7.2 Radiosity

Reading

Recommended:

- Foley, et al., Section 16.13

Optional:

- F. E. Nicodemus, et al. *Geometric Considerations and Nomenclature for Reflectance.* NBS Monograph 160. 1977.
- M. F. Cohen and J. R. Wallace. *Radiosity and Realistic Image Synthesis*. Academic Press. 1993.
- F. X. Sillion and C. Puech. *Radiosity and Global Illumination*. Morgan Kaufmann. 1994.
- C. M. Goral, K. E. Torrance, D. P. Greenberg, et al. Modeling the interaction of light between diffuse surfaces. SIGGRAPH '84, 213-222.
- J. Kajiya. The rendering equation. SIGGRAPH '86, 143–150.
- P. Hanrahan, P. Salzman, and L. Aupperle. A rapid hierarchical radiosity algorithm. SIGGRAPH '91, 197–206.

Outline

1. Radiometric and photometric units
2. U-Derive Radiosity
3. Methods for solving the radiosity equation

Radiometry and photometry

Two related disciplines:

Radiometry	**Photometry**
The measurement of electromagnetic radiation.	The measurement of visible light.

Very similar, but each has its own names and units.

Radiative units

Let Q denote *stuff*, like volume, mass, charge, radiant energy, or luminous energy.

The flow rate of stuff dQ/dt is called *flux*, denoted by Φ.

Radiometry		**Photometry**	
Radiant energy	[joule (J)]	*Luminous energy*	[talbot (T)]
Radiant flux	[watt (W)]	*Luminous flux*	[lumen (lm)]

Conversions:

$$\begin{aligned} watt &= joule/second \\ lumen &= talbot/second \\ erg &= 10^{-7}\ joule \\ calorie &\approx 4.184\ joule \end{aligned}$$

Converting from radiometry to photometry

The *relative spectral luminous efficiency curve* can be used to convert from radiometric to photometric units:

To convert, just use the integral

$$\Phi_v = \int \Phi_e(\lambda) V(\lambda) d\lambda \times [680\ lumen/watt]$$

Flux density

Flux density is the derivative of flux with respect to surface area $d\Phi/dA$, denoted by E for *incoming*, or M or B for *outgoing*.

Radiometry		**Photometry**	
Irradiance	$[W/m^2]$	*Illumination* $\equiv$ *Illuminance*	[*lux* (lx)]
Radiant exitance $\equiv$ *Radiosity*	$[W/m^2]$	*Luminous exitance*	[*lx*]

Conversions:

$$\begin{aligned} lux &= lumen/meter^2 \\ footcandle &= lumen/foot^2 \\ phot &= 10^4\ lux \\ nox &= 10^{-3}\ lux \end{aligned}$$

Angles and solid angles

$$\text{angle } \alpha = \frac{subtended\ arc\ length}{radius}\ [radians]$$

$$\text{solid angle } \omega = \frac{projected\ surface\ area}{radius^2}\ [steradians\ (sr)]$$

Example: Solid angle of the sun is 6×10^{-5} *steradians*.

Intensity

Intensity I is the derivative of flux with respect to solid angle $d\Phi/d\omega$.

Radiometry	**Photometry**
Radiant intensity $[W/sr]$	*Luminous intensity* [candela (cd)] $\equiv$ *Candlepower*

Conversions:

$$\begin{aligned} candela &= lumen/steradian \\ carcel &\approx 9.6\ candela \\ hefner &\approx 0.9\ candela \end{aligned}$$

Useful for point sources, or extended sources at a sufficient distance.

Angular flux density

Angular flux density L is the derivative of flux density with respect to "projected solid angle" $d\Phi/(d\overline{A} \cdot d\overline{\omega})$.

Radiometry	**Photometry**
Radiance $[W/m^2 \cdot sr]$	*Luminance* [*nit* (nt)]

Angular flux density, cont.

Conversions:

$$\begin{aligned} nit &= cd/m^2 = lx/sr = lm/m^2 sr \\ stilb &= cd/cm^2 \\ apostilb = blondel &= cd/\pi m^2 \\ lambert &= cd/\pi cm^2 \\ foot\text{-}lambert &= cd/\pi\ ft^2 \\ skot &= 10^{-3} apostilb \\ glim &= 10^{-3} foot\text{-}lambert \end{aligned}$$

Importance of radiance

Radiance is fundamental to surface-to-surface light transport for two reasons:

1. A sensor's response is proportional to total integrated flux, and therefore radiance
2. Radiance is preserved along a ray

Proof: Left as an exercise.

Summary of terminology

Radiometry		**Photometry**	
Radiant energy	[J]	*Luminous energy*	[T]
Radiant flux	[W]	*Luminous flux*	[lm]
Irradiance	$[W/m^2]$	*Illumination*	[lx]
Radiant exitance	$[W/m^2]$	*Luminous exitance*	[lx]
Radiant intensity	$[W/sr]$	*Luminous intensity*	[cd]
Radiance	$[W/m^2 \cdot sr]$	*Luminance*	[nt]

Reflection

Suppose we fix incoming and outgoing directions ω and ω', and look at how radiance varies with irradiance:

$$dL(x, \omega') \propto dE(x, \omega)$$

The constant of proportionality is the BRDF.

Definition of BRDF

The *bidirectional reflected-radiance-distribution function* (BRRDF) or *bidirectional reflectance-distribution function* (BRDF) for short (!) is given by

$$f_r(x, \omega \to \omega') = \frac{dL_r(x, \omega')}{dE_i(x, \omega)} = \frac{dL_r(x, \omega')}{L_i(x, \omega) \cos\theta_i \, d\omega}$$

It has units [_____].

The BRDF

The BRDF of a surface depends on:

- Material properties
- Surface finish

The BRDF is a function of wavelength.

In general, as roughness increases, reflection goes from specular to diffuse:

Intermediate type known as "directional diffuse," "rough specular," or "gloss."

Example: Lambertian reflector

The BRDF of a Lambertian reflector is uniform in all directions:

$$f_r(x, \omega \to \omega') \;=\; k_d$$

Models reflectance from a very rough surface.

Reflectance

The *reflectance* ρ is defined as the ratio of reflected flux to incident flux:

$$\rho(x, \omega \to \omega') \;=\; \frac{d\Phi_r(x, \omega')}{d\Phi_i(x, \omega)} \;=\; \frac{\int_{\omega'} L_r(x, \omega') \cos\theta' d\omega'}{\int_{\omega} L_i(x, \omega) \cos\theta d\omega}$$

The terms *directional* (for $d\omega$), *conical* (for $\Delta\omega$), and *hemispherical* (for 2π) describe the limits of integration of the denominator and numerator, respectively.

Conservation of energy is ensured if the bihemispherical reflectance is less than 1.

Diffuse reflectance

For Lambertian reflectors,

$$f_r(x, \omega \to \omega') \;=\; f_r(x) \;=\; \frac{L_r(x)}{E_i(x)}$$

So the reflectance and BRDF are related by

$$\rho(x) \;=\; \frac{d\Phi_r(x)}{d\Phi_i(x)}$$

$$=\; \pi f_r(x)$$

Group experience: U-Derive Radiosity

Assumptions:

- n polygonal "patches"
- No occlusion
- Radiosity constant over each patch
- All reflection ideal diffuse (Lambertian):

"Intensity" $= k$ $\qquad$ *Energy* $\propto \cos\theta$

Note: $\int_\Omega \cos\theta d\omega = \pi$

Definitions

Symbol	Meaning
A_i	Area (m^2) *Area of surface i*
E_i	Emitted radiosity (W/m^2) *Emitted flux density of surface i*
B_i	Radiosity (W/m^2) *Total flux density of surface i*
ρ_i	Reflectance (dimensionless) *Fraction of incident light reflected back into environment*
F_{ij}	Form factor (dimensionless) *Fraction of energy leaving surface i that lands on surface j*

The form factor

F_{ij} = *Fraction of energy leaving surface i that lands on surface j*

$$F_{ij} = \frac{\quad}{\quad}$$

The form factor

$$F = \frac{\quad}{\quad}$$

$$A_i F_{ij} = A_j F_{ji}$$

The radiosity equation

$$B_i A_i = \frac{\quad}{\quad} + \frac{\quad}{\quad} \sum_j \frac{\quad}{\quad}$$

Last step of derivation

Rewriting radiosity equation using reciprocity gives:

$$B_i \;=\; E_i \;+\; \rho_i \sum_j B_j F_{ij}$$

...a thorougly confusing formula!

In matrix form:

$$\mathbf{K}B \;=\; E$$

Solution methods

Setting up and solving this system of equations is known as the "global illumation problem."

The process involves two steps:

1. Estimating the form factors
2. Solving the resulting system of equations

We'll just look at the second step here. . . .

Solving the system of equations

Gauss-Seidel iteration (1984)

Formulation:

- n equations and n unknowns
- $O(n^2)$ interactions
- $O(n^3)$ solution time

"Advantages":

- View independence
- Can change ρ and S without recomputing form factors

Solving the system of equations, cont.

Adaptive meshing (1986)

Adaptive meshing (Cohen & Wallace, fig 6.24)

- Meshes adaptively in areas of high intensity gradient
- Good for capturing shadows

Solving the system of equations, cont.

Progressive radiosity (1988)

Gathering vs. shooting (Cohen & Wallace, figs. 5.2, 5.5)

- "Shooting" instead of "gathering"
- Still $O(n^2)$ interactions in all
- But useable solutions with just $O(n)$ interactions

Solving the system of equations, cont.

Hierarchical radiosity (1991)

Hierarchical interactions (Hanrahan et al., fig 3)

- $O(m^2 + n)$ interactions, where:
  - m is number of initial patches
  - n is total number of elements

Summary

What to take home from this lecture:

1. The relationship between radiometry and photometry
2. Definitions of basic radiometric units (p15), BRDF's, and reflectance
3. Definition of form factors
4. The radiosity equation:
   - What it represents
   - How it's derived
   - Methods for solving it