Paxos
The Part-Time Parliament

Parliament determines laws by passing sequence of numbered decrees.
Legislators can leave and enter the chamber at arbitrary times.
No centralized record of approved decrees—instead, each legislator carries a ledger.
Government 101

- No two ledgers contain contradictory information.

- If a majority of legislators were in the Chamber and no one entered or left the Chamber for a sufficiently long time, then:
  - any decree proposed by a legislator would eventually be passed.
  - any passed decree would appear on the ledger of every legislator.
Paxos legislature is non-partisan, progressive, and well-intentioned

Legislators only care that something is agreed to, not what is agreed to

We’ll take care of Byzantine legislators later
Back to the future

- A set of processes that can propose values
- Processes can crash and recover
- Processes have access to stable storage
- Asynchronous communication via messages
- Messages can be lost and duplicated, but not corrupted
The Game: Consensus

**SAFETY**
- Only a value that has been proposed can be chosen
- Only a single value is chosen
- A process never learns that a value has been chosen unless it has been chosen

**LIVENESS**
- Some proposed value is eventually chosen
- If a value is chosen, a process eventually learns it
The Players

- Proposers
- Acceptors
- Learners
Choosing a value

Use a single acceptor
What if the acceptor fails?

Choose only when a "large enough" set of acceptors accepts

Using a majority set guarantees that at most one value is chosen

6 is chosen!
Accepting a value

Suppose only one value is proposed by a single proposer.

That value should be chosen!

First requirement:

P1: An acceptor must accept the first proposal that it receives

...but what if we have multiple proposers, each proposing a different value?
P1 + multiple proposers

5

7

6

2

\( a_1 \)

\( a_2 \)

\( a_3 \)
P1 + multiple proposers

5

7

6

2

6 \quad a_1

5 \quad a_2

2 \quad a_3
P1 + multiple proposers

No value is chosen!
Handling multiple proposals

- Acceptors must accept more than one proposal.

- To keep track of different proposals, assign a natural number to each proposal.
  - A proposal is then a pair \((psn, value)\).
  - Different proposals have different \(psn\).

- A proposal is chosen when it has been accepted by a majority of acceptors.

- A value is chosen when a single proposal with that value has been chosen.
Choosing a unique value

“Any acceptor can accept as many proposals as he wants, so long as they all propose the same value”

Leslie Lamport

P2. If a proposal with value $\nu$ is chosen, then every higher-numbered proposal that is chosen has value $\nu$
It’s up to the Acceptors!

P2. If a proposal with value $v$ is chosen, then every higher-numbered proposal that is chosen has value $v$

We strengthen it to:

P2a. If a proposal with value $v$ is chosen, then every higher-numbered proposal accepted by any acceptor has value $v$
What about P1?

- Do we still need P1? YES, to ensure that some proposal is accepted.
- How well do P1 and P2a play together? Asynchrony is a problem...

6 is chosen!
It's up to the Proposers!

Recall P2a:

P2a. If a proposal with value $\nu$ is chosen, then every higher-numbered proposal accepted by any acceptor has value $\nu$

We strengthen it to:

P2b. If a proposal with value $\nu$ is chosen, then every higher-numbered proposal issued by any proposer has value $\nu$
What to propose

Suppose \( p \) wants to issue a proposal numbered \( n \).

If \( p \) can be certain that no proposal numbered \( n' < n \) has been chosen then \( p \) can propose any value!

If a proposal numbered \( n' < n \) has been chosen, then it has been accepted by a majority set \( S \).

Any majority set \( S' \) must intersect \( S \).

If \( p \) can find one \( S' \) in which no acceptors has accepted a proposal numbered \( n' < n \), then no such proposal can have yet been chosen!

If no such \( S' \), a proposal numbered \( n' < n \) may have been chosen...

Then what?

**P2b:** If a proposal with value \( v \) is chosen, then every higher-numbered proposal issued by any proposer has value \( v \).
What to propose

Suppose $p$ wants to issue a proposal numbered $n$. If $p$ can be certain that no proposal numbered $n' < n$ has been chosen then $p$ can propose any value! If not, $p$ should propose the chosen value. But how?

$P2b$: If a proposal with value $v$ is chosen, then every higher-numbered proposal issued by any proposer has value $v$. 

Suppose $p$ wants to issue a proposal numbered $n$.

- If $p$ can be certain that no proposal numbered $n' < n$ has been chosen then $p$ can propose any value!
- If not, $p$ should propose the chosen value. But how?
What to propose

P2b: If a proposal with value \( v \) is chosen, then every higher-numbered proposal issued by any proposer has value \( v \)

Suppose \( p \) wants to issue a proposal numbered \( n \).

- If \( p \) can be certain that no proposal numbered \( n' < n \) has been chosen then \( p \) can propose any value!

- If not, \( p \) should propose the chosen value. But how?

  - What about using induction...
  - Say proposal numbered \( m \) with value \( v \) is chosen: some majority-set \( C \) of acceptors has accepted it
  - Suppose every proposal issued with number \( m \ldots n-1 \) has value \( v \)
  - Consider proposal \( n \): since every majority set \( S' \) intersects with \( C \) and every proposal accepted by any acceptor with sequence number in \( m \ldots n-1 \) has value \( v \), then

  - \( p \) should propose the highest numbered proposal among all proposals, numbered less than \( n \), accepted by some majority set \( S \)
It’s up to an invariant!

P2b: If a proposal with value \( \nu \) is chosen, then every higher-numbered proposal issued by any proposer has value \( \nu \)

Achieved by enforcing the following invariant

P2c: For any \( \nu \) and \( n \), if a proposal with value \( \nu \) and number \( n \) is issued, then there is a set \( S \) consisting of a majority of acceptors such that either:

- no acceptor in \( S \) has accepted any proposal numbered less than \( n \), or
- \( \nu \) is the value of the highest-numbered proposal among all proposals numbered less than \( n \) accepted by the acceptors in \( S \)
P2c in action

$v$ is the value of the highest-numbered proposal among all proposals numbered less than $n$ and accepted by the acceptors in $S$. 

$v(18,2) a_1^{(4,8)} a_2^{(3,2)} a_3^{(5,2)}$
P2c in action

The value of the highest-numbered proposal among all proposals numbered less than \( n \) and accepted by the acceptors in \( S \).
P2c in action

\( v \) is the value of the highest-numbered proposal among all proposals numbered less than \( n \) and accepted by the acceptors in \( S \).

The invariant is violated.
Future telling?

\( p \) must learn the highest-numbered proposal with number less than \( n \), if any, that has been or will be accepted by each acceptor in some majority of acceptors.

Avoid predicting the future by extracting a promise from a majority of acceptors not to subsequently accept any proposals numbered less than \( n \).
The proposer’s protocol (I)

A proposer chooses a new proposal number $n$ and sends a request to each member of some set of acceptors, asking it to respond with:

- A promise never again to accept a proposal numbered less than $n$, and
- The accepted proposal with highest number less than $n$ if any.

...call this a prepare request with number $n$
The proposer’s protocol (II)

- If the proposer receives a response from a majority of acceptors, then it can issue a proposal with number $n$ and value $v$, where $v$ is
  
  - the value of the highest-numbered proposal among the responses, or
  
  - is any value selected by the proposer if responders returned no proposals

A proposer issues a proposal by sending, to some set of acceptors, a request that the proposal be accepted. 
...call this an accept request.
The acceptor’s protocol

- An acceptor receives **prepare** and **accept** requests from proposers.
- It can always respond to a **prepare** request.
- It can respond to an **accept** request, accepting the proposal, iff it has not promised not to, e.g.

  P1a: An acceptor can accept a proposal numbered \( n \) iff it has not responded to a prepare request having number greater than \( n \).

  ...which subsumes P1.
Small optimizations

If an acceptor receives a prepare request $r$ numbered $n$ when it has already responded to a prepare request for $n' > n$, then the acceptor can simply ignore $r$.

...so an acceptor needs only remember the highest numbered proposal it has accepted and the number of the highest-numbered prepare request to which it has responded.
Choosing a value:
Phase 1

- A proposer chooses a new $n$ and sends $<\text{prepare}, n>$ to a majority of acceptors.

- If an acceptor $a$ receives $<\text{prepare}, n'>$ where $n' > n$ of any $<\text{prepare}, n>$ to which it has responded, then it responds to $<\text{prepare}, n'>$ with:
  - a promise not to accept any more proposals numbered less than $n'$
  - the highest numbered proposal (if any) that it has accepted.
Choosing a value: Phase 2

- If the proposer receives a response to \(<\text{prepare},n>\)
  from a majority of acceptors, then it sends to each
  \(<\text{accept},n,v>\), where \(v\) is either
  - the value of the highest numbered proposal
    among the responses
  - any value if the responses reported no proposals

- If an acceptor receives \(<\text{accept},n,v>\), it accepts the
  proposal unless it has in the meantime responded to
  \(<\text{prepare},n'>\), where \(n' > n\)
Learning chosen values (I)

Once a value is chosen, learners should find out about it. Many strategies are possible:

i. Each acceptor informs each learner whenever it accepts a proposal.

ii. Acceptors inform a distinguished learner, who informs the other learners

iii. Something in between (a set of not-quite-as-distinguished learners)
Questions

What are the liveness properties of Paxos?
Question

What do you do when nodes fail?
Question

Are there any advantages/disadvantages to having a designated leader?
Implementing State Machine Replication

- Implement a sequence of separate instances of consensus, where the value chosen by the $i^{th}$ instance is the $i^{th}$ message in the sequence.

- Each server assumes all three roles in each instance of the algorithm.

- Assume that the set of servers is fixed
The role of the leader

- In normal operation, elect a single server to be a leader. The leader acts as the distinguished proposer in all instances of the consensus algorithm.

- Clients send commands to the leader, which decides where in the sequence each command should appear.

- If the leader, for example, decides that a client command is the $k^{th}$ command, it tries to have the command chosen as the value in the $k^{th}$ instance of consensus.
A new leader $\lambda$ is elected...

Since $\lambda$ is a learner in all instances of consensus, it should know most of the commands that have already been chosen. For example, it might know commands 1-10, 13, and 15.

$\square$ It executes phase 1 of instances 11, 12, and 14 and of all instances 16 and larger.

$\square$ This might leave, say, 14 and 16 constrained and 11, 12 and all commands after 16 unconstrained.

$\square$ $\lambda$ then executes phase 2 of 14 and 16, thereby choosing the commands numbered 14 and 16
Stop-gap measures

All replicas can execute commands 1-10, but not 13-16 because 11 and 12 haven't yet been chosen.

λ can either take the next two commands requested by clients to be commands 11 and 12, or can propose immediately that 11 and 12 be no-op commands.

λ runs phase 2 of consensus for instance numbers 11 and 12.

Once consensus is achieved, all replicas can execute all commands through 16.
To infinity, and beyond

- $\lambda$ can efficiently execute phase 1 for infinitely many instances of consensus! (e.g. command 16 and higher)

- $\lambda$ just sends a message with a sufficiently high proposal number for all instances

- An acceptor replies non trivially only for instances for which it has already accepted a value
Question

What are the costs to using Paxos? Is it practical?