Wait-free registers
Randomized Consensus

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(Slides from: Ellis Michael)

Drawbacks of Paxos

- Leader is a single bottleneck, processes $O(n)$ messages on every request.
- FLP means that liveness not guaranteed.
- More practically, Paxos can have bad availability during failure scenarios (e.g., if a leader fails, it takes time to elect a new one).

Alternatives

- Weaken the safety guarantees and accept weaker consistency (at your own peril).
- Constrain the problem (wait-free registers)
- Allow randomness (randomized algorithms)

Registers

- Hold a single value. Want multiple values? Use multiple registers.
- Allows reads and writes only. Does not allow appends or other read-modify-write operations.
- Provides **safe**, **regular**, and **atomic/linearizable** semantics

- **safe:** a read not concurrent with any write obtains the previously written value
- **regular:** safe + a read that overlaps a write obtains either the old or new value
- **atomic:** safe + reads and writes behave as if they occur in some definite order

Implementing a Register

- We will use the **client/server** model, where servers are replicas storing the value and clients send **reads** and **writes**.
- We want linearizability of reads and writes.
- As usual, we want to tolerate up to f server *crash failures*. Clients can also fail by crashing, no limit on number of client crashes.

Non-Blocking Algorithms

- **Lock-free** algorithms guarantee system-wide progress.
- **Wait-free** algorithms guarantee per-client progress. That is, no matter what steps other processes take, a correct client's operations are always completed in a finite number of steps.

Why No Appends?

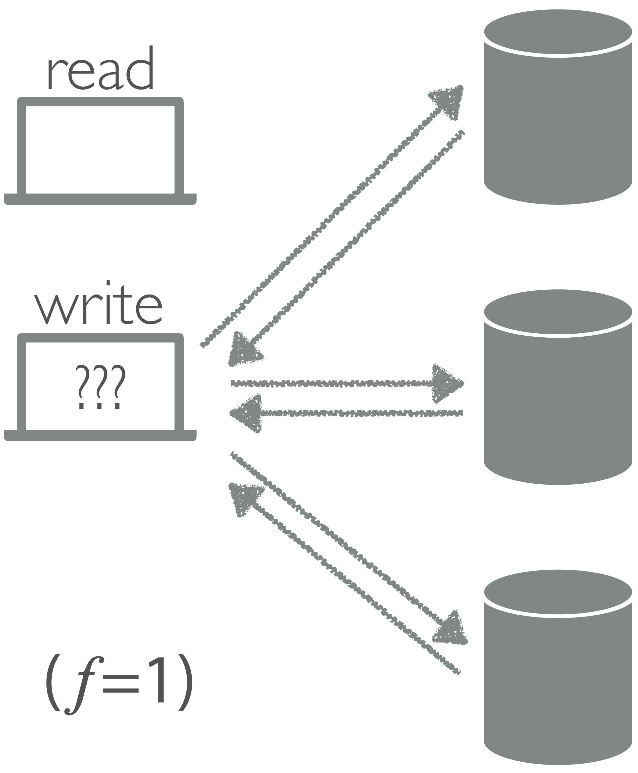
Simple way to implement consensus:

- All processes append their input value.
- All processes read the value.
- They all decide the first value that was appended.

If you can wait-free implement an appendable register, you can solve consensus (providing safety and liveness), which is impossible.

How Many Servers Do We Need?

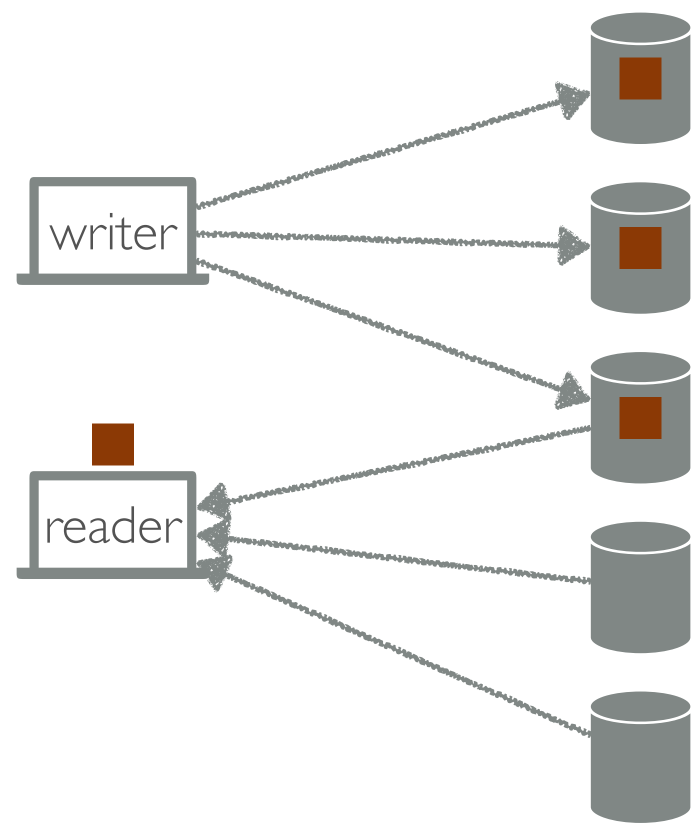
- If we want to make progress even when f servers crash, we can wait for at most n - f responses.
- We need to send writes to > f replicas, otherwise they could get lost forever.
- So we need *at least* 2 f+1 servers. And, in fact, we will use 2 f+1.
- Read quorum size plus write quorum size should be greater than n (i.e., they should overlap). We'll use simple majorities.



First Step: Single Reader, Single Writer (SWSR)

- Writer sends value to a majority.
- Reader reads value from a majority.
- Since majorities intersect, reader reads writer's value.

Does this work?



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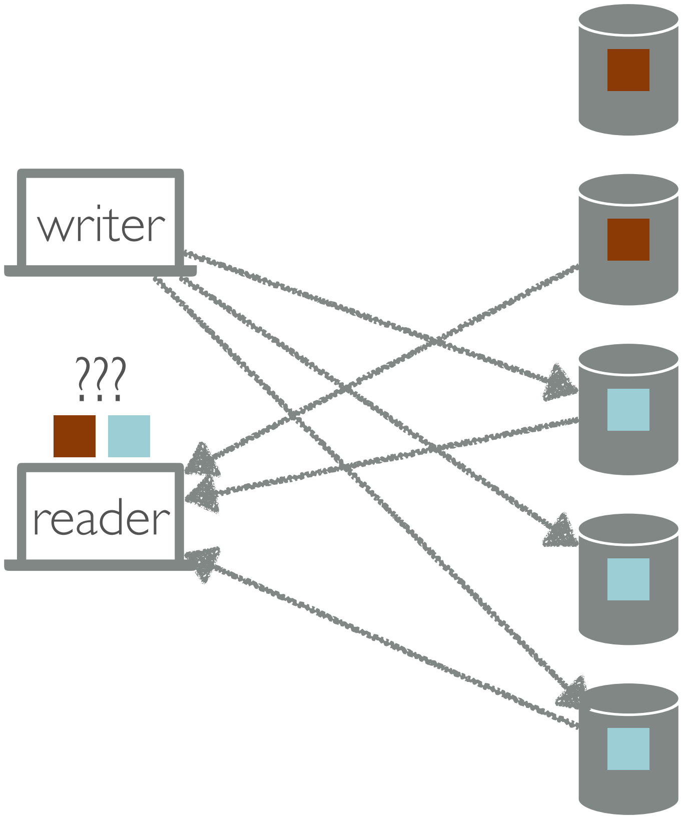

SWSR II

- Writer sends a **timestamped** value to a majority.
- Reader reads value from a majority, takes the one with the **highest timestamp**.
- Since majorities intersect, reader reads writer's latest written value.

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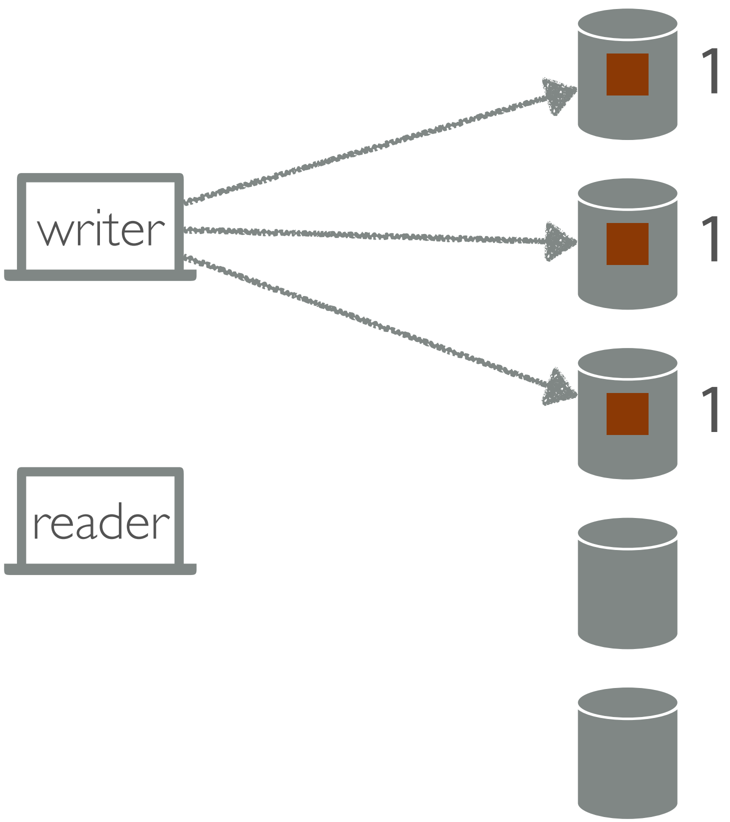

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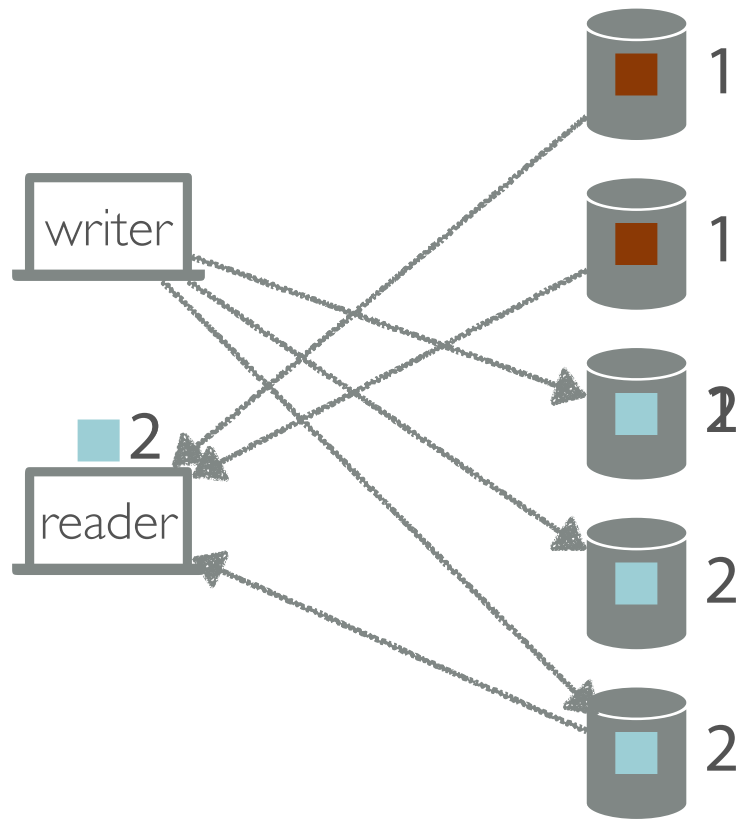
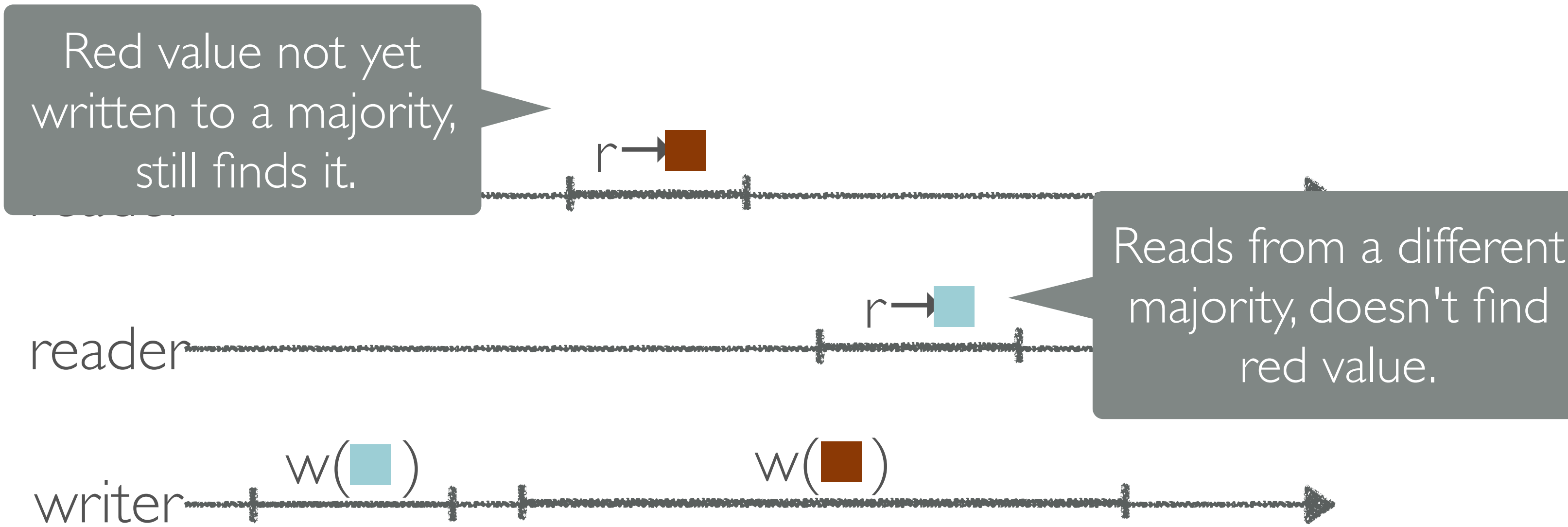


Multiple Readers, Single Writer (MRSW)

Does this previous solution just work?

What happens if there are multiple reads by **different processes** overlapping the same write?

MRSW: Inconsistent Reads

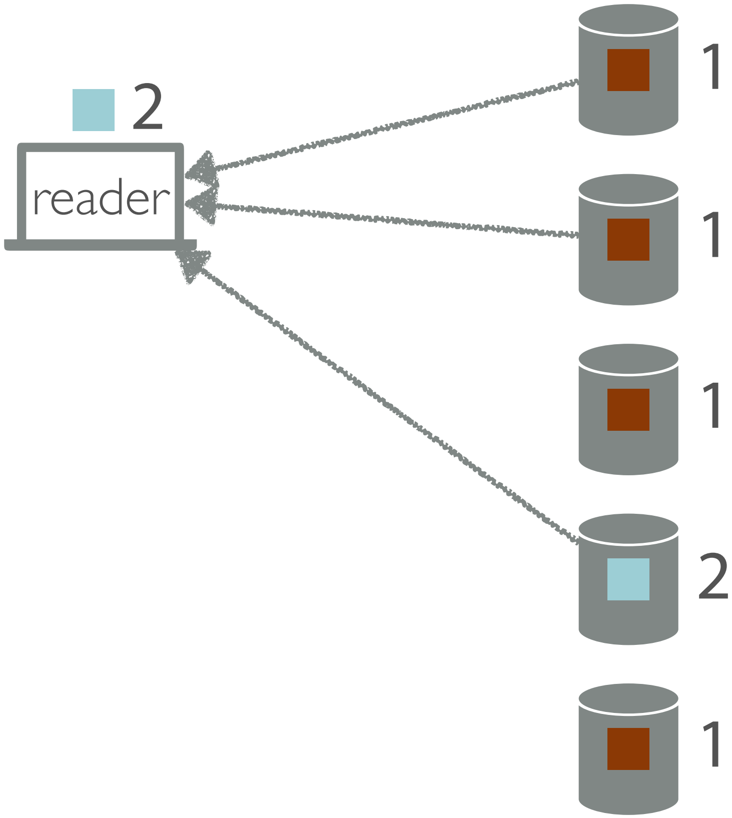


Not linearizable!

MRSW II

Suppose a write is ongoing (or the writer died).

- Reader reads value from a majority, takes the one with the highest timestamp.
- Reader then performs a **write-back**, writing the value to a majority (not necessarily the same one). Only returns from read after write-back is complete.
- Later readers are guaranteed to read a value **at least as new** as the previous returned one.



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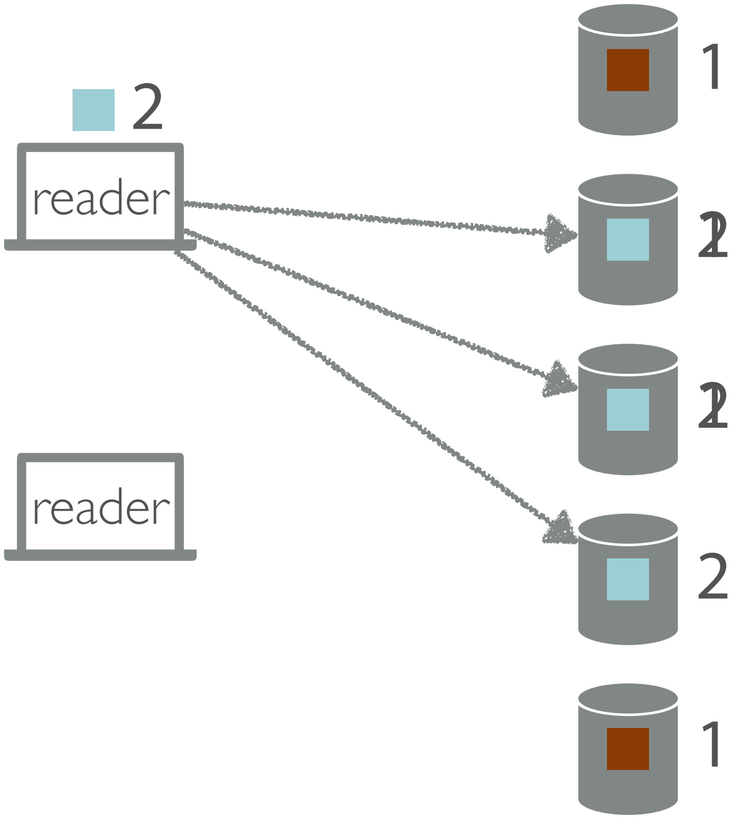
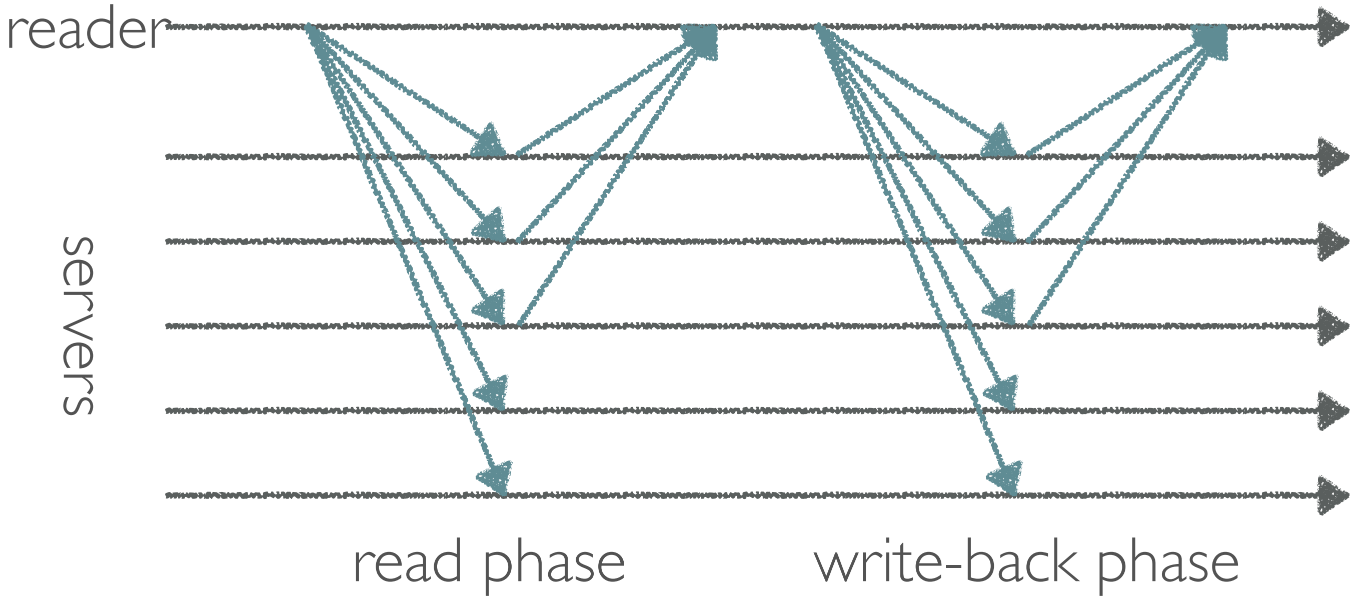


MRSW III



Do we always need to execute the write-back phase?

Putting It All Together: MRMW

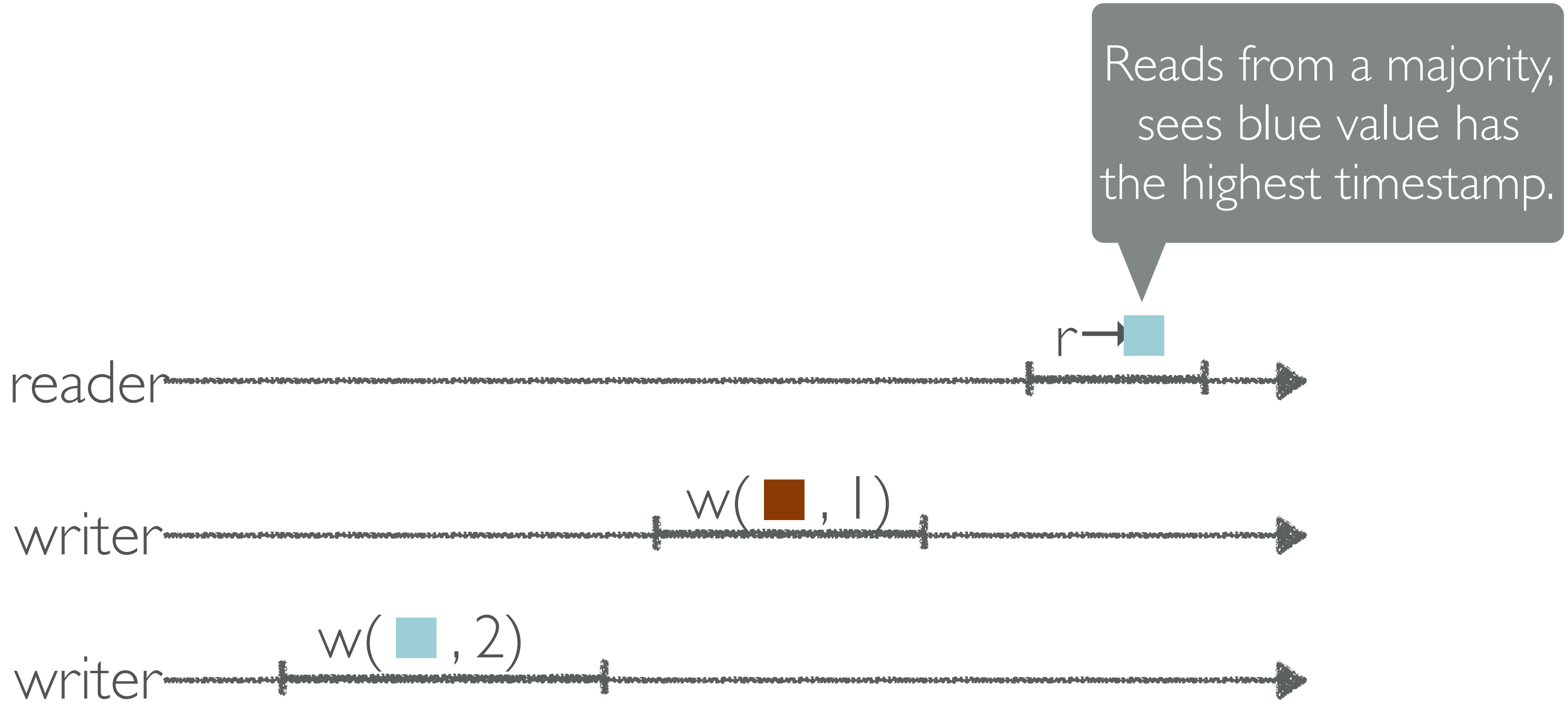
Does the previous solution just work?

What if writers use the **same timestamp**?

> Prevented by breaking ties using writers ID, same as PMMC.

What if a write that starts after a previous write ended uses a **smaller timestamp**?

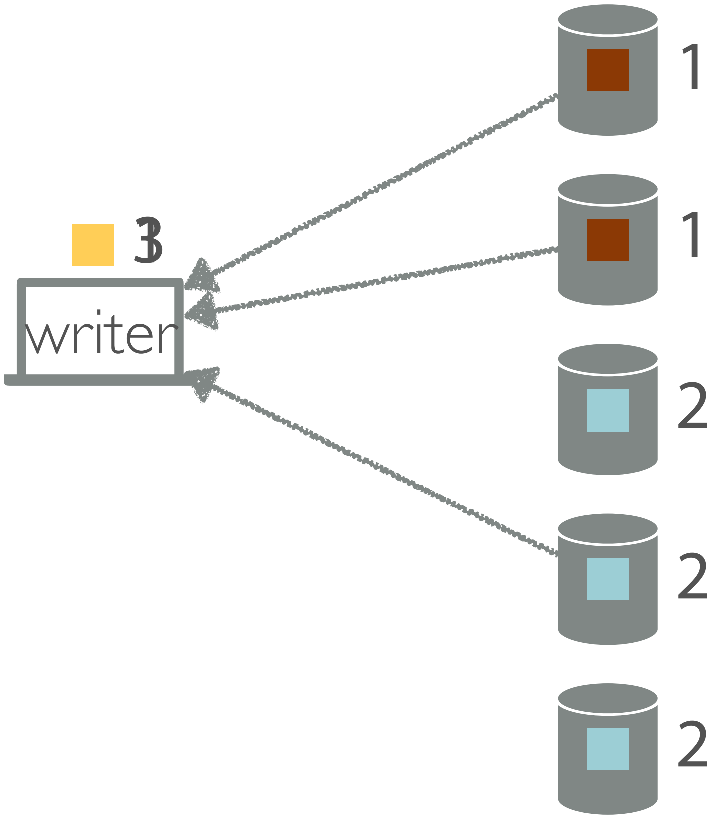
MRMW: Untimely Timestamps



Not linearizable!

MRMW II: Ensuring Timestamp Ordering

- Writer first **queries** a majority, updates its timestamp to be larger than largest timestamp found.
- Writer then writes value to majority as usual.
- Written value guaranteed to have a timestamp larger than previously written values, readers will read latest value (again, writer IDs break timestamp ties).



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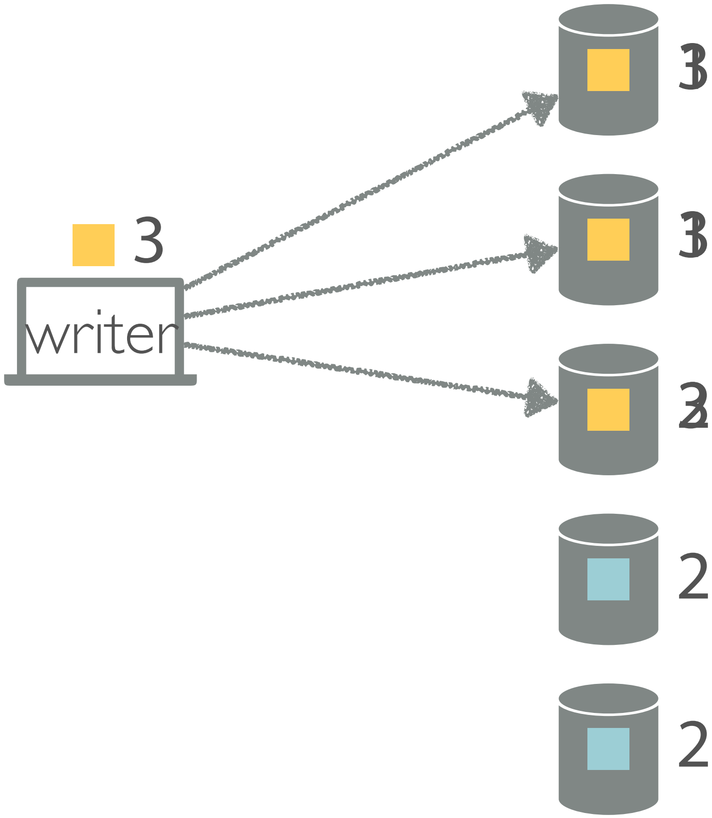

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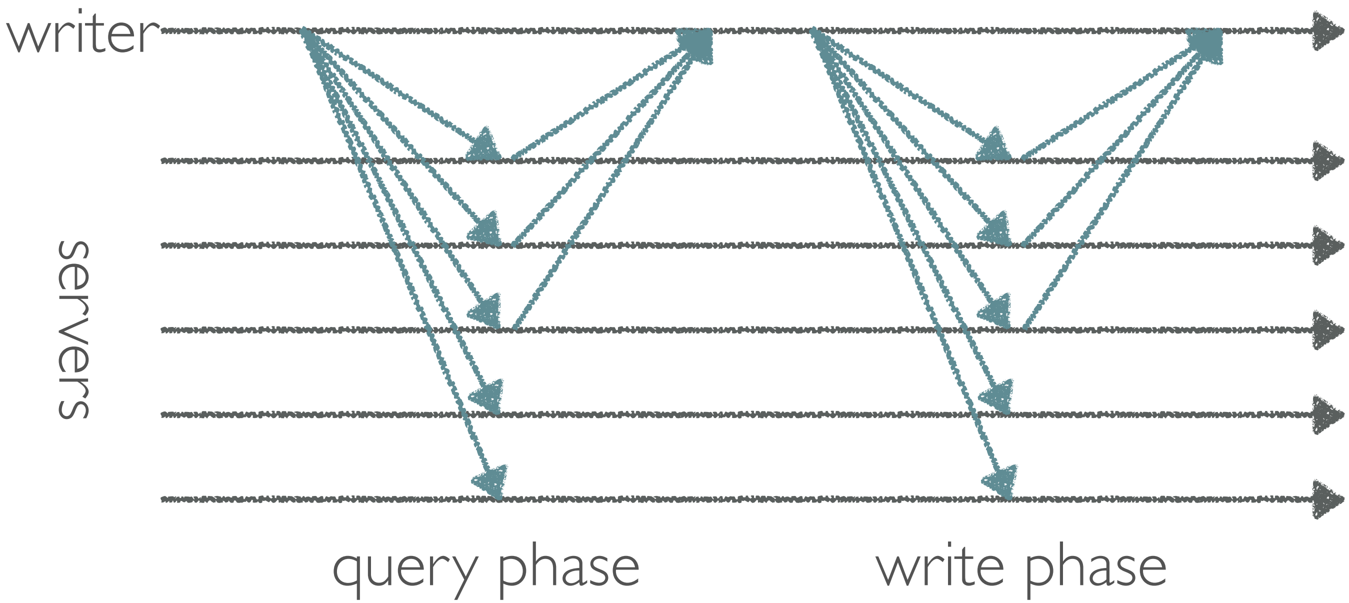


Read/writes almost the same!

- The methods for reading and writing are now the same.
- The only difference is that a read writes and returns the value that was read, but a write writes the value to be written.
- Also, for the record, there's no reason that clients can't be both readers and writers.

ABD vs. Paxos

- Paxos doesn't guarantee liveness when the network is asynchronous. ABD guarantees wait-freedom, even when there are multiple writers.
- Paxos-based state-machine replication (SMR) can support arbitrary state machines. The ABD algorithm only allows a read/write interface.
- ABD removes the leader bottleneck.
- How does its cost compare to leader-based Paxos?

What Can We Do With Registers?

- Implement a read/write key-value store
- Emulate shared memory

Consensus isn't always the right problem! Don't solve it if you don't have to!

Randomized Consensus

FLP Impossibility

Theorem: *In an asynchronous environment in which a single process can fail by crashing, there does not exist a protocol which solves binary consensus.*

Paxos doesn't save us. It doesn't guarantee liveness.

Result assumed a **deterministic** computation model.

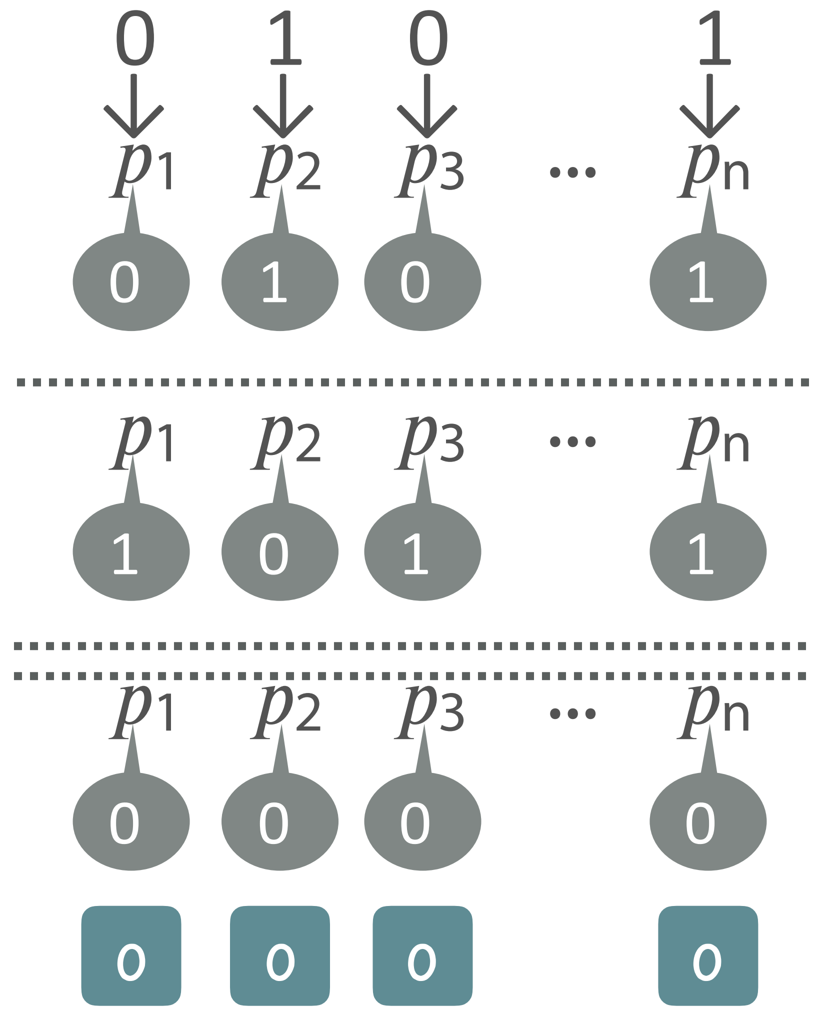
Let's go random!

Ben-Or's algorithm uses randomization to ***guarantee consensus*** for crash failures when $f < n/2$.

A variant even works for Byzantine faults!

Intuition

- At first every process proposes their input value.
- After that, they propose random values.
- When enough processes propose the same value, the value is chosen.
- Eventually, that will happen!



Setup

- Again, we're considering binary consensus.
- Protocol proceeds in **asynchronous rounds**, where each round has two phases.
- For each phase, processes broadcast their input values and wait for $n - f$ messages from the other processes.
- Each message is tagged with the round and phase number. (And messages can be resent to deal with a lossy network. But once a message is sent, that value is locked in for that process for that phase/round.)

Ben-Or Algorithm

Processes send proposals for each phase and then block and wait for the requisite $n - f$ messages (including their own).

During the first phase, processes make a preliminary proposal.

If they receive matching responses from a majority in the first phase, they propose that value in the second phase. Otherwise, they propose ⊥ (a special null value).

If they get enough non-⊥ responses from the second phase, they decide.

```
a←input
loop:
    send_phase1(a)
    A←receive_phase1()
    if (∃a′∈ A : |A_a′| > n/2):
        b←a′
    else:
        b←⊥

    send_phase2(b)
    B←receive_phase2()
    if (∃b′∈ B : b′≠⊥ ∧ |B_b′| > f):
        decide(b′)
    if (∃b′∈ B : b′≠⊥):
        a←b′
    else:
        a←choose_random({0,1})
```

Do We Have Consensus?

- **Agreement:** No two processes decide different values.
- **Integrity:** Every process decides at most one value, and if a process decides a value, some process had it as its input.
- **Termination:** Every correct process eventually decides a value.

$a \leftarrow$ *input*

loop:

    send_phase1(a)

    $A \leftarrow$ *receive_phase1*()

    if ($\exists a' \in A : |A_{a'}| > n/2$):

        $b \leftarrow a'$

    else:

        $b \leftarrow \perp$

    send_phase2(b)

    $B \leftarrow$ *receive_phase2*()

    if ($\exists b' \in B : b' \neq \perp \wedge |B_{b'}| > f$):

        decide(b')

    if ($\exists b' \in B : b' \neq \perp$):

        $a \leftarrow b'$

    else:

        $a \leftarrow$ *choose_random({0,1})*

Integrity I

If both 0 and 1 are input values to processes, integrity is trivially satisfied.

Suppose all processes have the same input value.

- Then, they all send the same phase 1 value in round 1.
- So they all send that same value in phase 2.
- So they all decide that value at the end of round 1.

```
a←input
loop:
    send_phase1(a)
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    else:
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    send_phase2(b)
    B←receive_phase2()
    if (∃b′∈ B : b′≠⊥ ∧ |B_b′| > f):
        decide(b′)
    if (∃b′∈ B : b′≠⊥):
        a←b′
    else:
        a←choose_random({0,1})
```

Fun Fact

Lemma: *No two processes receive different non-⊥ phase 2 values in the same round.*

Suppose they did. That means that one process received 0s from a majority in phase 1 and another received 1s.

But majorities intersect!

```
a←input
loop:
    send_phase1(a)
    A←receive_phase1()
    if (∃a′∈ A : |A_a′| > n/2):
        b←a′
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```

Agrement + Integrity II

Let round r be the first round any process decides a value, 0 w.l.o.g.

If a process decided a value, it must have received $> f$ 0s in phase 2.

Which means that every process received at least one 0 because they all wait for $n - f$ messages. No process received a 1 by the previous lemma.

Therefore, on round $r + 1$ (and all subsequent rounds), all processes propose 0 and all processes decide 0.

```
a←input
loop:
    send_phase1(a)
    A←receive_phase1()
    if (∃a′∈ A : |A_a′| > n/2):
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    else:
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    send_phase2(b)
    B←receive_phase2()
    if (∃b′∈ B : b′≠⊥ ∧ |B_b′| > f):
        decide(b′)
    if (∃b′∈ B : b′≠⊥):
        a←b′
    else:
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```

Termination

We know that if all processes propose the same value for a round, they all decide that value that round.

At worst, the probability of this happening on any particular round is $1/2^n$.

Why? By the previous lemma, all the non-random values are identical.

Over time, the probability of this happening on **at least one round** converges to 1.

$a \leftarrow$ *input*

loop:

    send_phase1(a)

    $A \leftarrow$ *receive_phase1*()

    if ($\exists a' \in A : |A_{a'}| > n/2$):

        $b \leftarrow a'$

    else:

        $b \leftarrow \perp$

    send_phase2(b)

    $B \leftarrow$ *receive_phase2*()

    if ($\exists b' \in B : b' \neq \perp \wedge |B_{b'}| > f$):

        decide(b')

    if ($\exists b' \in B : b' \neq \perp$):

        $a \leftarrow b'$

    else:

        $a \leftarrow$ *choose_random({0,1})*

Other Values?

Binary consensus is conceptually simple but not as useful. However, the algorithm can be to support larger domains, even when the processes don't know the domains *a priori* and even when some processes don't receive input values.

- Processes without input values start by proposing ⊥.
- Instead of randomly choosing from {0,1}, processes randomly choose from all non-⊥ values they've seen so far (in any message). Only choose ⊥ as a last resort.

$a \leftarrow$ *input*

loop:

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Takeaways

- Randomization can actually solve consensus*
- You can structure an asynchronous protocol using rounds. It's potentially useful and certainly an interesting way to think about asynchronous computation.