Surfaces of Revolution

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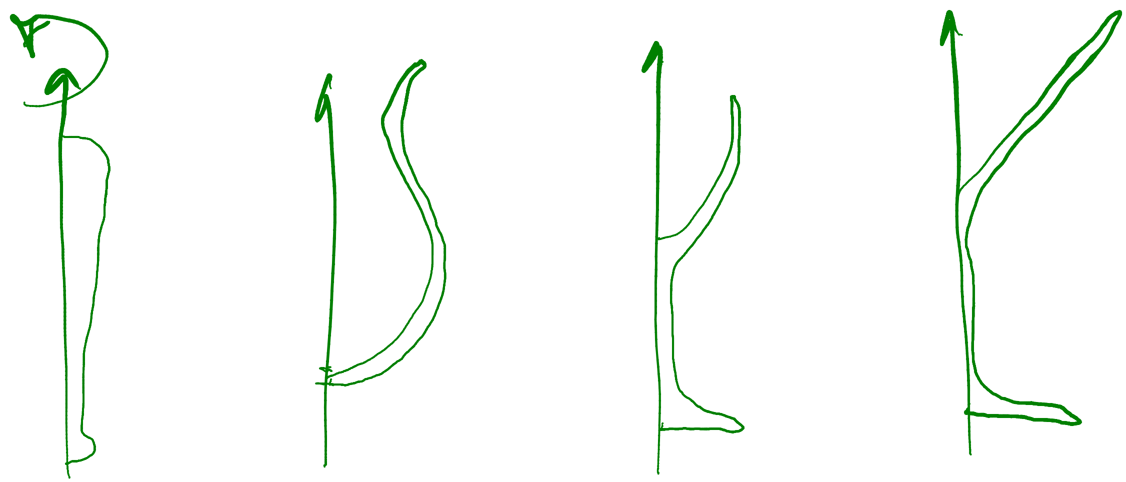
CSE 557
Fall 2014

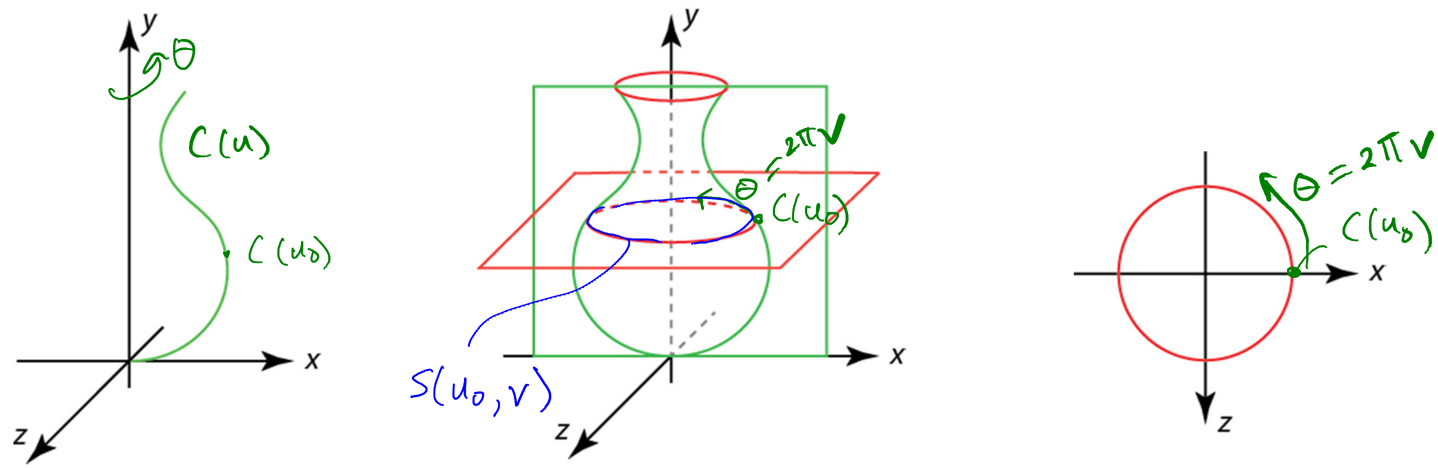
Surfaces of revolution



Idea: rotate a 2D **profile curve** around an axis.

What kinds of shapes can you model this way?

Constructing surfaces of revolution



Given: A curve $C(u)$ in the xy-plane:

$$C(u) = \begin{bmatrix} c_x(u) \\ c_y(u) \\ 0 \\ 1 \end{bmatrix}$$

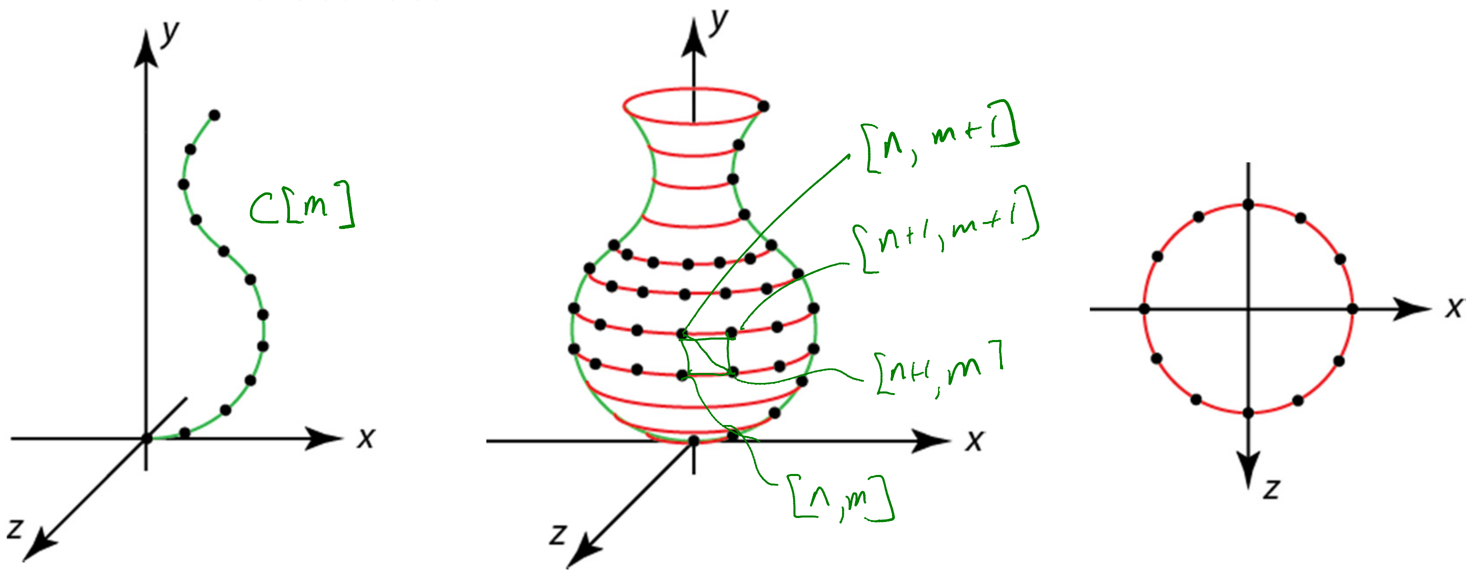
Let $R_y(\theta)$ be a rotation about the y-axis.

Find: A surface $S(u,v)$ which is $C(u)$ rotated about the y-axis, where $u, v \in [0, 1]$.

Solution: $S(u,v) = R_y(2\pi v)\, C(u)$

Constructing surfaces of revolution

We can sample in *u* and *v* to get a grid of points over the surface.



Suppose we sample:

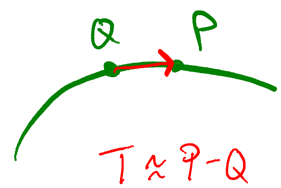
- in *u*, to give $C[m]$ where $m \in [0..M\text{-}1]$
- in *v*, to give rotation angle $\cancel{\phi}\ \theta[n] = 2\pi n / N$ where $n \in [0..N\text{-}1]$

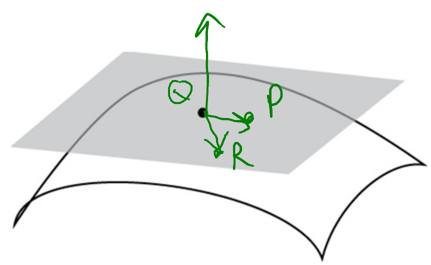
We can now write the surface as:

$$S[n,m] = R_y\left(\frac{2\pi n}{N}\right) C[m]$$

How would we turn this into a mesh of triangles?
How do we assign per-vertex normals?

Tangent vectors and tangent planes



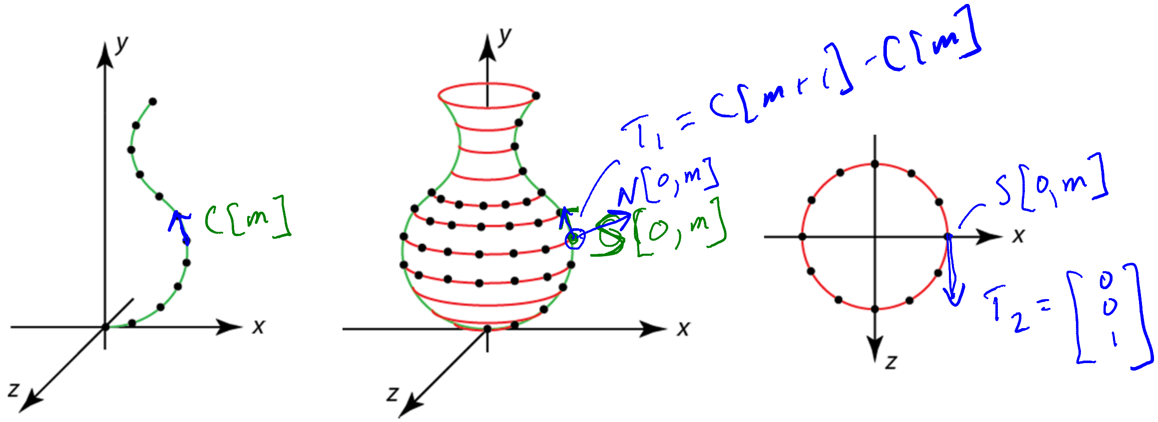


$T_1 \approx P - Q$

$T_2 \approx R - Q$

$N = T_2 \times T_1$

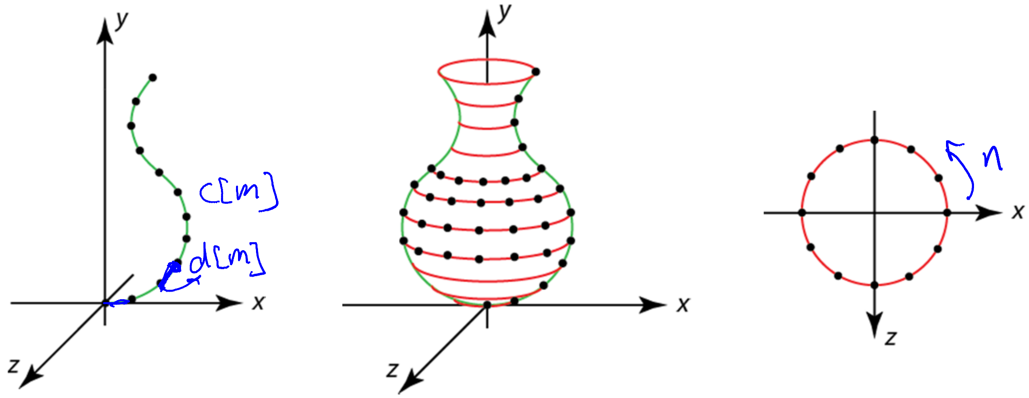
Normals on a surface of revolution



$$\hat{N}[0,m] = \frac{T_1 \times T_2}{\|T_1 \times T_2\|}$$

$$N[n,m] = R\left[\frac{2\pi n}{N}\right] \hat{N}[0,m]$$

Texture coordinates on a surface of revolution



$$\cancel{v = \frac{m}{M}}$$

$$u = \frac{n}{N}$$

$$v = \frac{\sum_{i=0}^{m-1} d[i]}{\sum_{i=0}^{M-1} d[i]} \sim \text{arc length}$$

Triangle meshes

How should we generally represent triangle meshes?