5.2 Color Theory

Reading

Suggested:

- Glassner, Sections 1.7, 2.1, 2.2, 3.6
- Foley et al., Sections 13.2–13.6

Further reading:

- Michael Wilcox. *Blue and Yellow Don't Make Green*. North Light Books, Cincinnati, 1987.
- Gerald S. Wasserman. *Color Vision: An Historical Introduction*. John Wiley & Sons, New York, 1978.
- John E. Kaufman, ed. *IES Lighting Handbook: Reference Volume*. Illuminating Engineering Society, New York, 1981.

Overview

1. Physics of light
2. Historical development of color theory
3. The CIE XYZ color system
4. Color spaces for computer graphics

Theories of light

Many theories have been advanced:

- Wave theory [Hooke, Huygens, 1600's]
  Light transmitted through the "aether" in waves.
- Corpuscular theory [Newton, 1600's]
  Light propagated as minute particles, ejected intermittently in straight lines.
- Electromagnetic theory [Maxwell, 1888]
  Waves again, propagated by coupled electric and magnetic fields: $\nu = c\lambda$
- Quantum theory [Planck, Einstein, 1900]
  Particles again: $Q = h\nu$
- Unified theory [De Broglie, Heisenberg, 1913]
  Both: Waves *and* particles!

The radiant energy spectrum

The wave theory allows a nice arrangement of electromagnetic radiation (EMR) according to wavelength:

The EMR spectrum (IES, 2-1)

Note:

- All EMR travels at same speed in a given medium.
- Energy proportional to frequency.

Emission spectra

A light source can be characterized by an emission spectrum:

The spectrum describes the energy at each wavelength.

Newton's experiments on color

Newton was the first to perform scientific experiments on color, in 1666.

Built a simple colorimeter:

- Hole in a shutter
- Prism to disperse white light into spectrum
- Comb-shaped aperture to manipulate spectrum
- Converging lens to recombine spectrum

Newton's experiments, cont.

Newton defined two types of light:

- Simple: Light that cannot be further dispersed by a prism (now called monochromatic).
- Compound: Light that can be dispersed.

He called the colors of simple lights primaries.

[This term has many other meanings today.]

Newton's barocentric model

Newton's barcentric model (Wasserman, 2-1)

Newton proposed a "barocentric model of color vision":

- Factored out brightness
- Identified the endpoints of the spectrum
- Placed white in the center
- Gave a qualitative model of light mixture

Q: Why did Newton choose seven divisions, or "tones"?

Subtractive mixtures

Two examples of subtractive color mixture (Wasserman, 2-2)

Newton also characterized the difference between additive and subtractive color mixtures.

Subtractive color mixtures:

- Due to selective absorption by pigments.
- Difficult to characterize in general.

Additive mixtures

Newton posed a fundamental question that took almost two centuries to answer:

Q: How many primaries does it take to produce a perfect white?

A: (Newton) Definitely more than 2; and 4 or 5 suffice.

In 1852, Helmholtz answered a generalized form of this question:

Q: How many primaries does it take to produce the *entire* spectrum?

A: (Newton) 7.

A: (Young, 1802) 3.

A: (Helmholtz) 5.

Trivariant mixtures

Illustration of trivariant color mixing (Wasserman, 2-3)

In 1856, Maxwell rescued the notion of trivariant color mixing by applying Newton's barocentric mixing more rigorously.

Q: How many primaries are required by Maxwell's model?

Q: In what sense can three primaries produce all colors?

Maxwell's imaginary primaries

A color space associated with imaginary primaries (Wasserman, 2-5)

Maxwell observed that with three "imaginary" primaries, you no longer need to use one as a desaturant.

Q: But how do you select these imaginary primaries? (*Last* 100 years devoted to answering this.)

Wright's experiments

In the late 20's, Wright found that the colors of all wavelengths could be reproduced with combinations of 3 primaries at 460, 530, and 650nm:

Relative luminances for metameric matches (Wasserman, 3-3)

Q: Why is there a double peak for the red primary?

Q: Suppose you change the set of primaries. Is there a way to avoid repeating the experiment?

Transforming primaries

Transforming primaries (Wasserman, 3-6)

A transformation of primaries can be described as a simple change of basis. . . .

Given: Color X in the ABC system:

$$X = aA + bB + cC$$

Find: Color X in the DEF system:

$$X = dD + eE + fF$$

Transforming primaries, cont.

Solution: Write A, B, C in terms of D, E, F:

The XYZ system

In 1931, the CIE (Commission Internationale de l'Eclairage) adopted the XYZ system, which has the following properties:

- Every color can be made with *positive* combinations of primaries called $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{Z}$.
- The $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$ primaries are imaginary.

The CIE color-matching functions

Here are the color-matching functions $\overline{x}, \overline{y}, \overline{z}$:

The color-matching functions (1.41)

Note: By design, the $\overline{y}$ curve is just the "luminous efficiency curve."

Computing CIE coordinates

The X, Y, Z coordinates are computed by:

$$X = \int I(\lambda)\overline{x}(\lambda)d\lambda$$
$$\vdots$$

From these, the chromaticity coordinates x, y, z are computed by:

$$x = \frac{X}{X+Y+Z}$$
$$\vdots$$

Note: x, y, z are all on the $X+Y+Z=1$ plane.

The CIE chromaticity diagram

The CIE chromaticity diagram is the projection of the $X+Y+Z=1$ plane onto the (X,Y) plane:

The chromaticity diagram (1.42)

Each point of the diagram gives a chromaticity value, which depends on:

- the "dominant wavelength" or "hue" — red, yellow, etc.;
- the "excitation purity" or "saturation" — closeness to grey.

The CIE diagram, cont.

The CIE diagram (1.42)

- *Dominant wavelengths* go around the outside of the diagram.

- *Excitation purity* is given by the ratio AC/BC, where
  - C is white light;
  - A is color being tested;
  - B is extrapolation of CA to curve — i.e., the dominant wavelength.

The CIE diagram, cont.

The CIE diagram (1.42)

- "Complementary colors"
  - Colors that can be mixed to produce white light.
  - Lie on opposite sides of C.

- "Nonspectral colors"
  - Lie on wedge that does not project to any dominant wavelength.
  - Reds, magentas, purples

Perceptual uniformity

The XYZ space is not perceptually uniform:

Areas of constant color (enlarged) (2.1)

Perceptually-uniform color spaces

Some perceptually-uniform spaces include:

- $L^*u^*v^*$
- $L^*a^*b^*$
- "Farnsworth's non-linear transformation"

The first two of these involve taking cube roots, etc.

Formulas for $L^*u^*v^*$ and $L^*a^*b^*$ in terms of XYZ are given in [Glassner].

Main point is that Euclidean distance is supposed to work better in these spaces.

Other color spaces

Other color spaces commonly used in computer graphics include:

- RGB — most natural for CRTs
- HSV — more intuitive for users
- CMY — used for hardcopy
- YIQ — used for TV broadcasting

RGB

You're already familiar with this one:

Q: Where are the greys?

HSV

A more intuitive color space.

Think of shower controls. . .

Most naturally represented by a cone:

- "Hue" — red, yellow, etc.
- "Saturation" — Distance from grey
- "Value" — Distance from black

Note: Value is not perceptually-based, so colors of the same value may have slightly different brightnesses.

CMY

A subtractive color space, used for printing.

Involves three subtractive primaries:

- Cyan — subtracts red
- Magenta — subtracts green
- Yellow — subtracts blue

Mixing two pigments subtracts their opposites from white.

Printers often use *black* ink rather than equal amounts of all three.

YIQ

Used in TV broadcasting. Exploits useful properties of the visual system:

- Uses most bandwidth for Y, the luminance
  - Exploits ability to judge relative luminance better than hue and saturation changes
  - Allows b&w TVs to just look at Y channel
- I chosen as major axis of remaining visual color space
- Q is the remaining axis
- Relative bandwidth ratios are 8:3:1