17. Subdivision surfaces

Reading

Recommended:


Building complex models

We can extend the idea of subdivision from curves to surfaces...

Subdivision surfaces

Chaikin’s use of subdivision for curves inspired similar techniques for subdivision surfaces.

Iteratively refine a control polyhedron (or control mesh) to produce the limit surface

\[ \sigma = \lim_{j \to \infty} M_j \]

using splitting and averaging steps.
Triangular subdivision

There are a variety of ways to subdivide a polygon mesh.

A common choice for triangle meshes is 4:1 subdivision – each triangular face is split into four subfaces:

Original

After splitting

Loop averaging step

Once again we can use masks for the averaging step:

\[
Q \leftarrow \frac{\alpha(n)Q_0 + Q_1 + \cdots + Q_n}{\alpha(n) + n}
\]

where

\[
\alpha(n) = \frac{n(1 - \beta(n))}{\beta(n)} \quad \beta(n) = \frac{5}{4} \left( \frac{3 + 2\cos(2\pi/n)}{32} \right)
\]

These values, due to Charles Loop, are carefully chosen to ensure smoothness – namely, tangent plane or normal continuity.

Note: tangent plane continuity is also known as $G^1$ continuity for surfaces.

Recipe for subdivision surfaces

As with subdivision curves, we can now describe a recipe for creating and rendering subdivision surfaces:

- Subdivide (split+average) the control polyhedron a few times. Use the averaging mask.
- Compute two tangent vectors using the tangent masks.
- Compute the normal from the tangent vectors.
- Push the resulting points to the limit positions. Use the evaluation mask.
- Render!
Adding creases without trim curves

In some cases, we want a particular feature such as a crease to be preserved. With NURBS surfaces, this required the use of trim curves.

For subdivision surfaces, we can just modify the subdivision mask:

This gives rise to $G^0$ continuous surfaces (i.e., having positional but not tangent plane continuity)

Creases without trim curves, cont.

Here’s an example using Catmull-Clark surfaces (based on subdividing quadrilateral meshes):

Face schemes

4:1 subdivision of triangles is sometimes called a face scheme for subdivision, as each face begets more faces.

An alternative face scheme starts with arbitrary polygon meshes and inserts vertices along edges and at face centroids:

Catmull-Clark subdivision:

Note: after the first subdivision, all polygons are quadrilaterals in this scheme.

Vertex schemes

In a vertex scheme, each vertex begets more vertices. In particular, a vertex surrounded by $n$ faces is split into $n$ sub-vertices, one for each face:

Doo-Sabin subdivision:

The number edges (faces) incident to a vertex is called its valence. Edges with only once incident face are on the boundary. After splitting in this subdivision scheme, all non-boundary vertices are of valence 4.
Interpolating subdivision surfaces

Interpolating schemes are defined by

- splitting
- averaging only new vertices

The following averaging mask is used in butterfly subdivision:

![Diagram of butterfly subdivision](image)

Setting $t=0$ gives the original polyhedron, and increasing small values of $t$ makes the surface smoother, until $t=1/8$ when the surface is provably $G^1$. 