Announcements:
- Homework late periods
  - Two late periods across four homeworks
  - No(!) credit if late a 3rd time. Submit on time ☺
- Colab 8 – Extra time until Tue June 2 to cover submodular optimization topic
- Tue May 26 – Extra Project Office Hours (optional)
  - Sign up on Ed
  - Only helpful if prepared and on time
  - This replaces lecture and Tim's OH on May 25

Mining Data Streams (Part 1)

CS547 Machine Learning for Big Data

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New Topic: Infinite Data

High dim. data	Graph data	Infinite data	Machine learning	Apps
Locality sensitive hashing	PageRank, SimRank	Sampling data streams	Decision Trees	Recommender systems
Clustering	Community Detection	Filtering data streams	SVM	Association Rules
Dimensionality reduction	Spam Detection	Queries on streams	Parallel SGD	Duplicate document detection

Data Streams

- **In many data mining situations, we do not know the entire data set in advance**
- **Stream Management** is important when the input rate is controlled **externally:**
  - Google queries
  - Twitter or Facebook status updates
- We can think of the **data** as **infinite** and **non-stationary** (the distribution changes over time)
  - This is the fun part and why interesting algorithms are needed

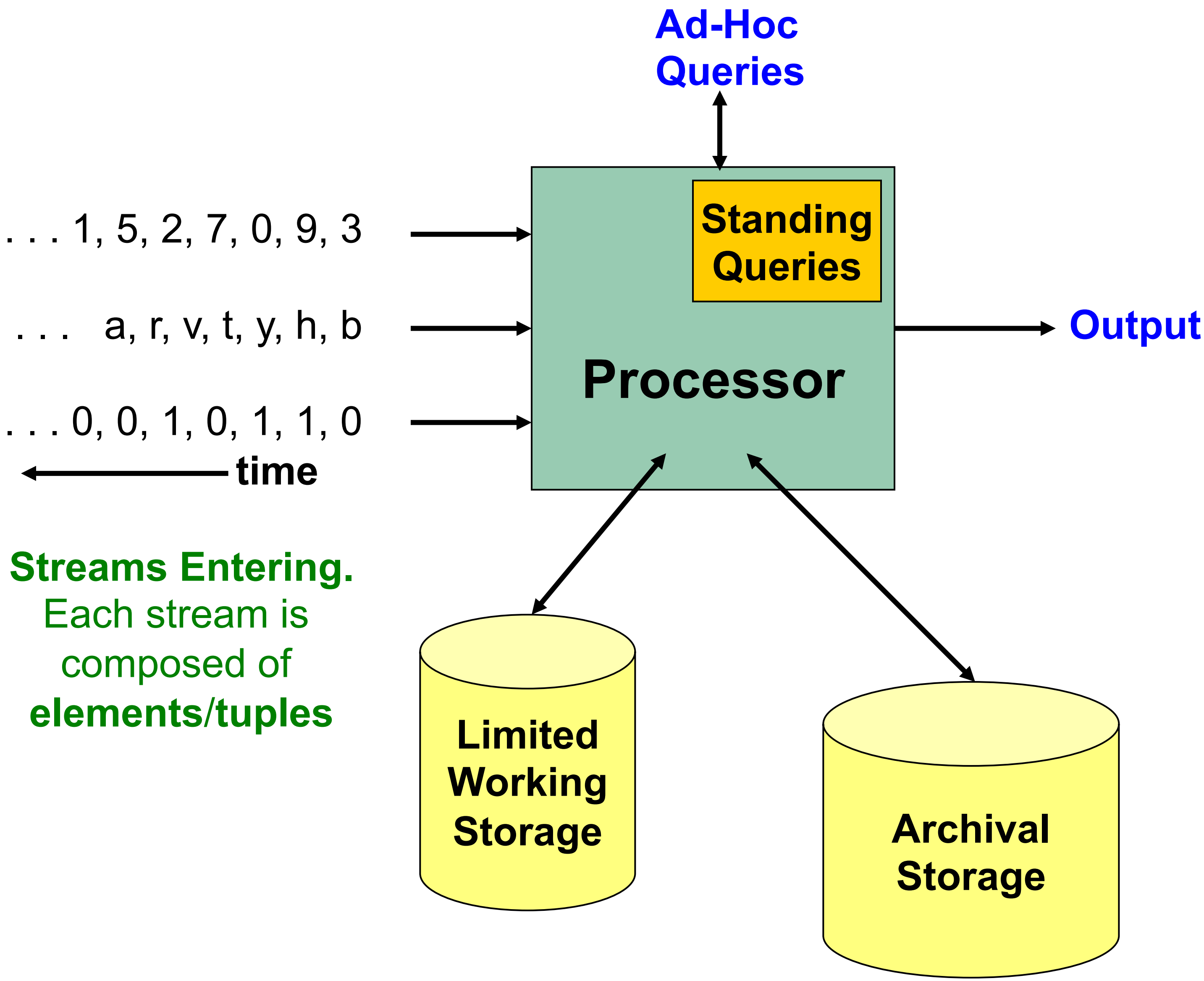
The Stream Model

- Input **elements** enter at a rapid rate, at one or more input ports (i.e., **streams**)
  - **We call elements of the stream tuples**
- **The system cannot store the entire stream accessibly**
- **Q: How do you make critical calculations about the stream using a limited amount of (secondary) memory?**

Side note: SGD is a Streaming Alg.

- **Stochastic Gradient Descent (SGD) is an example of a stream algorithm**
- **In Machine Learning we call this: Online Learning**
  - Allows for modeling problems where we have a continuous stream of data
  - We want an algorithm to learn from it and slowly adapt to the changes in data
- **Idea: Do small updates to the model**
  - **SGD** (SVM, Perceptron) makes small updates
  - **So:** First train the classifier on training data
  - **Then:** For every example from the stream, we slightly update the model (using small learning rate)

General Stream Processing Model



Streams Entering. Each stream is composed of **elements/tuples**

Problems on Data Streams

- **Types of queries one wants on answer on a data stream:** (we'll do these today)
  - **Sampling data from a stream**
    - Construct a random sample
  - **Queries over sliding windows**
    - Number of items of type ***x*** in the last ***k*** elements of the stream

Problems on Data Streams

- **Types of queries one wants on answer on a data stream:** (we'll do these on Thu)
  - **Filtering a data stream**
    - Select elements with property ***x*** from the stream
  - **Counting distinct elements**
    - Number of distinct elements in the last ***k*** elements of the stream
  - **Estimating moments**
    - Estimate avg./std. dev. of elements in stream
  - **Finding frequent elements**

Applications (1)

- **Mining query streams**
  - Google wants to know what queries are most frequent today
- **Mining click streams**
  - Wikipedia wants to know which of its pages are getting an unusual number of hits in the past hour
- **Mining social network news feeds**
  - Look for trending topics on Twitter, Facebook

Applications (2)

- **Sensor Networks**
  - Many sensors feeding into a central controller
- **Telephone call records**
  - Data feeds into customer bills as well as settlements between telephone companies
- **IP packets monitored at a switch**
  - Gather information for optimal routing
  - Detect denial-of-service attacks
- **Large-scale machine learning models**
  - Get summary statistics of data for candidate splits in decision tree model (e.g. Xgboost)

Sampling from a Data Stream: Sampling a fixed proportion

As the stream grows the sample also gets bigger

Sampling from a Data Stream

- Since **we can not store the entire stream**, one obvious approach is to store a **sample**
- **Two different problems:**
  - **(1)** Sample a **fixed proportion** of elements in the stream (say 1 in 10)
  - **(2)** Maintain a **random sample of fixed size** over a potentially infinite stream
    - At any "time" ***k*** we would like a random sample of ***s*** elements
      - **What is the property of the sample we want to maintain?** For all time steps ***k***, each of ***k*** elements seen so far has equal prob. of being sampled

Sampling a Fixed Proportion

- **Problem 1: Sampling fixed proportion**
- **Scenario:** Search engine query stream
  - **Stream of tuples:** (user, query, time)
  - **Answer questions such as: How often did a user run the same query in a single day**
  - Have space to store **1/10th** of query stream
- **Naïve solution:**
  - Generate a random integer in **[0..9]** for each query
  - Store the query if the integer is **0**, otherwise discard

Problem with Naïve Approach

- **Simple question: What fraction of unique queries by an average search engine user are duplicates?**
  - Suppose each user issues ***x*** queries once and ***d*** queries twice (total of ***x*+2*d*** query instances)
    - **Correct answer: *d*/(*x*+*d*)**
  - **Proposed solution: We keep 10% of the queries**
    - Sample will contain ***x*/10** of the singleton queries and **2*d*/10** of the duplicate queries at least once
    - But only ***d*/100** pairs of duplicates
      - **d/100 = 1/10 · 1/10 · d**
    - Of ***d*** "duplicates" ***18d/100*** appear exactly once
      - **18d/100 = ((1/10 · 9/10)+(9/10 · 1/10)) · d**
  - **So the sample-based answer is** $\frac{\frac{d}{100}}{\frac{x}{10}+\frac{d}{100}+\frac{18d}{100}} = \frac{d}{10x+19d}$

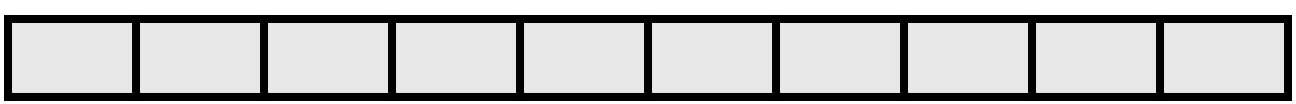
Solution: Sample Users

Solution:

- Pick **1/10th** of **users** and take **all** their searches in the sample
- Use a hash function that hashes the user name or user id uniformly into 10 buckets

Generalized Solution

- **Stream of tuples with keys:**
  - Key is some subset of each tuple's components
    - e.g., tuple is (user, search, time); key is **user**
  - Choice of key depends on application
- **To get a sample of *a/b* fraction of the stream:**
  - Hash each tuple's key uniformly into ***b*** buckets
  - Pick the tuple if its hash value is at most ***a***



Hash table with **b** buckets, pick the tuple if its hash value is at most **a.**
How to generate a 30% sample?
Hash into b=10 buckets, take the tuple if it hashes to one of the first 3 buckets

Sampling from a Data Stream: Sampling a fixed-size sample

As the stream grows, the sample is of fixed size

Maintaining a fixed-size sample

- **Problem 2: Fixed-size sample**
- **Suppose we need to maintain a random sample *S* of size exactly *s* tuples**
  - E.g., main memory size constraint
- **Why?** Don't know length of stream in advance
- **Suppose by time *n* we have seen *n* items**
  - **Each item is in the sample *S* with equal prob. *s/n***

How to think about the problem: say s = 2

Stream: a x c y z k c d e g…

At **n= 5,** each of the first 5 tuples is included in the sample **S** with equal prob.

At **n= 7,** each of the first 7 tuples is included in the sample **S** with equal prob.

Impractical solution would be to store all the *n* tuples seen so far and out of them pick *s* at random

Solution: Fixed Size Sample

- **Algorithm (a.k.a. Reservoir Sampling)**
  - Store all the first ***s*** elements of the stream to ***S***
  - Suppose we have seen ***n-1*** elements, and now the ***n****th* element arrives ($\boldsymbol{n > s}$)
    - With probability ***s/n***, keep the ***n****th* element, else discard it
    - If we picked the ***n****th* element, then it replaces one of the ***s*** elements in the sample ***S***, picked uniformly at random
- **Claim:** This algorithm maintains a sample ***S*** with the desired property:
  - After ***n*** elements, the sample contains each element seen so far with probability ***s/n***

Proof: By Induction

- **We prove this by induction:**
  - Assume that after ***n*** elements, the sample contains each element seen so far with probability ***s/n***
  - We need to show that after seeing element ***n+1*** the sample maintains the property
    - Sample contains each element seen so far with probability ***s/(n+1)***
- **Base case:**
  - After we see **n=s** elements the sample **S** has the desired property
    - Each out of **n=s** elements is in the sample with probability ***s/s = 1***

Proof: By Induction

- **Inductive hypothesis:** After ***n*** elements, the sample ***S*** contains each element seen so far with prob. ***s/n***
- **Now element *n+1* arrives**
- **Inductive step:** For elements already in ***S***, probability that the algorithm keeps it in ***S*** is:

$$\underbrace{\left(1-\frac{s}{n+1}\right)}_{\text{Element } \mathbf{n+1} \text{ discarded}}+\underbrace{\left(\frac{s}{n+1}\right)}_{\text{Element } \mathbf{n+1} \text{ not discarded}}\underbrace{\left(\frac{s-1}{s}\right)}_{\text{Element in the sample not picked}}=\frac{n}{n+1}$$

- So, at time ***n,*** tuples in ***S*** were there with prob. **s/n**
- Time ***n→n+1***, tuple stayed in ***S*** with prob. **n/(n+1)**
- So prob. tuple is in ***S*** at time ***n+1*** $= \frac{s}{n} \cdot \frac{n}{n+1} = \frac{s}{n+1}$

Queries over a (long) Sliding Window

Sliding Windows

- A useful model of stream processing is that queries are about a ***window*** of length ***N*** – the ***N*** most recent elements received
- **Interesting case:** ***N*** is so large that the data cannot be stored in memory, or even on disk
  - Or, there are so many streams that windows for all cannot be stored
- **Amazon example:**
  - For every product **X** we keep 0/1 stream of whether that product was sold in the **n**-th transaction
  - We want answer queries, how many times have we sold **X** in the last **k** sales

Sliding Window: 1 Stream

- **Sliding window on a single stream:**

N = 6

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

← Past Future →

Counting Bits (1)

- **Problem:**
  - Given a stream of **0**s and **1**s
  - Be prepared to answer queries of the form
    How many 1s are in the last *k* bits? For any $k \le N$

- **Obvious solution:**
  Store the most recent ***N*** bits
  - When new bit comes in, discard the ***N*+1st** bit

0 1 0 0 1 1 0 1 1 1 0 1 0 1 0 1 1 0 $\boxed{1\ 1\ 0\ 1\ 1\ 0}$ Suppose N=6

← Past Future →

Counting Bits (2)

- **You can not get an exact answer without storing the entire window**

- **Real Problem:**
  What if we cannot afford to store *N* bits?
  - Say we're processing many such streams and for each *N*=1 billion

0 1 0 0 1 1 0 1 1 1 0 1 0 1 0 1 1 0 ~~1 1 0 1 1 0~~

← Past   Future →

- **But we are happy with an approximate answer**

An attempt: Simple solution

- Q: **How many 1s are in the last *N* bits?**
- A simple solution that does not really solve our problem: **Uniformity assumption**

← *N* →

0 1 0 0 1 1 1 0 0 0 1 0 1 0 0 1 0 0 0 1 0 1 1 0 1 1 0 1 1 1 0 0 1 0 1 0 1 1 0 0 1 1 0 1 0

← Past Future →

- **Maintain 2 counters:**
  - ***S***: number of 1s from the beginning of the stream
  - ***Z***: number of 0s from the beginning of the stream
- **How many 1s are in the last N bits?** $N \cdot \frac{S}{S+Z}$
- **But, what if stream is non-uniform?**
  - What if distribution changes over time?

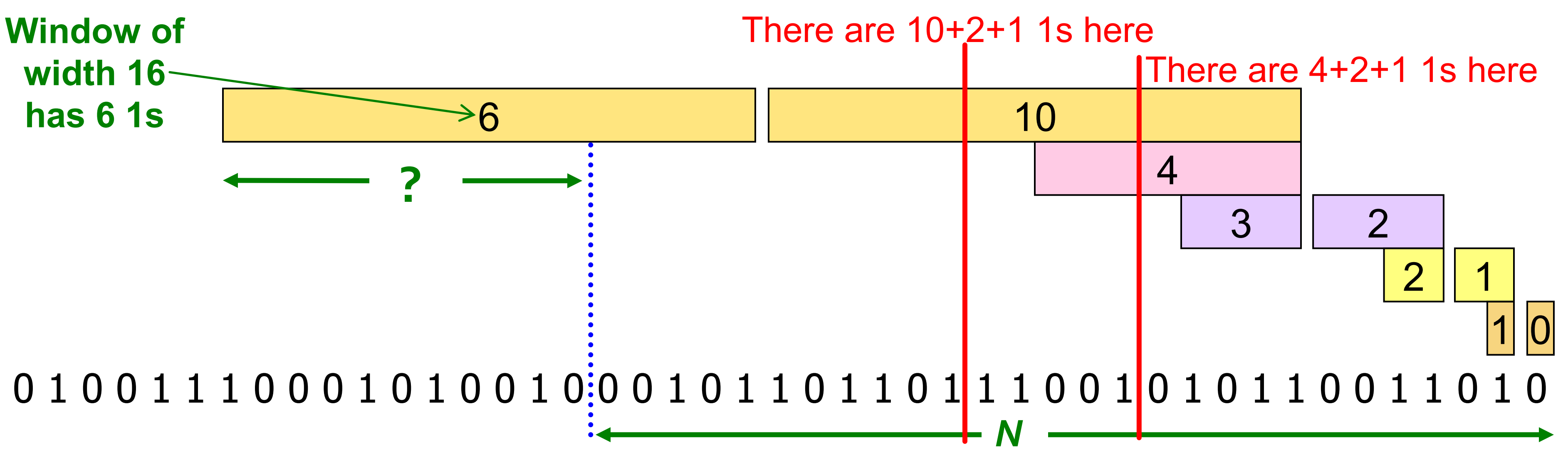
[Datar, Gionis, Indyk, Motwani]

DGIM Method

- **DGIM solution that does <u>not</u> assume uniformity**
- We store $O(\log^2 N)$ bits per stream
- **Solution gives approximate answer, never off by more than 50%**
  - Error factor can be reduced to any fraction > 0, with more complicated algorithm and proportionally more stored bits
    - Error: If we have 10 1s then 50% error means 10 +/- 5

Idea: Exponential Windows

- **Solution that doesn't (quite) work:**
  - Summarize **exponentially increasing** regions of the stream, looking backward
  - Drop small regions if they begin at the same point as a larger region



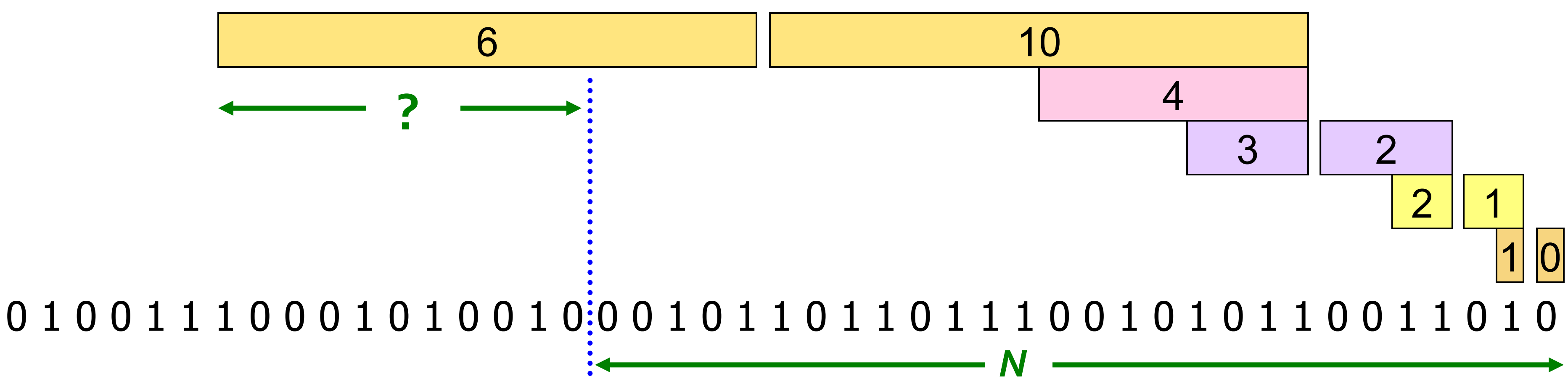
We can reconstruct the count of the last ***N*** bits, except we are not sure how many of the last **6 1s** are included in the ***N***

What's Good?

- **Stores only O($\log^2 N$) bits**
  - $\boldsymbol{O}(\log \boldsymbol{N})$ counts of $\log_{\mathbf{2}} \boldsymbol{N}$ bits each
- **Easy update as more bits enter**
- Error in count no greater than the number of **1s** in the "**unknown**" area

What's Not So Good?

- As long as the **1s** are fairly evenly distributed, the error due to the unknown region is small – **no more than 50%**
- But it could be that all the **1s** are in the unknown area at the end
- In that case, **the relative error is unbounded!**



[Datar, Gionis, Indyk, Motwani]

Fixup: DGIM method

- **Idea:** Instead of summarizing fixed-length blocks, summarize blocks with specific number of **1s**:
  - Let the block ***sizes*** (number of **1s**) increase exponentially

- **When there are few 1s in the window, block sizes stay small, so errors are small**

1001010110001011 0 10101010101011 0 1010101010111 0 1010101 110101 000 101 1 00 1 0

← *N* →

DGIM: Timestamps

- Each bit in the stream has a ***timestamp***, starting **1**, **2,** …
- Record timestamps modulo ***N*** (**the window size**), so we can represent any **relevant** timestamp in $\boldsymbol{O(log_2 N)}$ bits

DGIM: Buckets

- A ***bucket*** in the DGIM method is a record consisting of:
  - **(A) The timestamp of its end** [O(log *N*) bits]
  - **(B) The number of 1s between its beginning and end** [O(log log *N*) bits]
- **Constraint on buckets:**
  Number of **1s** must be a power of **2**
  - That explains the **O(log log *N*)** in **(B) above**

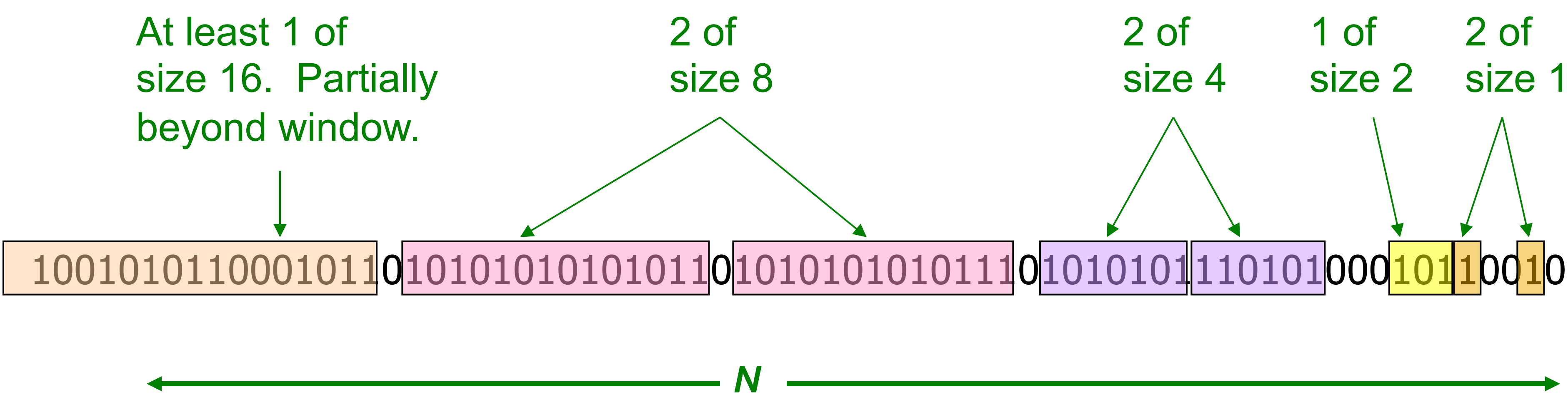
1001010110001011 0 1010101010101011 0 1010101010101110 1010101 110101 000 101 1 00 1 0

← *N* →

Representing a Stream by Buckets

- Either **one** or **two** buckets with the same **power-of-2 number** of **1s**
- **Buckets do not overlap in timestamps**
- **Buckets are sorted by size**
  - Earlier buckets are not smaller than later buckets
- Buckets disappear when their end-time is **>** ***N*** time units in the past

Example: Bucketized Stream



Three properties of buckets that are maintained:

- Either **one** or **two** buckets with the same **power-of-2** number of **1s**
- Buckets do not overlap in timestamps
- Buckets are sorted by size

Updating Buckets (1)

- When a new bit comes in, drop the last (oldest) bucket if its end-time is prior to ***N*** time units before the current time
- **2 cases:** Current bit is **0** or **1**
- **If the current bit is 0:**
  no other changes are needed

Updating Buckets (2)

- **If the current bit is 1:**
  - **(1)** Create a new bucket of size **1**, for just this bit
    - **End timestamp = current time**
  - **(2)** If there are now **three buckets of size 1**, **combine the oldest two into a bucket of size 2**
  - **(3)** If there are now **three buckets of size 2**, **combine the oldest two into a bucket of size 4**
  - **(4) And so on …**

Example: Updating Buckets

Current state of the stream:

1001010110001011 0 10101010101011 0 10101010101111 0 1010101 110101 000 101 1 00 1 0

Bit of value 1 arrives

001010110001011 0 10101010101011 0 10101010101111 0 1010101 110101 000 101 1 00 1 0 **1**

Two orange buckets get merged into a yellow bucket

001010110001011 0 10101010101011 0 10101010101111 0 1010101 110101 000 101 1001 0 **1**

Next bit 1 arrives, new orange bucket is created, then 0 comes, then 1:

010110001011 0 10101010101011 0 10101010101111 0 1010101 110101 000 101 1001 0 1 **1** **0** **1**

Buckets get merged…

010110001011 0 10101010101011 0 10101010101111 0 1010101 110101 000 101 1001 0 1**1** **0** **1**

State of the buckets after merging

010110001011 0 1010101010101110101010101111 0 1010101110101 000 1011001 0 11 0 1

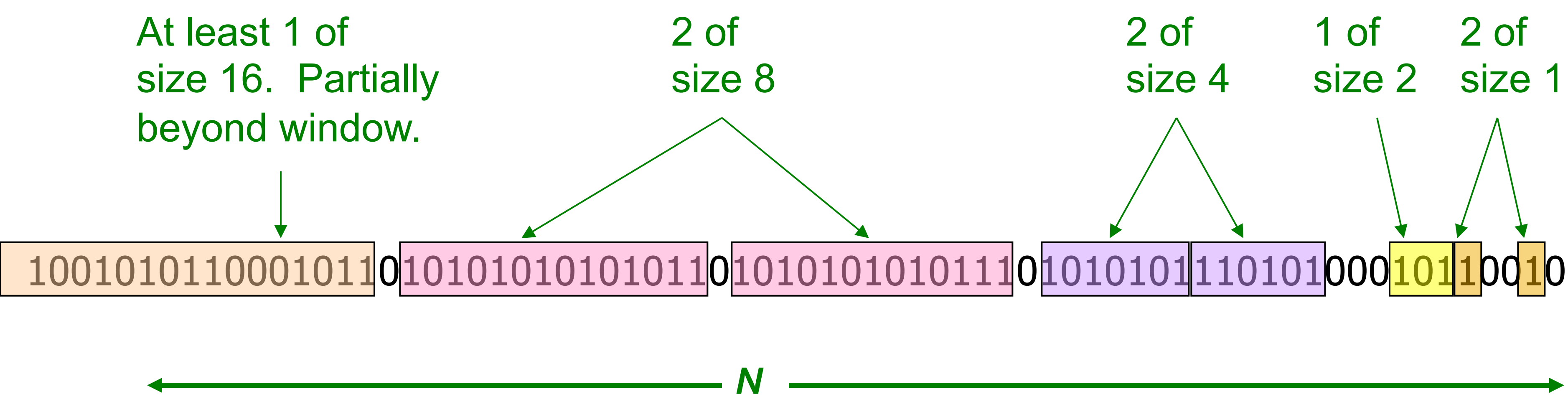
How to Query?

- **To estimate the number of 1s in the most recent *N* bits:**
  1. **Sum the sizes of all buckets but the last**

     (note "size" means the number of 1s in the bucket)
  2. **Add half the size of the last bucket**

- **Remember:** We do not know how many **1s** of the last bucket are still within the wanted window

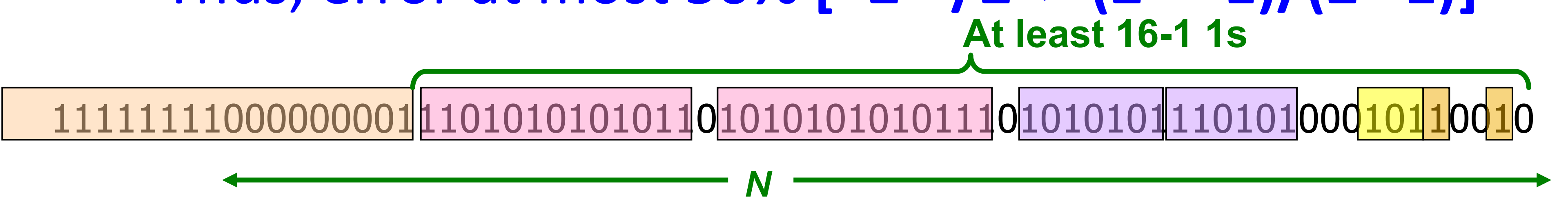
Example: Bucketized Stream



Estimate for the number of ones in window of size N is:
1 + 1 + 2 + 4 + 4 + 8 + 8 + 16/2

Error Bound: Proof Sketch

- **Why is error at most 50%? Let's prove it!**
- Suppose the last bucket has size $\mathbf{2^r}$
- Worst case overestimate: All the **1s** in the bucket are outside of window (except rightmost) - we make an error of at most $\mathbf{2^{r-1}-1}$
- Since there is at least one bucket of each of the sizes less than $\mathbf{2^r}$, the true sum is at least
  $\mathbf{1 + 2 + 4 + .. + 2^{r-1} = 2^r - 1}$
- Thus, error at most **50%** $[=2^{r-1}/2^r > (2^{r-1}-1)/(2^r-1)]$



Further Reducing the Error

- Instead of maintaining **1** or **2** of each size bucket, we allow either ***r*-1** or ***r*** buckets (***r* > 2**)
  - Except for the largest size buckets; we can have any number between **1** and ***r*** of those
- **Error is at most *O*(1/*r*)**
  - see MMDS book for details
- By picking ***r*** appropriately, we can tradeoff between number of bits we store and the error

Extensions

- Can we use the same trick to answer queries **How many 1's in the last *k*?** where ***k*** **<** ***N***?
  - **A:** Find earliest bucket **B** that at overlaps with ***k***. Number of **1s** is the **sum of sizes of more recent buckets + ½ size of B**

1001010110001011 0 1010101010101011 0 1010101010101111 0 1010101 110101 000 101 1 00 1 0

← *k* →

- **How can we handle the case where the stream is not bits, but integers, and we want the sum of the last *k* elements?**

[Distributed Streams. Algorithms for Sliding Windows, Gibbons & Tirthapura]

Extensions

- **Stream of positive integers**
- **We want the sum of the last *k* elements**
  - **Amazon:** Avg. price of last **k** sales
- **Solution:**
  - **(1) If you know all have at most *m* bits**
    - Treat ***m*** bits of each integer as a separate stream
    - Use DGIM to count **1s** in each integer/stream
    - The sum is $= \sum_{i=0}^{m-1} c_i 2^i$
  - **(2) Use buckets to keep partial sums**
    - **Sum of elements in size *b* bucket is at most 2^b**

c_i …estimated count for **i-th** bit

2 5 7 1 3 8 4 6 7 9 1 3 7 [6 5] [3 5] [7 1] [3] [3] [1] [2] [2] [3]

2 5 7 1 3 8 4 6 7 9 1 3 7 6 5 [3 5 7 1] [3 3] [1 2] [2] [3] [3]

2 5 7 1 3 8 4 6 7 9 1 3 7 6 5 [3 5 7 1] [3 3] [1 2] [2] [3] [3] [2]

2 5 7 1 3 8 4 6 7 9 1 3 7 6 5 [3 5 7 1] [3 3] [1 2] [2] [3] [3] [2] [5]

Idea: Sum in each bucket is at most 2^b (unless bucket has only **1** integer)

Max bucket sum:

16	8	4	2	1

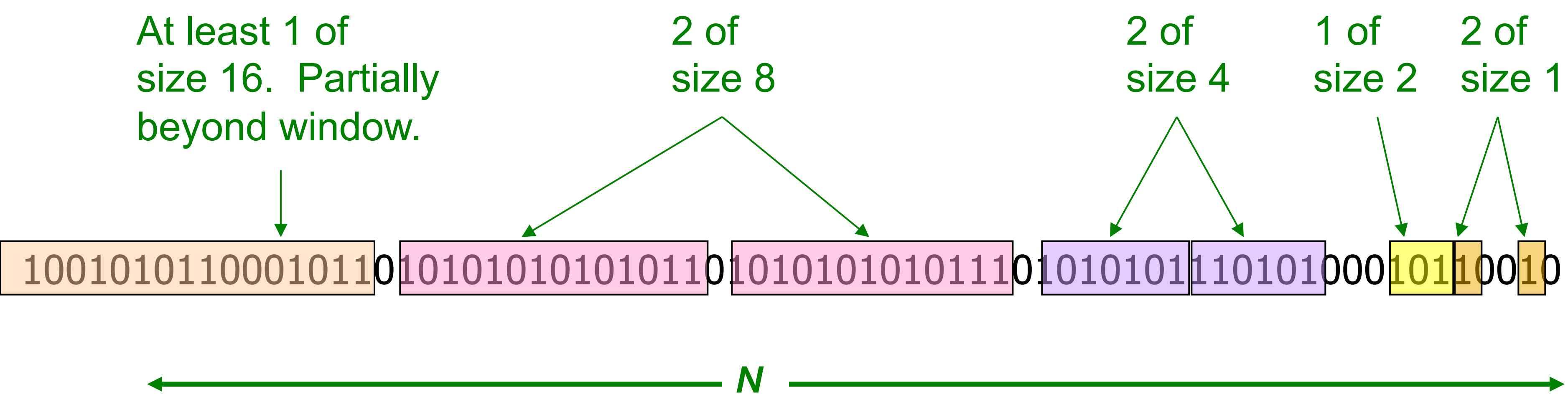
Summary

- **Sampling a fixed proportion of a stream**
  - Sample size grows as the stream grows
- **Sampling a fixed-size sample**
  - Reservoir sampling
- **Counting the number of 1s in the last N elements**
  - Exponentially increasing windows
  - Extensions:
    - Number of 1s in any last k (k < N) elements
    - Sums of integers in the last N elements

Counting Itemsets

Counting Itemsets

- **New Problem:** **Given a stream, which items appear more than *s* times in the window?**
- **Possible solution:** Think of the stream of baskets as one binary stream per item
  - **1** = item present; **0** = not present
  - Use **DGIM** to estimate counts of **1**s for all items



Extension to Itemsets

- **In principle, you could count frequent pairs or even larger sets the same way**
  - **One stream per itemset**
- **Drawbacks:**
  - Only approximate
  - **Number of itemsets is way too big**

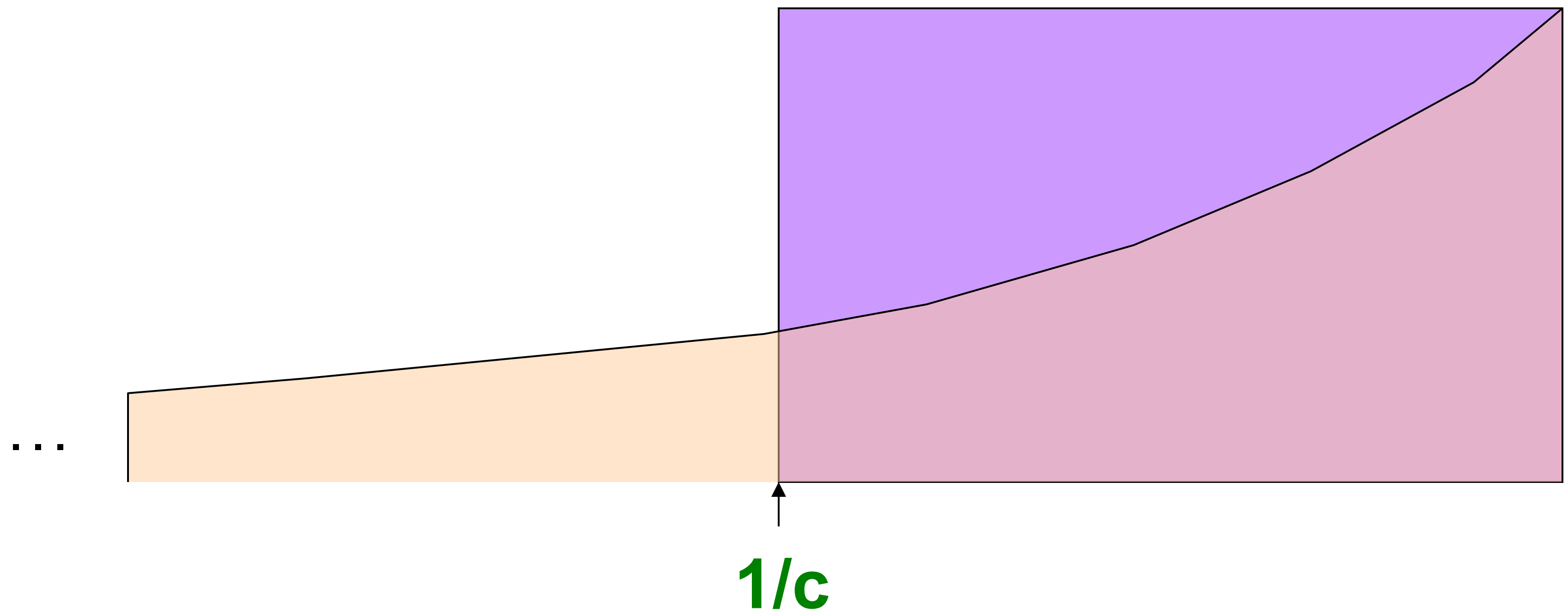
Exponentially Decaying Windows

- **Exponentially decaying windows: A heuristic for selecting likely frequent item(sets)**
  - **What are "currently" most popular movies?**
    - Instead of computing the raw count in last ***N*** elements
    - Compute a **smooth aggregation** over the whole stream
- If stream is $\boldsymbol{a_1, a_2, \ldots}$ and we are taking the sum of the stream, take the answer at time $\boldsymbol{t}$ to be:
  $$= \sum_{i=1}^{t} a_i (1-c)^{t-i}$$
    - **c** is a constant, presumably tiny, like $\mathbf{10^{-6}}$ or $\mathbf{10^{-9}}$
- **When new a_{t+1} arrives:**
  Multiply current sum by **(1-c)** and add $\mathbf{a_{t+1}}$

Example: Counting Items

- If each a_i is an "item" we can compute the **characteristic function** of each possible item ***x*** as an Exponentially Decaying Window
  - That is: $\sum_{i=1}^{t} \delta_i \cdot (1-c)^{t-i}$
    where $\boldsymbol{\delta_i}$**=1** if $\mathbf{a_i}$**=x**, and **0** otherwise
  - Imagine that for each item ***x*** we have a binary stream (**1** if ***x*** appears, **0** if ***x*** does not appear)
  - New item ***x*** arrives:
    - Multiply all counts by **(1-c)**
    - Add **+1** to count for element ***x***
- **Call this sum the "weight" of item *x***

Sliding Versus Decaying Windows



- **Important property:** Sum over all weights $\sum_t (1-c)^t$ **is 1/[1 – (1 – c)] = 1/c**

$$\sum_{k=0}^{n} z^k = \frac{1-z^{n+1}}{1-z}$$

Example: Counting Items

- **What are "currently" most popular movies?**
- **Suppose we want to find movies of weight > ½**
  - **Important property:** Sum over all weights $\sum_t (1-c)^t$ is 1/[1 – (1 – c)] **= 1/c**
- **Thus:**
  - There cannot be more than **2/c** movies with weight of ½ or more
- So, **2/c** is a limit on the number of movies being counted at any time

Extension to Itemsets

- **Count (some) itemsets in an E.D.W.**
  - **What are currently "hot" itemsets?**
    - **Problem:** Too many itemsets to keep counts of all of them in memory
- **When a basket B comes in:**
  - Multiply all counts by **(1-c)**
  - For uncounted items in **B**, create new count
  - Add **1** to count of any item in **B** and to any **itemset** contained in **B** that is already being counted
  - **Drop counts < ½**
  - Initiate new counts (next slide)

Initiation of New Counts

- Start a count for an itemset $S \subseteq B$ if every proper subset of ***S*** had a count prior to arrival of basket ***B***
  - **Intuitively:** If all subsets of **S** are being counted this means they are "**frequent/hot**" and thus ***S*** has a potential to be "**hot**"
- **Example:**
  - Start counting ***S*=**{i, j}** iff both **i** and **j** were counted prior to seeing ***B***
  - Start counting ***S*=**{i, j, k}** iff **{i, j}**, **{i, k}**, and **{j, k}** were all counted prior to seeing ***B***

Summary: Counting Itemsets

- **Task: Which were the most popular recent items?**
  - Can keep exponentially decaying counts for items and potentially larger itemsets
- **Number of larger itemsets is very large**
- **But we are conservative about starting counts of large sets**
  - All subsets need to be counted currently
  - If we counted every set we saw, one basket of **20** items would initiate **1M** counts (2^20)