# Shading

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**CSE 557**
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## Reading

Optional:

- Angel and Shreiner: chapter 5.
- Marschner and Shirley: chapter 10, chapter 17.

Further reading:

- OpenGL red book, chapter 5.

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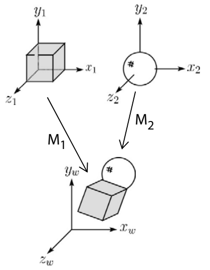
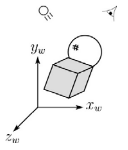
## Basic 3D graphics

With affine matrices, we can now transform virtual 3D objects in their local coordinate systems into a global (world) coordinate system:



To synthesize an image of the scene, we also need to add light sources and a viewer/camera:



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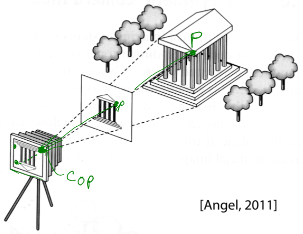
## Pinhole camera

To create an image of a virtual scene, we need to define a camera, and we need to model lighting and shading. For the camera, we use a **pinhole camera**.



[Angel, 2011]

The image is rendered onto an **image plane** (usually in front of the camera).

Viewing rays emanate from the **center of projection** (COP) at the center of the pinhole.

The image of an object point **P** is at the intersection of the viewing ray through **P** and the image plane.

But is **P** visible? This the problem of **hidden surface removal** (a.k.a., **visible surface determination**). We'll consider this problem later.

## Shading

Next, we'll need a model to describe how light interacts with surfaces.

Such a model is called a **shading model**.

Other names:

- Lighting model
- Light reflection model
- Local illumination model
- Reflectance model
- BRDF

## An abundance of photons

Given the camera and shading model, properly determining the right color at each pixel is *extremely hard.*

Look around the room. Each light source has different characteristics. Trillions of photons are pouring out every second.

These photons can:

- interact with molecules and particles in the air ("participating media")
- strike a surface and
  - be absorbed
  - be reflected (scattered)
  - cause fluorescence or phosphorescence.
- interact in a wavelength-dependent manner
- generally bounce around and around

## Our problem

We're going to build up to *approximations* of reality called the **Phong** and **Blinn-Phong illumination models**.

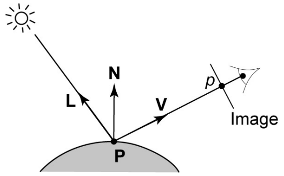
They have the following characteristics:

- *not* physically correct
- gives a "first-order" *approximation* to physical light reflection
- very fast
- widely used

In addition, we will assume **local illumination**, i.e., light goes: light source -> surface -> viewer.

No interreflections, no shadows.

## Setup…



Given:

- a point $\mathbf{P}$ on a surface visible through pixel *p*
- The normal $\mathbf{N}$ at $\mathbf{P}$
- The lighting direction, $\mathbf{L}$, and (color) intensity, $I_L$, at $\mathbf{P}$
- The viewing direction, $\mathbf{V}$, at $\mathbf{P}$
- The shading coefficients at $\mathbf{P}$

Compute the color, $I$, of pixel $p$.

Assume that the direction vectors are normalized:

$$\|\mathbf{N}\| = \|\mathbf{L}\| = \|\mathbf{V}\| = 1$$

## “Iteration zero”

The simplest thing you can do is…

Assign each polygon a single color:

$$I = k_e$$

where

- $I$ is the resulting intensity
- $k_e$ is the **emissivity** or intrinsic shade associated with the object

This has some special-purpose uses, but not really good for drawing a scene.

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## “Iteration one”

Let’s make the color at least dependent on the overall quantity of light available in the scene:

$$I = k_e + k_a I_{La}$$

- $k_a$ is the **ambient reflection coefficient**.
    - really the reflectance of ambient light
    - “ambient” light is assumed to be equal in all directions
- $I_{La}$ is the **ambient light intensity**.

Physically, what is “ambient” light?

Poor man's interreflection

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## Wavelength dependence

Really, $k_e$, $k_a$, and $I_{La}$ are functions over all wavelengths $\lambda$.

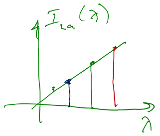
Ideally, we would do the calculation on these functions. For the ambient shading equation, we would start with:

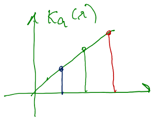
$$I(\lambda) = k_a(\lambda) I_{La}(\lambda)$$

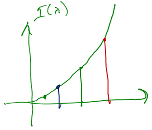
then we would find good RGB values to represent the spectrum $I(\lambda)$.

Traditionally, though, $k_a$ and $I_{La}$ are represented as RGB triples, and the computation is performed on each color channel separately:

$$I^R = k_a^R \, I_{La}^R$$
$$I^G = k_a^G \, I_{La}^G$$
$$I^B = k_a^B \, I_{La}^B$$







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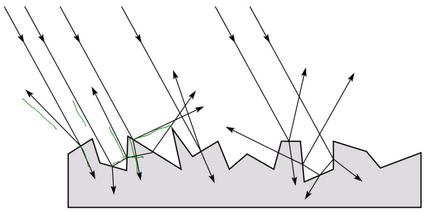
## Diffuse reflectors

Emissive and ambient reflection don’t model realistic lighting and reflection. To improve this, we will look at **diffuse** (a.k.a., **Lambertian**) reflection.

Diffuse reflection can occur from dull, matte surfaces, like latex paint, or chalk.

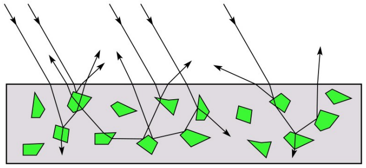
These diffuse reflectors reradiate light equally in all directions.

Picture a rough surface with lots of tiny **microfacets**.



## Diffuse reflectors

…or picture a surface with little pigment particles embedded beneath the surface (neglect reflection at the surface for the moment):



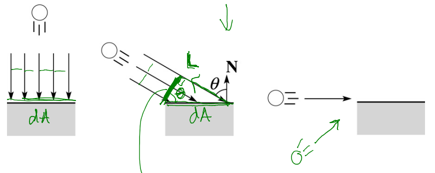
The microfacets and pigments distribute light rays in all directions.

Embedded pigments are responsible for the coloration of diffusely reflected light in plastics and paints.

Note: the figures in this and the previous slide are intuitive, but not strictly (physically) correct.

## Diffuse reflectors, cont.

The reflected intensity from a diffuse surface does not depend on the direction of the viewer. The incoming light, though, does depend on the direction of the light source:



$I \sim dA \cos\theta$

$I \sim \cancel{\cos\theta = N \cdot L}$

$B \cos\theta = B(N \cdot L)$

$$B = \begin{cases} 1 & N \cdot L > 0 \\ 0 & N \cdot L \le 0 \end{cases}$$

## "Iteration two"

The incoming energy is proportional to $\underline{\cos\theta}$, giving the diffuse reflection equations:

$$I = k_e + k_a I_{La} + k_d I_L B \, \underline{\cos\theta}$$

$$= k_e + k_a I_{La} + k_d I_L B(N \cdot L)$$

where:

- $k_d$ is the **diffuse reflection coefficient**
- $I_L$ is the (color) intensity of the light source
- $\mathbf{N}$ is the normal to the surface (unit vector)
- $\mathbf{L}$ is the direction to the light source (unit vector)
- $B$ prevents contribution of light from below the surface:

$$B = \begin{cases} 1 & \text{if } N \cdot L > 0 \\ 0 & \text{if } N \cdot L \le 0 \end{cases}$$

## Specular reflection

**Specular reflection** accounts for the highlight that you see on some objects.

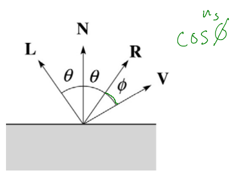
It is particularly important for *smooth, shiny* surfaces, such as:

- metal
- polished stone
- plastics
- apples
- skin

Properties:

- Specular reflection depends on the viewing direction $\mathbf{V}$.
- For non-metals, the color is determined solely by the color of the light.
- For metals, the color may be altered (e.g., brass)

## Specular reflection “derivation”



For a perfect mirror reflector, light is reflected about **N**, so

$$I = \begin{cases} I_L & \text{if } \mathbf{V} = \mathbf{R} \\ 0 & \text{otherwise} \end{cases}$$

For a near-perfect reflector, you might expect the highlight to fall off quickly with increasing angle $\phi$.

Also known as:

- **“rough specular” reflection**
- **“directional diffuse” reflection**
- **“glossy” reflection**

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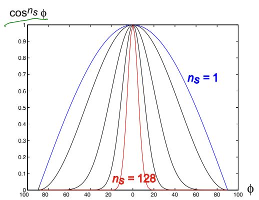
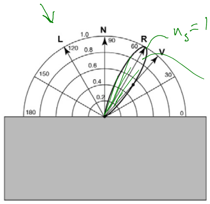
## Phong specular reflection

goniometric diagram





$n_s = 128$

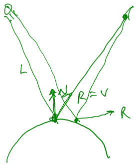
specular lobe

One way to get this effect is to take (**R·V**), raised to a power $n_s$.

Phong specular reflection is proportional to:

$$\underline{I}_{\text{specular}} \sim B\underbrace{(\mathbf{R} \cdot \mathbf{V})}_{}{}_{+}^{n_s}$$

where $\underline{(x)_+ \equiv \max(0, x)}$.



**Q**: As $n_s$ gets larger, does the highlight on a curved surface get smaller or larger?

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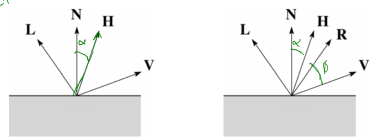
## Blinn-Phong specular reflection

A common alternative for specular reflection is the **Blinn-Phong model** (sometimes called the **modified Phong model**.)

We compute the vector halfway between **L** and **V** as:

$$H \sim \frac{1}{2}(L+V)$$

$$H = \frac{\frac{1}{2}(L+V)}{\|\frac{1}{2}(L+V)\|} = \frac{L+V}{\|L+V\|}$$



Analogous to Phong specular reflection, we can compute the specular contribution in terms of (**N·H**), raised to a power $n_s$:

$$I_{\text{specular}} \sim B\underbrace{(\mathbf{N} \cdot \mathbf{H})}_{}{}_{+}^{n_s}$$

where, again, $(x)_+ \equiv \max(0, x)$.

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## “Iteration three”

The next update to the Blinn-Phong shading model is then:

$$I = k_e + k_a I_{La} + k_d I_L B(\mathbf{N} \cdot \mathbf{L}) + k_s I_L B(\mathbf{N} \cdot \mathbf{H})_+^{n_s}$$

$$= k_e + k_a I_{La} + I_L B\left[k_d(\mathbf{N} \cdot \mathbf{L}) + k_s(\mathbf{N} \cdot \mathbf{H})_+^{n_s}\right]$$

where:

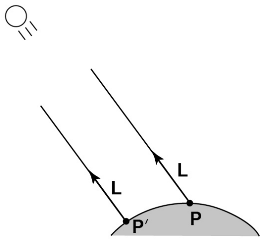
- $k_s$ is the **specular reflection coefficient**
- $n_s$ is the **specular exponent** or **shininess**
- **H** is the unit halfway vector between **L** and **V**, where **V** is the viewing direction.

## Directional lights

The simplest form of lights supported by renderers are ambient, directional, and point. Spotlights are also supported often as a special form of point light.

We've seen ambient light sources, which are not really geometric.

**Directional light** sources have a single direction and intensity associated with them.



Using affine notation, what is the homogeneous coordinate for a directional light? 0

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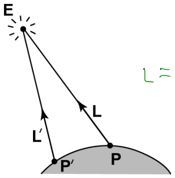
## Point lights

The direction of a **point light** sources is determined by the vector from the light position to the surface point.



$L = \frac{E-P}{\|E-P\|}$

Physics tells us the intensity must drop off inversely with the square of the distance:

$$f_{\text{atten}} = \frac{1}{r^2}$$

Sometimes, this distance-squared dropoff is considered too "harsh." A common alternative is:

$$f_{\text{atten}} = \frac{1}{a + br + cr^2}$$

with user-supplied constants for $a$, $b$, and $c$.

Using affine notation, what is the homogeneous coordinate for a point light? 1

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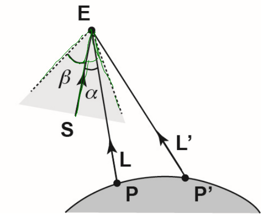
## Spotlights

We can also apply a *directional attenuation* of a point light source, giving a **spotlight** effect.



A common choice for the spotlight intensity is:

$$f_{\text{spot}} = \begin{cases} \dfrac{(\mathbf{L}\cdot\mathbf{S})^e}{a + br + cr^2} & \alpha \le \beta \\ 0 & \text{otherwise} \end{cases}$$

where

- $\mathbf{L}$ is the direction to the point light.
- $\mathbf{S}$ is the center direction of the spotlight.
- $\alpha$ is the angle between $\mathbf{L}$ and $\mathbf{S}$
- $\beta$ is the cutoff angle for the spotlight
- $e$ is the angular falloff coefficient

Note: $\alpha \le \beta \Leftrightarrow \cos^{-1}(\mathbf{L}\cdot\mathbf{S}) \le \beta \Leftrightarrow \mathbf{L}\cdot\mathbf{S} \ge \cos\beta$.

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## "Iteration four"

Since light is additive, we can handle multiple lights by taking the sum over every light.

Our equation is now (for spotlight lighting):

$$I = k_e + \sum_j k_a I_{La,j} + \frac{(\mathbf{L}_j \cdot \mathbf{S}_j)^{e_j}_{\beta_j}}{a_j + b_j r_j + c_j r_j^{\,2}} I_{L,j} B_j \left[ k_d (\mathbf{N}\cdot\mathbf{L}_j) + k_s (\mathbf{N}\cdot\mathbf{H}_j)_+^{n_s} \right]$$

This is the Blinn-Phong illumination model (for spotlights). Note that, in practice, we usually set $k_a = \underset{\smile}{k_d}$.

Which quantities are spatial vectors?

Which are RGB triples?

Which are scalars?

## Choosing Blinn-Phong shading parameters

Experiment with different parameter settings. To get you started, here are a few suggestions:

- Try $n_s$ in the range $[0, 100]$
- Try $k_a + k_d + k_s < 1$
- Use a small $k_a$ $(\sim 0.1)$

|  | $n_s$ | $k_d$ | $k_s$ |
|---|---|---|---|
| Metal | large | Small, color of metal | Large, color of metal |
| Plastic | medium | Medium, color of plastic | Medium, white |
| Planet | 0 | varying | 0 |

## BRDF

For more physical correctness, we would also weight the specular part by $\mathbf{N}\cdot\mathbf{L}$:

$$I = I_L B\left[k_d(\mathbf{N}\cdot\mathbf{L}) + k_s(\mathbf{N}\cdot\mathbf{L})\left(\mathbf{N}\cdot\frac{\mathbf{L}+\mathbf{V}}{\|\mathbf{L}+\mathbf{V}\|}\right)_+^{n_s}\right]$$

$$= I_L B(\mathbf{N}\cdot\mathbf{L})\left[k_d + k_s\left(\mathbf{N}\cdot\frac{\mathbf{L}+\mathbf{V}}{\|\mathbf{L}+\mathbf{V}\|}\right)_+^{n_s}\right]$$

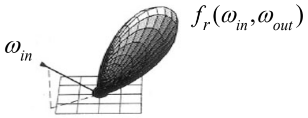
$$= I_L B(\mathbf{N}\cdot\mathbf{L})\, f_r(\mathbf{L},\mathbf{V})$$

The function $f_r$ maps incoming (light) directions $\omega_{in}$ to outgoing (viewing) directions $\omega_{out}$:

$$f_r(\omega_{in},\omega_{out}) \quad \text{or} \quad f_r(\omega_{in} \rightarrow \omega_{out})$$

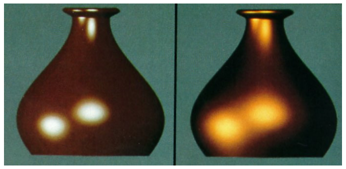
This function is called the **Bi-directional Reflectance Distribution Function** (**BRDF**).

Here's a plot with $\omega_{in}$ held constant:

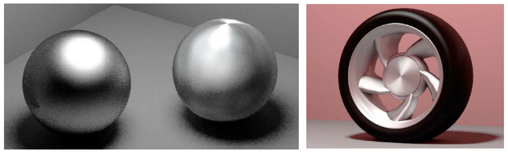


BRDF's can be quite sophisticated…

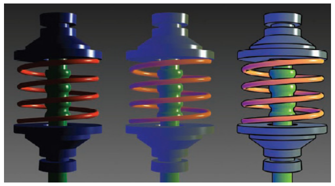
## More sophisticated BRDF's



[Cook and Torrance, 1982]

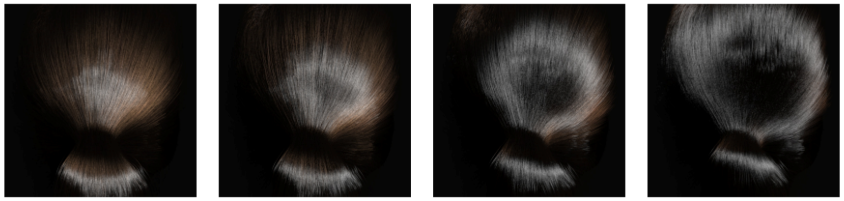


Anisotropic BRDFs [Westin, Arvo, Torrance 1992]

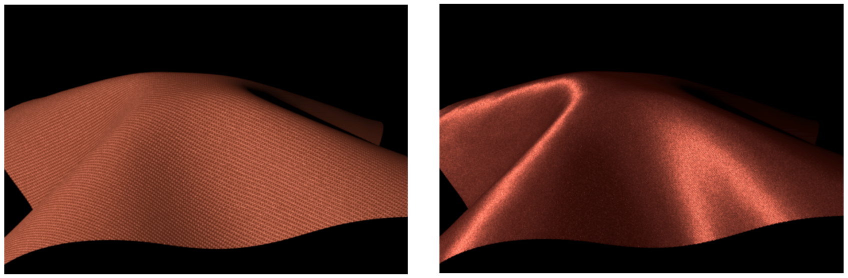


Artistics BRDFs [Gooch]

## More sophisticated BRDF's (cont'd)

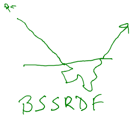


Hair illuminated from different angles [Marschner et al., 2003]



Wool cloth and silk cloth [Irawan and Marschner, 2012]

# BSSRDFs for subsurface scattering



[Jensen et al., 2001]