2. Color

Reading

Required

- Brian Wandell, *Foundations of Vision*. Sinauer Associates, Sunderland, MA, 1995, Chapter 4.

Suggested:

- Foley et al., Sections 13.2–13.6
- Glassner, Sections 1.7, 2.1, 2.2, 3.6

Further reading:

- Gerald S. Wasserman. *Color Vision: An Historical Introduction*. John Wiley & Sons, New York, 1978.
- Michael Wilcox. *Blue and Yellow Don't Make Green*. North Light Books, Cincinnati, 1987.
- John E. Kaufman, ed. *IES Lighting Handbook: Reference Volume*. Illuminating Engineering Society, New York, 1981.

Scotopic matching

Recall that scotopic vision is low light vision initiated by the rods.

For the scotopic matching experiment, an observer compares two lights:

1. A test light, $\mathbf{t}$
2. A primary light, $\mathbf{p}$

The observer can adjust the emissive power, e, of the primary.

The key results of this experiment are:

1. Only one primary is required.
2. Any primary will work, thus, no wavelength discrimination
3. Matching is linear
   - If $\mathbf{t}$ matches $e\mathbf{p}$, then $c\mathbf{t}$ matches $ce\mathbf{p}$.
   - If $\mathbf{t_1}$ matches $e_1\mathbf{p_1}$ and $\mathbf{t_2}$ matches $e_2\mathbf{p_2}$, then $\mathbf{t_1}+\mathbf{t_2}$ matches $e_1\mathbf{p_1}+e_2\mathbf{p_2}$.

Scotopic matching, cont'd

Due to linearity, we can write a matrix equation to describe scotopic matching:

$$\begin{bmatrix} R \end{bmatrix} = \begin{bmatrix} r_1 & r_2 & \cdots & r_{n_\lambda} \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n_\lambda} \end{bmatrix}$$

or:

$$\begin{bmatrix} R \end{bmatrix} = \begin{bmatrix} \quad \mathbf{r}^T \quad \end{bmatrix} \begin{bmatrix} \\ \mathbf{a} \\ \\ \end{bmatrix} = \mathbf{r} \cdot \mathbf{a}$$

where $t_1, t_2, \cdots, t_{n_\lambda}$ are samples of the test spectrum and $r_1, r_2, \cdots, r_{n_\lambda}$ are samples of the *scotopic spectral sensitivity function*.

By applying monochromatic test lights, $\mathbf{m_i} = [0 \cdots 010 \cdots 0]$, we can determine $\mathbf{r}$.

Connection to rods

The result can be drawn as a continuous curve:

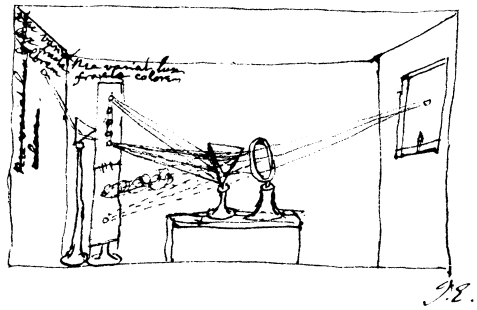
Biological basis: this curve corresponds exactly to the absorption characteristics of *rhodopsin*, the photopigment in rods.

We can in general determine the matching coefficients by integrating:

$$R = \int r(\lambda)a(\lambda)d\lambda$$

Newton's experiments on color

Newton was the first to perform scientific experiments on color, in 1666.



Newton's sketch of his experiment (Wandell, 4.1)

Built a simple colorimeter:

- Hole in a shutter
- Prism to disperse white light into spectrum
- Comb-shaped aperture to manipulate spectrum
- Converging lens to recombine spectrum

Newton's experiments, cont.

Newton defined two types of light:

- Simple: Light that cannot be further dispersed by a prism (now called monochromatic).
- Compound: Light that can be dispersed.

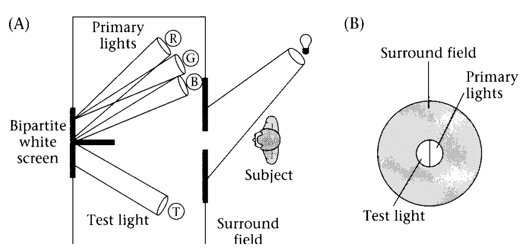
He called the colors of simple lights primaries.

[This term has many other meanings today.]

Newton's experiments, cont.

To study the appearance of colors, Newton recombined primaries to create new colors.

A modern day version of this kind of experiment looks like:



The color matching experiment (Wandell, 4.10)

Cones and color matching

As with rods, the cones contain photopigments that characterize there responses to light.

We can write the cone respones equations as integrals:

$$
\begin{aligned}
L &= \int l(\lambda)t(\lambda)d\lambda \\
M &= \int m(\lambda)t(\lambda)d\lambda \\
S &= \int s(\lambda)t(\lambda)d\lambda
\end{aligned}
$$

We can also use matrix notation, which will prove useful in a moment:

$$
\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \begin{bmatrix} \mathbf{l}^T \\ \mathbf{m}^T \\ \mathbf{s}^T \end{bmatrix} \begin{bmatrix} \mathbf{t} \end{bmatrix}
$$

Q: When do two lights look the same?

Q: How many different spectra will look the same?

How many primaries?

Newton posed a fundamental question that took almost two centuries to answer:

Q: How many primaries does it take to produce a perfect white?

A: (Newton) Definitely more than 2; and 4 or 5 suffice.

In 1852, Helmholtz proposed a generalized form of this question:

Q: How many primaries does it take to produce the *entire* spectrum?

A: (Newton) 7.

A: (Young, 1802) 3.

A: (Helmholtz) 5.

A: (Maxwell) 3.

Who do you think was right? Why?

Photopic matching

Let's assume the answer to be 3 and perform the photopic matching experiment. (Recall that photopic vision is high light level vision initiated by cones.)

Consider three primaries, $\mathbf{p_1}, \mathbf{p_2}, \mathbf{p_3}$, with three emissive power knobs, e_1, e_2, e_3.

The three knobs allow us to create spectra of the form:

How do we set the knobs to match test spectrum, $\mathbf{t}$?

Photopic matching (cont'd)

First, we compute the response to the primaries:

$$
\begin{aligned}
\begin{bmatrix} L_p \\ M_p \\ S_p \end{bmatrix} &= \begin{bmatrix} \mathbf{l}^T \\ \mathbf{m}^T \\ \mathbf{s}^T \end{bmatrix} \begin{bmatrix} e_1\mathbf{p_1} + e_2\mathbf{p_2} + e_3\mathbf{p_3} \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{l}^T \\ \mathbf{m}^T \\ \mathbf{s}^T \end{bmatrix} \begin{bmatrix} \mathbf{p_1} & \mathbf{p_2} & \mathbf{p_3} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{l}\cdot\mathbf{p_1} & \mathbf{l}\cdot\mathbf{p_2} & \mathbf{l}\cdot\mathbf{p_3} \\ \mathbf{m}\cdot\mathbf{p_1} & \mathbf{m}\cdot\mathbf{p_2} & \mathbf{m}\cdot\mathbf{p_3} \\ \mathbf{s}\cdot\mathbf{p_1} & \mathbf{s}\cdot\mathbf{p_2} & \mathbf{s}\cdot\mathbf{p_3} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}
\end{aligned}
$$

Photopic matching (cont'd)

In order for the primaries to match the test, we require the cone responses to be identical:

$$\begin{bmatrix} L_t \\ M_t \\ S_t \end{bmatrix} = \begin{bmatrix} \mathbf{l}^T \\ \mathbf{m}^T \\ \mathbf{s}^T \end{bmatrix} \begin{bmatrix} \mathbf{t} \end{bmatrix} = \begin{bmatrix} L_p \\ M_p \\ S_p \end{bmatrix}$$

This gives us:

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \mathbf{l}\cdot\mathbf{p_1} & \mathbf{l}\cdot\mathbf{p_2} & \mathbf{l}\cdot\mathbf{p_3} \\ \mathbf{m}\cdot\mathbf{p_1} & \mathbf{m}\cdot\mathbf{p_2} & \mathbf{m}\cdot\mathbf{p_3} \\ \mathbf{s}\cdot\mathbf{p_1} & \mathbf{s}\cdot\mathbf{p_2} & \mathbf{s}\cdot\mathbf{p_3} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{l}^T \\ \mathbf{m}^T \\ \mathbf{s}^T \end{bmatrix} \begin{bmatrix} \mathbf{t} \end{bmatrix}$$

and finally:

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{p_1}}^T \\ \overline{\mathbf{p_2}}^T \\ \overline{\mathbf{p_3}}^T \end{bmatrix} \begin{bmatrix} \mathbf{t} \end{bmatrix}$$

Photopic matching (cont'd)

Key observations:

1. Three primaries are "sufficient" for color matching.
2. We can compute the knob settings using three vectors (functions), $\overline{\mathbf{p_1}}, \overline{\mathbf{p_2}}, \overline{\mathbf{p_3}}$. These are called *color matching functions*.
3. Color matching functions are determined by the primaries and the cone responses. These functions are linear transforms of the cone responses.
4. All sets of color matching functions are linear transforms of each other.
5. The resulting knob settings can take on negative values.

Negative light

What does it mean to use a negative amount of a primary?

Consider:

$$\begin{bmatrix} \mathbf{l}^T \\ \mathbf{m}^T \\ \mathbf{s}^T \end{bmatrix} \begin{bmatrix} \mathbf{t} \end{bmatrix} = \begin{bmatrix} \mathbf{l}^T \\ \mathbf{m}^T \\ \mathbf{s}^T \end{bmatrix} \begin{bmatrix} 0.5\mathbf{p_1} - 0.3\mathbf{p_2} + 0.4\mathbf{p_3} \end{bmatrix}$$

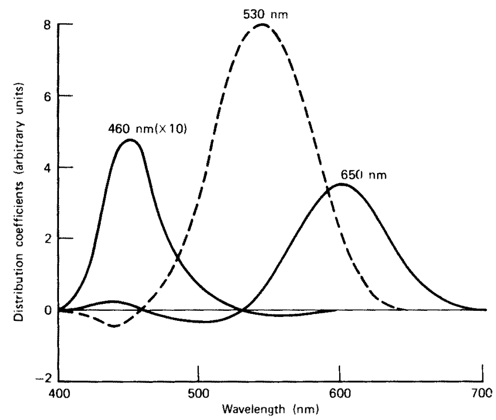
To make e_2 behave like a "real" (i.e., positive values only) knob, we have to move it over to the other side:

$$\begin{bmatrix} \mathbf{l}^T \\ \mathbf{m}^T \\ \mathbf{s}^T \end{bmatrix} \begin{bmatrix} \mathbf{t} + 0.3\mathbf{p_2} \end{bmatrix} = \begin{bmatrix} \mathbf{l}^T \\ \mathbf{m}^T \\ \mathbf{s}^T \end{bmatrix} \begin{bmatrix} 0.5\mathbf{p_1} + 0.4\mathbf{p_3} \end{bmatrix}$$

So, if we are allowed to move a primary to the other side, we will be able to match *any* color.

Example: Wright's experiments

In the late 20's, Wright found that the colors of all wavelengths could be reproduced with combinations of 3 primaries at 460, 530, and 650nm:

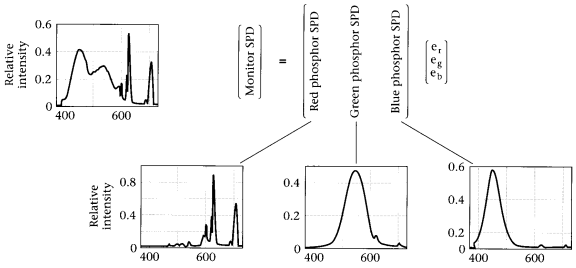


Relative luminances for color matches (Wasserman, 3-3)

These functions are color-matching functions for the given primaries.

R,G,B phosphors as primaries

Here are the spectra for typical R,G,B phosphors in a color monitor:

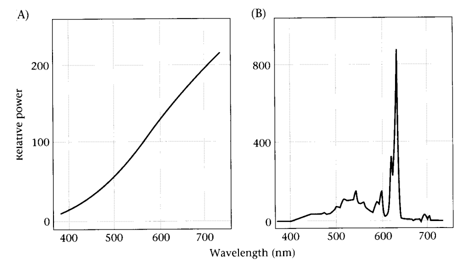


Emission spectra for RGB monitor phosphors (Wandell B.3)

Metamers

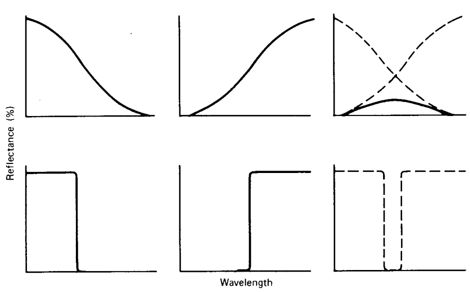
After integrating a spectrum against a set of color matching functions, you are left with 3 numbers.

Spectra that yield the same three numbers will be *indistinguishable*. Such spectra are called metamers.



Metamers: dim tungsten bulb and RGB monitor with suitably chosen R,G,B values (Wandell, 4.11)

Subtractive mixtures



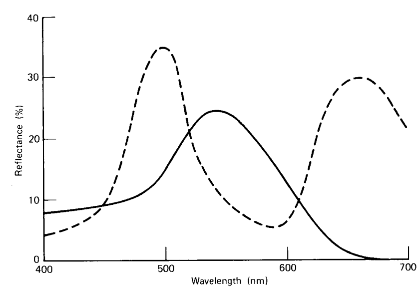
Two examples of subtractive color mixture (Wasserman, 2-2)

Newton also characterized the difference between additive and subtractive color mixtures.

Subtractive color mixtures:

- Due to selective absorption by pigments.
- Difficult to characterize in general.

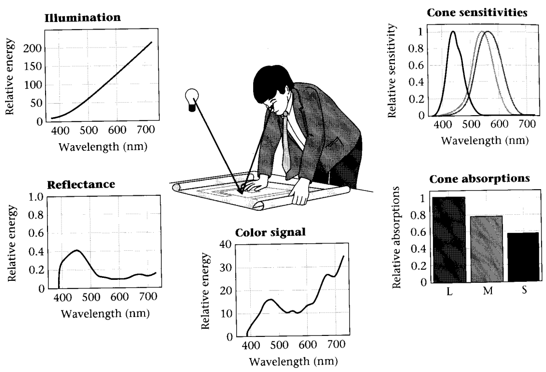
Subtractive metamers



Reflectances of two surfaces that are metamers in daylight, but appear to have different colors under tungsten light (Wasserman, 3.9)

- The solid curve appears green both indoors and out.
- The dashed curve looks green outdoors, but brown under yellowish incandescent light.

Illustration of color appearance



From illumination to cone responses (Wandell, 9.2)

Imaginary primaries

Given mathematical freedom, we can also dream up primaries that have negative power at some wavelengths. Such primaries are "imaginary primaries."

This is actually a necessity if we want to devise strictly positive matching functions. Why would we want to do this?

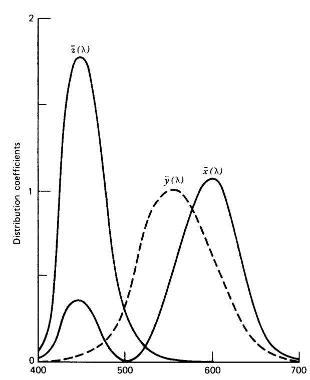
The XYZ system

In 1931, the CIE (Commission Internationale de l'Eclairage) adopted the XYZ system, which has the following properties:

- Every color can be made with *positive* combinations of primaries called **X**, **Y**, **Z**.
- The **X**, **Y**, **Z** primaries are imaginary.

The CIE color-matching functions

Here are the color-matching functions $\overline{x}, \overline{y}, \overline{z}$:



The XYZ color-matching functions (Wasserman 3-8)

Note: By design, the $\overline{y}$ curve is just the "luminous efficiency curve."

Computing CIE coordinates

The X, Y, Z coordinates are computed by:

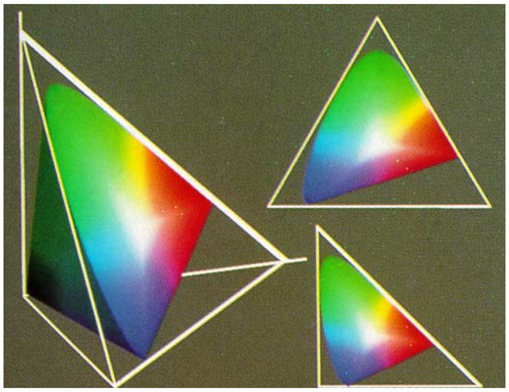
$$X = \int I(\lambda)\overline{x}(\lambda)d\lambda$$
$$\vdots$$

From these, the chromaticity coordinates x, y, z are computed by:

$$x = \frac{X}{X+Y+Z}$$
$$\vdots$$

Note: x, y, z are all on the $X + Y + Z = 1$ plane.

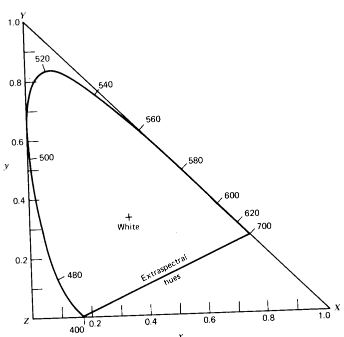
Visualizations of the CIE color space



Visualizations fo the CIE color space (Foley, II-1)

The CIE chromaticity diagram

The CIE chromaticity diagram is the projection of the $X + Y + Z = 1$ plane onto the (X, Y) plane:

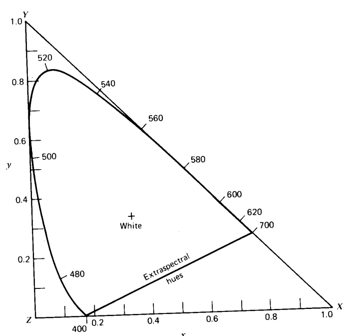


The chromaticity diagram (Wasserman 3-7)

Each point of the diagram gives a chromaticity value, which depends on:

- the "dominant wavelength" or "hue" — red, yellow, etc.;
- the "excitation purity" or "saturation" — closeness to grey.

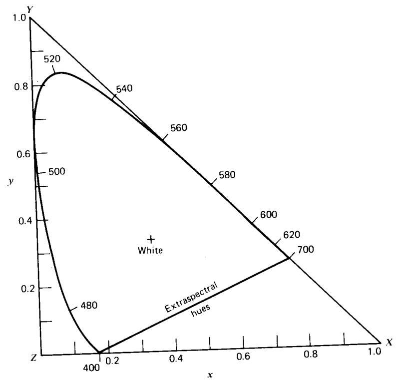
The CIE diagram, cont.



The chromaticity diagram (Wasserman 3-7)

- *Dominant wavelengths* go around the outside of the diagram.
- *Excitation purity* is given by the ratio AC/BC, where
  - · C is white light;
  - · A is color being tested;
  - · B is extrapolation of CA to curve — i.e., the dominant wavelength.

The CIE diagram, cont.



The chromaticity diagram (Wasserman 3-7)

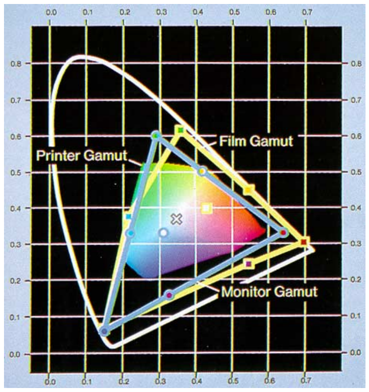
- "Complementary colors"
  - Colors that can be mixed to produce white light.
  - Lie on opposite sides of C.

- "Nonspectral colors"
  - Lie on wedge that does not project to any dominant wavelength.
  - Reds, magentas, purples

Gamuts

A display device has a range of reproducible colors that depend on the spectra reproduced by the phosphors/pigments/etc.

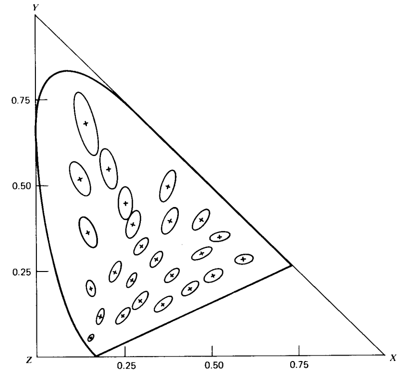
This range or reproducible colors is called the gamut of the device.



Gamuts of a monitor and a printer illustrated in CIE coordinates (Foley, II-2)

Perceptual uniformity

The XYZ space is not perceptually uniform:



Areas of constant color (enlarged) (Wasserman 3-10)

Perceptually-uniform color spaces

Some perceptually-uniform spaces include:

- $L^*u^*v^*$
- $L^*a^*b^*$
- "Farnsworth's non-linear transformation"

The first two of these involve taking cube roots, etc.

Formulas for $L^*u^*v^*$ and $L^*a^*b^*$ in terms of XYZ are given in [Glassner].

Main point is that Euclidean distance is supposed to work better in these spaces.