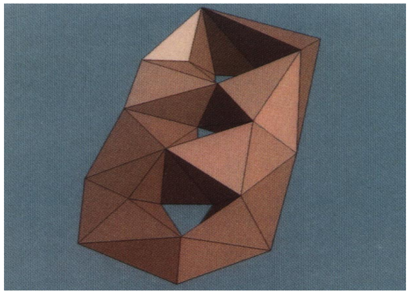
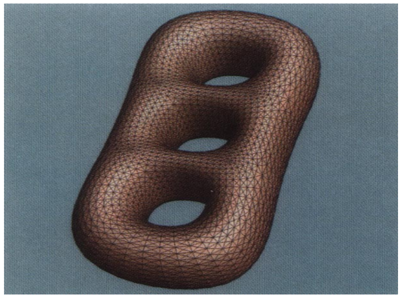
Subdivision surfaces

Reading

Stollnitz, DeRose, and Salesin. *Wavelets for Computer Graphics: Theory and Applications*, 1996, section 10.2.

Building complex models





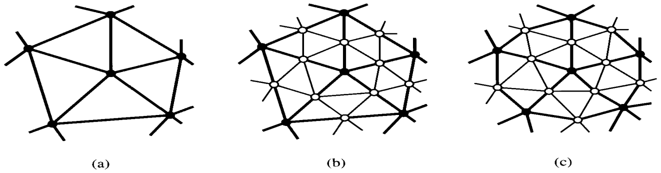
Subdivision surfaces

Chaikin's use of subdivision for curves inspired similar techniques for subdivision.

Iteratively refine a **control polyhedron** (or **control mesh**) to produce the limit surface

$$\sigma = \lim_{j \to \infty} M^j$$

using splitting and averaging steps.

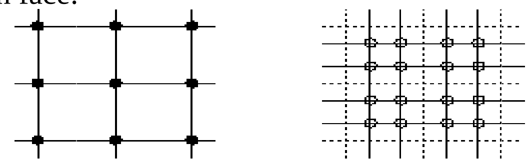


There are two types of splitting steps:

- **vertex schemes**
- **face schemes**

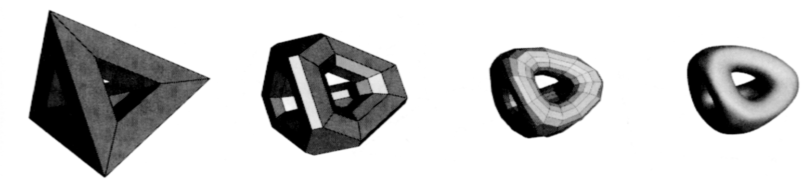
Vertex schemes

A vertex surrounded by n faces is split into n subvertices, one for each face:



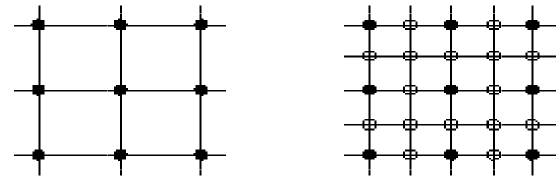
Original   After splitting

Doo-Sabin subdivision:



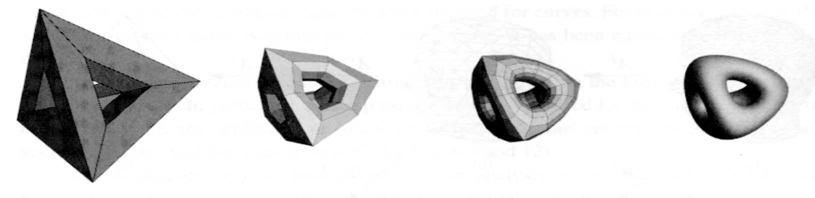
Face schemes

Each quadrilateral face is split into four subfaces:



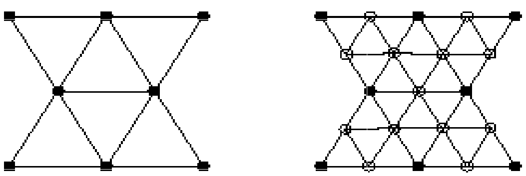
Original   After splitting

Catmull-Clark subdivision:



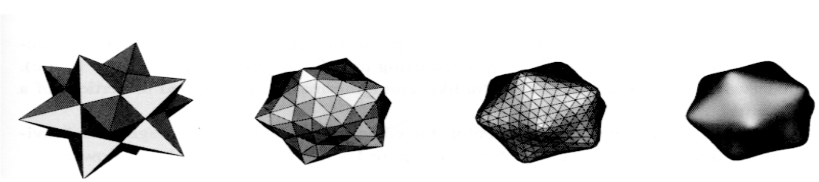
Face schemes, cont.

Each triangular face is split into four subfaces:



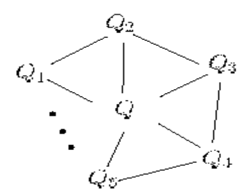
Original   After splitting

Loop subdivision:

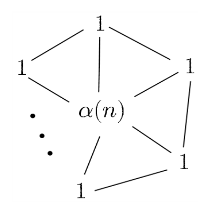


Averaging step

Once again we can use **masks** for the averaging step:



Vertex labeling



Averaging mask

$$Q \leftarrow \frac{\alpha(n) + Q_1 + \cdots + Q_n}{\alpha(n) + n}$$

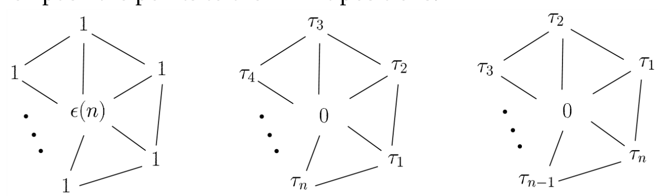
where

$$\alpha(n) = \frac{n(1-\beta(n))}{\beta(n)} \quad \beta(n) = \frac{5}{4} - \frac{(3 + 2\cos(2\pi/n))^2}{32}$$

(carefully chosen to ensure smoothness.)

Loop evaluation and tangent masks

As with subdivision curves, we can split and average a number of times and then push the points to their limit positions.



Evaluation mask   Tangent masks

$$Q^{\infty} = \frac{\varepsilon(n) + Q_1 + \cdots + Q_n}{\varepsilon(n) + n}$$

where

$$\varepsilon(n) = \frac{3n}{\beta(n)} \qquad \tau_i(n) = \cos(2\pi i / n)$$

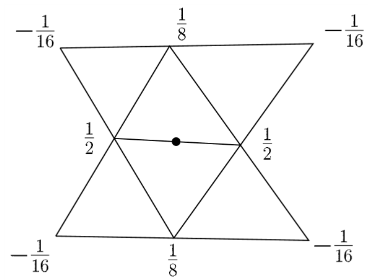
How do we compute the normal?

Interpolation

Interpolating schemes are defined by:

- Splitting
- Averaging only new vertices

Averaging mask for odd vertices in the "modified butterfly scheme":



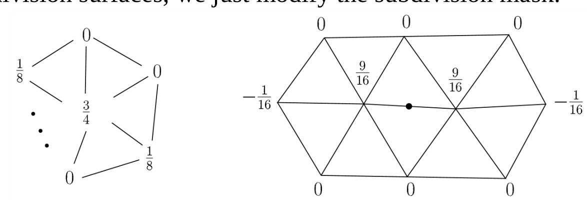
Recipe for subdivision surfaces

As with subdivision curves, we can now describe a recipe for creating and rendering subdivision surfaces:

- Subdivide (split+average) the control polyhedron a few times. Use the averaging mask.
- Push the resulting points to the limit positions. Use the evaluation mask.
- Compute the tangents using the tangent masks.
- Compute the normal from the tangent vectors.
- Render!

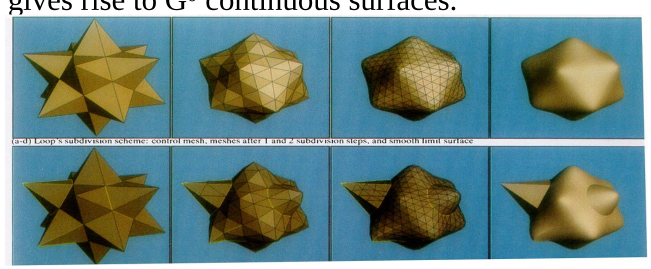
Adding creases without trim curves

Sometimes, particular feature such as a crease should be preserved. With NURBS surfaces, this required the use of trim curves.
For subdivision surfaces, we just modify the subdivision mask:



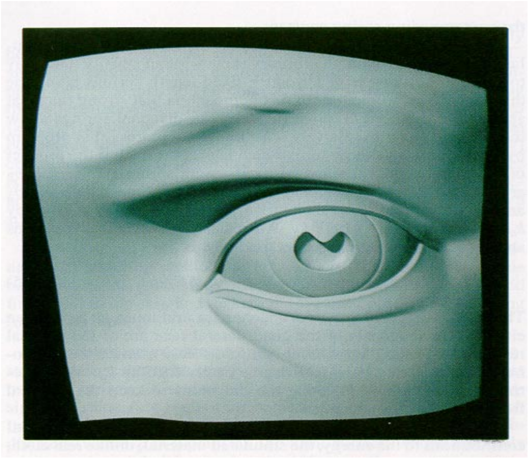
Loop crease/boundary edge   Buttery crease/boundary edge

This gives rise to G^0 continuous surfaces.



Creases without trim curves, cont.

Here's an example using Catmull-Clark surfaces of the kind found in Geri's Game:



Interpolating subdivision surfaces

Interpolating schemes are defined by

- splitting
- averaging only new vertices

Summary

What to take home:

- The various kinds of splitting steps, especially Loop
- How to construct subdivision surfaces from their averaging masks, evaluation masks, and tangent masks