Lenses

The human eye employs a lens to focus light.

To quantify lens properties, we'll need some terms from optics (the study of sight and the behavior of light):

- **Focal point** - the point where parallel rays converge when passing through a lens.
- **Focal length** - the distance from the lens to the focal point.

By tracing rays through a lens, we can generally tell where an object point will be focused to an image point:

This construction leads to the Gaussian lens formula:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$
### Compound lenses

A compound lens is a sequence of simple lenses.

When simple, thin lenses are stacked on top of each other, it focuses much like a single lens. We can compute the focal length of the resulting compound lens as follows:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

It is convenient to define the **diopter** of a simple lens as the reciprocal of the focal length, measured in meters.

**Example:** A lens with a “power” of 10D has a focal length of 0.1m.

Why is this convenient?

### Structure of the eye

The most important structural elements of the eye are:

- **Cornea** - a clear coating over the front of the eye:
  - Protects eye against physical damage.
  - Provides initial focusing (40D).
- **Crystalline lens** – provides additional focusing
- **Retina** – layer of photosensitive cells lining the back of the eye.

We can treat the cornea + crystalline lens as a compound lens, which roughly follows the Gaussian lens formula. Again, this is:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Q: Given these three parameters, how does the human eye keep the world in focus?

Q: As an object moves closer, do the ciliary muscles contract or relax to keep the object in focus?
Structure of the eye

The remaining important elements are:

- Iris - Colored annulus with radial muscles.
- Pupil - The hole whose size is controlled by the iris.

The iris adjusts the size of the pupil according to the light levels in front of the subject.

Eye geometry

Eye geometry can account for near- and far- sightedness.

- Emmetropic eye - resting eye has focal point on retina.
- Myopic eye - eye too long.
- Hyperopic eye - eye too short.

Near- and far-sightedness can also result from deficiencies in focusing at the cornea or through the lens.

Presbyopia is loss of flexibility in the lens, reducing up-close focusing power. This happens naturally with age.

Q: Myopia and hyperopia and worse under low light. Why?

Retina

- Retina - a layer of photosensitive cells covering 200° on the back of the eye.
  - Cones - responsible for color perception.
  - Rods - Limited to intensity (but 10x more sensitive).
- Fovea - Small region (1 or 2°) at the center of the visual axis containing the highest density of cones (and no rods).

The human retina

Photomicrographs at increasing distances from the fovea. The large cells are cones; the small ones are rods. (Glassner, 1.5 and Wandell, 3.4)
The human retina, cont’d

Photomicrograph of a cross-section of the retina near the fovea (Wandell, 5.1).

Neuronal connections

Even though the retina is very densely covered with photoreceptors, we have much more acuity in the fovea than in the periphery.

In the periphery, the outputs of the photoreceptors are averaged together before being sent to the brain, decreasing the spatial resolution. As many as 1000 rods may converge to a single neuron.

Accuity across visual field

With one eye shut, look at the center dot with the other eye. At the right distance, all of these letters should appear equally legible (Glassner, 1.7).

High resolution imaging?

Given that our vision is only high resolution over a very small range of our visual field…

…how do we manage to see “everything” at high resolution?

Blind spot

Close your left eye and focus on the “+” with your right eye. At the right distance with the right head rotation, the black dot disappears.
Fixations and saccades

By scanning our eyes over a scene, we build a composite, high resolution image in our brain.

**Fixations**: our eyes pause at certain location to see the detail; these pauses are called **fixations**. **Saccades**: between fixations, we scan rapidly with very jittery motion.

Through gaze tracking, scientists can study how we look at the world.

---

Perceptual light intensity

The human eye is highly adaptive to allow us a wide range of flexibility.

One consequence is that we perceive light intensity as we do sound, i.e., on a relative or logarithmic scale.

**Example**: The perceived difference between 0.20 and 0.22 is the same as between 0.80 and ______.

---

Lightness contrast and constancy

The apparent brightness of a region depends largely on the surrounding region.

The **lightness contrast** phenomenon makes a constant colored region seem lighter or darker depending on the surround:

The **lightness constancy** phenomenon makes a surface look the same under widely varying lighting conditions.
Lightness contrast and constancy

Checker Shadow Effect (Edward Adelson, 1995)

Adaptation

Adaptive processes can adjust the base activity ("bias") and scale the response ("gain").

Through adaptation, the eye can handle a large range of illumination:

<table>
<thead>
<tr>
<th>Background</th>
<th>Luminance (cd/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moonless overcast night</td>
<td>0.00003</td>
</tr>
<tr>
<td>Moonlit overcast night</td>
<td>0.003</td>
</tr>
<tr>
<td>Twilight</td>
<td>3</td>
</tr>
<tr>
<td>Overcast day</td>
<td>300</td>
</tr>
<tr>
<td>Day with sunlit clouds</td>
<td>30,000</td>
</tr>
</tbody>
</table>

Some of our ability to handle this range comes from our ability to control the iris (aperture) of our eyes, and the fact that we have different types of photoreceptors.

However, much of the range comes from the adaptability of the photoreceptors themselves.

Mach bands

Mach bands were first discussed by Ernst Mach, an Austrian physicist.

Appear when there are rapid variations in intensity, especially at C₀ intensity discontinuities:

And at C₁ intensity discontinuities:

Mach bands, cont.

Possible cause: lateral inhibition of nearby cells.

Q: What image processing filter does this remind you of?
The radiant energy spectrum

We can think of light as waves, instead of rays. Wave theory allows a nice arrangement of electromagnetic radiation (EMR) according to wavelength:

![Diagram of EMR spectrum]

Emission spectra

A light source can be characterized by an emission spectrum:

![Emission spectra for daylight and a tungsten lightbulb (Wandell, 4.4)]

The spectrum describes the energy at each wavelength.

What is color?

The eyes and brain turn an incoming emission spectrum into a discrete set of values. The signal sent to our brain is somehow interpreted as color.

Color science asks some basic questions:

- When are two colors alike?
- How many pigments or primaries does it take to match another color?

One more question: why should we care?

Photopigments

Photopigments are the chemicals in the rods and cones that react to light. Can respond to a single photon!

Rods contain rhodopsin, which has peak sensitivity at about 500nm.
Univariance

**Principle of univariance**: For any single photoreceptor, no information is transmitted describing the wavelength of the photon.

![Univariance](Image)

Photocurrents measured for two light stimuli: 550nm (solid) and 659 nm (gray). The brightnesses of the stimuli are different, but the shape of the response is the same. (Wandell 4.17)

What rods measure

A rod responds to a spectrum through its spectral sensitivity function, $p(\lambda)$.

![Rods Sensitivity](Image)

The response to a test light, $t(\lambda)$, is simply:

$$P_r = \int t(\lambda) p(\lambda) d\lambda$$

Suppose a rod sees three light spots:
455nm blue laser of amplitude 1.0
505nm green laser of amplitude 0.5
550nm yellow laser of amplitude 1.0
Will these spots look different?

Cone photopigments

Cones come in three varieties: L, M, and S.

![Cone Photopigments](Image)

Cones are active under high light levels, i.e., they are responsible for photopic vision.

What cones measure

Color is perceived through the responses of the cones to light.

The response of each cone can be written simply as:

$$L_r = \int t(\lambda) L(\lambda) d\lambda$$

$$M_r = \int t(\lambda) M(\lambda) d\lambda$$

$$S_r = \int t(\lambda) S(\lambda) d\lambda$$

These are the only three numbers used to determine color.
What cones measure

Consider the sensitivity spectra again:

Suppose we show three light spots with unit intensity lasers at 460nm, 540nm, and 620nm. What will the cones measure?

What if I put all of these together in one spot?

The color matching experiment

We can actually distinguish all of the individual wavelengths as different colors. Does this mean our eyes are full spectral sensors?

Unfortunately, no. To show this, we can perform a color matching experiment.

The idea is to see if we can match a given test light using a finite number of lights called primaries with power control knobs.

The primary spectra are \( a(\lambda), b(\lambda), c(\lambda), \ldots \)

The power knob settings are A, B, C, …

Matching the test light

With the knob settings, we can produce spectra of the form:

\[ Aa(\lambda) + Bb(\lambda) + Cc(\lambda) \]

Can we match the \( L, M, S \) responses of the test light?

First some notation:

\[ \langle f, h \rangle = \int f(\lambda) h(\lambda) d\lambda \]

\[ \langle \alpha f, h \rangle = \int (\alpha f(\lambda)) h(\lambda) d\lambda \]

\[ \langle f + g, h \rangle = \int (f(\lambda) + g(\lambda)) h(\lambda) d\lambda \]

\[ \langle \alpha f + \beta g, h \rangle = \]

Thus, choosing the primary knobs to match a test light amounts to multiplying a matrix!
Choosing Primaries

The primaries could be three color (monochromatic) lasers.

But, they can also be non-monochromatic, e.g., monitor phosphors from an old CRT:

\[ e(\lambda) = Rr(\lambda) + Gg(\lambda) + Bb(\lambda) \]

Emission spectra for RGB monitor phosphors (Wandell 8.3)

Emission Spectrum is not color

Clearly, information is lost in this projection step…

Different light sources can evoke exactly the same colors. Such lights are called metamers.

A dim tungsten bulb and an RGB CRT monitor set up to emit a metameric spectrum (Wandell 4.31)

Colored Surfaces

So far, we've discussed the colors of lights. How do surfaces acquire color?

A surface's reflectance, \( \rho(\lambda) \), is its tendency to reflect incoming light across the spectrum.

Reflectance is combined "subtractively" with incoming light. Actually, the process is multiplicative:

\[ I(\lambda) = \rho(\lambda)I(\lambda) \]

Subtractive Metamers

Reflectance adds a whole new dimension of complexity to color perception.

The solid curve appears green indoors and out. The dashed curve looks green outdoors, but brown under incandescent light.
Illustration of Color Appearance

- Illumination
  - Relative energy
  - Wavelength (nm)

- Reflectance
  - Relative energy
  - Wavelength (nm)

- Color signal

- Cone sensitivities
  - Relative sensitivity
  - Wavelength (nm)

- Cone absorptions
  - Relative absorption
  - Wavelength (nm)

How light and reflectance become cone responses (Wandell, 1992)

Human vision, perspective, and 3D

The human visual system uses a lens to collect light more efficiently, but records perspectively projected images much like a pinhole camera.

Q: Why did nature give us eyes that perform perspective projections?

Q: Do our eyes “see in 3D”?

3D Displays

So-called 3D displays are all the rage now for movies and soon for televisions.

Much of our perception of 3D comes from stereo vision: each eye sees a different view of the world.

So, to create the illusion of 3D, we only need to show each eye an image of a scene created from that eye’s point of view!