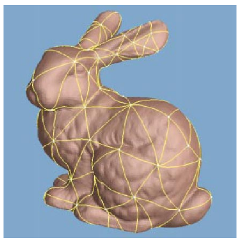
3. Parametrization of meshes

(Eck et al, SIGGRAPH 95; Lee et al, SIGGRAPH 98)

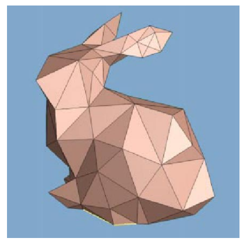
3.1 Introduction

Given: Mesh M, possibly with vertex colors or other attributes.

Goal: Construct simple *base mesh* M_0 and a homeomorphism (continuous $1-1$ map) $\Pi : M \mapsto M_0$.



Piecewise linear 1-to-1 continuous map



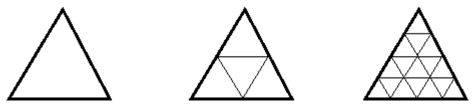
Motivation 1: Texture mapping

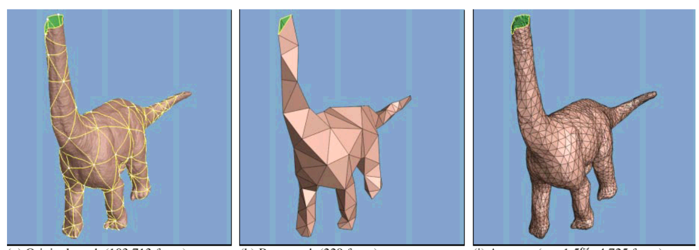
Each vertex of M is mapped to some point on the base mesh M_0. Can interpolate vertex colors to a pixel grid on each base mesh face $\Rightarrow$ texture map

Motivation 2: Remeshing

Can perform (repeated) 4-1 subdivisions of base mesh faces. The map Π^{-1} maps each vertex of subdivided base mesh to $R^3 \Rightarrow$ obtain approximations of M by meshes with subdivision connectivity.

Subdivision connectivity essential for multiresolution analysis (Lounsbery et al)



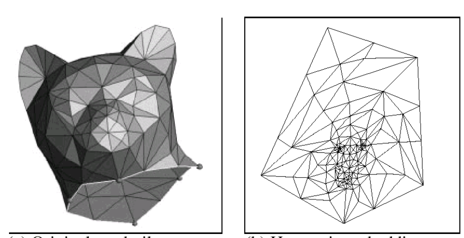


(g) Original mesh (103,713 faces) (h) Base mesh (229 faces) (i) Approx. (ϵ = 1.5%; 4,725 faces)

3.2 Basic approaches

Eck et al:

- Triangulate original mesh M (yellow curves on dino). Tricky! Each triangular region corresponds to a face of the base mesh M_0.
- Construct piecewise linear homeomorphism from each triangular region to corresponding base mesh face, making sure that map is continuous accross base mesh edges.
  How: *Harmonic maps*



(a) Original mesh tile (b) Harmonic embedding

Lee et al:

- Perform sequence of vertex removals, thereby obtaining sequence of meshes $M = M_L, M_{L-1}, \ldots, M_0$
- Construct homeomorphisms $\Pi_l : M_l \mapsto M_{l-1}$. Key step!
  Then $\Pi = \Pi_1 \circ \Pi_2 \circ \cdots \circ \Pi_L$
  How: *Conformal maps*

Advantages of Lee et al:

- No need to find initial triangulation of M.
- Can be adapted so that base mesh edges and vertices align with sharp features of original mesh.

Will discuss Lee et al.

3.3 MAPS (Lee et al)

Selecting vertices for removal

Lee et al construct hierarchy of simplified meshes, making sure that at each level at least some fixed fraction of vertices is removed. *Does not seem essential.*

Will not further discuss this aspect and instead describe sequential procedure.

Associate with each vertex a loss combining measure of mesh curvature and area of *star* surrounding vertex.

At each step remove vertex with smallest loss.

Constructing homeomorphisms

M_{l-1}: mesh obtained from M_l by removing a vertex p and re-triangulating the hole.

- How do we re-triangulate the hole?
- How do we construct homeomorphism $\Pi_l : M_l \to M_{l-1}$?
  Note: Π_l is the identity outside of $star(p)$

$p_1, \ldots, p_k$: vertices surrounding p, enumerated cyclically; $p_0 = p_k$.

Define

$$\theta_i = \sum_{j=1}^{i} \angle(p_{j-1}, p, p_j) \qquad i = 1, \ldots, k$$

For locally planar mesh, $\theta_k = 2\pi$

Define

$$\begin{aligned} r_j &= \|p - p_j\| \\ a &= 2\pi/\theta_k \end{aligned}$$

Map p to the origin of the (complex) plane.
Map p_j to $r_j^a \exp(i\,\theta_j\,a)$
This defines piecewise linear map from $star(p)$ to complex plane.

Remove origin and re-triangulate hole using constrained Delauney triangulation.

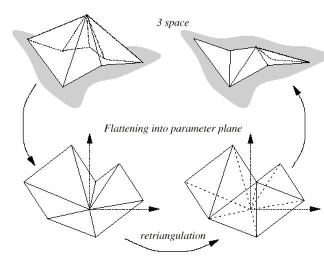


Figure 5: *In order to remove a vertex p_i, its star (i) is mapped from 3-space to a plane using the map z^a. In the plane the central vertex is removed and the resulting hole retriangulated (bottom right).*

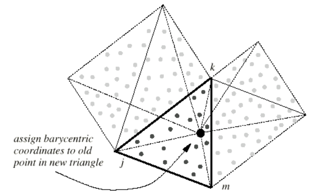


Figure 6: *After retriangulation of a hole in the plane (see Figure 5), the just removed vertex gets assigned barycentric coordinates with respect to the containing triangle on the coarser level. Similarly, all the finest level vertices that were mapped to a triangle of the hole now need to be reassigned to a triangle of the coarser level.*

Embellishments

Can deal with sharp features.

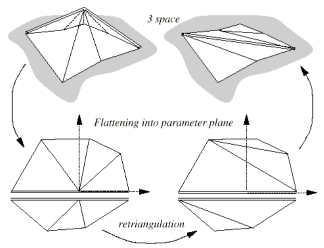


Figure 9: *When a vertex with two incident feature edges is removed, we want to ensure that the subsequent retriangulation adds a new feature edge to replace the two old ones.*

Can smooth parametrization.

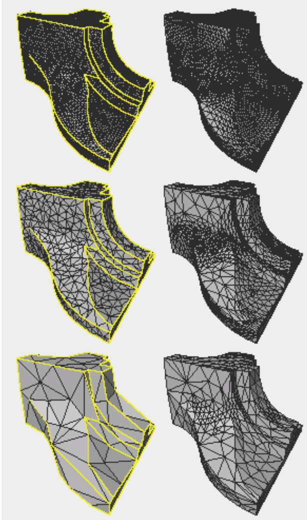


Figure 13: *Left (top to bottom): three levels in the DK pyramid, finest ($L = 15$) with 12946, intermediate ($l = 8$) with 1530, and coarsest ($l = 0$) with 168 triangles. Feature edges, dart and corner vertices survive on the base domain. Right (bottom to top): adaptive mesh with $\epsilon = 5\%$ and 1120 triangles (bottom), $\epsilon = 1\%$ and 3430 triangles (middle), and uniform level 3 (top). (Original dataset courtesy University of Washington.)*