

## 4. Image processing

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## Reading

Required:

- Jain, Kasturi, Schunck, *Machine Vision*.  
McGraw-Hill, 1995. Chapter 4 and Sections 5.1-5.4.

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## Image processing

**Defintion:** *Image processing* is an operation that takes an image into an image.

Useful for:

- Enhancing contrast
- Removing noise
- Enhancing edges
- Warping (rotating, scaling, shearing, ...)

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## Image analysis

**Defintion:** *Image analysis* is an operation that takes an image into anything else.

Examples:

- Extracting measurements
- Recovering edges or geometry
- Image segmentation

(The term “image processing” is sometimes loosely used to cover both processing and analysis.)

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## What is an image?

We can think of an *image* as a continuous function over two dimensions:

$$f(x, y)$$

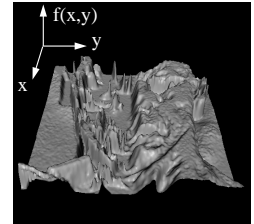
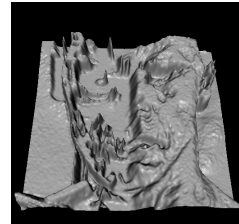
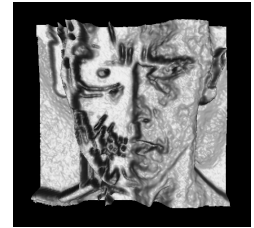
The image may represent light captured by a real or synthetic camera. It could also represent a drawing or painting or visualization of data.

A color image is a “vector-valued” function:

$$\mathbf{f}(x, y) = \begin{pmatrix} r(x, y) \\ g(x, y) \\ b(x, y) \end{pmatrix}$$

For now, we will concentrate on grayscale images,  $f(x, y)$ .

## Images as functions

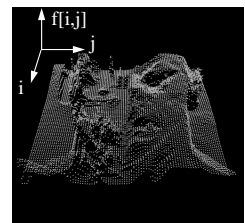
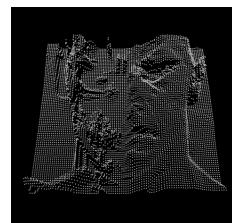
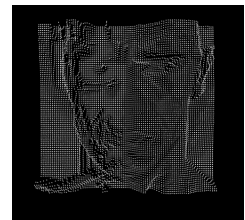
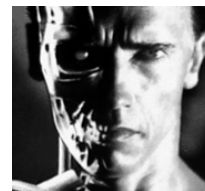


## What is a digital image?

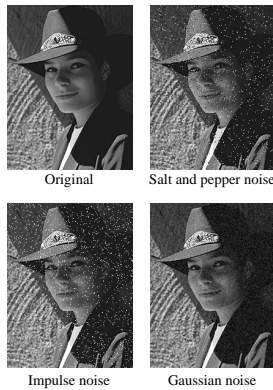
A *digital image* results from sampling and quantizing an image.

$$f[i, j] = \text{Quantize}\{f(i\Delta, j\Delta)\}$$

We will use the term “image” to refer to either continuous or digital images.



## Noise



Some common types of noise include:

- *Salt and pepper noise*: contains random occurrences of black and white pixels.
- *Impulse noise*: contains random occurrences of white pixels.
- *Gaussian noise*: contains variations in intensity drawn from a Gaussian normal distribution.

## Mean filters

One of the simplest filters we can design is the *mean filter*. It takes the form:

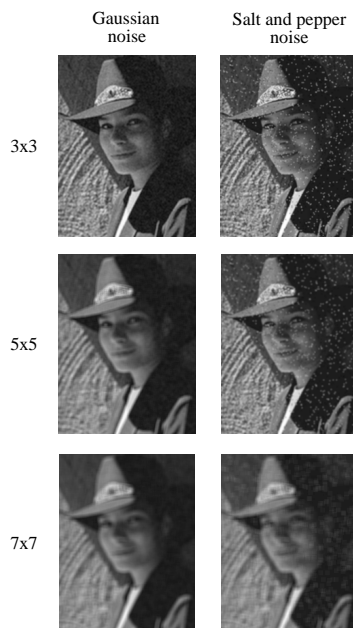
$$\frac{1}{m^2} \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & 1 & 1 & \dots & 1 \\ 1 & 1 & 1 & \dots & 1 \\ & & & \ddots & \\ 1 & 1 & 1 & \dots & 1 \end{bmatrix}$$

**Q:** To which continuous domain filter does this correspond?

**Q:** What are mean filters good for?

**Q:** What is the effect of varying  $m$ ?

## Effect of mean filters



Mean filtering of various filter sizes applied to two types of noisy images

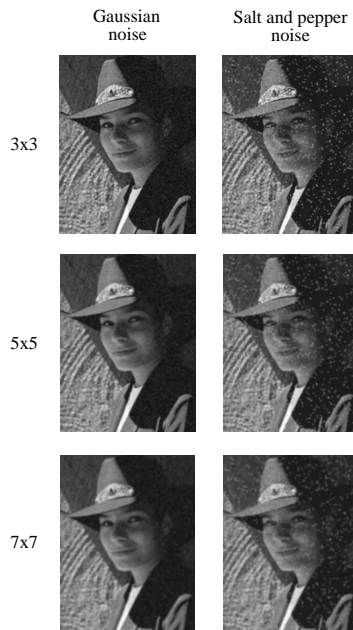
## Gaussian filters

Choosing weights according to a Gaussian often does a nicer job than mean filters:

$$g[i, k] = e^{-(i^2 + k^2)/(2\sigma^2)}$$

Gaussians provide a nice trade-off between blurring and preserving features in the image.

### Effect of Gaussian filters



*Gaussian filtering of various filter sizes applied to two types of noisy images*

### Median filters

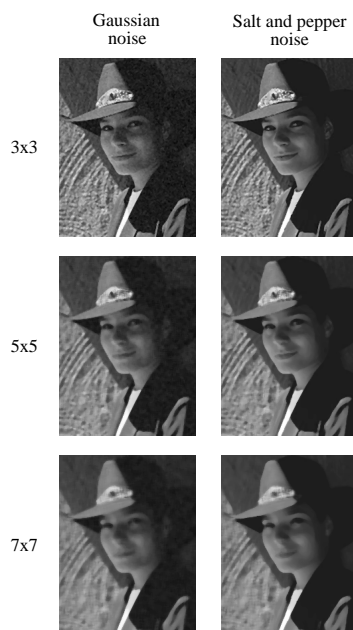
A *median filter* operates over an  $m \times m$  window by selecting the median pixel in the window.

**Q:** What advantage does a median filter have over a mean filter?

**Q:** Is a median filter an LSI filter?

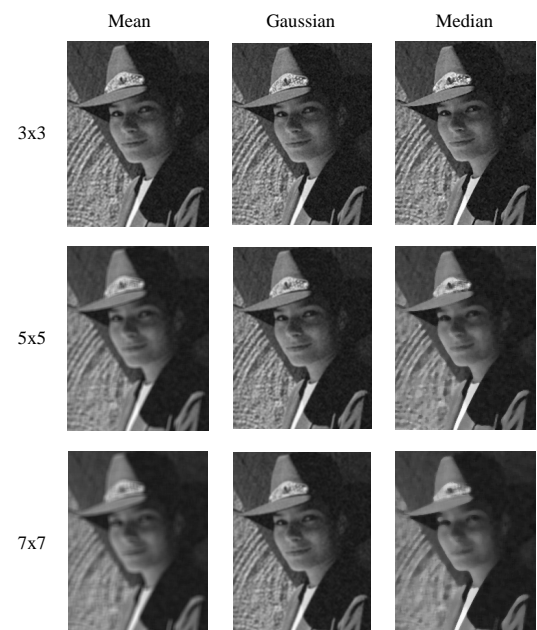
**Q:** Can we write a median filter as a convolution filter?

### Effect of median filters

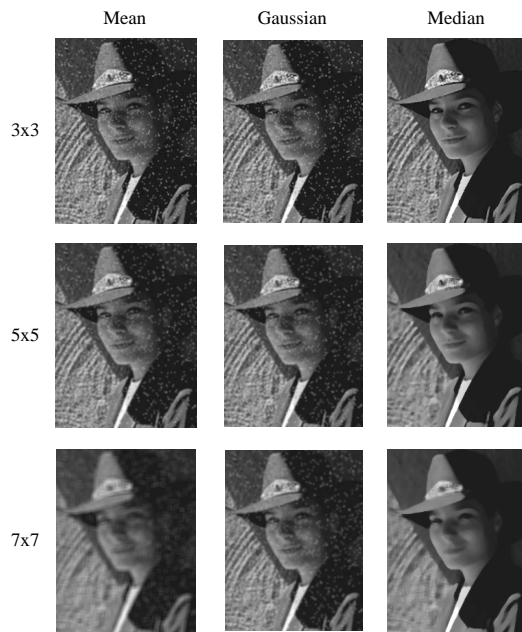


*Median filtering of various filter sizes applied to two types of noisy images*

### Comparison of filters: Gaussian noise



## Comparison of filters: Salt and pepper noise



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## Edge detection

Edge detection is

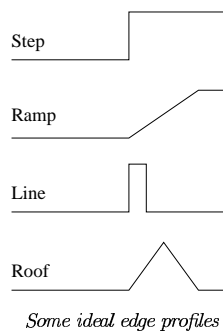
- Easy for humans
- Not so easy for computers

It's also

- Fundamental in computer vision
- Pretty useful in graphics too

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## What is an edge?



**Q:** How might you detect an edge in 1D?

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## Gradients

The *gradient* is the equivalent of the first derivative:

$$\nabla f(x, y) = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$$

Properties of the gradient:

- Points in the direction of maximum increase of  $f$
- Magnitude gives the rate of increase along this direction

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## Derivatives of discrete signals

How can we take the derivative of a discrete function?  
Consider the 1D case:

$$g(x) = \left[ \frac{d}{dx} \{ [f(x) \cdot \text{III}(x)] * r(x) \} \right] \text{III}(x)$$

Going through the steps:

$$\begin{aligned} \frac{d}{dx} \{ [f(x) \cdot \text{III}(x)] * r(x) \} &= \frac{d}{dx} \sum_i f[i] r(x-i) \\ &= \sum_i f[i] \frac{d}{dx} r(x-i) \\ &= \sum_i f[i] r'(x-i) \end{aligned}$$

$$\left[ \frac{d}{dx} \{ [f(x) \cdot \text{III}(x)] * r(x) \} \right] \text{III}(x) = \sum_{k=-\infty}^{\infty} \{ f[k] * r'[k] \} \delta(x-k)$$

we arrive at:

$$g[k] = f[i] * r'[i]$$

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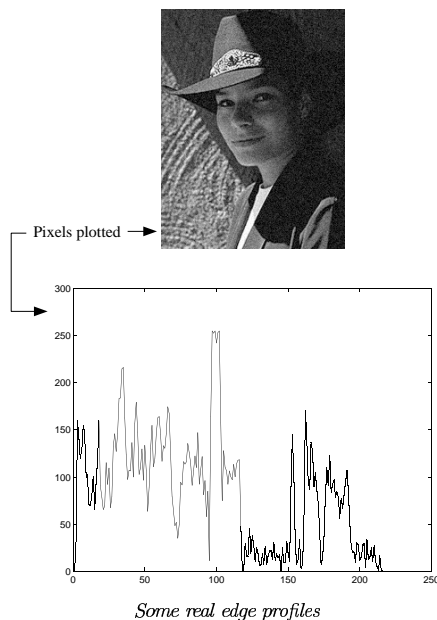
## Image gradients

**Q:** What is the 1D derivative filter assuming piecewise linear reconstruction (interpolation)?

**Q:** What is the 2D gradient filter for images, assuming bilinear reconstruction (interpolation)?

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## Less than ideal edges



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## Steps in edge detection

Algorithms for edge detection contain 3 or 4 steps:

1. Filtering

**Q:** Why?

2. Enhancement

Usually through gradient magnitude

3. Detection

Usually via thresholding

4. Localization (optional)

Estimating subpixel resolution and orientation.

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### Enhancement step

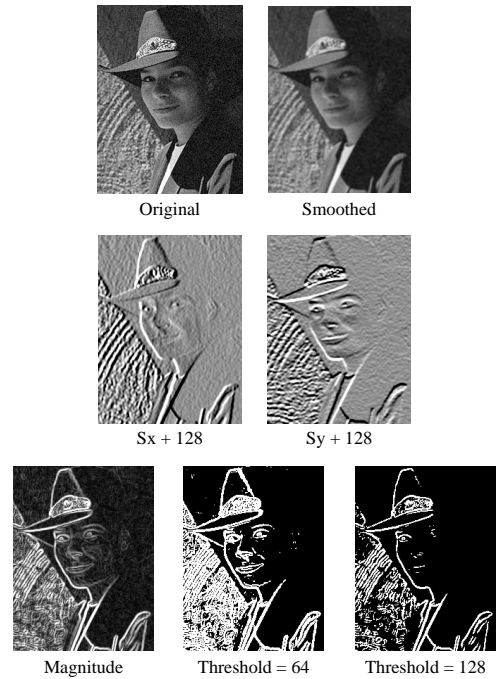
One of the most common measures of gradient magnitude is the *Sobel operator*. The partials  $s_x$  and  $s_y$  are computed by:

$$s_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

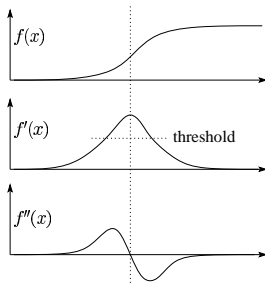
$$s_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

(Gives more emphasis to pixels near the center of the mask.)

### Results of Sobel edge detection



### Second derivative operators



Comparison of first and second derivative approaches

Note that the Sobel operator gives thick edges anywhere the derivative is above threshold.

A better approach might be to look for peaks in the first derivative.

**Q:** A peak in the first derivative corresponds to what in the second derivative?

### Localization with the Laplacian

An equivalent measure of the second derivative in 2D is the *Laplacian*:

$$\nabla^2 f(x, y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

A common choice for the Laplacian is:

$$\nabla^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Zero crossings of this filter correspond to positions of maximum gradient. These zero crossings can be used to localize edges.