CSE 562: Mobile Systems & Applications

Quals Course – Systems Area
Shyam Gollakota
The SIGMOBILE Outstanding Contribution Award is given for significant and lasting contributions to the research on mobile computing and communications and wireless networking.

2020 Recipient

Marty Cooper

For seminal contributions to the conception, practice and adoption of portable telephony.
Goal of this course

• Have an understanding of state of the art mobile systems research

• Explore applications that are capable with mobile devices
Course material

1. Signal processing fundamentals
2. Acoustic device and device-free tracking
3. Physiological sensing using phones and speakers
4. IMW tracking and GPS localization
5. Wi-Fi localization and sensing
6. Designing and building IoT device hardware
Course material

7. Backscatter systems

8. Mobile privacy and security

9. Robotics mobile systems
Grading

3 hands-on assignments (20+20+20% in all)
- One every two weeks
- Requires programming phones, microcontroller, etc.

Class presentation of one paper (10%)

Final research project (30%)
- Proposal due on May 1
- 2-3 person project
Signal processing basics

(Slides by Nirupam Roy)
Model for a signal (frequency, amplitude, and phase)
Model for a signal (frequency, amplitude, and phase)

\[ A \cdot \sin(\theta) \]

\[ A \cdot \cos(\theta) \]
Model for a signal (frequency, amplitude, and phase)

$f$ cycles per second

$2\pi$ angles per cycle

$\theta = 2\pi ft$
Frequency, Amplitude, and Phase

\[ A \cdot \sin(\theta) = A \cdot \sin(2\pi ft) \]

\[ A \cdot \sin(2\pi ft + \phi) \quad \text{-- with initial/additional phase } \phi \]
Frequency, Amplitude, and Phase

Frequency: 2Hz

Amplitude

Frequency: 4Hz

Amplitude
Frequencies of an arbitrary signal
The concept of the Fourier series
Time Domain and Frequency Domain

\[ \sin(2\pi ft) \]

Approx. square wave

\[ \frac{1}{3} \sin(2\pi \cdot 3ft) \]

\[ \frac{1}{5} \sin(2\pi \cdot 5ft) \]

\[ \frac{1}{7} \sin(2\pi \cdot 7ft) \]
Time Domain and Frequency Domain

Time domain view

Frequency domain view
Analogy: Food coloring chart

Basis for food colors

- RED
- YELLOW
- GREEN
- BLUE
**Analogy: Food coloring chart**

**Basis for food colors**

<table>
<thead>
<tr>
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<th>RED</th>
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<td><strong>ORANGE</strong></td>
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## Analogy: Food coloring chart

**Basis for food colors**

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### Analogy: Food coloring chart

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<tr>
<td>PURPLE</td>
<td>3</td>
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<td>1</td>
</tr>
<tr>
<td>DARK GREEN</td>
<td>1</td>
<td>1</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>LIME GREEN</td>
<td></td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>AQUA</td>
<td></td>
<td></td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>FLESH</td>
<td>2</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BROWN</td>
<td>6</td>
<td>6</td>
<td></td>
<td>4</td>
</tr>
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\[
\begin{align*}
    &= A_1 \sin(2\pi f_1 t) + B_1 \cos(2\pi f_1 t) \\
    &\quad+ A_2 \sin(2\pi f_2 t) + B_2 \cos(2\pi f_2 t) \\
    &\quad+ A_3 \sin(2\pi f_3 t) + B_3 \cos(2\pi f_3 t) \\
    &\quad+ \ldots
\end{align*}
\]
Time Domain and Frequency Domain

Amplitude

Time

FFT = Fast Fourier Transform
IFFT = Inverse Fast Fourier Transform
A 4 kHz frequency band starting at 2 kHz

What is bandwidth?
What is center frequency?
Spectrogram
Spectrogram

FFT of overlapping windows of samples (Spectrogram)
Spectrogram
Physical signal (voice)

Time varying voltage signal

Spectrogram plot on computer

FFT

A collection of numbers
Analog vs Digital World

Temperature

Thermo-couple

Analog

Digital

Voltage

Time
Analog

Physical signal (voice)

Time varying voltage signal

Digital

Spectrogram plot on computer

FFT

A collection of numbers

?
Analog signal (voice) to Digital signal

Physical signal (voice) → Time varying voltage signal → ADC → Spectrogram plot on computer → FFT → A collection of numbers

Analog-to-Digital Converter
Sampling theorem

\[
\begin{bmatrix}
0.34 & 0.22 & 0.09 & 0.21 & 0.30 & 0.08 & 0.09
\end{bmatrix}
\]
Sampling theorem

$T = \text{Sampling interval}$

$f_s = \frac{1}{T} = \text{Sample rate (or sampling frequency)}$

Clock
1-dimensional sampling
1-dimensional sampling

2-dimensional sampling
1-dimensional sampling

2-dimensional sampling

3-dimensional sampling
Sampling theorem
Sampling theorem

Aliasing: Two signals become indistinguishable after sampling
Aliasing
Aliasing
Aliasing in real life

https://www.youtube.com/watch?v=QOwzkND_o0U
How to find a good sample rate?
How to find a good sample rate?

Nyquist sampling theorem: 
In order to uniquely represent a signal $F(t)$ by a set of samples, the sampling rate must be more than twice the highest frequency component present in $F(t)$.

If sample rate is $f_s$ and maximum frequency we want record is $f_{\text{max}}$, then

$$f_s > 2f_{\text{max}}$$
Nyquist frequency = Maximum alias-free frequency for a given sample rate.

Nyquist rate = Lower bound of sample rate for a signal.
\[ x(t) = \sum_{n=-\infty}^{\infty} x(nT) \cdot \text{sinc} \left( \frac{t - nT}{T} \right), \]
A 4 kHz frequency band starting at 2 kHz

$f_{\text{max}} = 12 \text{ Hz}$

$f_{\text{max}} = 6000 \text{ Hz}$
Commonly, the maximum frequency in human voice is 4 kHz, what sample rate will you use in your audio recorder?
Aliasing: A real life scenario

Nyquist frequency

Frequency spectrum

Amplitude

10k  20k  30k  40k  50k  60k  70k  80k  90k  100k
Aliasing: A real life scenario

Nyquist frequency
Aliasing: A real life scenario

We need a “Low-pass filter” to remove unwanted high frequency signals.
Anti-aliasing filter

Nyquist frequency

Amplitude

Frequency spectrum

Sensor → Anti-aliasing Filter → ADC
Anti-aliasing filter

Frequency spectrum

Nyquist frequency

Amplitude

Sensor → Anti-aliasing Filter → ADC
Anti-aliasing filter

Frequency spectrum

Amplitude

Sensor → Anti-aliasing Filter → ADC
Model for a signal (frequency, amplitude, and phase)

\[ A \cdot \sin(\theta) \]

\[ A \cdot \cos(\theta) \]
How can we incorporate both Sine and Cosine in the equation?
Model for a signal (frequency, amplitude, and phase)

1. $\cos(\theta) + \sin(\theta)$
Model for a signal (frequency, amplitude, and phase)

1. \( \cos(\theta) + \sin(\theta) \)
2. \( \langle \cos(\theta), \sin(\theta) \rangle \)
Model for a signal (frequency, amplitude, and phase)

1. \( \cos(\theta) + \sin(\theta) \)
2. \( \langle \cos(\theta), \sin(\theta) \rangle \)
3. \( \cos(\theta) + j \sin(\theta) \)
Complex numbers

$j = \sqrt{-1}$

Imaginary
Complex numbers

This point represents the real number $a = -2.2$

This point represents the complex number $c = 2.5 + j2$
Complex numbers

The diagram illustrates a complex number $c = a + jb$ in the complex plane, where $a$ is the real part and $b$ is the imaginary part. The point $M$ represents the magnitude of the complex number, and $\phi$ represents the argument (angle) of the complex number.
Complex numbers

Figure 4. What happens to the real number 8 when you start multiplying it by j.

Multiplying any number on the real axis by j results in an imaginary product that lies on the imaginary axis. The example in Figure 4 shows that if +8 is represented by the dot lying on the positive real axis, multiplying +8 by j results in an imaginary number, +j8, whose position has been rotated 90° counterclockwise (from +8) putting it on the positive imaginary axis. Similarly, multiplying +j8 by j results in another 90° rotation yielding the -8 lying on the negative real axis because j² = -1. Multiplying -8 by j results in a further 90° rotation giving the -j8 lying on the negative imaginary axis.

Whenever any number represented by a dot is multiplied by j, the result is a counterclockwise rotation of 90°. (Conversely, multiplication by -j results in a clockwise rotation of -90° on the complex plane.)

If we let I = S/2 in Eq. 7, we can say that:

\[
\cos\left(\frac{S}{2}\right) + jsin\left(\frac{S}{2}\right) = 0 + j1,
\]

or

\[
e^{jS/2} = j.
\]

Here’s the point to remember. If you have a single complex number, represented by a point on the complex plane, multiplying that number by j or by \(e^{jS/2}\) will result in a new complex number that’s rotated 90° counterclockwise (CCW) on the complex plane. Don’t forget this, as it will be useful as you begin reading the literature of quadrature processing systems!

Let’s pause for a moment here to catch our breath. Don’t worry if the ideas of imaginary numbers and the complex plane seem a little mysterious. It’s that way for everyone at first—you’ll get comfortable with them the more you use them. (Remember, the j-operator puzzled Europe’s heavyweight mathematicians for hundreds of years.) Granted, not only is the mathematics of complex numbers a bit strange at first, but the terminology is almost bizarre. While the term imaginary is an unfortunate one to use, the term complex is downright weird. When first encountered, the phrase complex numbers makes us think ‘complicated numbers’. This is regrettable because the concept of complex numbers is not really all that complicated. Just know that the purpose of the above mathematical rigmarole was to validate Eqs. (2), (3), (7), and (8). Now, let’s (finally!) talk about time-domain signals.

Representing Real Signals Using Complex Phasors

OK, we now turn our attention to a complex number that is a function of time. Consider a number whose magnitude is one, and whose phase angle increases with time. That complex number is the \(e^{j2\pi f t}\) point shown in Figure 5(a). (Here the Complex numbers...
Complex numbers and Natural exponential

\[ e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \frac{z^4}{4!} + \frac{z^5}{5!} + \frac{z^6}{6!} + \]
Eqs. (3) and (4) remind us that \( c \) can also be considered the tip of a phasor on the complex plane, with magnitude \( M \), oriented in the direction of \( I \) radians relative to the positive real axis as shown in Figure 2. Keep in mind that \( c \) is a complex number and the variables \( a, b, M, \) and \( I \) are all real numbers.

The magnitude of \( c \), sometimes called the \textit{modulus} of \( c \), is

\[
M = |c| = a^2 + b^2 \quad (5)
\]

[Trivia question: In what 1939 movie, considered by many to be the greatest movie ever made, did a main character attempt to quote Eq. (5)?]

OK, back to business. The phase angle \( I \), or \textit{argument}, is the arctangent of the ratio

\[
\frac{\text{imaginary part}}{\text{real part}} = \frac{b}{a} \quad (6)
\]

If we set Eq. (3) equal to Eq. (2),

\[
Me^jI = M[\cos(I) + jsin(I)]
\]

we can state what's named in his honor and now called one of Euler's identities as:

\[
e^{jI} = \cos(I) + jsin(I) \quad (7)
\]

The suspicious reader should now be asking, “Why is it valid to represent a complex number using that strange expression of the base of the natural logarithms, \( e \), raised to an imaginary power?”

We can validate Eq. (7) as did the world's greatest expert on infinite series, Herr Leonard Euler, by plugging \( jI \) in for \( z \) in the series expansion definition of \( e^z \) in the top line of Figure 3. That substitution is shown on the second line.

\[
e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \frac{z^4}{4!} + \frac{z^5}{5!} + \frac{z^6}{6!} + \ldots
\]

\[
e^{j\phi} = 1 + j\phi + \frac{(j\phi)^2}{2!} + \frac{(j\phi)^3}{3!} + \frac{(j\phi)^4}{4!} + \frac{(j\phi)^5}{5!} + \ldots
\]
Complex numbers and Natural exponential

\[ e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \frac{z^4}{4!} + \frac{z^5}{5!} + \frac{z^6}{6!} + \ldots \]

\[ e^{j\phi} = 1 + j\phi + \frac{(j\phi)^2}{2!} + \frac{(j\phi)^3}{3!} + \frac{(j\phi)^4}{4!} + \frac{(j\phi)^5}{5!} + \frac{(j\phi)^6}{6!} + \ldots \]

\[ = 1 + j\phi - \frac{\phi^2}{2!} - j \frac{\phi^3}{3!} + \frac{\phi^4}{4!} + j \frac{\phi^5}{5!} - \frac{\phi^6}{6!} + \ldots \]
Complex numbers and Natural exponential

\[ e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \ldots \]

\[ e^{j\phi} = 1 + j\phi + \frac{(j\phi)^2}{2!} + \frac{(j\phi)^3}{3!} + \ldots \]

\[ = 1 + j\phi - \frac{\phi^2}{2!} - j \frac{\phi^3}{3!} + \ldots \]

\[ \cos(\phi) \]

\[ j \sin(\phi) \]
$e^{i\varphi} = \cos \varphi + i \sin \varphi$
Model for a signal (frequency, amplitude, and phase)

\[ \cos(\theta) + j \sin(\theta) = e^{j\theta} \]
Model for a signal (frequency, amplitude, and phase)

\[ \cos(\theta) + j \sin(\theta) = e^{j\theta} = e^{j \cdot 2\pi ft} \]
Model for a signal (frequency, amplitude, and phase)

\[ e^{j 2\pi ft} \]
Model for a signal (frequency, amplitude, and phase)

Figure 6. The motion of the $ej2\pi ft$ phasor (a), and phasor ‘s tip (b).

Return to Figure 5(b) and ask yourself: “Self, what’s the vector sum of those two phasors as they rotate in opposite directions?”

Think about this for a moment... That’s right, the phasors’ real parts will always add constructively, and their imaginary parts will always cancel. This means that the summation of these $ej2\pi ft$ and $e^{-j2\pi ft}$ phasors will always be a purely real number.

Implementations of modern-day digital communications systems are based on this property!

To emphasize the importance of the real sum of these two complex sinusoids we’ll draw yet another picture. Consider the waveform in the three-dimensional Figure 7 generated by the sum of two half-magnitude complex phasors, $ej2\pi ft/2$ and $e^{-j2\pi ft/2}$, rotating in opposite directions around, and moving down along, the time axis.

Figure 7. A cosine represented by the sum of two rotating complex phasors.

Thinking about these phasors, it’s clear now why the cosine wave can be equated to the sum of two complex exponentials by

$$\cos(2\pi ft) = e^{j2\pi ft} + e^{-j2\pi ft}$$

$$= e^{j2\pi ft/2} e^{j\pi ft/2} + e^{-j2\pi ft/2} e^{-j\pi ft/2}$$

$$= e^{j\pi ft/2} (e^{j\pi ft/2} + e^{-j\pi ft/2})$$

$$= e^{j\pi ft/2} (2\cos(\pi ft))$$

$$= e^{j\pi ft/2} \cos(2\pi ft)$$

Eq. (10), a well-known and important expression, is also called one of Euler’s identities. We could have derived this identity by solving Eqs. (7) and (8).

$e^{j2\pi ft}$

$e^{-j2\pi ft}$
Model for a signal (frequency, amplitude, and phase)

\[ e^{j\theta} = \cos(\theta) + j \sin(\theta) \]
\[ e^{-j\theta} = \cos(\theta) - j \sin(\theta) \]

How about real sinusoids?

\[ \cos(\theta) = ? \]
\[ \sin(\theta) = ? \]
Presenting real signal with the complex model

\[ e^{j\theta} = \cos(\theta) + j \sin(\theta) \]
\[ e^{-j\theta} = \cos(\theta) - j \sin(\theta) \]

\[ \cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2} \]

\[ \sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j} \]
Presenting real signal with the complex model

\[
e^{j 2\pi ft} = \cos(2\pi ft) + j \sin(2\pi ft)
\]

\[
e^{-j 2\pi ft} = \cos(2\pi ft) - j \sin(2\pi ft)
\]

\[
\cos(2\pi ft) = \frac{e^{j 2\pi ft} + e^{-j 2\pi ft}}{2}
\]

\[
\sin(2\pi ft) = \frac{e^{j 2\pi ft} - e^{-j 2\pi ft}}{2j}
\]
Time Domain and Frequency Domain

Fourier Transform

FFT = Fast Fourier Transform
IFFT = Inverse Fast Fourier Transform
Time Domain and Frequency Domain

FFT = Fast Fourier Transform
IFFT = Inverse Fast Fourier Transform
Time Domain and Frequency Domain

Fourier Transform

$\cos(2\pi ft)$

FFT = Fast Fourier Transform

IFFT = Inverse Fast Fourier Transform
Time Domain and Frequency Domain

Fourier Transform

\[
cos(2\pi ft) = \frac{e^{j \cdot 2\pi ft} + e^{-j \cdot 2\pi ft}}{2}
\]

Time domain

FFT = Fast Fourier Transform

IFFT = Inverse Fast Fourier Transform

Frequency domain
**Fourier Transform**

\[ \cos(2\pi ft) = \frac{e^{j2\pi ft} + e^{-j2\pi ft}}{2} \]

**Time domain**

**Frequency domain**

**FFT** = Fast Fourier Transform

**IFFT** = Inverse Fast Fourier Transform
Time Domain and Frequency Domain

**Fourier Transform**

$$\cos(2\pi 4t)$$

$$= \frac{e^{j 2\pi ft} + e^{-j 2\pi ft}}{2}$$

**Time domain**

**Frequency domain**

FFT = Fast Fourier Transform

IFFT = Inverse Fast Fourier Transform
Plotting the DFT spectrum

Magnitude \(|z_m|\)

Frequency

m = 0 1 2 3 4 ... N-2 N-1
DFT (Discrete Fourier Transform)

\[ X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{i2\pi}{N}kn} \]

\[ = \sum_{n=0}^{N-1} x_n \cdot \left[ \cos\left(\frac{2\pi}{N}kn\right) - i \cdot \sin\left(\frac{2\pi}{N}kn\right) \right] \]
Plotting the DFT spectrum

Magnitude $|z_m|$

Phase $\angle z_m$

m = 0 1 2 3 4 ... N-2 N-1 Frequency

Frequency
The Curious Case of “Negative frequency”

Positive rotation with $\frac{2\pi}{N}$ radian angle per step

Negative rotation with $\frac{2\pi}{N}$ radian angle per step
The Curious Case of “Negative frequency”

\[ X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{i2\pi}{N} kn} \]
The Curious Case of “Negative frequency”

\[ X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{i2\pi}{N}kn} \]
The Curious Case of “Negative frequency”

Real signal’s magnitude spectrum is symmetric.

Why?

\[ X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{i2\pi}{N}kn} \]
The Curious Case of “Negative frequency”

Complex signal’s magnitude spectrum may or may not be symmetric.

Why?

\[ X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{i2\pi kn}{N}} \]
Sampling frequency = $f_s$ (i.e., $f_s$ samples per second)

Slowest frequency ($\frac{2\pi}{N}$ radians per step) = N samples per rotation

$$= (N/ f_s) \text{ seconds per rotation}$$

Therefore, the slowest frequency = $(f_s / N)$ Hz

Higher frequencies are integer multiple of $(f_s / N)$ Hz

$$0, \frac{f_s}{N}, \frac{2f_s}{N}, \frac{3f_s}{N}, \frac{4f_s}{N}, \ldots ,$$
The resolution and the highest frequency

Resolution
= minimum observable frequency difference = \( \frac{f_s}{N} \)

What if the actual frequency falls in between two frequency bins?
The resolution and the highest frequency

Highest frequency = $\frac{f_s}{2}$
How can we increase the resolution?

\[ \frac{f_s}{N} = \frac{\text{sample rate}}{\# \text{ of FFT points}} \]

How can we increase the range of the spectrum?

\[ \left[ -\frac{f_s}{2}, \frac{f_s}{2} \right] \]
What should be the sample rate?
What should be the sample rate?

Generally, bandwidth of the signal determines the sample rate.