5. Shading

Introduction

Affine transformations help us to place objects into a scene.

Before creating images of these objects, we'll look at models for how light interacts with their surfaces.

Such a model is called a shading model.

Other names:
• Lighting model
• Light reflection model
• Local illumination model
• Reflectance model

An abundance of photons

Properly determining the right color is really hard.

Look around the room. Each light source has different characteristics. Trillions of photons are pouring out every second.

These photons can:
• interact with the atmosphere, or with things in the atmosphere
• strike a surface and
  • be absorbed
  • be reflected
  • cause fluorescence or phosphorescence.
• interact in a wavelength-dependent manner
• generally bounce around and around

Reading

Required:
• Watt, sections 6.2-6.3

Optional:
• Watt, chapter 7.
Our problem

We’re going to build up to an approximation of reality called the Phong illumination model.

It has the following characteristics:

- not physically based
- gives a first-order approximation to physical light reflection
- very fast
- widely used

In addition, we will assume local illumination, i.e., light goes: light source -> surface -> viewer.

No interreflections, no shadows.

Setup…

Given:

- a point \( P \) on a surface visible through pixel \( p \)
- The normal \( N \) at \( P \)
- The lighting direction, \( L \), and intensity, \( I_a \), at \( P \)
- The viewing direction, \( V \), at \( P \)
- The shading coefficients at \( P \)

Compute the color, \( I \), of pixel \( p \).

Assume that the direction vectors are normalized:

\[
\|N\| = \|L\| = \|V\| = 1
\]

Iteration zero

The simplest thing you can do is…

Assign each polygon a single color:

\[
I = k_e
\]

where

- \( I \) is the resulting intensity
- \( k_e \) is the emissivity or intrinsic shade associated with the object

This has some special-purpose uses, but not really good for drawing a scene.

[Note: \( k_e \) is omitted in Watt.]

Iteration one

Let’s make the color at least dependent on the overall quantity of light available in the scene:

\[
I = k_e + k_a I_a
\]

- \( k_a \) is the ambient reflection coefficient.
  - really the reflectance of ambient light
  - “ambient” light is assumed to be equal in all directions
- \( I_a \) is the ambient intensity.

Physically, what is “ambient” light?
Wavelength dependence

Really, \( k_\alpha \), \( k_\omega \), and \( I_\alpha \) are functions over all wavelengths \( \lambda \).

Ideally, we would do the calculation on these functions. For the ambient shading equation, we would start with:

\[
I(\lambda) = k_\alpha(\lambda)I_\alpha(\lambda)
\]

then we would find good RGB values to represent the spectrum \( I(\lambda) \).

Traditionally, though, \( k_\alpha \) and \( I_\alpha \) are represented as RGB triples, and the computation is performed on each color channel separately:

\[
\begin{align*}
I_R &= k_{R_\alpha} I_{R_\alpha} \\
I_G &= k_{G_\alpha} I_{G_\alpha} \\
I_B &= k_{B_\alpha} I_{B_\alpha}
\end{align*}
\]

Diffuse reflectors

Diffuse reflection occurs from dull, matte surfaces, like latex paint, or chalk.

These diffuse or Lambertian reflectors reradiate light equally in all directions.

Picture a rough surface with lots of tiny microfacets.

Diffuse reflectors, cont.

...or picture a surface with little pigment particles embedded beneath the surface (neglect reflection at the surface for the moment):

The microfacets and pigments distribute light rays in all directions.

Embedded pigments are responsible for the coloration of diffusely reflected light in plastics and paints.

Note: the figures above are intuitive, but not strictly (physically) correct.
Iteration two

The incoming energy is proportional to \( I \), giving the diffuse reflection equations:

\[
I = k_d I_A + k_d I_i + k_d I_r \]

where:

- \( k_d \) is the **diffuse reflection coefficient**
- \( I_A \) is the intensity of the light source
- \( N \) is the normal to the surface (unit vector)
- \( L \) is the direction to the light source (unit vector)
- \( (x)_+ \) means max \( (0,x) \)

[Note: Watt uses \( I_i \) instead of \( I_A \).]

Specular reflection

**Specular reflection** accounts for the highlight that you see on some objects.

It is particularly important for *smooth, shiny* surfaces, such as:

- metal
- polished stone
- plastics
- apples
- skin

Properties:

- Specular reflection depends on the viewing direction \( V \).
- For non-metals, the color is determined solely by the color of the light.
- For metals, the color may be altered (e.g., brass)

Specular reflection “derivation”

For a perfect mirror reflector, light is reflected about \( N \), so

\[
I = \begin{cases} 
I_i & \text{if } V = R \\
0 & \text{otherwise} 
\end{cases}
\]

For a near-perfect reflector, you might expect the highlight to fall off quickly with increasing angle \( \phi \).

Also known as:

- “rough specular” reflection
- “directional diffuse” reflection
- “glossy” reflection

Derivation, cont.

One way to get this effect is to take \( (R \cdot V) \), raised to a power \( n_s \).

As \( n_s \) gets larger,

- the dropoff becomes (more,less) gradual
- gives a (larger,smaller) highlight
- simulates a (more,less) mirror-like surface
Iteration three

The next update to the Phong shading model is then:

\[ I = k_e + k_s I_s + k_d I_d (N \cdot L)_s + k_s I_s (V \cdot R)_{1/2} \]

where:

- \( k_s \) is the **specular reflection coefficient**
- \( n_s \) is the **specular exponent** or shininess
- \( R \) is the reflection of the light about the normal (unit vector)
- \( V \) is viewing direction (unit vector)

[Note: Watt uses \( n \) instead of \( n_s \).]

Intensity drop-off with distance

OpenGL supports different kinds of lights: point, directional, and spot.

For point light sources, the laws of physics state that the intensity of a point light source must drop off inversely with the square of the distance.

We can incorporate this effect by multiplying \( I \) by \( 1/d^2 \).

Sometimes, this distance-squared dropoff is considered too "harsh." A common alternative is:

\[ f_{atten}(d) = \frac{1}{a + bd + cd^2} \]

with user-supplied constants for \( a \), \( b \), and \( c \).

[Note: not discussed in Watt.]

Iteration four

Since light is additive, we can handle multiple lights by taking the sum over every light.

Our equation is now:

\[ I = k_e + k_s I_s + \sum f_{atten}(d)_j I_{\mathit{j}} \left[ k_d (N \cdot L)_j + k_s (V \cdot R)_j \right] \]

This is the Phong illumination model.

Choosing the parameters

Experiment with different parameter settings. To get you started, here are a few suggestions:

- Try \( n_s \) in the range \([0,100]\)
- Try \( k_a + k_d + k_s < 1 \)
- Use a small \( k_a \) (~0.1)

<table>
<thead>
<tr>
<th>Medium</th>
<th>Large, color of metal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal</td>
<td>Small, color of metal</td>
</tr>
<tr>
<td>Plastic</td>
<td>Medium, color of plastic</td>
</tr>
<tr>
<td>Planet</td>
<td>Medium, white</td>
</tr>
<tr>
<td>Large, color of metal</td>
<td>Metal</td>
</tr>
<tr>
<td>Medium, color of plastic</td>
<td>Plastic</td>
</tr>
<tr>
<td>Medium, white</td>
<td>Planet</td>
</tr>
<tr>
<td>varying</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
**BRDF**

The Phong illumination model is really a function that maps light from incoming (light) directions to outgoing (viewing) directions:

\[ f_r(\omega_{in}, \omega_{out}) \]

This function is called the **Bi-directional Reflectance Distribution Function (BRDF)**.

Here’s a plot with \( \omega_{in} \) held constant:

![Plot of BRDF](image1)

Physically valid BRDF’s obey Helmholtz reciprocity:

\[ f_r(\omega_{in}, \omega_{out}) = f_r(\omega_{out}, \omega_{in}) \]

and should conserve energy (no light amplification).

---

**Surface reflection equation**

To compute the reflection from a surface, we would actually solve the **surface reflection equation**:

\[ l(\omega_{out}) = \int \int f_r(\omega_{in}, \omega_{out}) \, d\omega_{in} \]

How might we represent light from a single direction?

We can plot the reflected light as a function of viewing angle for multiple light source contributions:

![Plot of reflected light](image2)

---

**Cook-Torrance-Sparrow model**

Cook and Torrance, 1982

**Anisotropic reflection**

Westin, Arvo, Torrance 1992

Poulin and Fournier 1990

Cook and Torrance, 1982
Weird BRDF: the moon

Gouraud vs. Phong interpolation

Now we know how to compute the color at a point on a surface using the Phong lighting model.

Does graphics hardware do this calculation at every point? Unfortunately not…

Smooth surfaces are often approximated by polygonal facets, because:

- Graphics hardware generally wants polygons (esp. triangles).
- Sometimes it easier to write ray-surface intersection algorithms for polygonal models.

How do we compute the shading for such a surface?

Faceted shading

Assume each face has a constant normal:

For a distant viewer and a distant light source, how will the color of each triangle vary?

Result: faceted, not smooth, appearance.

Gouraud interpolation

To get a smoother result that is easily performed in hardware, we can do Gouraud interpolation.

Here’s how it works:

1. Compute normals at the vertices.
2. Shade only the vertices.
3. Interpolate the resulting vertex colors.
Gouraud interpolation, cont'd

Gouraud interpolation has significant limitations.

1. If the polygonal approximation is too coarse, we can miss specular highlights.

2. We will encounter Mach banding (derivative discontinuity enhanced by human eye).

Alas, this is usually what graphics hardware supports.

Maybe someday soon we’ll get…

Phong interpolation

To get an even smoother result with fewer artifacts, we can perform Phong interpolation.

Here’s how it works:

1. Compute normals at the vertices.
2. Interpolate normals and normalize.
3. Shade using the interpolated normals.

Summary

The most important thing to take away from this lecture is the final equation for the Phong model.

- What is the physical meaning of each variable?
- How are the terms computed?
- What effect does each term contribute to the image?
- What does varying the parameters do?

You should also understand the differences between faceted, Gouraud, and Phong interpolated shading.