13. Radiometry

Reading

Recommended:

- Cohen and Wallace, *Radiosity and Realistic Image Synthesis*, Chapter 2.

Physically-based rendering

Basic optics

- Physics of light and color
- Geometrical optics
  - Ray "metaphor"
  - Reflection and transmission
- Radiative transfer
  - Measurement: radiometry and photometry
  - Transport theory and integral equations

Physically-based rendering, cont'd

Why physically based?

- Insights into problem of rendering
- Illumination engineering
- Quest for realism
  - A "grand challened" problem in graphics
  - The light holodeck...someday

Transport theory

Transport theory is concerned with calculating how much stuff, Q, flows through the environment.

Stuff can be:

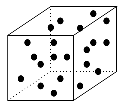
- Mass, m
- Charge, q
- Radiant energy, Φ

We'll think about transport theory in terms of particles, namely photons.

Wheat are some of the key quantities to think about?

Partical density and flow

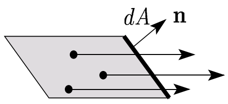
We need a notion of particle density:

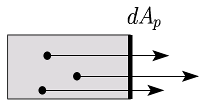


$$Q(\mathbf{x}) = N(\mathbf{x})d^3x$$

Consider particles flowing with constant velocity.

Q: How many particles cross a fictitious surface in time dt?





$$\begin{aligned} Q(\mathbf{x}) &= N(\mathbf{x})d^3x \\ &= \end{aligned}$$

Flux and flux density

Flux is stuff/time:

$$\Phi = \frac{dQ}{dt} =$$

Fluxdensity is stuff/(time $\cdot$ area):

$$\frac{d\Phi}{dA} = \frac{d^2Q}{dtdA} =$$

Directional dependence

How do we account for the fact that particles have different *velocities* (speed and directions)?

Photons all have same speed, so we're more concerned with directional distributions.

For this, we need a measure of directions.

Area measures are easy:

$$\begin{aligned} A &= \Delta x \cdot \Delta y \\ dA &= dx \cdot dy \end{aligned}$$

Directions are trickier...

Angles and differential angles

Angles:

$\theta = $ (arc length)/(radius)

Thus, there are 2π radians in a circle.

Angle is equal to arc length on the unit circle.

An arbitrary segment can be projected onto the unit circle to compute subtended angle.

Differential angles:

Solid angles and differential solid angles

Solid angles:

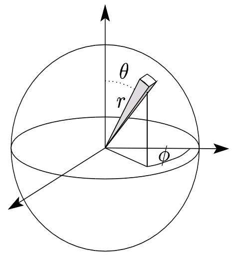
$\omega = $ (area)/(radius squared)

Thus, there are 4π steriadians in a sphere

Solid angle is equal to area on the unit sphere.

An arbitrary surface patch can be projected onto the unit circle to compute subtended solid angle.

Differential solid angle:



Angular flux density

We can now define a particle distribution based on position and directions, $n(\mathbf{x}, \omega)$:

$$dN(\mathbf{x}) = n(\mathbf{x}, \omega) d\omega$$

The *angular flux density* is stuff/(time · area · solid angle):

$$\frac{d^2\Phi}{dA d\omega} =$$

For the case of photons, we should also include wavelengths.

The fundamental photon distribution is then:

$$n(\mathbf{x}, \omega, \lambda)$$

From photons to light energy

Photons:

- Carry energy $hf = hc/\lambda$ per particle
- Do not interact with each other
- Travel with the same speed in different directions

This leads to a definition of *radiance*:

$$L(\mathbf{x}, \omega, \lambda) = \frac{hc}{\lambda} n(\mathbf{x}, \omega, \lambda)$$

Radiance is the most fundamental quantity in light transport.

Radiometry vs. photometry

Radiance and radiometry [power unit = watt]

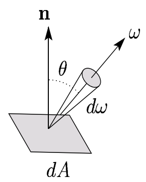
- Physical measurement of electromagnetic energy

Luminance and photometry [“power” unit = lumen]

- Perceptual measurement of relative subjective sensation to light of different wavelengths

Radiance

The radiance is the power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray:



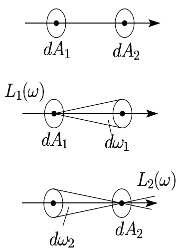
Radiance is the fundamental field quantity that characterizes the distribution of light in an environment. All other quantities derive from it.

Laws:

- Radiance is invariant along a ray.
- The response of a sensor is proportional to radiance.

First law of radiance

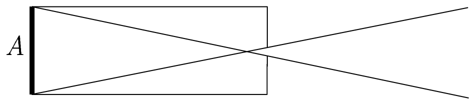
The radiance in the direction of a light ray remains constant as the ray propogates.



L is the numeric quantity that should be associated with rays in ray tracers.

Second law of radiance

The response of a sensor is proportional to the radiance of the surface visible to the sensor.

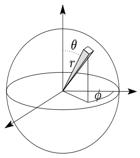


L is what should be computed and displayed.

If you’re standing in front of a uniformly lit wall, does the wall get brighter or darker as you move away?

Irradiance

The irradiance is the power per unit area incident on a surface. Equivalent to flux denisty.



Radiant intensity

The radiant intensity is the power per unit solid angle from a point.

Irradiance due to an isotrpoic point source:

Radiosity

The radiosity is the energy per unit area leaving a surface:

Officially referred to as the radiant exitance.

Consider a uniform diffuse source:

The BRDF

Bidirectional Reflectance Distribution Function

Properties:

- Helmholtz reciprocity
- Isotropic vs. anisotropic

The reflection equation

Outgoing radiance is computed by integrating over incoming radiance:

Linearity:

- Doubling the incident light doubles the reflected light.
- Reflection from several sources behaves like the sum of reflections from each source.

Examples of reflection

Ideal specular:

Ideal diffuse:

Bihemsipherical reflectance

It is sometimes convenient to compute ratios of incoming flux density to outgoing flux density over posrtions of incoming and outgoing hemispheres.

These are called the *biconical reflectances*.

One important ratio compares total incoming flux desnity (irradiance) to total outgoing flux density (radiosity):

This is the *bihemsipherical reflectance*.

By conservation of energy:

Diffuse reflection

In the case of a diffuse surface:

1. The reflected radiance L is constant and equal in all reflected directions.
2. Thus, $L = B/\pi$.
3. The magnitude of the reflected radiance equals $f_{r,d}E$.
4. The bihemsipherical reflectance is now: