15. Subdivision surfaces

Chaikin’s use of subdivision for curves inspired similar techniques for subdivision surfaces.

Iteratively refine a control polyhedron (or control mesh) to produce the limit surface

\[ \sigma = \lim_{j \to \infty} M_j \]

using splitting and averaging steps.

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Reading

Recommended:


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Triangular subdivision

There are a variety of ways to subdivide a polygon mesh.

A common choice for triangle meshes is 4:1 subdivision – each triangular face is split into four subfaces:

- [Diagram showing original and subdivided triangle mesh]
Loop averaging step

Once again we can use masks for the averaging step:

\[ Q \leftarrow \alpha(n)Q + Q_1 + \ldots + Q_n \]

where

\[ \alpha(n) = \frac{n(1 - \beta(n))}{\beta(n)} \quad \beta(n) = \frac{5}{4} \left( \frac{3 + 2\cos(2\pi/n)}{32} \right) \]

These values, due to Charles Loop, are carefully chosen to ensure smoothness – namely, tangent plane or normal continuity.

Note: tangent plane continuity is also known as \( G^1 \) continuity.

Recipe for subdivision surfaces

As with subdivision curves, we can now describe a recipe for creating and rendering subdivision surfaces:

- Subdivide (split+average) the control polyhedron a few times. Use the averaging mask.
- Compute two tangent vectors using the tangent masks.
- Compute the normal from the tangent vectors.
- Push the resulting points to the limit positions. Use the evaluation mask.
- Render!

Adding creases without trim curves

In some cases, we want a particular feature such as a crease to be preserved. With NURBS surfaces, this required the use of trim curves.

For subdivision surfaces, we can just modify the subdivision mask:

This gives rise to \( G^0 \) continuous surfaces (i.e., having positional but not tangent plane continuity).
Creases without trim curves, cont.

Here's an example using Catmull-Clark surfaces (based on subdividing quadrilateral meshes):

Face schemes

There are other kinds of subdivision techniques for surfaces besides 4:1 triangular subdivision.

For example, quadrilateral faces can be split into four subfaces:

Catmull-Clark subdivision:

Vertex schemes

A vertex surrounded by $n$ faces is split into $n$ subvertices, one for each face:

Doo-Sabin subdivision:

Summary

What to take home:

- The meanings of all the **boldfaced** terms.
- How to construct and render Loop subdivision surfaces from the averaging masks, evaluation masks, and tangent masks.