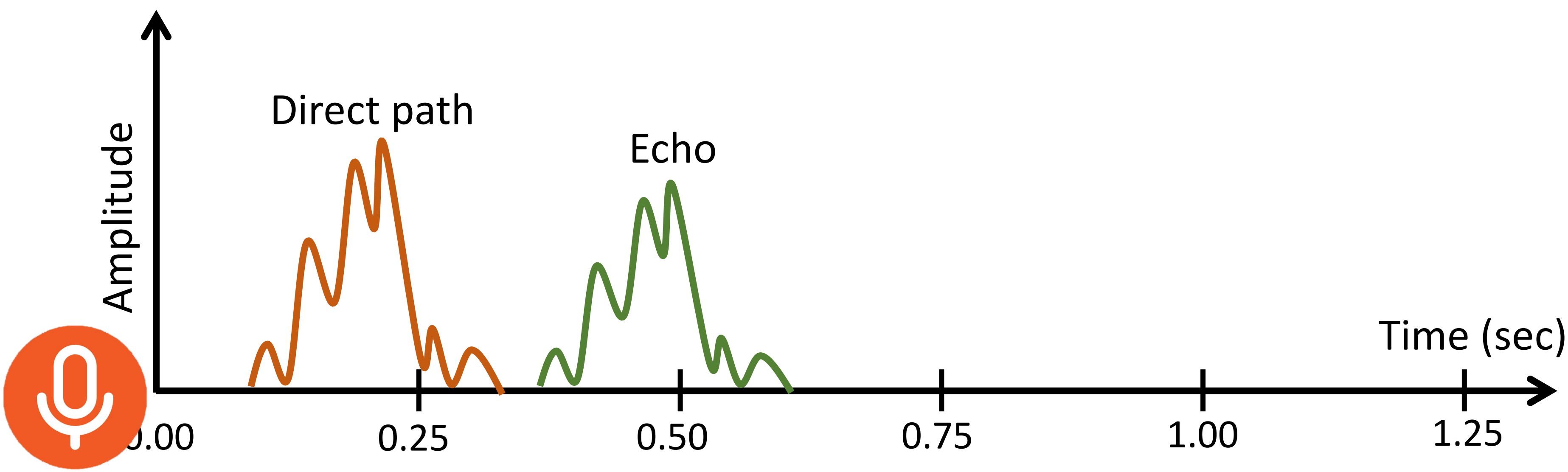
Multipath: Convolution



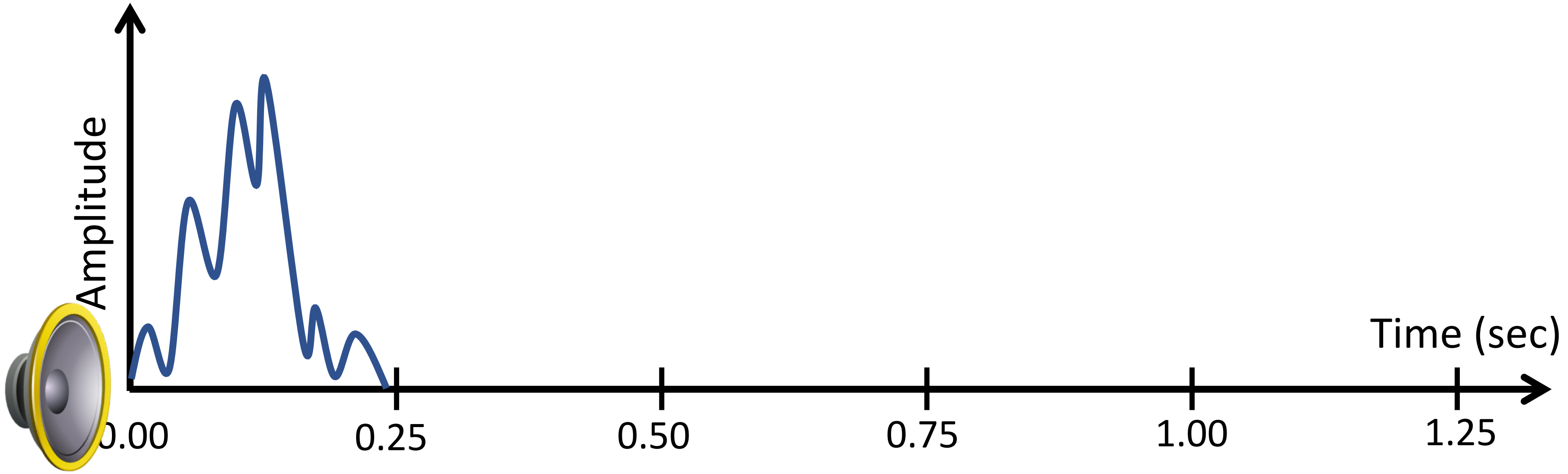


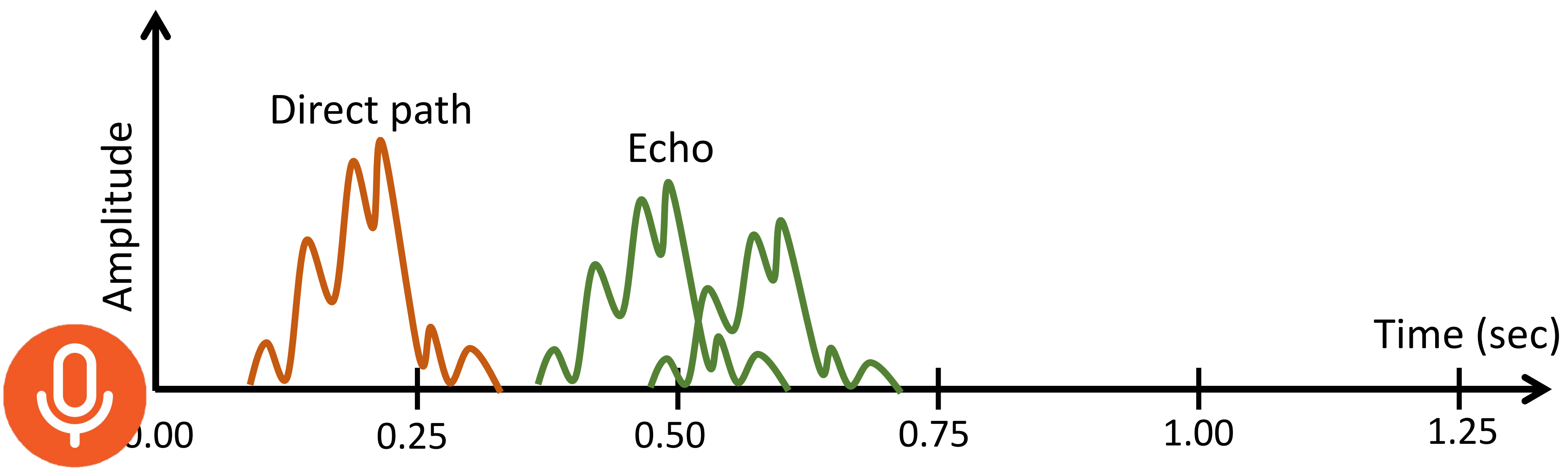
Multipath: Convolution



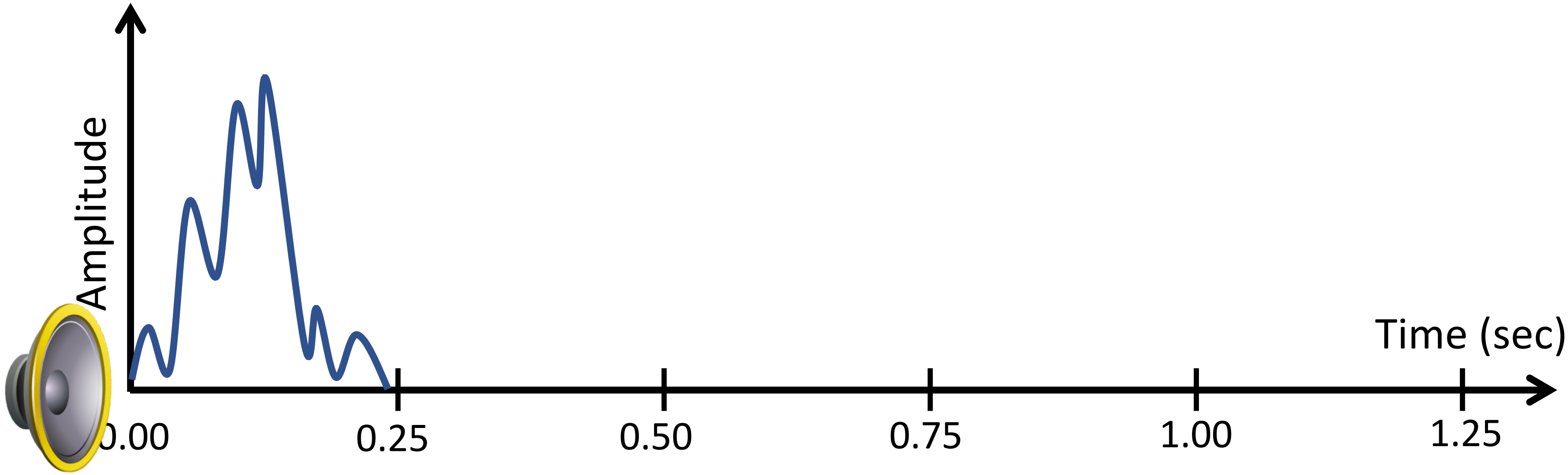


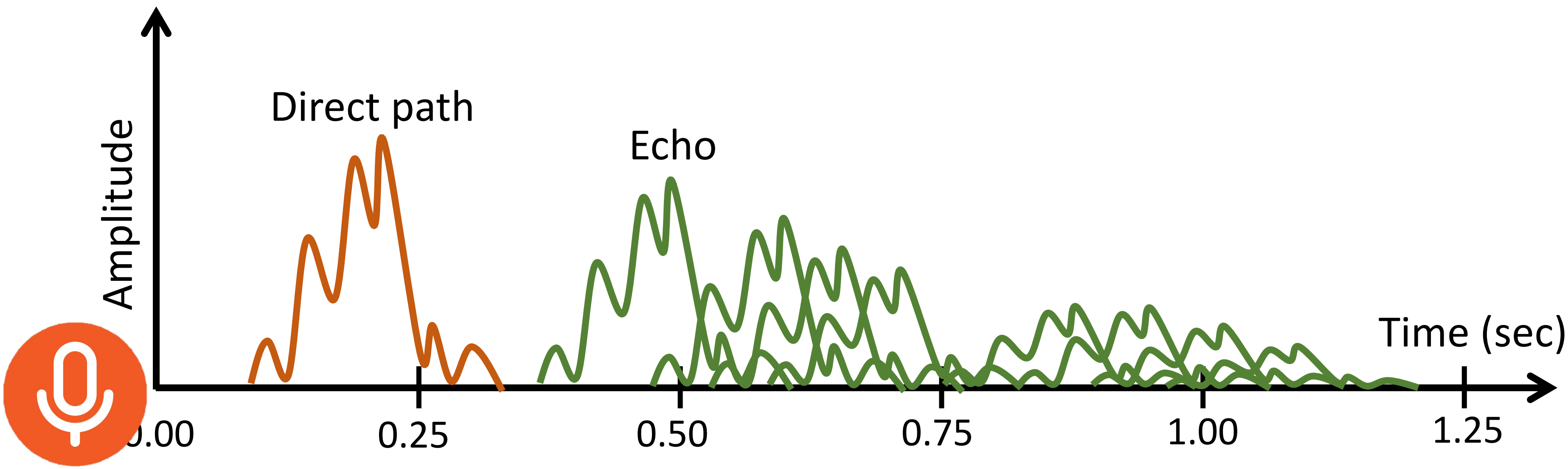
Multipath: Convolution



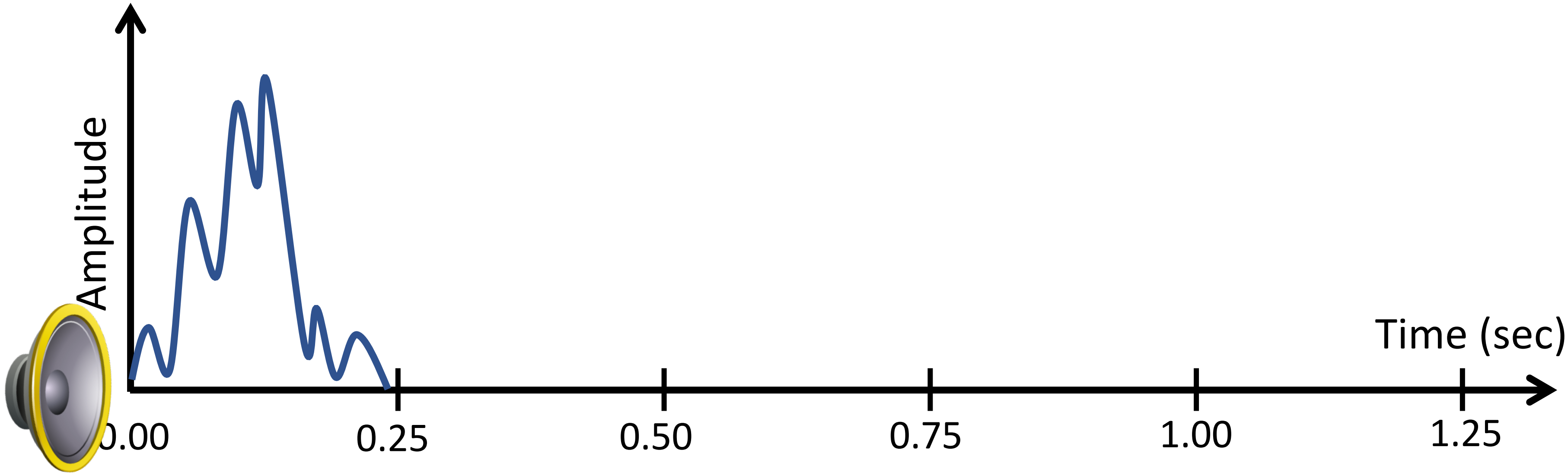


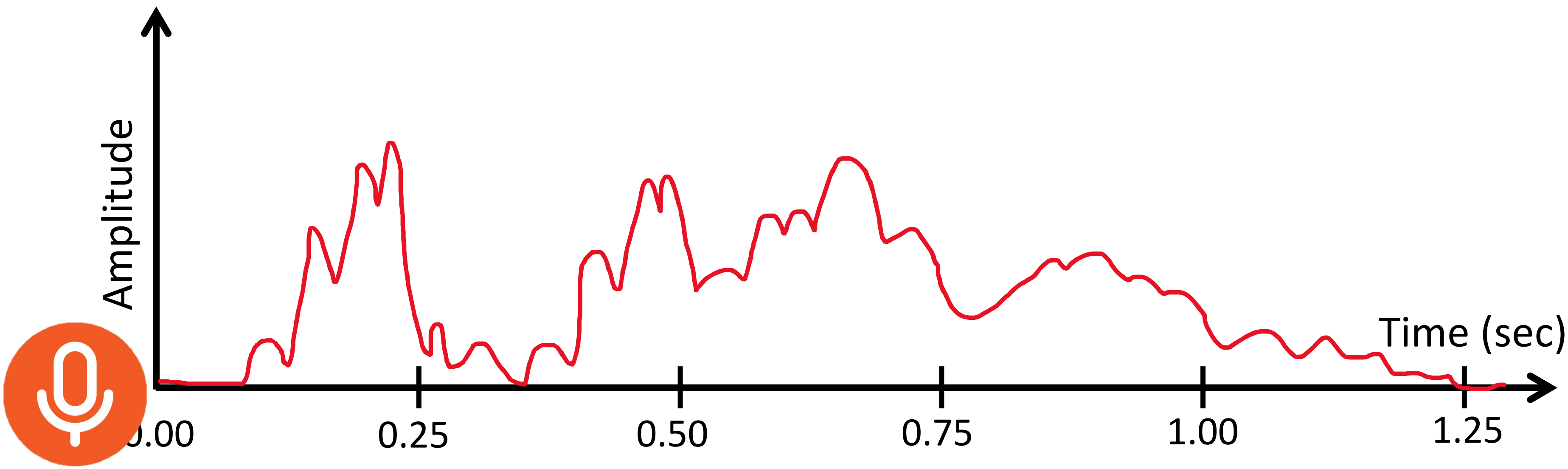
Multipath: Convolution



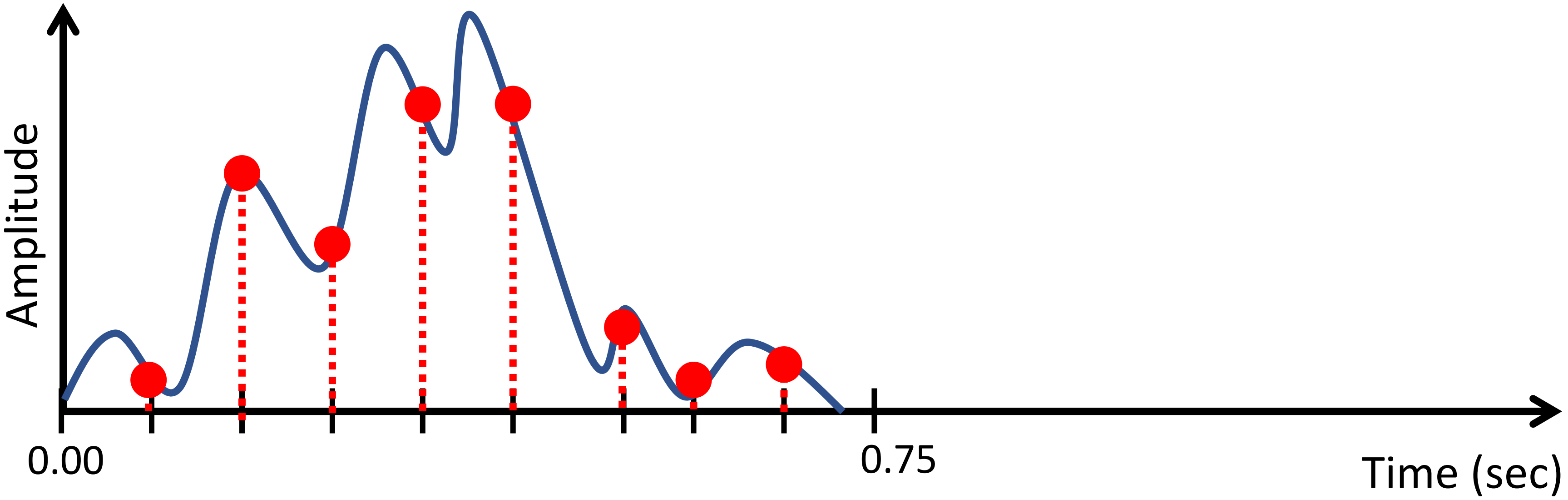


Multipath: Convolution

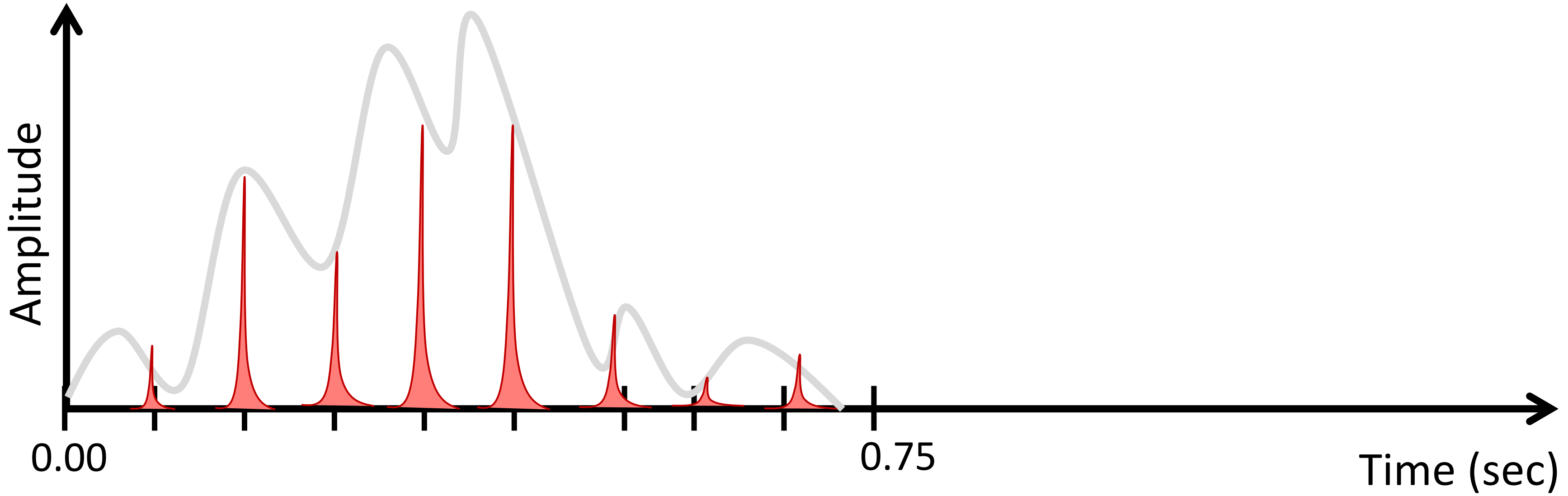




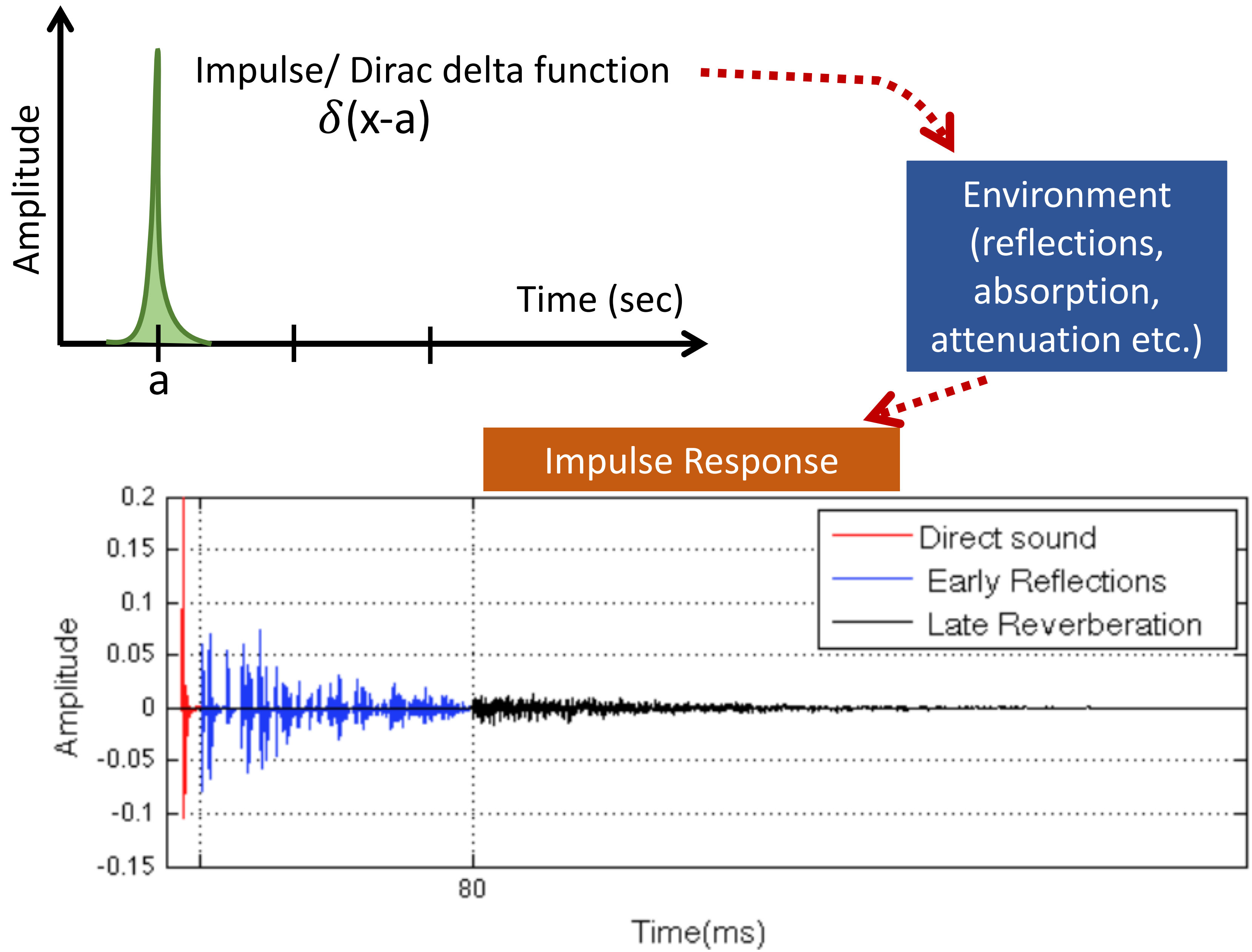
Multipath: Convolution



Multipath: Convolution

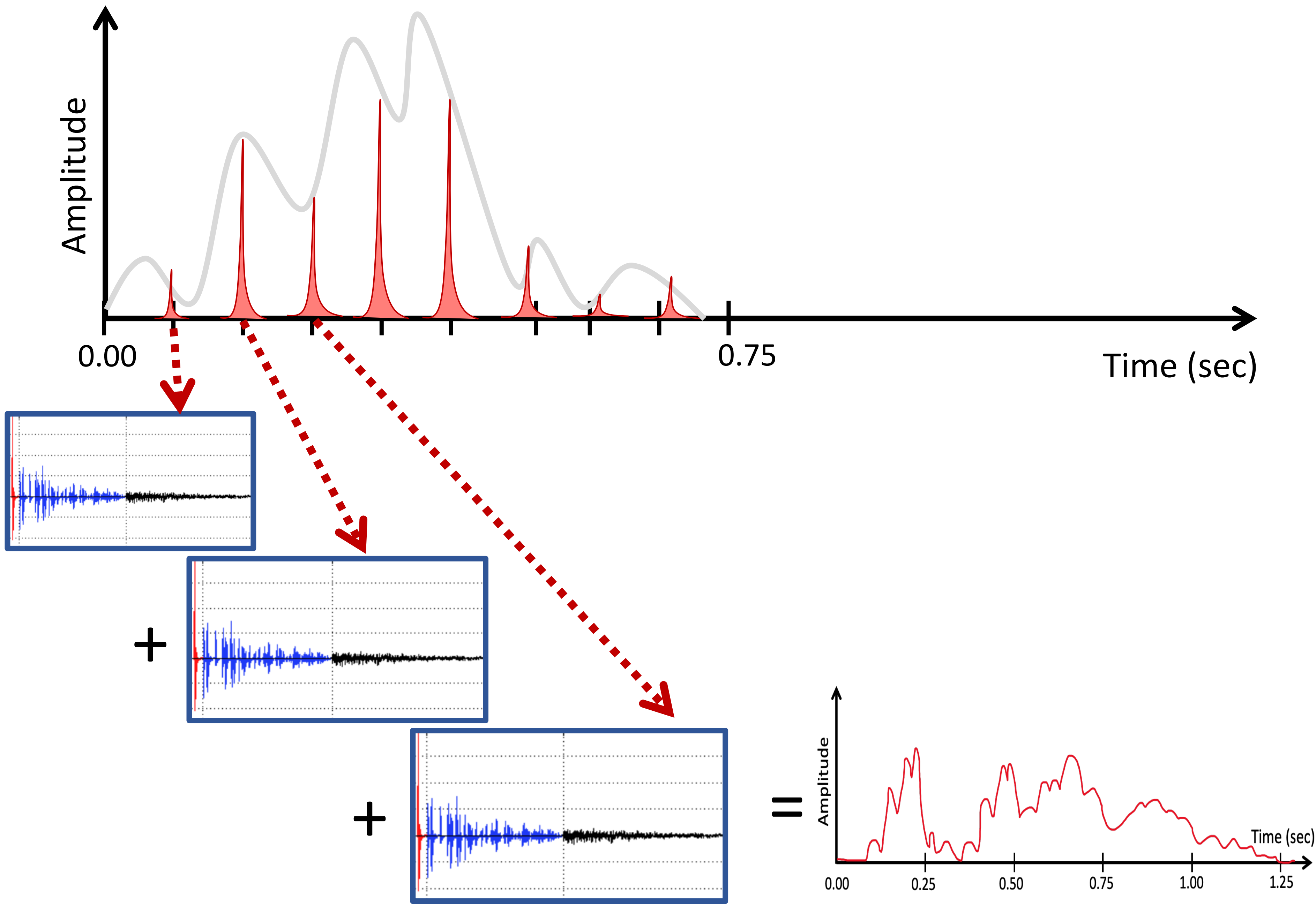


Impulse response

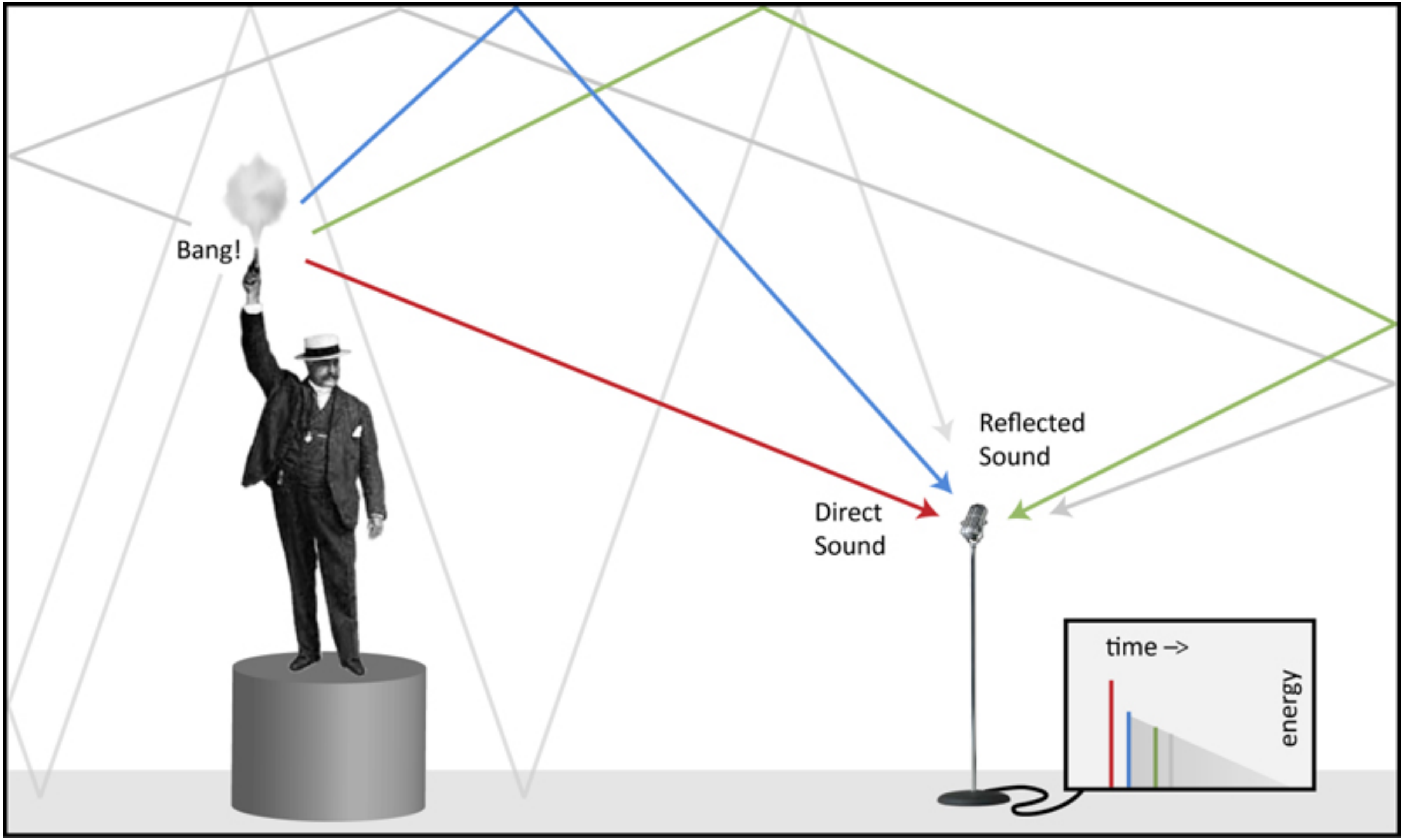


Linear Time Invariant (LTI) System

Linear Time Invariant (LTI) System

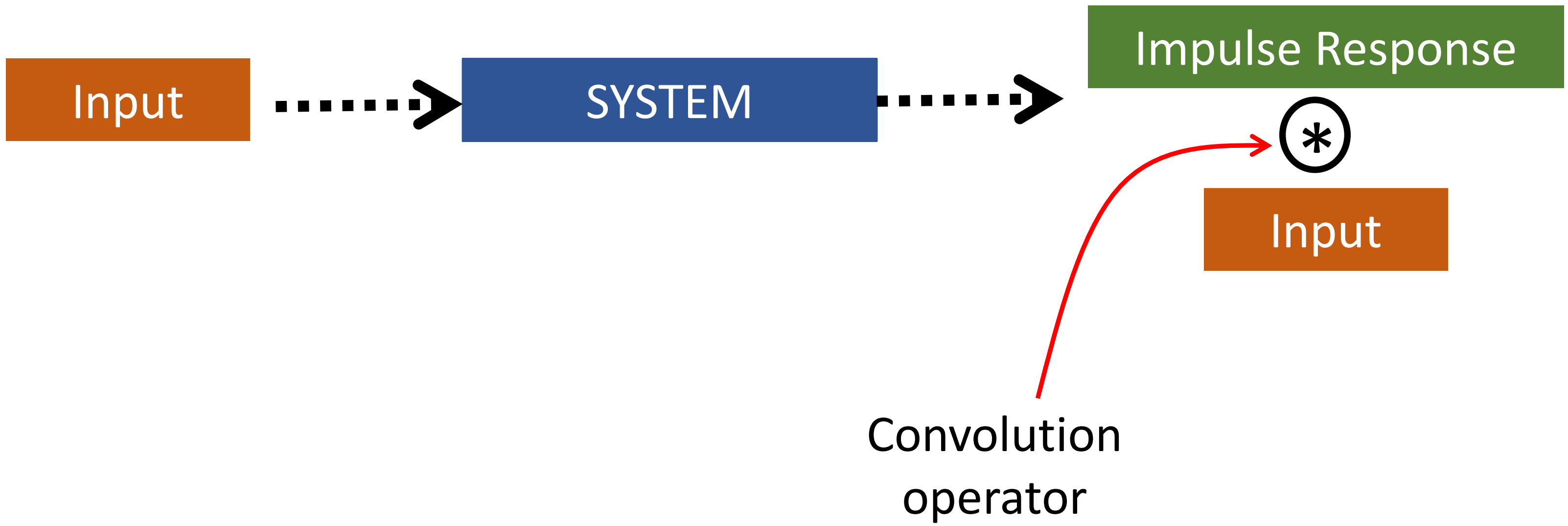


Impulse response



Impulse response

Impulse response



Convolution operator

Convolution operator: Definition

$$y(i) = x(i) * h(i)$$

Convolution operator: Properties

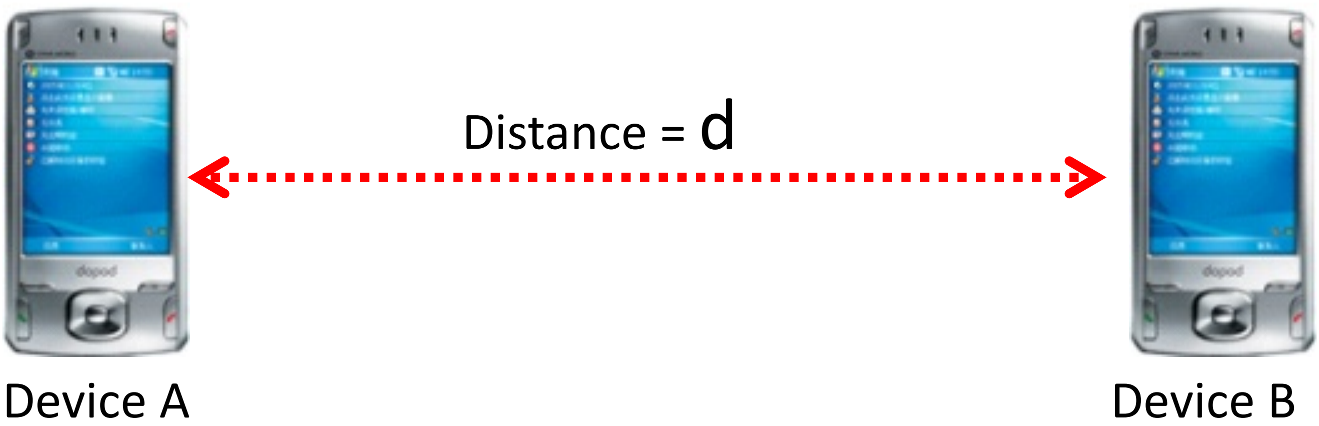
Commutative: $x(i) * h(i) = h(i) * x(i)$

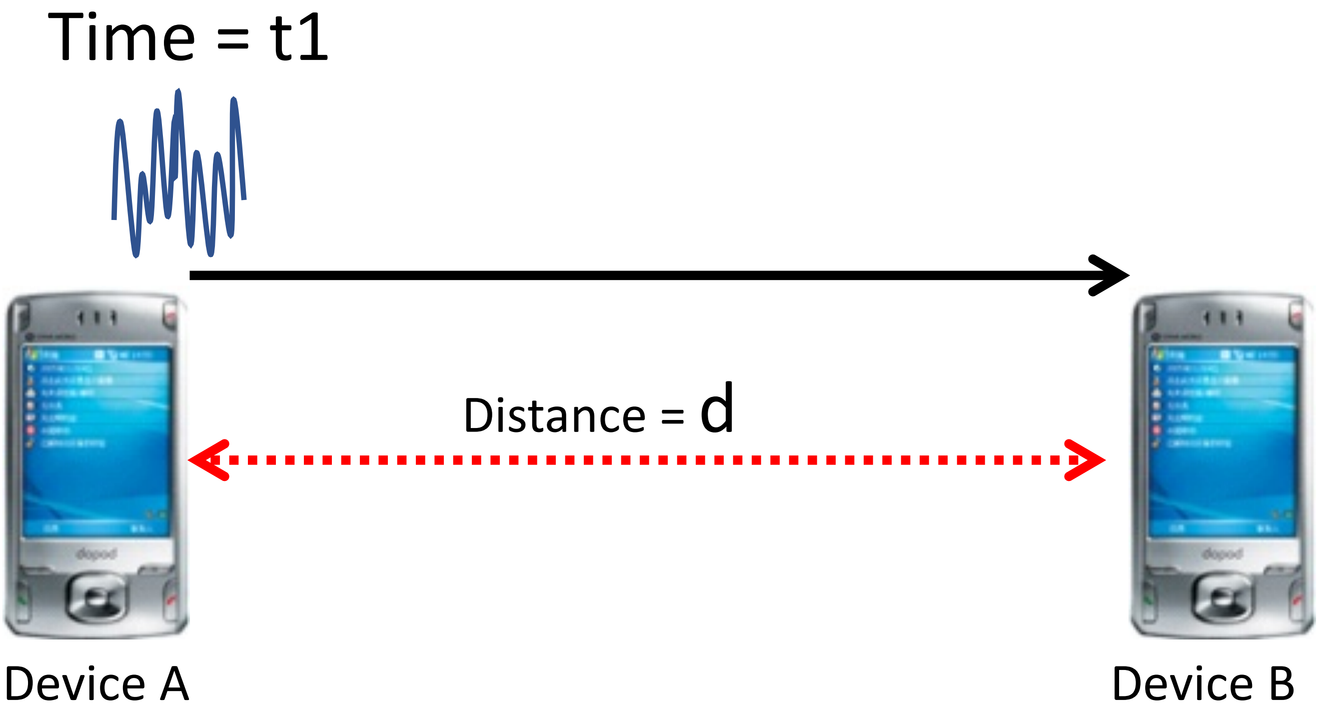
Convolution operator

Distributive : $x(i) * \{h_1(i) + h_2(i)\} = x(i) * h_1(i) + x(i) * h_2(i)$

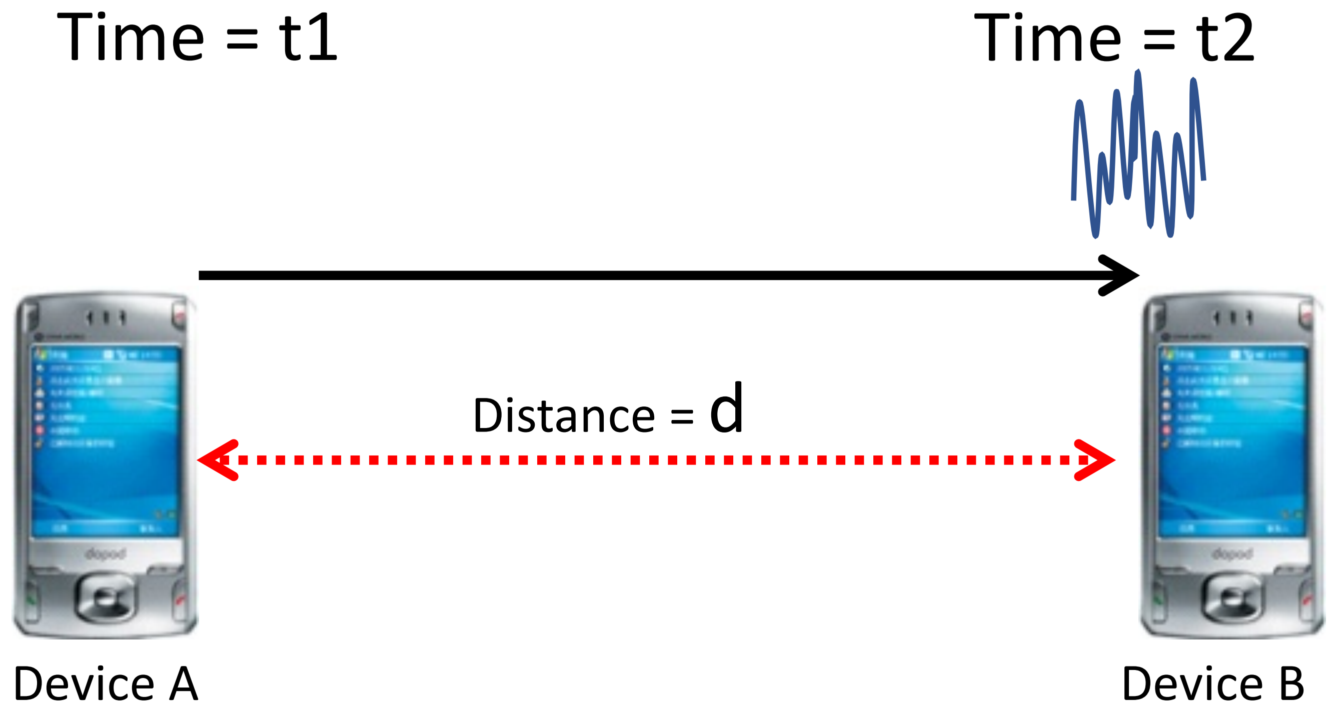
A simple acoustic ranging technique

BeepBeep – SenSys 2007

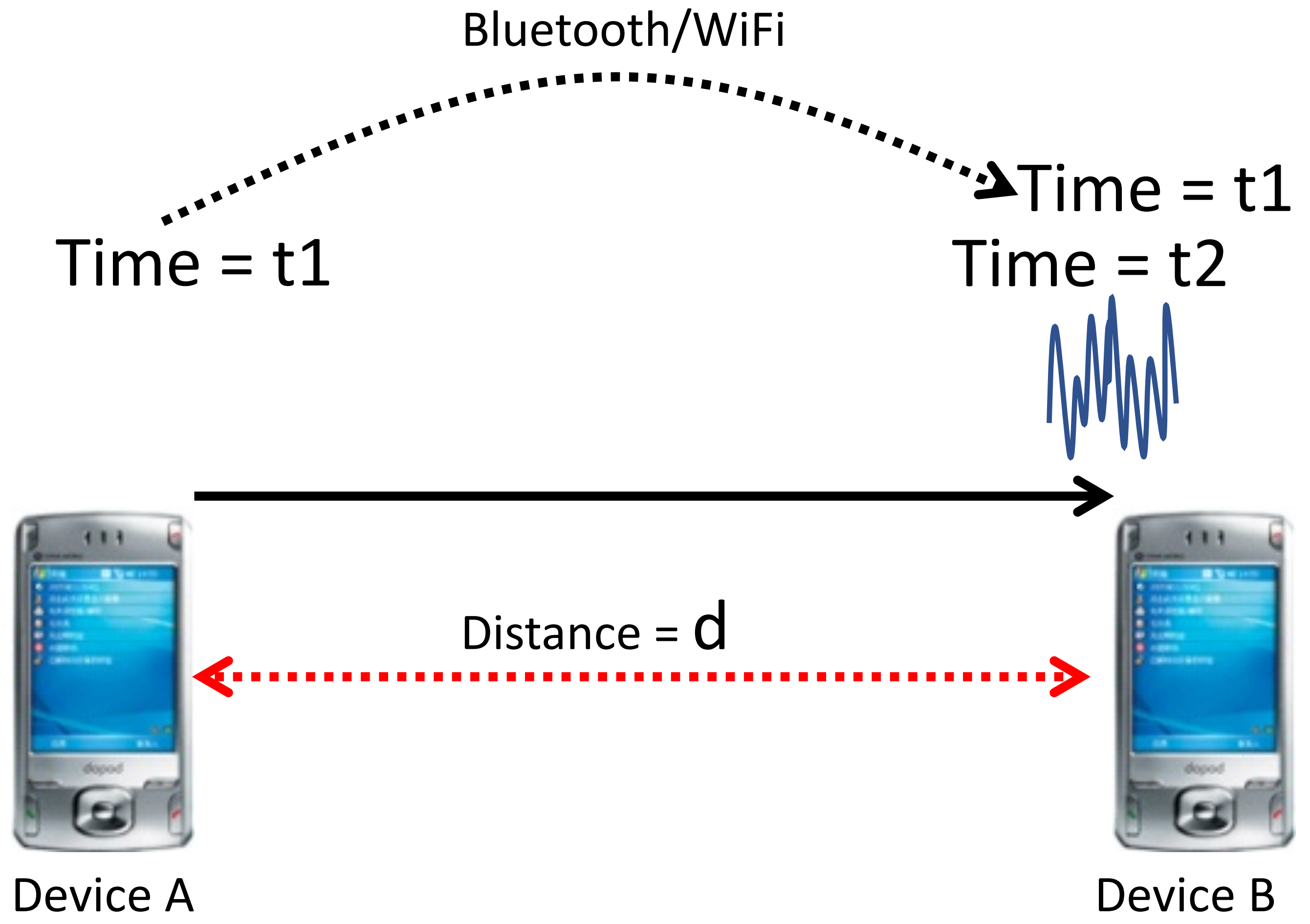

Distance = d
Device A
Device B

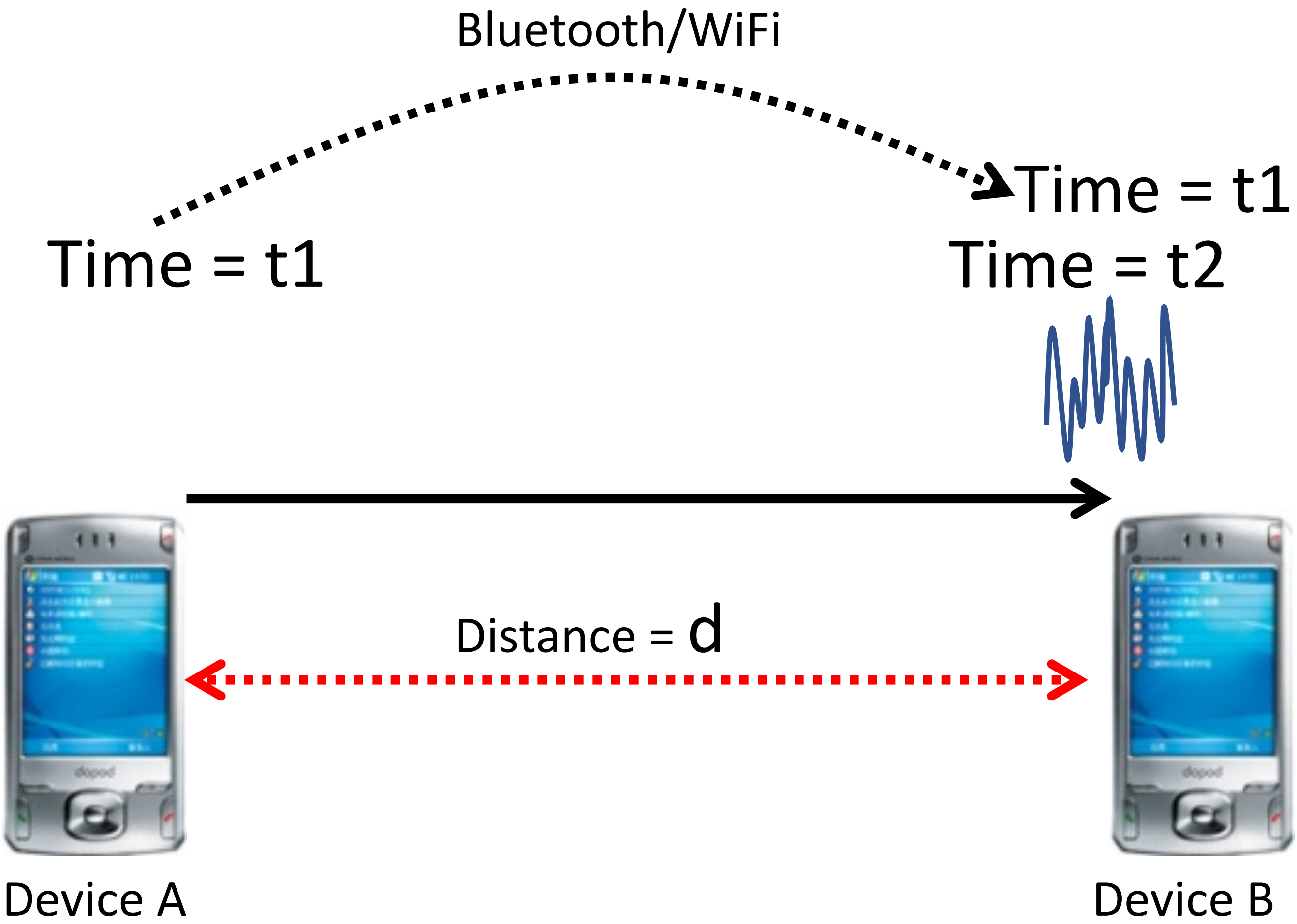


Time = t1
Distance = d
Device A
Device B



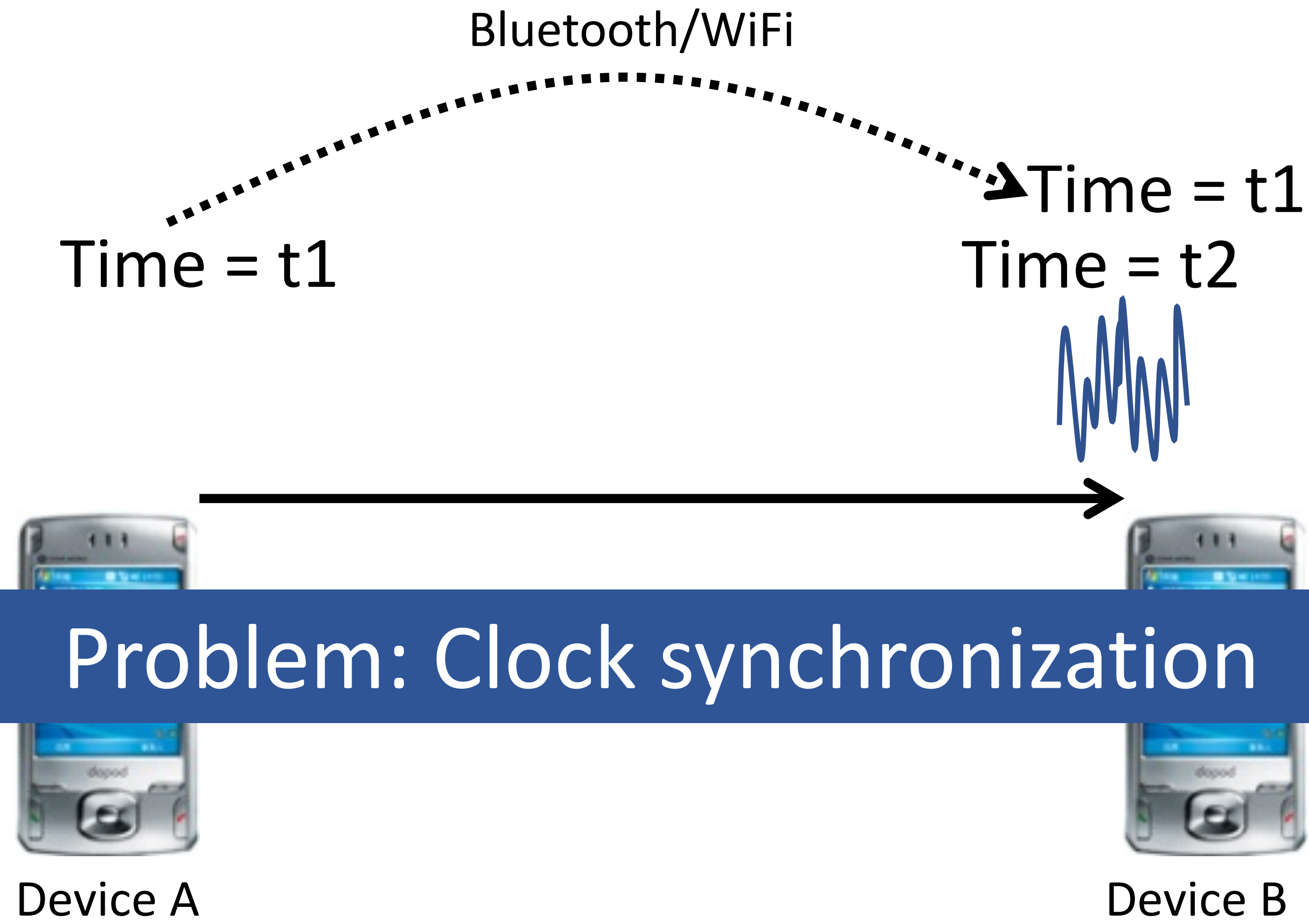
Time = t1
Time = t2
Distance = d
Device A
Device B


Bluetooth/WiFi
Time = t1
Time = t1
Time = t2
Distance = d
Device A
Device B


Bluetooth/WiFi
Time = t1
Time = t1
Time = t2
Distance = d
Device A
Device B

$$d = C \cdot (t2 - t1)$$

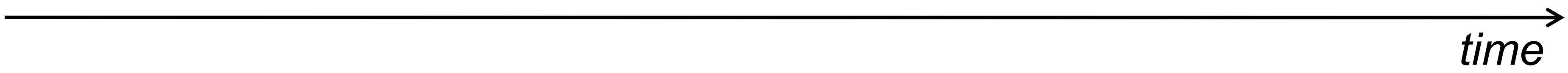
Problem: Clock synchronization



$$d = C \cdot (t2 - t1)$$

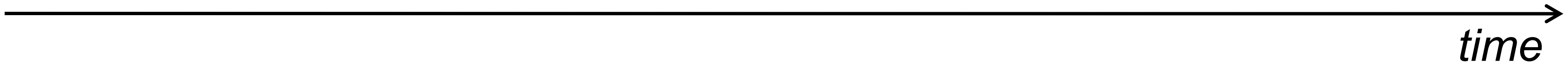
The root cause of inaccuracy – three uncertainties

- Clock synchronization uncertainty



The root cause of inaccuracy – three uncertainties

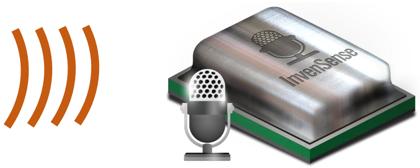
- Clock synchronization uncertainty
- Sending uncertainty



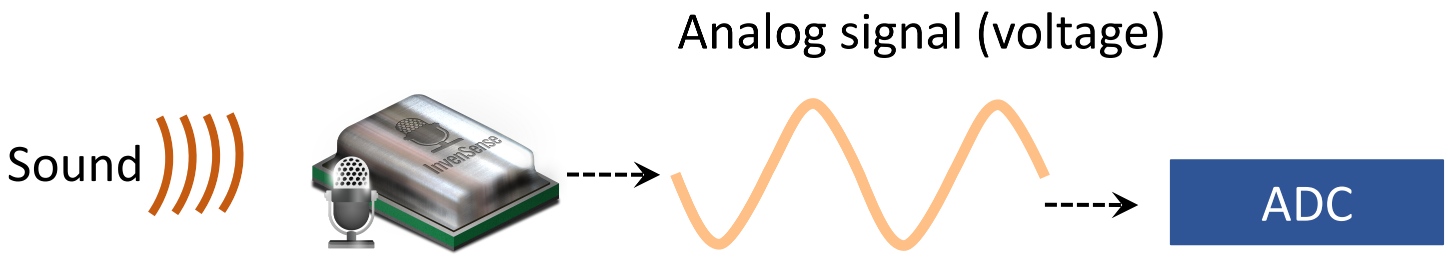
Sound production and recording

Sound recording with microphone

Sound

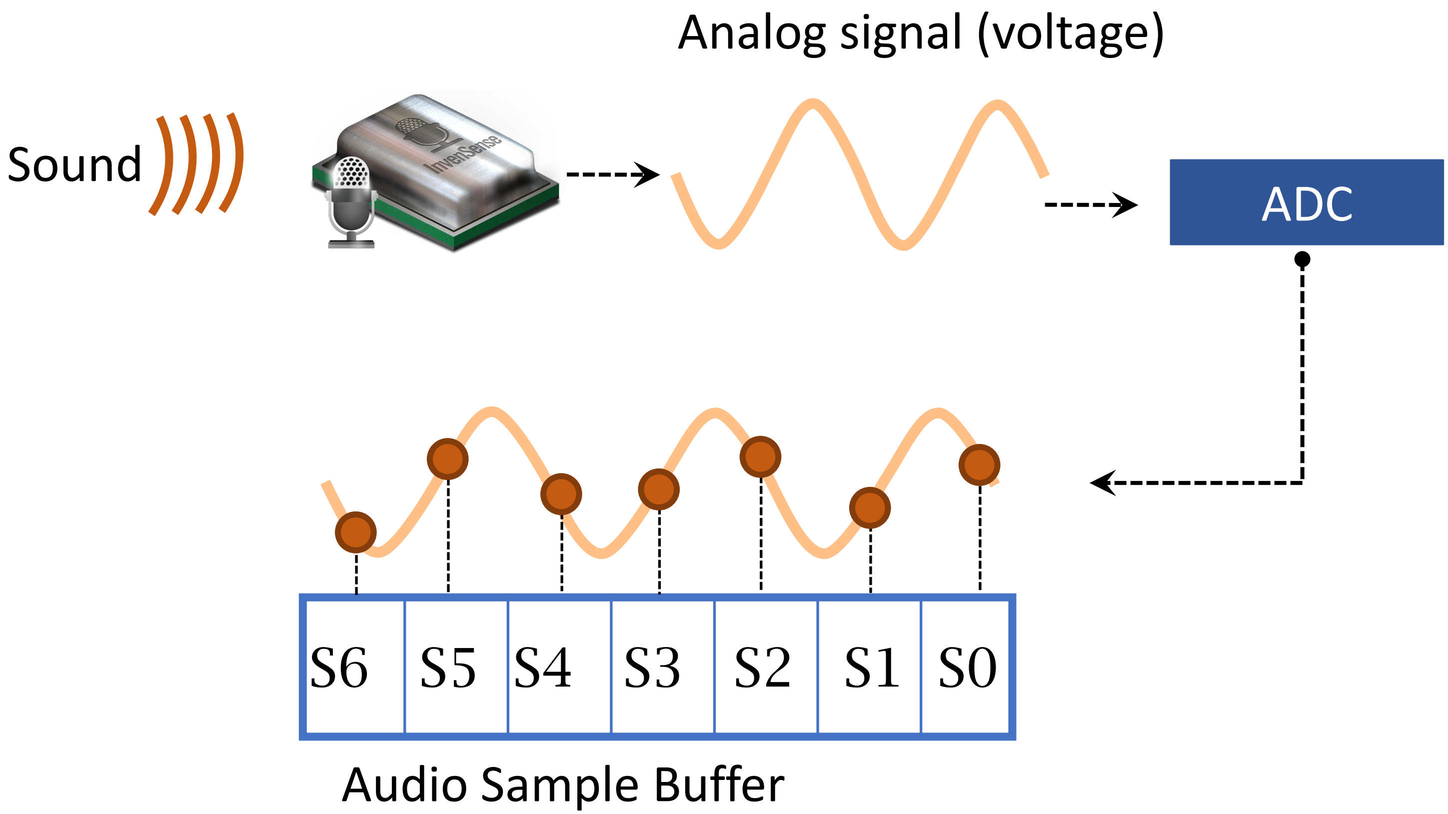


Sound recording with microphone



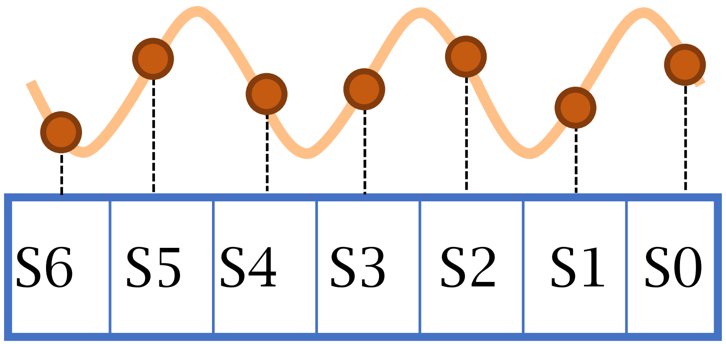
ADC = Analog-to-Digital Converter

Sound recording with microphone



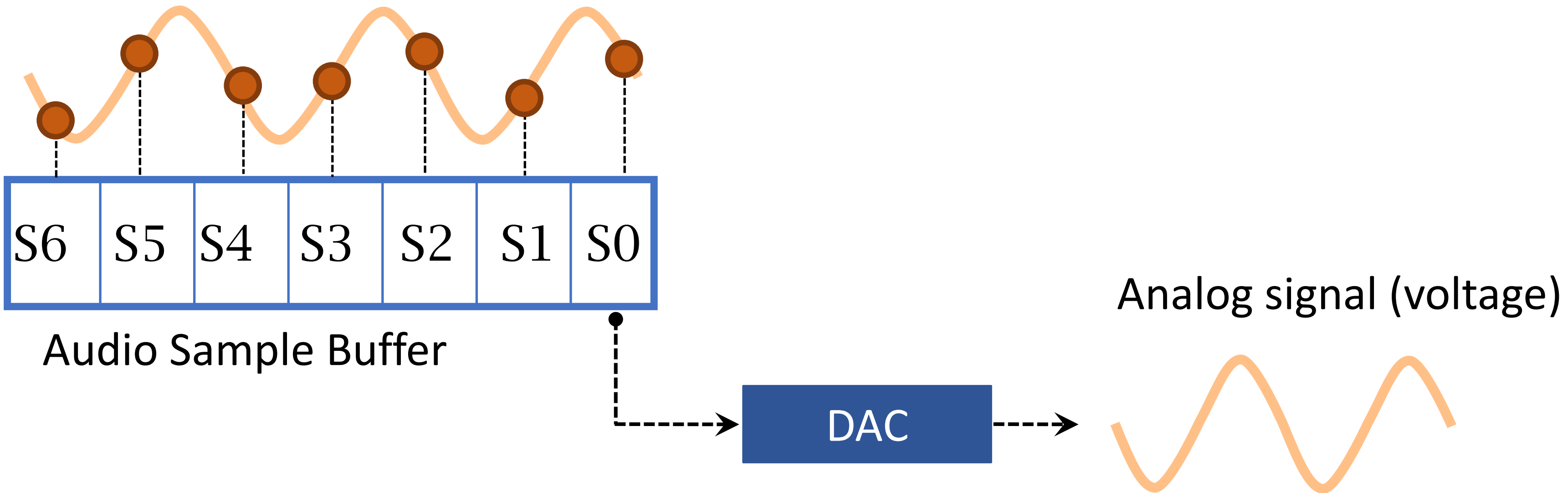
ADC = Analog-to-Digital Converter

Sound production with speaker



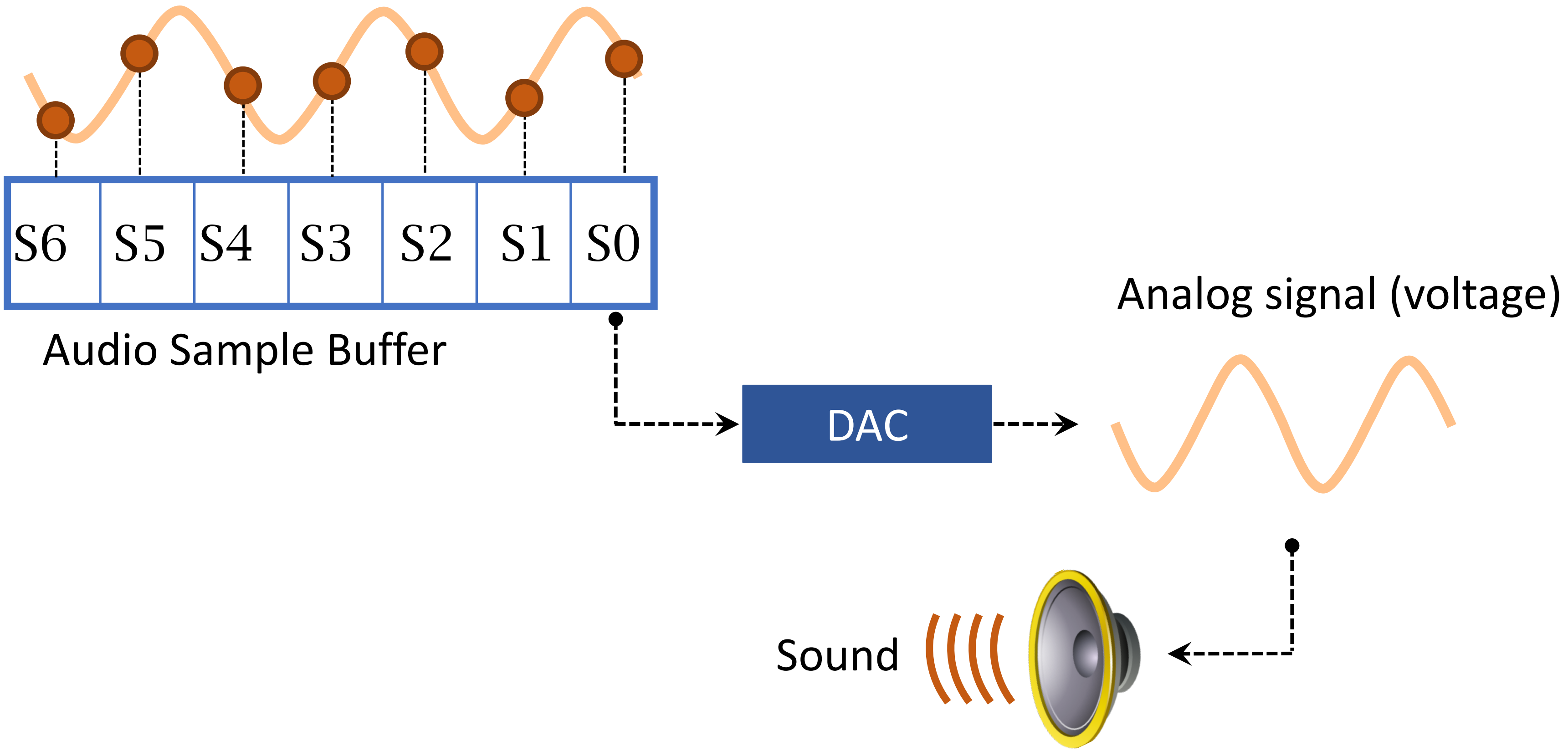
Audio Sample Buffer

Sound production with speaker



DAC = Digital-to-Analog Converter

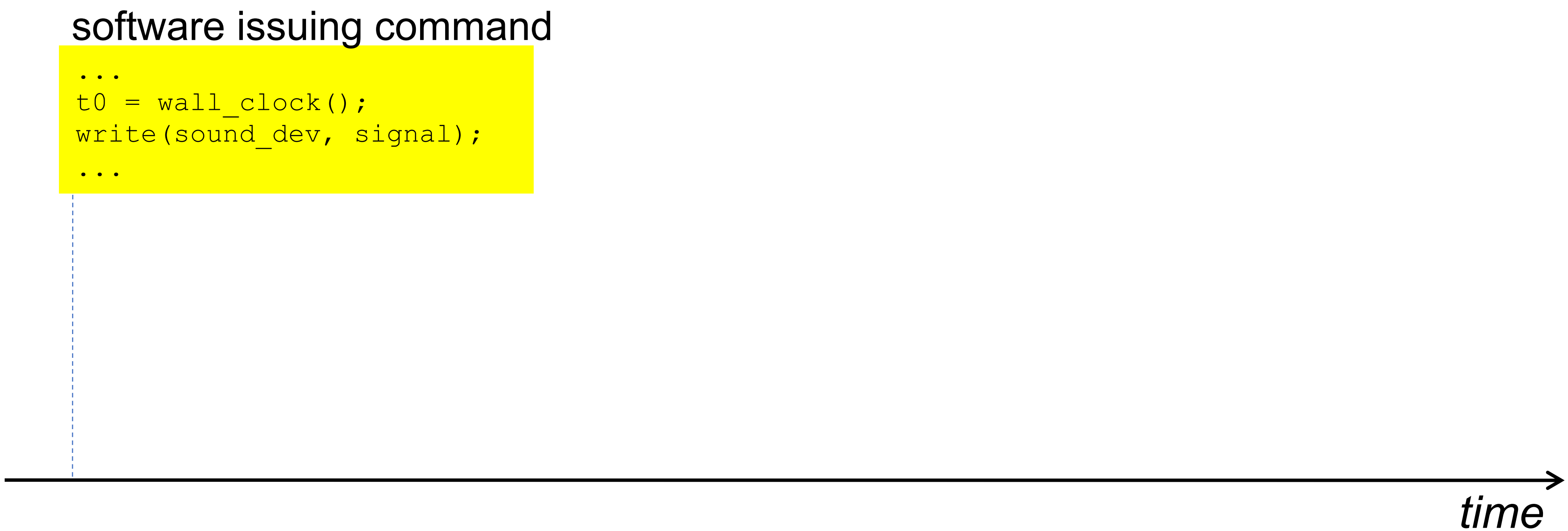
Sound production with speaker



DAC = Digital-to-Analog Converter

The root cause of inaccuracy – three uncertainties

- Clock synchronization uncertainty
- Sending uncertainty

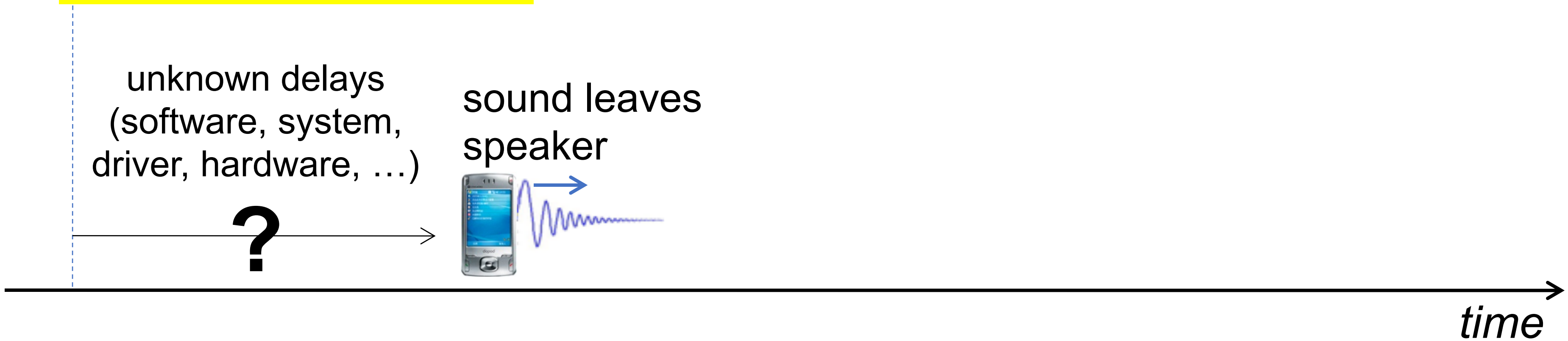


The root cause of inaccuracy – three uncertainties

- Clock synchronization uncertainty
- Sending uncertainty

software issuing command

```
...
t0 = wall_clock();
write(sound_dev, signal);
...
```

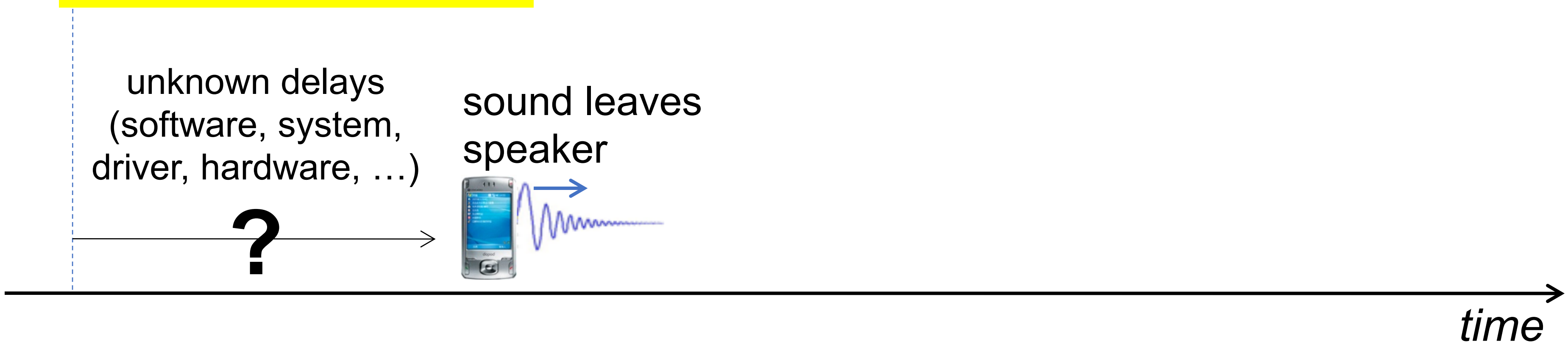


The root cause of inaccuracy – three uncertainties

- Clock synchronization uncertainty
- Sending uncertainty
- Receiving uncertainty

software issuing command

```
...
t0 = wall_clock();
write(sound_dev, signal);
...
```

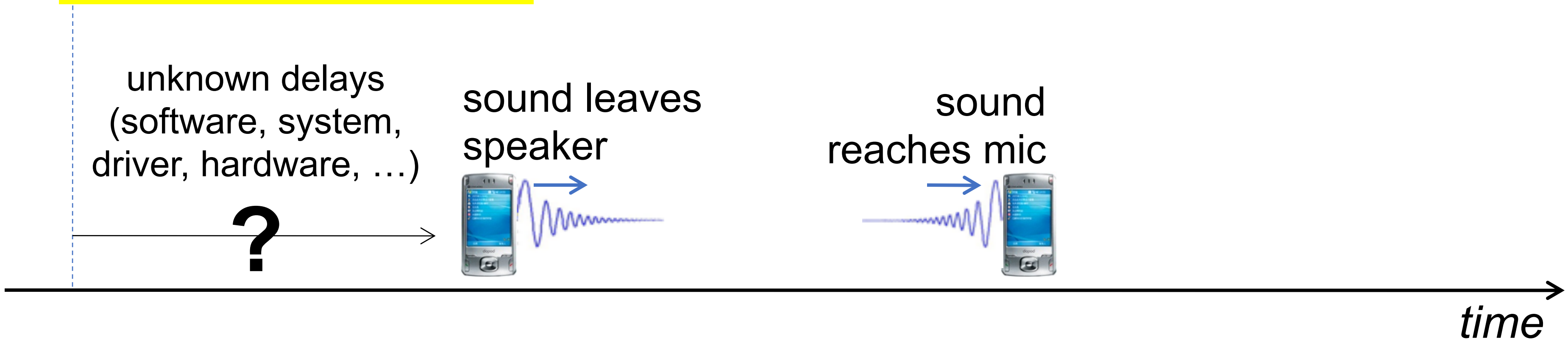


The root cause of inaccuracy – three uncertainties

- Clock synchronization uncertainty
- Sending uncertainty
- Receiving uncertainty

software issuing command

```
...
t0 = wall_clock();
write(sound_dev, signal);
...
```

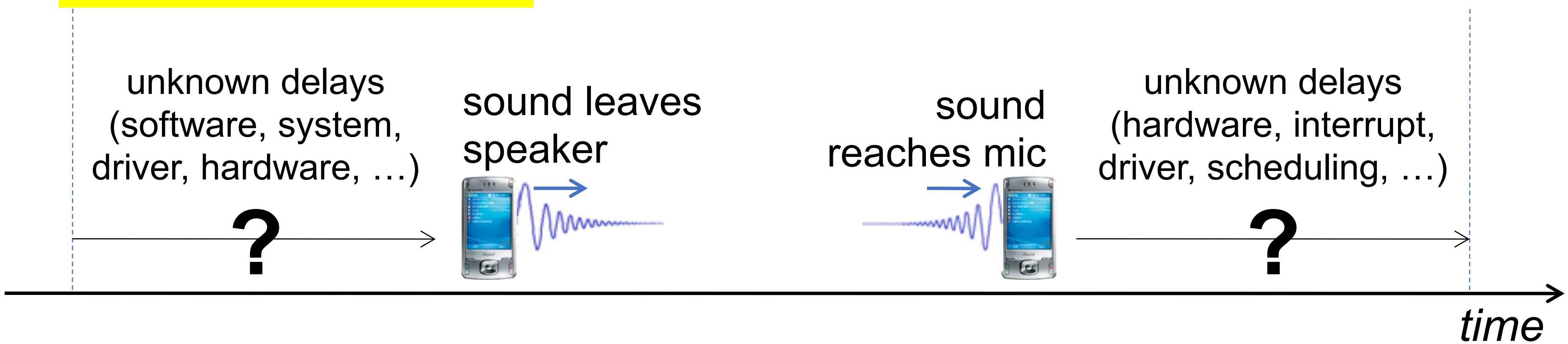


The root cause of inaccuracy – three uncertainties

- Clock synchronization uncertainty
- Sending uncertainty
- Receiving uncertainty

software issuing command

```
...
t0 = wall_clock();
write(sound_dev, signal);
...
```



The root cause of inaccuracy – three uncertainties

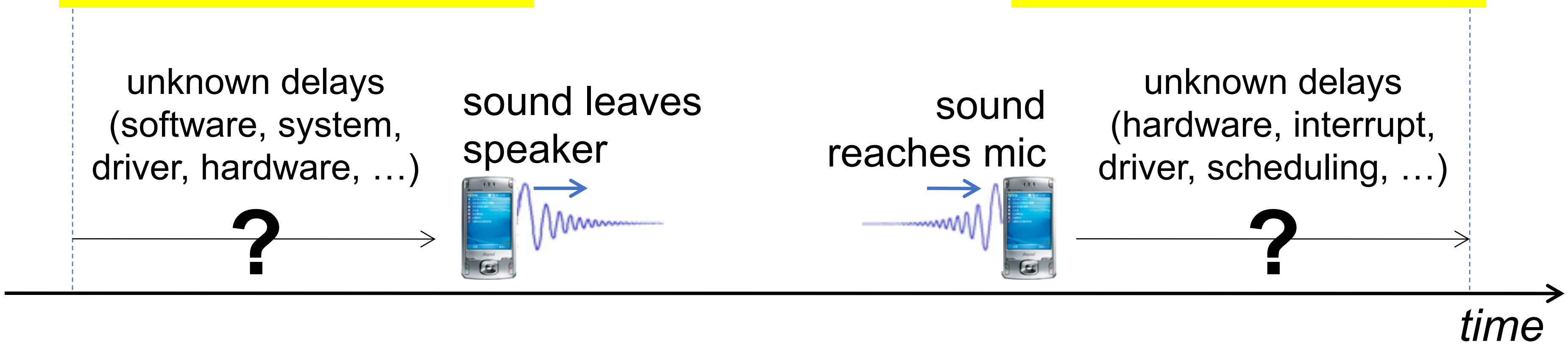
- Clock synchronization uncertainty
- Sending uncertainty
- Receiving uncertainty

software issuing command

```
...
t0 = wall_clock();
write(sound_dev, signal);
...
```

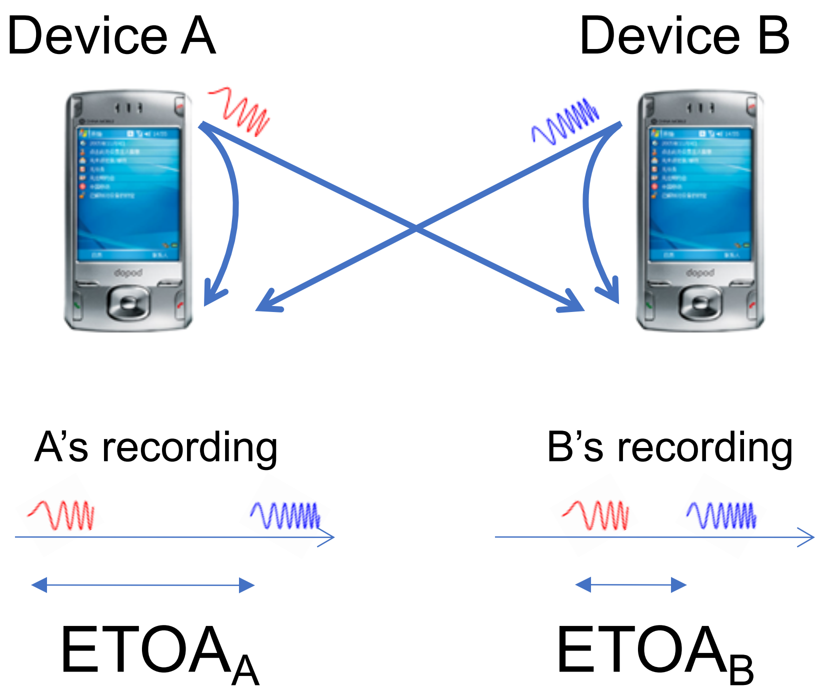
software aware of arrival

```
...
read(sound_dev, signal);
t1 = wall_clock();
...
```

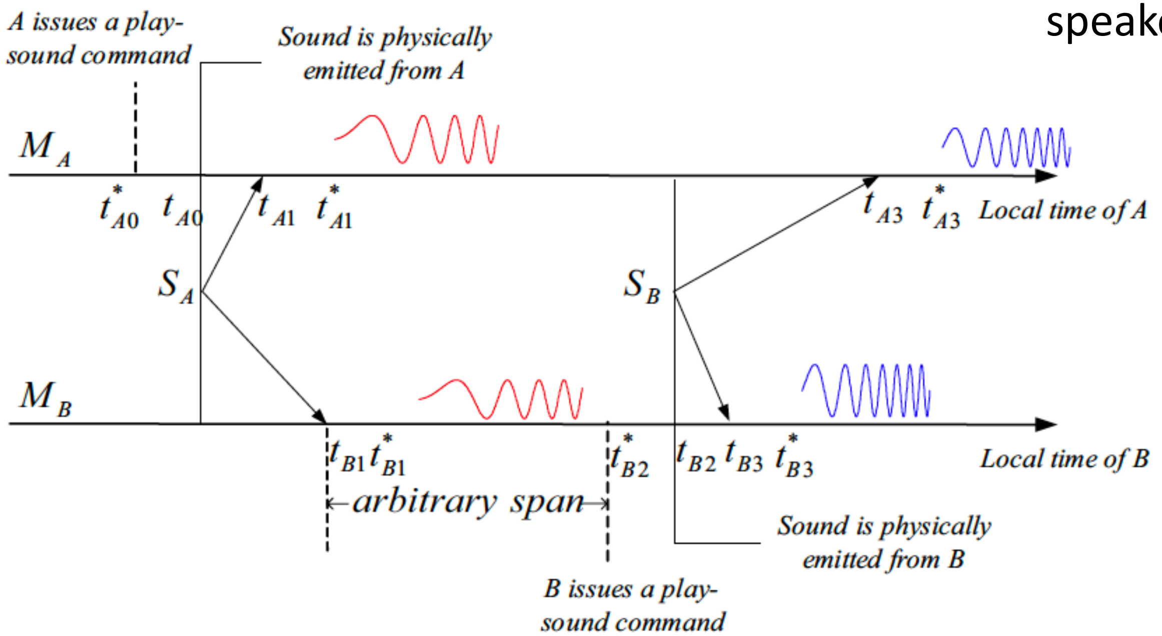


Beepbeep's basic procedure

1. Device A emits a beep while both recording
2. Device B emits another beep while both continue recording
3. Both devices detect TOA of the two beeps and obtain respective ETOAs
4. Exchange ETOAs and calculate the distance



Dx,y is distance between x's speaker to y's microphone



$$
\begin{aligned}
d_{A,A} &= c \cdot (t_{A1} - t_{A0}) \\
d_{A,B} &= c \cdot (t_{B1} - t_{A0}) \\
d_{B,A} &= c \cdot (t_{A3} - t_{B2}) \\
d_{B,B} &= c \cdot (t_{B3} - t_{B2})
\end{aligned}
$$

$$
\begin{aligned}
D &= \frac{1}{2} \cdot (d_{A,B} + d_{B,A}) \\
&= \frac{c}{2} \cdot ((t_{B1} - t_{A0}) + (t_{A3} - t_{B2})) \\
&= \frac{c}{2} \cdot (t_{B1} - t_{B2} + t_{B3} - t_{B3} + t_{A3} - t_{A0} + t_{A1} - t_{A1}) \\
&= \frac{c}{2} \cdot ((t_{A3} - t_{A1}) - (t_{B3} - t_{B1}) + \\
&\quad (t_{B3} - t_{B2}) + (t_{A1} - t_{A0})) \\
&= \frac{c}{2} \cdot ((t_{A3} - t_{A1}) - (t_{B3} - t_{B1})) + d_{B,B} + d_{A,A}
\end{aligned}
$$