Computer Graphics
CSE 557, Winter 1997

Study Sheet - Geometric Transformations

For the first two problems, your affine transformations should be compositions of translations, rotation (about the x, y or z axis), scalings, shears, and reflections (about the x, y, or z axis).

1. Find an affine transformation to rotate points in two dimensions about a point P other than the origin.
2. In two dimensions, we write a line as $y = mx + h$. Find an affine transformation to reflect 2-dimensional points about this line. Extend your result to reflection about a plane in 3 dimensions.
3. Give an example of an affine space other than that obtained by taking points and vectors in R^n.
4. Show that the following identities hold for all points P, Q and vectors $\vec{v}$ in an affine space. (Use the axioms relating points and vectors that we gave in class.)
   (a) $\vec{v} + (Q - R) = (Q + \vec{v}) - R$
   (b) $P = Q + (P - Q)$
5. Show that translation along a fixed vector $\vec{t}$, i.e.,
$$T(P) = P + \vec{t}$$
   preserves affine combinations. In other words, show that for every pair of points P_1, P_2, and for every α_1, α_2 such that $\alpha_1 + \alpha_2 = 1$
$$T(\alpha_1 P_1 + \alpha_2 P_2) = \alpha_1 T(P_1) + \alpha_2 T(P_2).$$
6. What might be an advantage of a barycentric coordinates representation in computer graphics?
7. Show that an affine map is completely determined if the image of an n-simplex is known.
8. Suppose that $S = (Q_0, \ldots, Q_n)$ is an arbitrary simplex in an affine space $\mathcal{A}$ and that $S' = (Q'_0, \ldots, Q'_n)$ is arbitrary collection of points in $\mathcal{B}$, not necessarily forming a simplex. Show that there is a unique affine map from $\mathcal{A}$ to $\mathcal{B}$ that carries S into S'.
9. Show that if Q lies on the line segment between Q_1 and Q_2 and breaks it into segments of relative ratios t:1-t, then $F(Q)$ lies on the line segment between $F(Q_1)$ and $F(Q_2)$ and breaks it into segments of relative ratios t:1-t, if F is an affine map.
10. Do affine transformations preserve ratios between the lengths of unrelated line segments? If so, prove it. If not, give a counterexample.

11. Do affine transformations preserve angles between line segments? If so, prove it. If not, give a counterexample.

12. Show that affine transformations preserve parallelism.

13. How does the fact that affine transformations map lines into lines help us in computer graphics?

14. Show that the definition of the action of F, an affine map, on vectors is well defined in the sense that if P, Q, P', Q' are two pairs of points such that $P - Q = P' - Q'$, then $F(P) - F(Q) = F(P') - F(Q')$. d

15. Show that orthographic projection preserves affine combinations and therefore is an affine map.

16. Give a matrix for doing projection normalization (i.e., performs perspective transformation, but maintains relative depth ordering of objects). Specify what assumptions you are making about the view volume.

17. Give three advantages of using homogeneous coordinates in computer graphics.

18. Show that projective transformations map hyperplanes into hyperplanes.

19. Explain the problems that arise when a normal vector (normal to a surface at some point) is transformed just like any other vector by an affine map.

20. Give the equation of the plane in 3 dimensions that passes through the point P and is perpendicular to the vector $\vec{v}$.

21. Just for fun: Another common model of the projective plane is the spherical model. In this model, the projective plane is the surface of a sphere with diametrically opposite points identified. The lines of the projective plane are represented by the great circles of the sphere, again with opposite points identified. The spherical model clearly shows that all lines and poitns are equivalent in their topological and incidence properties. (The special role that the line and points at infinity seem to play in the standard model is a mere artificat of their representation in that model.) A point (a, b) in the projective plane corresponds to the line (wa, wb, w) in homogeneous coordinates. Come up with a natural correspondence between points and lines in the projective plane and points and lines in the spherical model. (Hint: take the projective plane to be the plane at $w = 1$ in homogeneous coordinates space, and consider the sphere to be the sphere of diameter 1 whose center is at the origin.)

22. Do any problem in Foley et al, chapters 5 or 6.