10. Subdivision surfaces

Reading

Recommended:

- Stollnitz, DeRose, and Salesin, *Wavelets for Computer Graphics: Theory and Applications*, 1996. Section 10.2.
- Zorin, Course notes on subdivision surfaces, SIGGRAPH 1998.
- Heckbert, Errata for butterfly scheme in Zorin's course notes, 1999.
- Zorin, Sweldens, and Schroeder, "Interpolating subdivision for meshes with arbitrary topology," SIGGRAPH '96, pp. 189-192.
- DeRose and Kass, "Subdivision surfaces in character animation," SIGGRAPH '98, pp. 85-94.

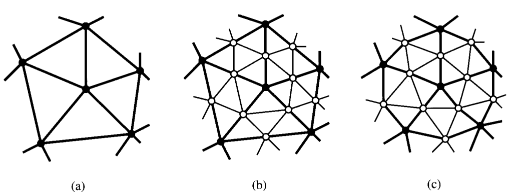
Subdivision surfaces

Chaikin's use of subdivision for curves inspired similar techniques for surfaces....

Idea: Iteratively refine a *control polyhedron* (or *control mesh*) $M^0 \to M^1 \to \cdots$ to a limit surface

$$\sigma = \lim_{j \to \infty} M^j$$

using splitting and averaging steps:



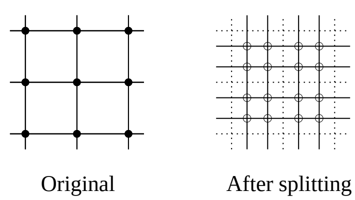
Splitting and averaging steps (Stollnitz et al., 10.4)

Two types of splitting steps:

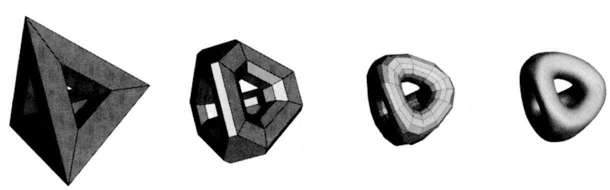
- *Vertex schemes*
- *Face schemes*

Vertex schemes

A vertex surrounded by n faces is split into n subvertices, one for each face:



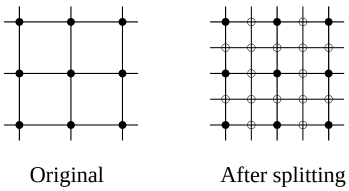
Splitting step for Doo-Sabin subdivision



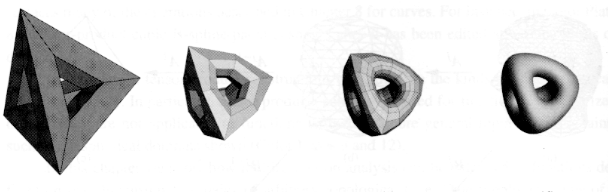
Doo-Sabin subdivision (Stollnitz et al., 10.3)

Face schemes

Each quadrilateral face is split into four subfaces:



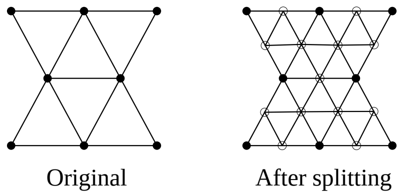
Splitting step for Catmull-Clark subdivision



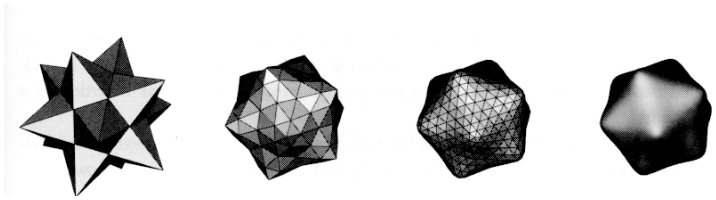
Catmull-Clark subdivision (Stollnitz et al., 10.2)

Face schemes, cont.

Each triangular face is split into four subfaces:



Splitting step for Loop subdivision



Loop subdivision (Stollnitz et al., 10.5)

Terms

Genus

Boundary vs. interior

Valence

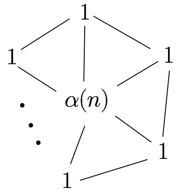
Extraordinary interior vertex

Extraordinary boundary vertex

Even vs. odd vertices

Loop averaging mask

Once again we can use *masks* for the averaging step:



$$Q \leftarrow \frac{\alpha(n)Q + Q_1 + \cdots + Q_n}{\alpha(n) + n}$$

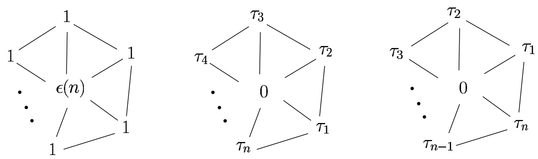
where

$$\alpha(n) = \frac{n\,(1-\beta(n))}{\beta(n)} \qquad \text{and} \qquad \beta(n) = \frac{5}{4} - \frac{(3 + 2\cos(2\pi/n))^2}{32}$$

(α carefully chosen to ensure smoothness.)

Loop evaluation and tangent masks

As with subdivision curves, we can split and average a number of times and then push the points to their limit positions.



Evaluation mask

Tangent masks

$$Q^\infty = \frac{\epsilon(n)Q + Q_1 + \cdots + Q_n}{\epsilon(n) + n}$$

where

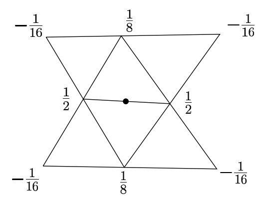
$$\epsilon(n) = \frac{3n}{4\beta(n)} \qquad \text{and} \qquad \tau_i(n) = \cos(2\pi i/n)$$

Interpolating subdivision surfaces

Interpolating schemes are defined by:

- Splitting
- Averaging only new vertices

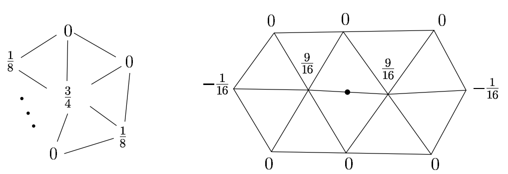
Here is the averaging mask for odd vertices in the "modified butterfly scheme":



See Zorin's notes and SIGGRAPH paper for more details.

Adding creases

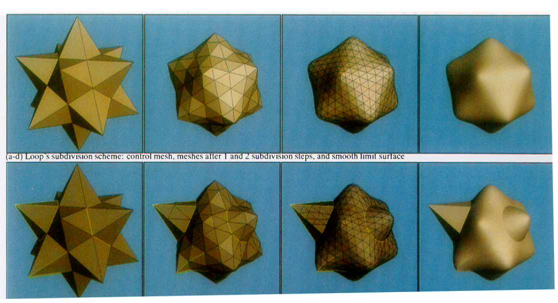
In some cases, we want a particular feature such as a crease to be preserved. For subdivision surfaces, we can just modify the subdivision mask:



Loop crease/boundary edge

Butterfly crease/boundary edge

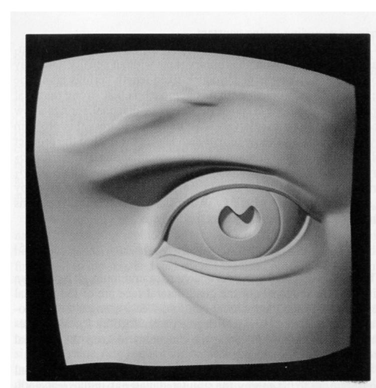
This gives rise to G^0 continuous surfaces:



From Hoppe, et al, SIGGRAPH '94.

Semi-sharp creases

Here's an example using Catmull-Clark surfaces of the kind found in Geri's Game:



From Derose and Kass, SIGGRAPH '98.

Creases of variable sharpness can be obtained:

- allow non-stationary, non-uniform subdivision rules for a few steps
- then, push to the limit