Study Sheet - Numerical Methods

1. What is the 1-dimensional Haar wavelet transform of the 8-pixel image [3 5 2 1 0 8 6 8]?

2. Suppose you construct the 1D Haar wavelet transform described in class on a 64 pixel one-dimensional image, and set to 0 all values below 0.01. Suppose you then took the resulting wavelet transform and reconstructed the image. Give the best upper bound you can on $\|c[i] - \tilde{c}[i]\|$, where $c[i]$ is the original color of pixel i and $\tilde{c}[i]$ is the reconstructed color of pixel i.

3. What is a scaling function? What is a wavelet? How do you use wavelet transforms to construct lower resolution versions of an object?

4. Explain why it's desirable to have all the wavelets in the basis orthogonal to all scaling functions.

5. Explain why it might be desirable for the wavelets to have vanishing moments. For example, why might it be desirable to have
$$\int_{-\infty}^{\infty} \Psi(x)dx = 0$$
for eeach wavelet $\Psi(x)$.

6. What is the direction of steepest descent for a function? Why is that direction chosen for line minimization?

7. Explain the principle underlying the conjugate gradient method.

8. Suppose you are told that a certain function takes on the value y_i when it's input is equal to x_i, for $1 \leq i \leq m$. Suppose you want to find a line $y = ax + b$ that best fits this data. Set the problem up as a least squares problem.

9. Explain how you might use simulated annealing to help with solving the mesh optimization problem.

10. What happens in simulated annealing if you run the simulation at a very low temperature for a very long time? Why do people use a cooling schedule instead of just starting out with a very low fixed temperature?

11. Prove that the solution of $A^T Ax = A^T b$ is the least squares solution of $Ax = b$.