Announcements:

- Homework late periods
  - Two late periods across four homeworks
  - No(!) credit if late a 3rd time. Submit on time ☺
- Project Milestone & Final Report: We expect that everyone in a group fairly contributes to the group project. We will have a description of individual contributions at the end of the report. We reserve the right to give different grades across the group. Discuss this in your groups and create a fair solution.
- Project Milestone grading and feedback: will be released today.
- Tue May 21 – Extra Project Office Hours with TAs (highly encouraged)
  - Only helpful if prepared and on time
  - This replaces lecture and Tim's OH on May 21
- Thu May 23: Guest lecture.

Mining Data Streams (Part 1)

CS547 Machine Learning for Big Data

Tim Althoff

PAUL G. ALLEN SCHOOL
OF COMPUTER SCIENCE & ENGINEERING

New Topic: Infinite Data

High dim. data
- Locality sensitive hashing
- Clustering
- Dimensionality reduction

Graph data
- PageRank, SimRank
- Community Detection
- Spam Detection

Infinite data
- Sampling data streams
- Filtering data streams
- Queries on streams

Machine learning
- Decision Trees
- SVM
- Parallel SGD

Apps
- Recommender systems
- Association Rules
- Duplicate document detection

Data Streams

- **In many data mining situations, we do not know the entire data set in advance**
- **Stream Management** is important when the input rate is controlled **externally:**
  - Google queries
  - Twitter or Facebook status updates
- We can think of the **data** as **infinite** and **non-stationary** (the distribution changes over time)
  - This is the fun part and why interesting algorithms are needed

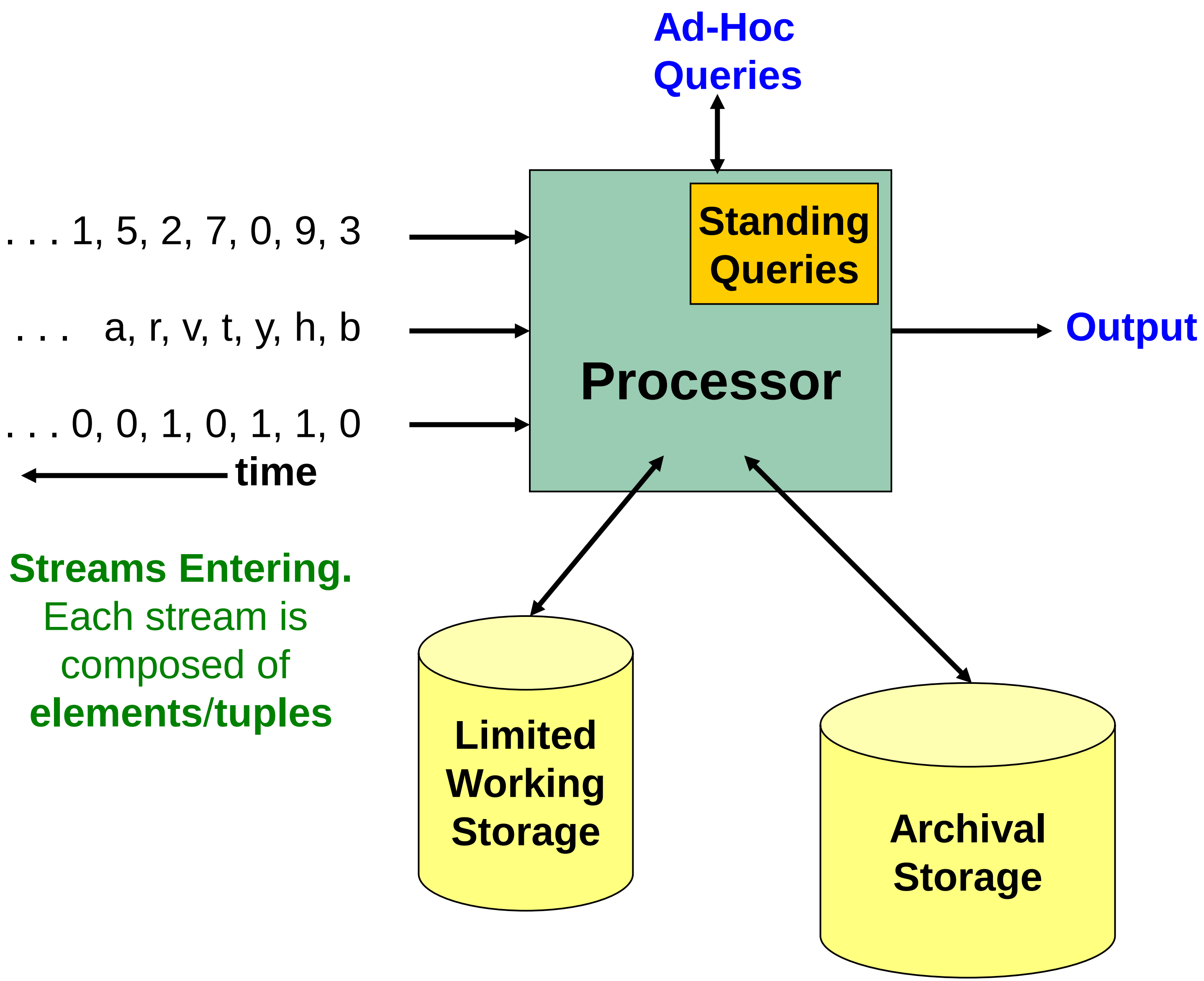
The Stream Model

- Input **elements** enter at a rapid rate, at one or more input ports (i.e., **streams**)
  - **We call elements of the stream tuples**
- **The system cannot store the entire stream accessibly**
- **Q: How do you make critical calculations about the stream using a limited amount of (secondary) memory?**

Side note: SGD is a Streaming Alg.

- **Stochastic Gradient Descent (SGD) is an example of a stream algorithm**
- **In Machine Learning we call this: Online Learning**
  - Allows for modeling problems where we have a continuous stream of data
  - We want an algorithm to learn from it and slowly adapt to the changes in data
- **Idea: Do small updates to the model**
  - **SGD** (SVM, Perceptron) makes small updates
  - **So:** First train the classifier on training data
  - **Then:** For every example from the stream, we slightly update the model (using small learning rate)

General Stream Processing Model



Streams Entering.
Each stream is composed of **elements/tuples**

Problems on Data Streams

- **Types of queries one wants to answer on a data stream:** (we'll do these today)
  - **Sampling data from a stream**
    - Construct a random sample
  - **Queries over sliding windows**
    - Number of items of type ***x*** in the last ***k*** elements of the stream

Problems on Data Streams

- **Types of queries one wants to answer on a data stream:** (we'll do these on Thu)
  - **Filtering a data stream**
    - Select elements with property ***x*** from the stream
  - **Counting distinct elements**
    - Number of distinct elements in the last ***k*** elements of the stream
  - **Estimating moments**
    - Estimate avg./std. dev. of elements in stream
  - **Finding frequent elements**

Applications (1)

- **Mining query streams**
  - Google wants to know what queries are most frequent today
- **Mining click streams**
  - Wikipedia wants to know which of its pages are getting an unusual number of hits in the past hour
- **Mining social network news feeds**
  - Look for trending topics on Twitter, Facebook

Applications (2)

- **Sensor Networks**
  - Many sensors feeding into a central controller
- **Telephone call records**
  - Data feeds into customer bills as well as settlements between telephone companies
- **IP packets monitored at a switch**
  - Gather information for optimal routing
  - Detect denial-of-service attacks
- **Large-scale machine learning models**
  - Get summary statistics of data for candidate splits in decision tree model (e.g. Xgboost)

Sampling from a Data Stream: Sampling a fixed proportion

As the stream grows the sample also gets bigger

Sampling from a Data Stream

- Since **we can not store the entire stream**, one obvious approach is to store a **sample**
- **Two different problems:**
  - **(1)** Sample a **fixed proportion** of elements in the stream (say 1 in 10)
  - **(2)** Maintain a **random sample of fixed size** over a potentially infinite stream
    - At any "time" ***k*** we would like a random sample of ***s*** elements
      - **What is the property of the sample we want to maintain?** For all time steps ***k***, each of ***k*** elements seen so far has equal prob. of being sampled

Sampling a Fixed Proportion

- **Problem 1: Sampling fixed proportion**
- **Scenario:** Search engine query stream
  - **Stream of tuples:** (user, query, time)
  - **Answer questions such as: How often did a user run the same query in a single day**
  - Have space to store **1/10th** of query stream
- **Naïve solution:**
  - Generate a random integer in **[0..9]** for each query
  - Store the query if the integer is **0**, otherwise discard

Problem with Naïve Approach

- **Simple question: What fraction of unique queries by an average search engine user are duplicates?**
  - Suppose each user issues ***x*** queries once and ***d*** queries twice (total of ***x*+2*d*** query instances)
    - **Correct answer: *d*/(*x*+*d*)**
  - **Proposed solution: We keep 10% of the queries**
    - Sample will contain ***x*/10** of the singleton queries and **2*d*/10** of the duplicate queries
    - But only ***d*/100** pairs of duplicates
      - **d/100 = 1/10 · 1/10 · d**
    - Of ***d*** "duplicates" in the full stream, ***18d/100*** queries appear exactly once
      - **18d/100 = ((1/10 · 9/10)+(9/10 · 1/10)) · d**
  - **So the sample-based answer is** $\frac{\frac{d}{100}}{\frac{x}{10}+\frac{d}{100}+\frac{18d}{100}} = \frac{d}{10x+19d}$

Solution: Sample Users

Solution:

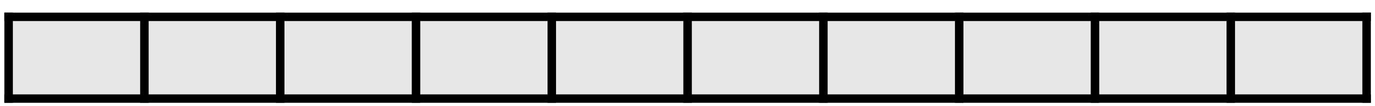
- Pick **1/10th** of **users** and take **all** their searches in the sample

- Use a hash function that hashes the user name or user id uniformly into 10 buckets (0-9). If the user hashes to bucket 0, then accept this search query for the sample, and if not, then not. (reading 4.2.2)

Generalized Solution

- **Stream of tuples with keys:**
  - Key is some subset of each tuple's components
    - e.g., tuple is (user, query, time); key is **user**
  - Choice of key depends on application

- **To get a sample of *a/b* fraction of the stream:**
  - Hash each tuple's key uniformly into ***b*** buckets
  - Pick the tuple if its hash value is at most ***a***



Hash table with **b** buckets, pick the tuple if its hash value is at most **a.**
How to generate a 30% sample?
Hash into b=10 buckets, take the tuple if it hashes to one of the first 3 buckets

Sampling from a Data Stream: Sampling a fixed-size sample

As the stream grows, the sample is of fixed size

Sampling from a Data Stream

- Since **we can not store the entire stream**, one obvious approach is to store a **sample**
- **Two different problems:**
  - **(1)** Sample a **fixed proportion** of elements in the stream (say 1 in 10)
  - **(2)** Maintain a **random sample of fixed size** over a potentially infinite stream
    - At any "time" ***k*** we would like a random sample of ***s*** elements
      - **What is the property of the sample we want to maintain?** For all time steps ***k***, each of ***k*** elements seen so far has equal prob. of being sampled

Maintaining a fixed-size sample

- **Problem 2: Fixed-size sample**
- **Suppose we need to maintain a random sample *S* of size exactly *s* tuples**
  - E.g., main memory size constraint
- **Why?** Don't know length of stream in advance
- **Suppose by time *n* we have seen *n* items**
  - **Each item is in the sample *S* with equal prob. *s/n***

How to think about the problem: say s = 2

Stream: a x c y z k c d e g…

At **n= 5,** each of the first 5 tuples is included in the sample **S** with equal prob.

At **n= 7,** each of the first 7 tuples is included in the sample **S** with equal prob.

Impractical solution would be to store all the *n* tuples seen so far and out of them pick *s* at random

Solution: Fixed Size Sample

- **Algorithm (a.k.a. Reservoir Sampling)**
  - Store all the first ***s*** elements of the stream to ***S***
  - Suppose we have seen ***n-1*** elements, and now the n^{th} element arrives ($n > s$)
    - With probability ***s/n***, keep the n^{th} element, else discard it
    - If we picked the n^{th} element, then it replaces one of the ***s*** elements in the sample ***S***, picked uniformly at random
- **Claim:** This algorithm maintains a sample ***S*** with the desired property:
  - After ***n*** elements, the sample contains each element seen so far with probability ***s/n***

Proof: By Induction

- **We prove this by induction (reading 4.5.5):**
  - Assume that after ***n*** elements, the sample contains each element seen so far with probability ***s/n***
  - We need to show that after seeing element ***n+1*** the sample maintains the property
    - Sample contains each element seen so far with probability ***s/(n+1)***
- **Base case:**
  - After we see **n=s** elements the sample **S** has the desired property
    - Each out of **n=s** elements is in the sample with probability ***s/s = 1***

Proof: By Induction

- **Inductive hypothesis:** After ***n*** elements, the sample ***S*** contains each element seen so far with prob. ***s/n***
- **Now element *n+1* arrives**
- **Inductive step:** For each element already in ***S***, probability that the algorithm keeps it in ***S*** is:

$$\underbrace{\left(1-\frac{s}{n+1}\right)}_{\text{Element } \mathbf{n+1} \text{ discarded}}+\underbrace{\left(\frac{s}{n+1}\right)}_{\text{Element } \mathbf{n+1} \text{ not discarded}}\underbrace{\left(\frac{s-1}{s}\right)}_{\text{Element in the sample not picked}}=\frac{n}{n+1}$$

- So, at time ***n,*** tuples in ***S*** were there with prob. **s/n**
- Time ***n→n+1***, tuple stayed in ***S*** with prob. **n/(n+1)**
- So prob. tuple is in ***S*** at time ***n+1*** $= \frac{s}{n} \cdot \frac{n}{n+1} = \frac{s}{n+1}$

Queries over a (long) Sliding Window

Sliding Windows

- A useful model of stream processing is that queries are about a ***window*** of length ***N*** – the ***N*** most recent elements received
- **Interesting case:** ***N*** is so large that the data cannot be stored in memory, or even on disk
  - Or, there are so many streams that windows for all cannot be stored
- **Amazon example:**
  - For every product **X** we keep 0/1 stream of whether that product was sold in the **n**-th transaction
  - We want answer queries, how many times have we sold **X** in the last **N** sales

Sliding Window: 1 Stream

- **Sliding window on a single stream:**

N = 6

q w e r t y u i o p $\boxed{\text{a s d f g h}}$ j k l z x c v b n m

q w e r t y u i o p a $\boxed{\text{s d f g h j}}$ k l z x c v b n m

q w e r t y u i o p a s $\boxed{\text{d f g h j k}}$ l z x c v b n m

q w e r t y u i o p a s d $\boxed{\text{f g h j k l}}$ z x c v b n m

← Past Future →

Counting Bits (1)

- **Problem (reading 4.6):**
  - Given a stream of **0**s and **1**s
  - Be prepared to answer queries of the form **How many 1s are in the last *k* bits?** For any $k \leq N$, ***where N is the window size.***

- **Obvious solution:**
  Store the most recent ***N*** bits
  - When new bit comes in, discard the **(*N*+1)st** bit

0 1 0 0 1 1 0 1 1 1 0 1 0 1 0 1 1 0 $\boxed{1\ 1\ 0\ 1\ 1\ 0}$ Suppose N=6

← Past Future →

Counting Bits (2)

- **You can not get an exact answer without storing the entire window (**details of proof is in reading 4.6.1**)**

- **Real Problem:**
  What if we cannot afford to store *N* bits?
  - Say we're processing many such streams and for each *N*=1 billion

0 1 0 0 1 1 0 1 1 1 0 1 0 1 0 1 1 0 ~~1 1 0 1 1 0~~

← Past Future →

- **But we are happy with an approximate answer**

An attempt: Simple solution

- Q: How many 1s are in the last *N* bits?
- A simple solution that does not really solve our problem: **Uniformity assumption**

← ***N*** →

0 1 0 0 1 1 1 0 0 0 1 0 1 0 0 1 0 0 0 1 0 1 1 0 1 1 0 1 1 1 0 0 1 0 1 0 1 1 0 0 1 1 0 1 0

← Past Future →

- **Maintain 2 counters:**
  - ***S***: number of 1s from the beginning of the stream
  - ***Z***: number of 0s from the beginning of the stream
- **How many 1s are in the last N bits?** $N \cdot \frac{S}{S+Z}$
- **But, what if stream is non-uniform?**
  - What if distribution changes over time?

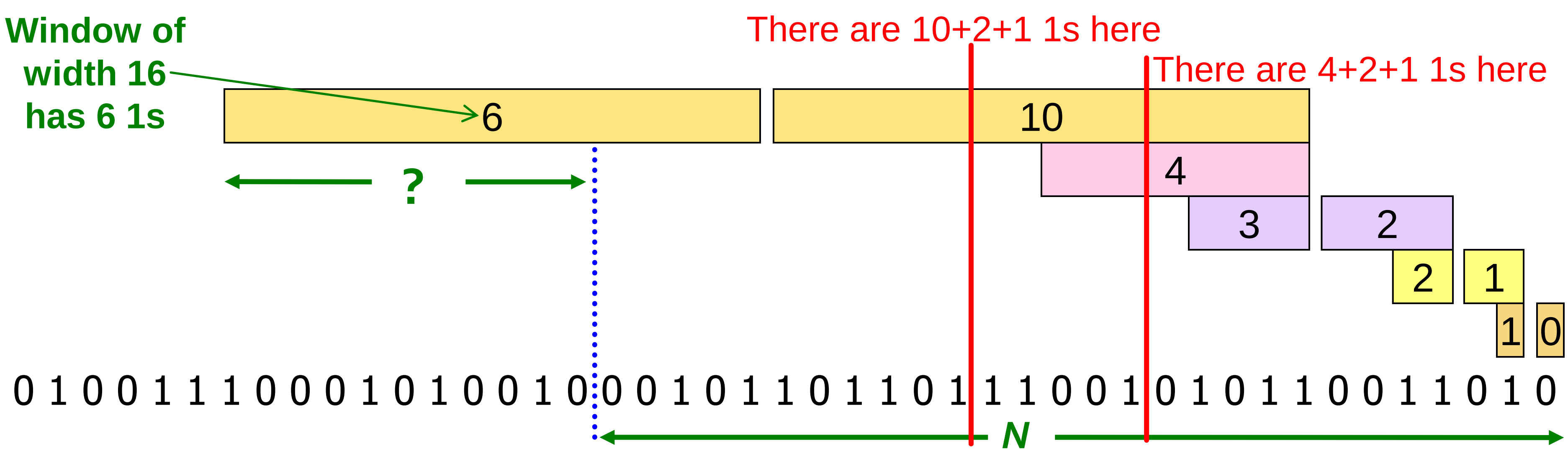
[Datar, Gionis, Indyk, Motwani]

DGIM Method

- **DGIM solution that does <u>not</u> assume uniformity**
- We store $\boldsymbol{O}(\log^2 \boldsymbol{N})$ bits per stream
- **Solution gives approximate answer, never off by more than 50%**
  - Error factor can be reduced to any fraction > 0, with more complicated algorithm and proportionally more stored bits
    - Error: If we have 10 1s then 50% error means 10 +/- 5

Idea: Exponential Windows

- **Solution that doesn't (quite) work:**
  - Summarize **exponentially increasing** regions of the stream, looking backward
  - Drop small regions if they begin at the same point as a larger region



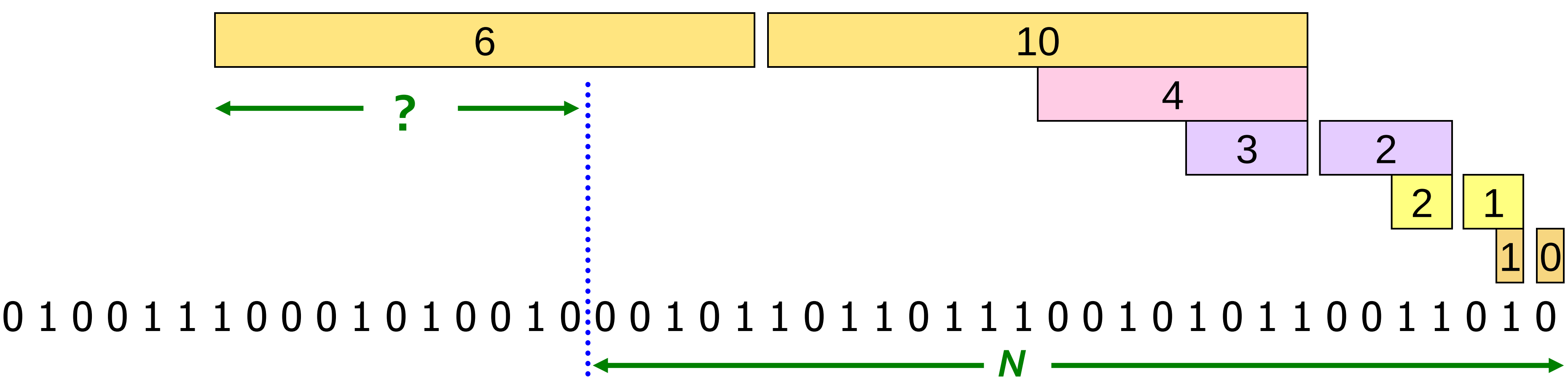
We can reconstruct the count of the last ***N*** bits, except we are not sure how many of the last **6 1s** are included in the ***N***

What's Good?

- **Stores only O($\log^2 N$) bits**
  - $\boldsymbol{O}(\log \boldsymbol{N})$ counts of $\log_{\mathbf{2}} \boldsymbol{N}$ bits each (similar to reading 4.6.3)
- **Easy update as more bits enter**
- Error in count no greater than the number of **1s** in the "**unknown**" area

What's Not So Good?

- As long as the **1s** are fairly evenly distributed, the error due to the unknown region is small – **no more than 50%**
- But it could be that all the **1s** are in the unknown area at the end
- In that case, **the relative error is unbounded!**



[Datar, Gionis, Indyk, Motwani]

Fixup: DGIM method

- **Idea:** Instead of summarizing fixed-length blocks, summarize blocks with specific number of **1s**:
  - Let the block ***sizes*** (number of **1s**) increase exponentially

- **When there are few 1s in the window, block sizes stay small, so errors are small**

1001010110001011010101010101011010101010101110101010111010100010110010

N

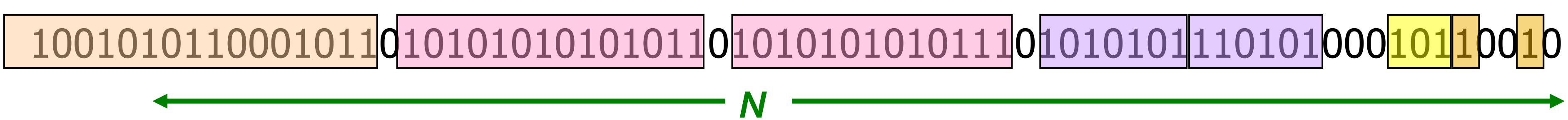
DGIM: Timestamps

- Each bit in the stream has a ***timestamp***, starting **1**, **2,** …

- Record timestamps modulo $\boldsymbol{N}$ (**the window size**), so we can represent any **relevant** timestamp in $\boldsymbol{O(log_2 N)}$ bits

DGIM: Buckets

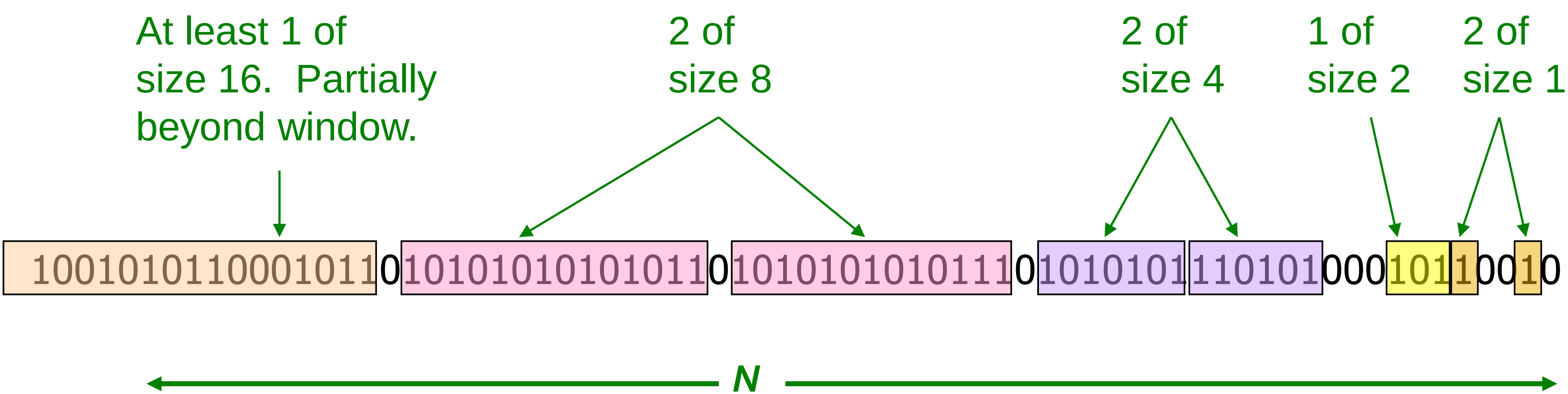
- A ***bucket*** in the DGIM method is a record consisting of:
  - **(A) The timestamp of its right (most recent) end** **[O(log *N*) bits]**
  - **(B) The number of 1s between its left beginning and right end** **[O(log log *N*) bits]**
- **Constraint on buckets:**
  Number of **1s** must be a power of **2**
  - That explains the **O(log log *N*)** in **(B) above**



Representing a Stream by Buckets

- Either **one** or **two** buckets with the same **power-of-2 number** of **1s**
- **Buckets do not overlap in timestamps**
- **Buckets are sorted by size**
  - Earlier buckets are not smaller than later buckets
- Buckets disappear when their end-time is **>** $\boldsymbol{N}$ time units in the past

Example: Bucketized Stream



Three properties of buckets that are maintained:

- Either **one** or **two** buckets with the same **power-of-2** number of **1s**
- Buckets do not overlap in timestamps
- Buckets are sorted by size

Updating Buckets (1)

- When a new bit comes in, drop the last (oldest) bucket if its end-time is prior to ***N*** time units before the current time

- **2 cases:** Current bit is **0** or **1**

- **If the current bit is 0:**
  no other changes are needed

Updating Buckets (2)

- **If the current bit is 1:**
  - **(1)** Create a new bucket of size **1**, for just this bit
    - **End timestamp = current time**
  - **(2)** If there are now **three buckets of size 1**, **combine the oldest two into a bucket of size 2**
  - **(3)** If there are now **three buckets of size 2**, **combine the oldest two into a bucket of size 4**
  - **(4) And so on …**

Example: Updating Buckets

Current state of the stream:

10010101100010110101010101010101101010101010101111010101011101010001011001 0

Bit of value 1 arrives

001010110001011010101010101010110101010101010111101010101110101000101100101

Two orange buckets get merged into a yellow bucket

001010110001011010101010101010110101010101010111101010101110101000101100101

Next bit 1 arrives, new orange bucket is created, then 0 comes, then 1:

010110001011010101010101010110101010101010111101010101110101000101100101101

Buckets get merged…

010110001011010101010101010110101010101010111101010101110101000101100101101

State of the buckets after merging

010110001011010101010101010110101010101010111101010101110101000101100101101

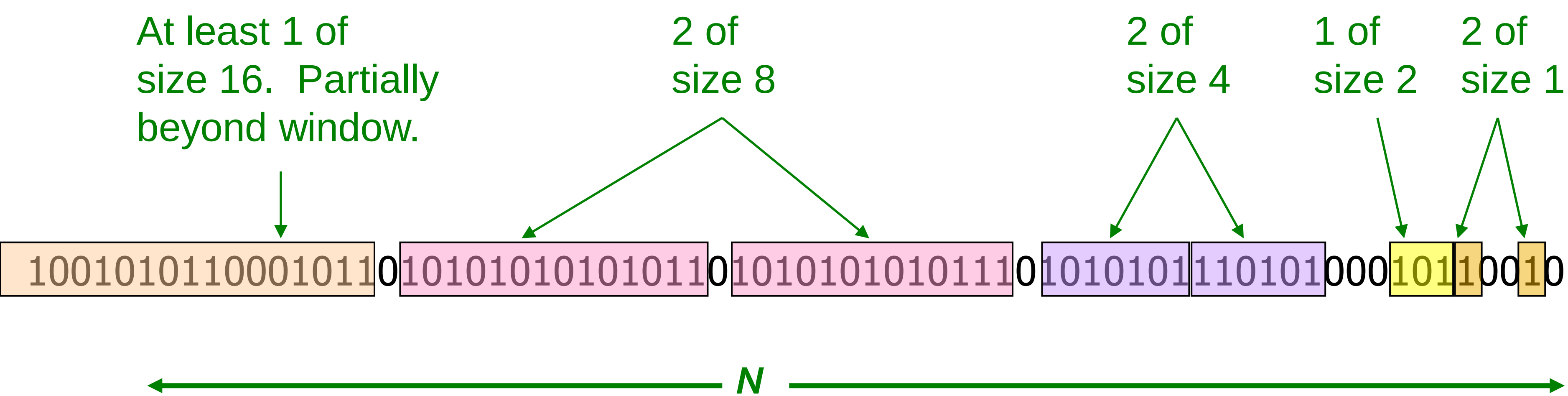
How to Query?

- **To estimate the number of 1s in the most recent *N* bits:**
  1. **Sum the sizes of all buckets but the last**

     (note "size" means the number of 1s in the bucket)
  2. **Add half the size of the last bucket**

- **Remember:** We do not know how many **1s** of the last bucket are still within the wanted window

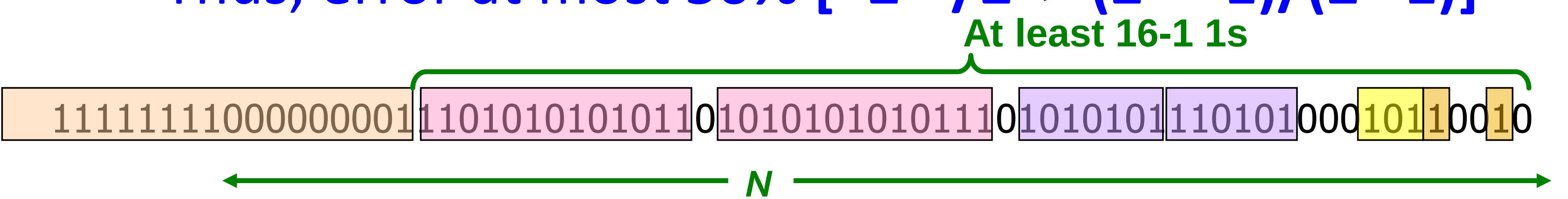
Example: Bucketized Stream



Estimate for the number of ones in window of size N is:
1 + 1 + 2 + 4 + 4 + 8 + 8 + 16/2

Error Bound: Proof Sketch

- **Why is error at most 50%? Let's prove it!**
- Suppose the last bucket has size $\mathbf{2^{r}}$
- Worst case overestimate: All the **1s** in the bucket are outside of window (except rightmost) - we make an error of at most $\mathbf{2^{r-1} - 1}$
- Since there is at least one bucket of each of the sizes less than $\mathbf{2^{r}}$, the true sum is at least
  $\mathbf{1 + 2 + 4 + .. + 2^{r-1} = 2^{r} - 1}$
- Thus, error at most **50%** $[= 2^{r-1}/2^{r} > (2^{r-1}-1)/(2^{r}-1)]$

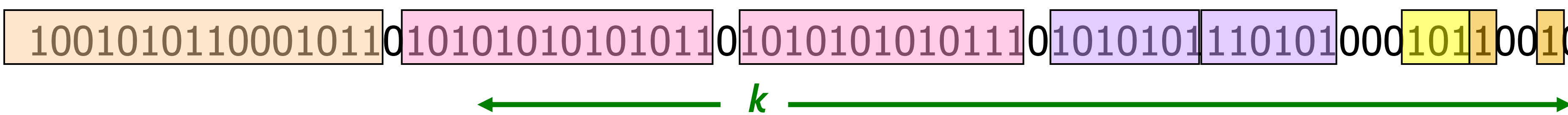


Further Reducing the Error

- Instead of maintaining **1** or **2** of each size bucket, we allow either ***r*-1** or ***r*** buckets (***r* > 2**)
  - Except for the largest size buckets; we can have any number between **1** and ***r*** of those
- **Error is at most *O*(1/*r*)**
  - see MMDS book for details (4.6.6)
- By picking ***r*** appropriately, we can tradeoff between number of bits we store and the error

Extensions

- Can we use the same trick to answer queries **How many 1's in the last *k*?** where ***k* < *N***?
  - **A:** Find earliest bucket **B** that at overlaps with ***k***. Number of **1s** is the **sum of sizes of more recent buckets + ½ size of B**



- **How can we handle the case where the stream is not bits, but integers, and we want the sum of the last *k* elements?**

[Distributed Streams. Algorithms for Sliding Windows, Gibbons & Tirthapura]

Extensions

- **Stream of positive integers**
- **We want the sum of the last *k* elements**
  - **Amazon:** Avg. price of last **k** sales
- **Solution:**
  - **(1) If you know all have at most *m* bits**
    - Treat ***m*** bits of each integer as a separate stream
    - Use DGIM to count **1s** in each integer/stream
    - The sum is $= \sum_{i=0}^{m-1} c_i 2^i$
  - **(2) Use buckets to keep partial sums**
    - **Sum of elements in size *b* bucket is at most 2^b**

c_i …estimated count for **i-th** bit

2 5 7 1 3 8 4 6 7 9 1 3 7 | 6 5 | 3 5 | 7 1 | 3 | 3 | 1 | 2 | 2 | 3

2 5 7 1 3 8 4 6 7 9 1 3 7 6 5 | 3 5 7 1 | 3 3 | 1 2 | 2 | 3 | **3**

2 5 7 1 3 8 4 6 7 9 1 3 7 6 5 | 3 5 7 1 | 3 3 | 1 2 | 2 | 3 | 3 | **2**

2 5 7 1 3 8 4 6 7 9 1 3 7 6 5 | 3 5 7 1 | 3 3 | 1 2 | 2 | 3 | 3 | 2 | **5**

Idea: Sum in each bucket is at most 2^b (unless bucket has only **1** integer)

Max bucket sum:

16	**8**	**4**	**2**	**1**

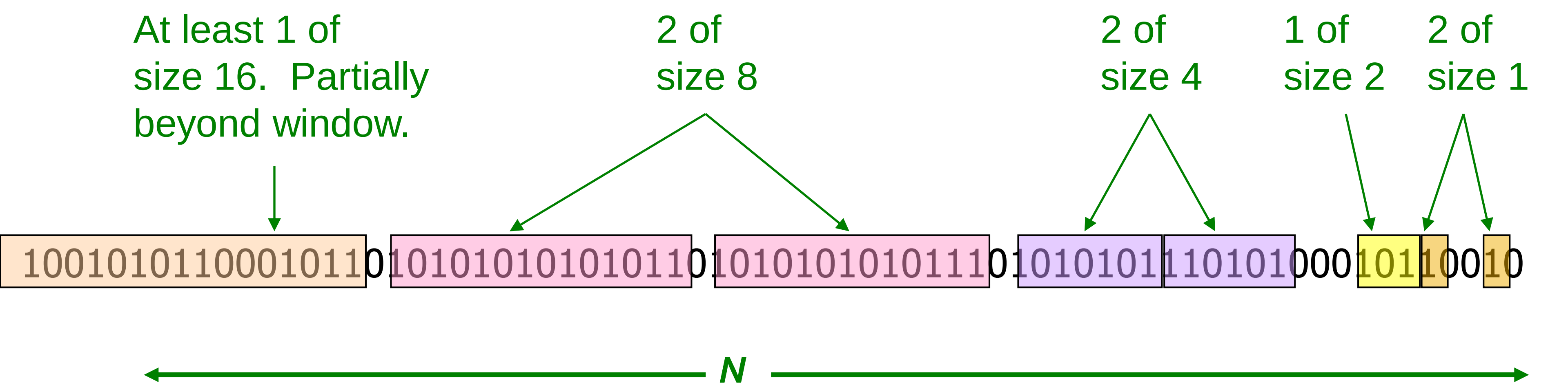
Summary

- **Sampling a fixed proportion of a stream**
  - Sample size grows as the stream grows
- **Sampling a fixed-size sample**
  - Reservoir sampling
- **Counting the number of 1s in the last N elements**
  - Exponentially increasing windows
  - Extensions:
    - Number of 1s in any last k (k < N) elements
    - Sums of integers in the last N elements

Counting Itemsets

Counting Itemsets

- **New Problem: Given a stream, which items appear more than *s* times in the window?**
- **Possible solution:** Think of the stream of baskets as one binary stream per item
  - **1** = item present; **0** = not present
  - Use **DGIM** to estimate counts of **1**s for all items



Extension to Itemsets

- **In principle, you could count frequent pairs or even larger sets the same way**
  - **One stream per itemset**
- **Drawbacks:**
  - Only approximate
  - **Number of itemsets is way too big**

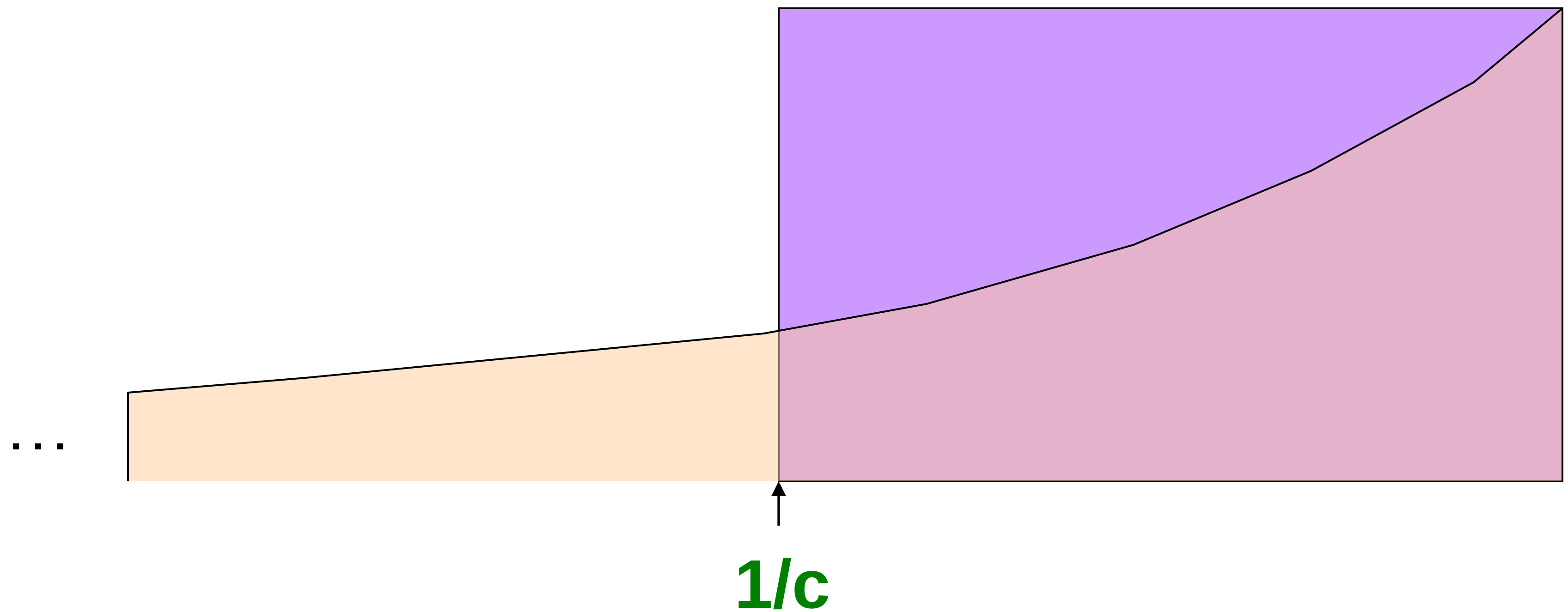
Exponentially Decaying Windows

- **Exponentially decaying windows: A heuristic for selecting likely frequent item(sets)**
  - **What are "currently" most popular movies?**
    - Instead of computing the raw count in last ***N*** elements
    - Compute a **smooth aggregation** over the whole stream
- If stream is $a_1, a_2, \ldots$ and we are taking the sum of the stream, take the answer at time t to be:
  $$= \sum_{i=1}^{t} a_i (1-c)^{t-i}$$
    - **c** is a constant, presumably tiny, like $\mathbf{10^{-6}}$ or $\mathbf{10^{-9}}$
- **When new a_{t+1} arrives:**
  Multiply current sum by **(1-c)** and add $\mathbf{a_{t+1}}$

Example: Counting Items

- If each a_i is an "item" we can compute the **characteristic function** of each possible item x as an Exponentially Decaying Window
  - That is: $\sum_{i=1}^{t} \delta_i \cdot (1-c)^{t-i}$ where $\delta_i = 1$ if $a_i = x$, and **0** otherwise
  - Imagine that for each item x we have a binary stream (**1** if x appears, **0** if x does not appear)
  - New item x arrives:
    - Multiply all counts by **(1-c)**
    - Add **+1** to count for element x
- **Call this sum the "weight" of item *x***

Sliding Versus Decaying Windows



- **Important property:** Sum over all weights $\sum_t (1-c)^t$ **is 1/[1 – (1 – c)] = 1/c**

$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

Example: Counting Items

- **What are "currently" most popular movies?**
- **Suppose we want to find movies of weight > ½**
  - **Important property:** Sum over all weights $\sum_t (1-c)^t$ is 1/[1 – (1 – c)] = **1/c**
- **Thus:**
  - There cannot be more than **2/c** movies with weight of ½ or more
- So, **2/c** is a limit on the number of movies being counted at any time

Extension to Itemsets

- **Count (some) itemsets in an E.D.W.**
  - **What are currently "hot" itemsets?**
    - **Problem:** Too many itemsets to keep counts of all of them in memory
- **When a basket B comes in:**
  - Multiply all counts by **(1-c)**
  - For uncounted items in **B**, create new count
  - Add **1** to count of any item in **B** and to any **itemset** contained in **B** that is already being counted
  - **Drop counts < ½**
  - Initiate new counts (next slide)

Initiation of New Counts

- Start a count for an itemset $S \subseteq B$ if every proper subset of ***S*** had a count prior to arrival of basket ***B***
  - **Intuitively:** If all subsets of **S** are being counted this means they are "**frequent/hot**" and thus ***S*** has a potential to be "**hot**"
- **Example:**
  - Start counting ***S*={i, j}** iff both **i** and **j** were counted prior to seeing ***B***
  - Start counting ***S*={i, j, k}** iff **{i, j}**, **{i, k}**, and **{j, k}** were all counted prior to seeing ***B***

Summary: Counting Itemsets

- **Task: Which were the most popular recent items?**
  - Can keep exponentially decaying counts for items and potentially larger itemsets
- **Number of larger itemsets is very large**
- **But we are conservative about starting counts of large sets**
  - All subsets need to be counted currently
  - If we counted every set we saw, one basket of **20** items would initiate **1M** counts (2^20)

Please give us feedback ☺

https://bit.ly/CSE547feedback2024