Announcements:

- Submit your project group **TODAY** (Ed Pinned Post)
- Project Proposal **due this Thursday** (no late periods)
- Upload homework on time (23:59pm)!

Recommender Systems: Content-based Systems & Collaborative Filtering

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CS547 Machine Learning for Big Data

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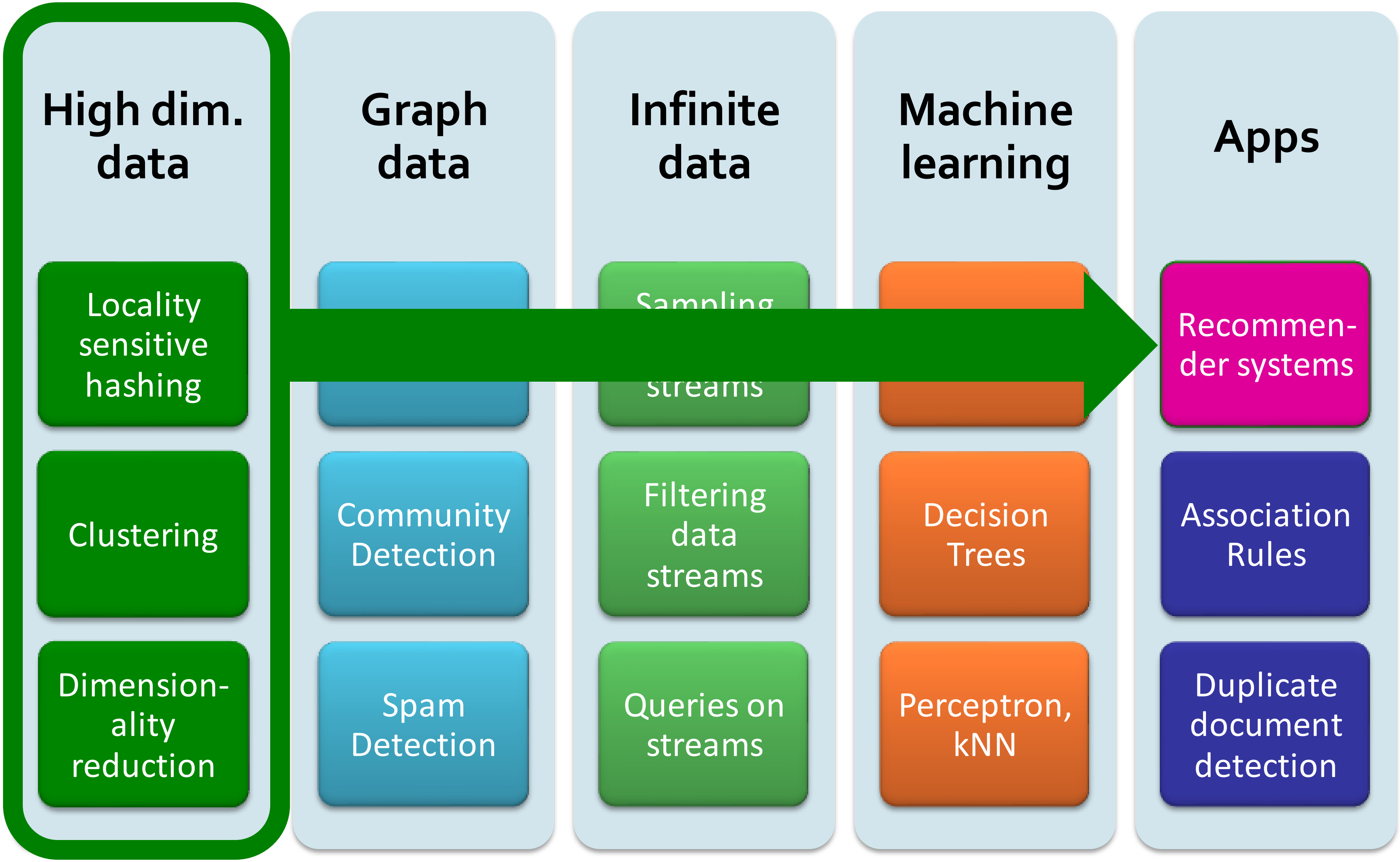
Clarification: SVD – Properties

It is **always** possible to decompose a real matrix $\boldsymbol{A}$ into $\boldsymbol{A} = \boldsymbol{U} \Sigma \boldsymbol{V}^T$, where

- $\boldsymbol{U}, \Sigma, \boldsymbol{V}$: unique*
- $\boldsymbol{U}, \boldsymbol{V}$: column orthonormal
  - $\boldsymbol{U}^T \boldsymbol{U} = \boldsymbol{I}; \boldsymbol{V}^T \boldsymbol{V} = \boldsymbol{I}$ (*I*: identity matrix)
  - (Columns are orthogonal unit vectors)
- Σ: diagonal
  - Entries (**singular values**) are positive, and sorted in decreasing order ($\boldsymbol{\sigma_1 \geq \sigma_2 \geq \ldots \geq 0}$)

* Up to permutations for redundant singular values and orientation of singular vectors (details)

High Dimensional Data



High dim. data
- Locality sensitive hashing
- Clustering
- Dimensionality reduction

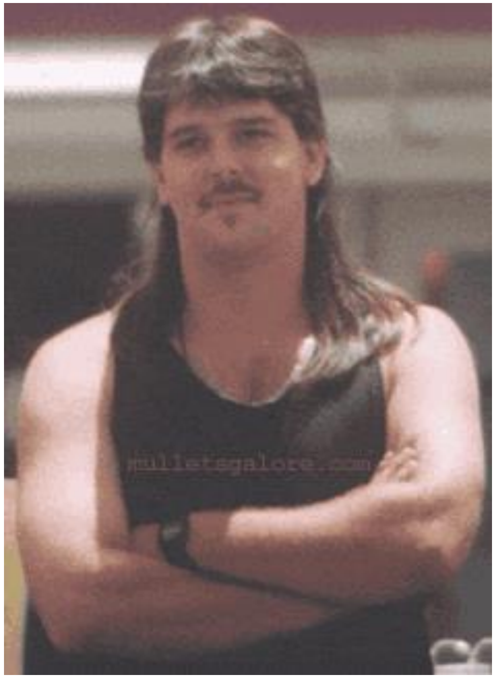
Graph data
- Community Detection
- Spam Detection

Infinite data
- Sampling
  streams
- Filtering data streams
- Queries on streams

Machine learning
- Decision Trees
- Perceptron, kNN

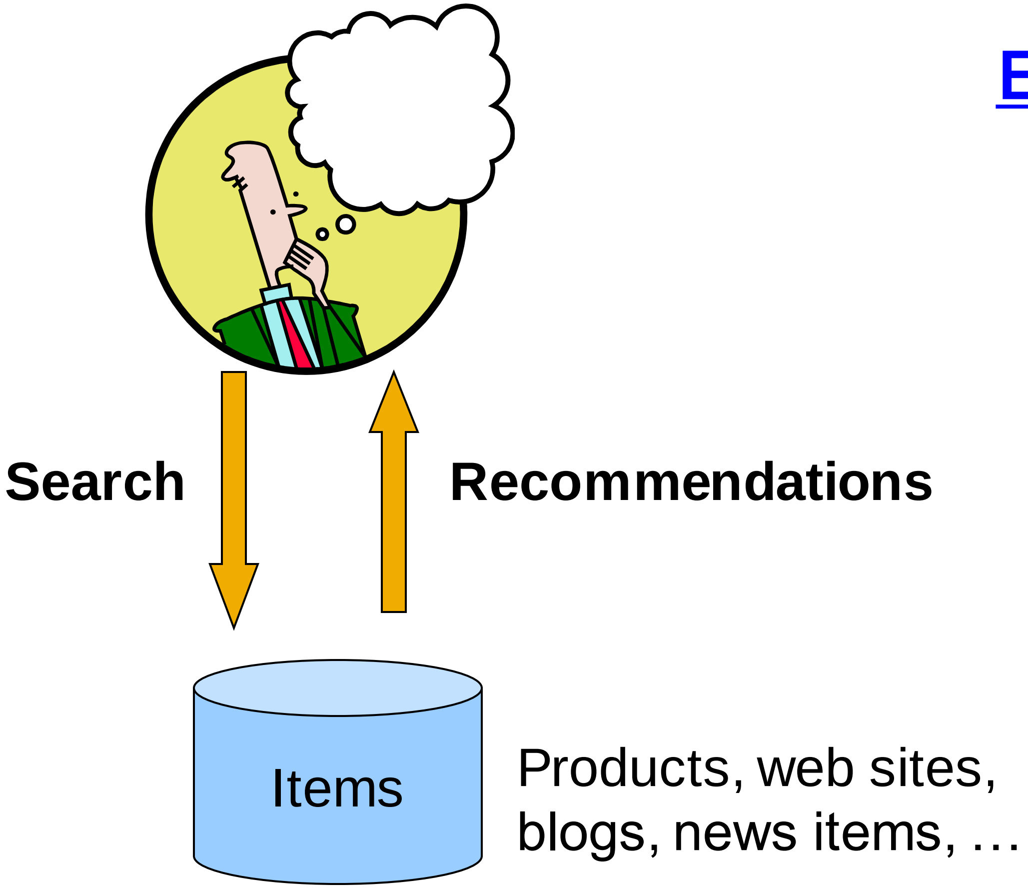
Apps
- Recommender systems
- Association Rules
- Duplicate document detection

Example: Recommender Systems



- **Customer X**
  - Buys Metallica CD
  - Buys Megadeth CD
- **Customer Y**
  - Does search on Metallica
  - Recommender system suggests Megadeth from data collected about customer **X**

Recommendations



Search

Recommendations

Items

Products, web sites, blogs, news items, …

Examples:

amazon.com

PANDORA

StumbleUpon

NETFLIX

Google News

last.fm the social music revolution

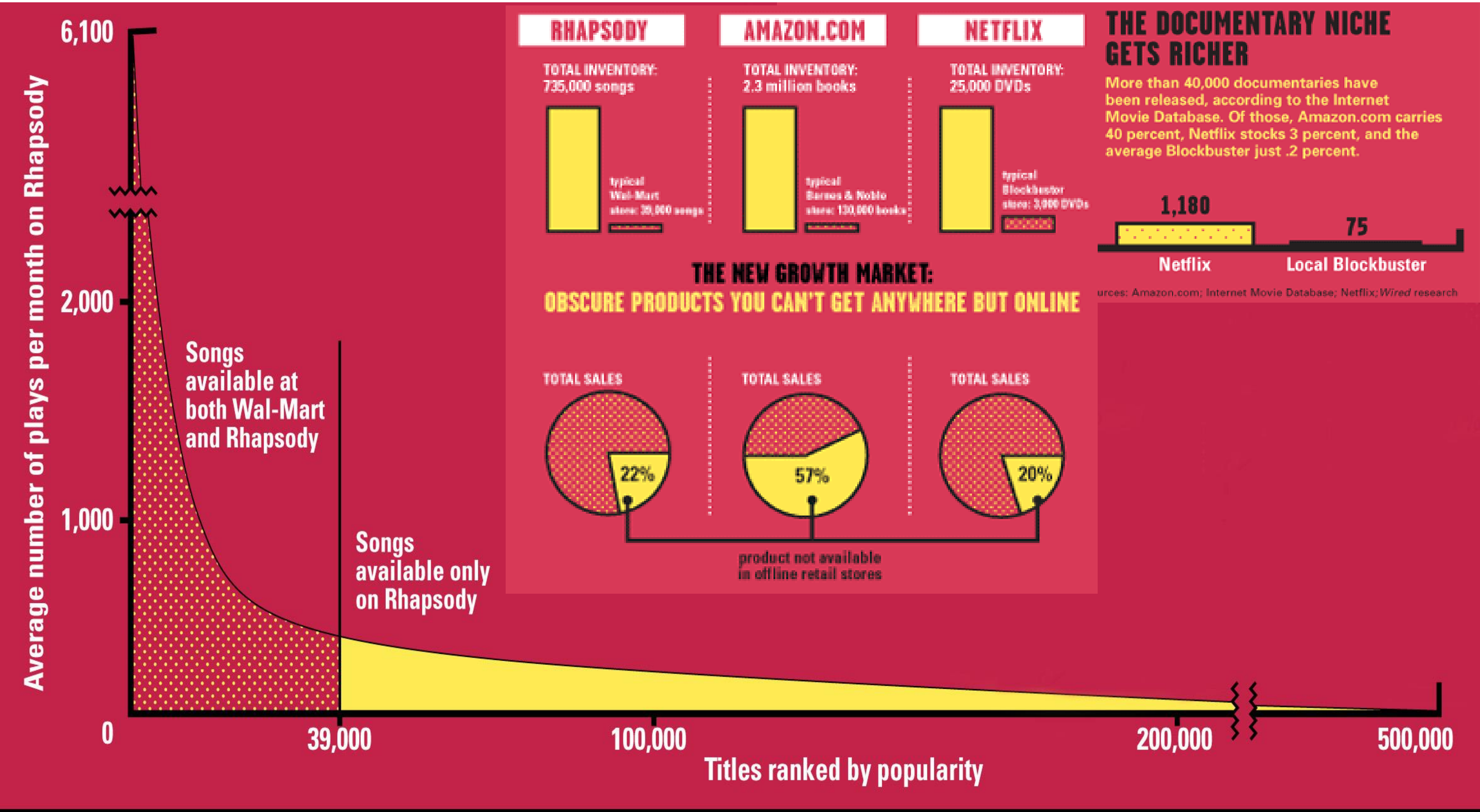
XBOX LIVE

YouTube

From Scarcity to Abundance

- **Shelf space is a scarce commodity for traditional retailers**
  - Also: TV networks, movie theaters,…
- **Web enables near-zero-cost dissemination of information about products**
  - From scarcity to abundance
- **More choice necessitates better filters:**
  - Recommendation engines
  - **Association rules:** How **Into Thin Air** made **Touching the Void** a bestseller: http://www.wired.com/wired/archive/12.10/tail.html

Sidenote: The Long Tail



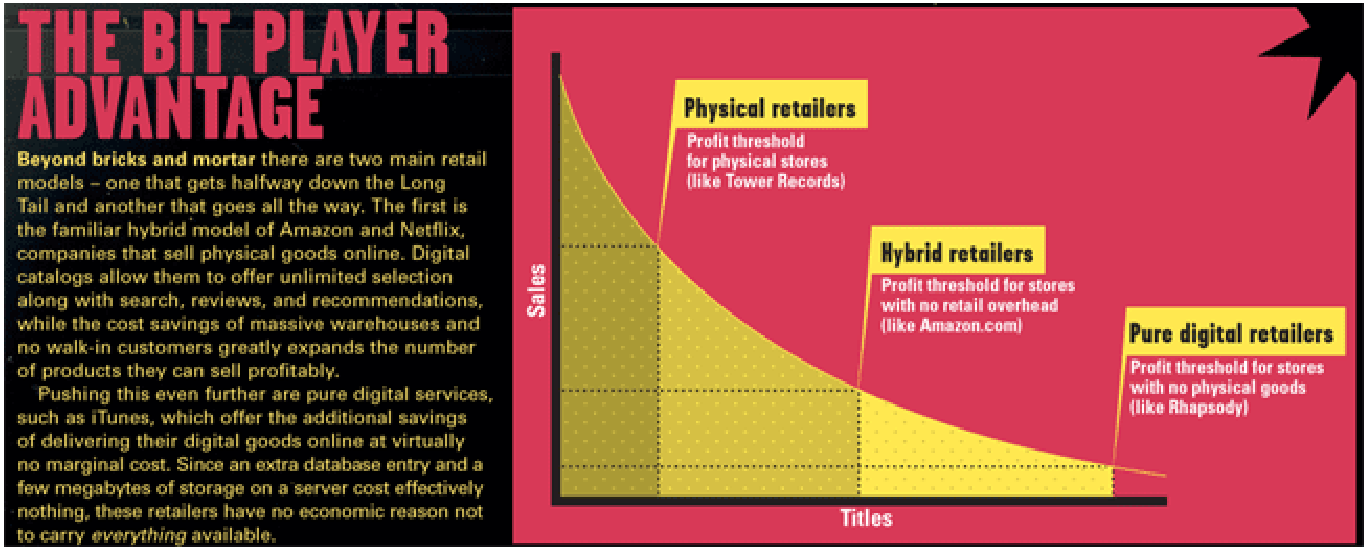
More than 40,000 documentaries have been released, according to the Internet Movie Database. Of those, Amazon.com carries 40 percent, Netflix stocks 3 percent, and the average Blockbuster just .2 percent.

urces: Amazon.com; Internet Movie Database; Netflix; *Wired* research

Sources: Erik Brynjolfsson and Jeffrey Hu, MIT, and Michael Smith, Carnegie Mellon; Barnes & Noble; Netflix; RealNetworks

Source: Chris Anderson (2004)

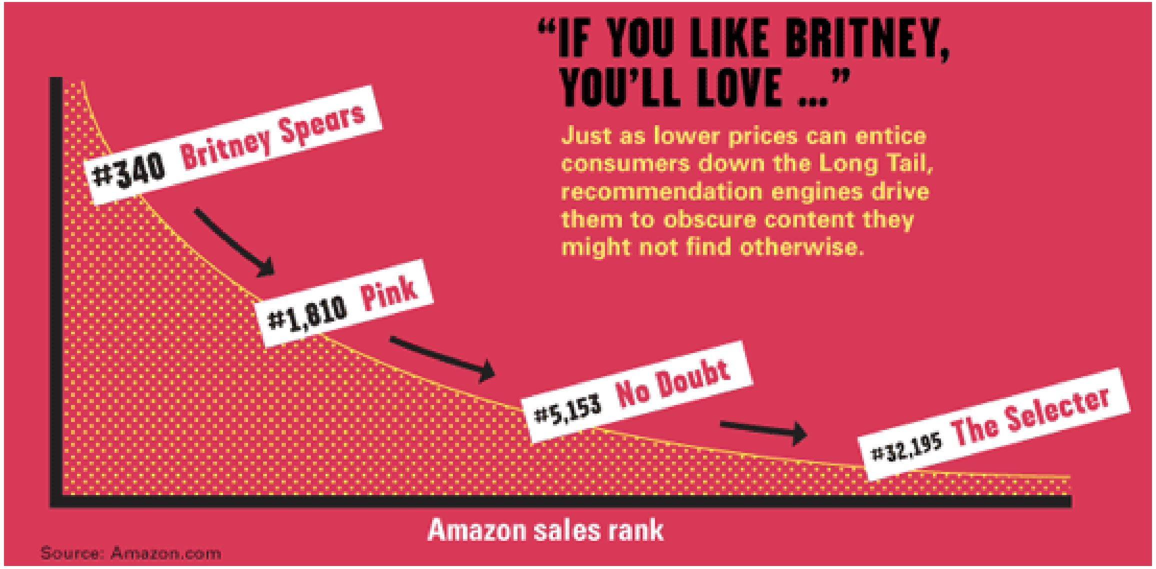
Physical vs. Online



THE BIT PLAYER ADVANTAGE

Beyond bricks and mortar there are two main retail models – one that gets halfway down the Long Tail and another that goes all the way. The first is the familiar hybrid model of Amazon and Netflix, companies that sell physical goods online. Digital catalogs allow them to offer unlimited selection along with search, reviews, and recommendations, while the cost savings of massive warehouses and no walk-in customers greatly expands the number of products they can sell profitably.

Pushing this even further are pure digital services, such as iTunes, which offer the additional savings of delivering their digital goods online at virtually no marginal cost. Since an extra database entry and a few megabytes of storage on a server cost effectively nothing, these retailers have no economic reason not to carry *everything* available.



"IF YOU LIKE BRITNEY, YOU'LL LOVE ..."

Just as lower prices can entice consumers down the Long Tail, recommendation engines drive them to obscure content they might not find otherwise.

Source: Amazon.com

Read http://www.wired.com/wired/archive/12.10/tail.html to learn more!

Types of Recommendations

- **Editorial and hand curated**
  - List of favorites
  - Lists of “essential” items
- **Simple aggregates**
  - Top 10, Most Popular, Recent Uploads
- **Tailored to individual users**
  - Amazon, Netflix, …

Today’s class

Formal Model

- ***X*** = set of **Customers**
- ***S*** = set of **Items**

- **Utility function *u*: *X* × *S* → *R***
  - ***R*** = set of ratings
  - ***R*** is a totally ordered set
  - e.g., **1-5** stars, real number in **[0,1]**

Utility Matrix

	Avatar	LOTR	Matrix	Pirates
Alice	1		0.2	
Bob		0.5		0.3
Carol	0.2		1	
David				0.4

Key Problems

- **(1) Gathering "known" ratings for matrix**
  - How to collect the data in the utility matrix
- **(2) Extrapolating unknown ratings from the known ones**
  - Mainly interested in **high** unknown ratings
    - We are not interested in knowing what you don't like but what you like
- **(3) Evaluating extrapolation methods**
  - How to measure success/performance of recommendation methods

(1) Gathering Ratings

- **Explicit**
  - Ask people to rate items
  - Doesn’t work well in practice – people don’t like being bothered
  - Crowdsourcing: Pay people to label items
- **Implicit**
  - Learn ratings from user actions
    - E.g., purchase implies high rating
    - E.g., add to playlist, play in full, skip song…
  - What about low ratings?

(2) Extrapolating Utilities

- **Key problem:** Utility matrix ***U*** is **sparse**
  - Most people have not rated most items
  - **Cold Start Problem:**
    - New items have no ratings
    - New users have no history

- **Three approaches to recommender systems:**
  - **1)** Content-based
  - **2)** Collaborative
  - **3)** Latent factor based

Today!

Content-based Recommender Systems

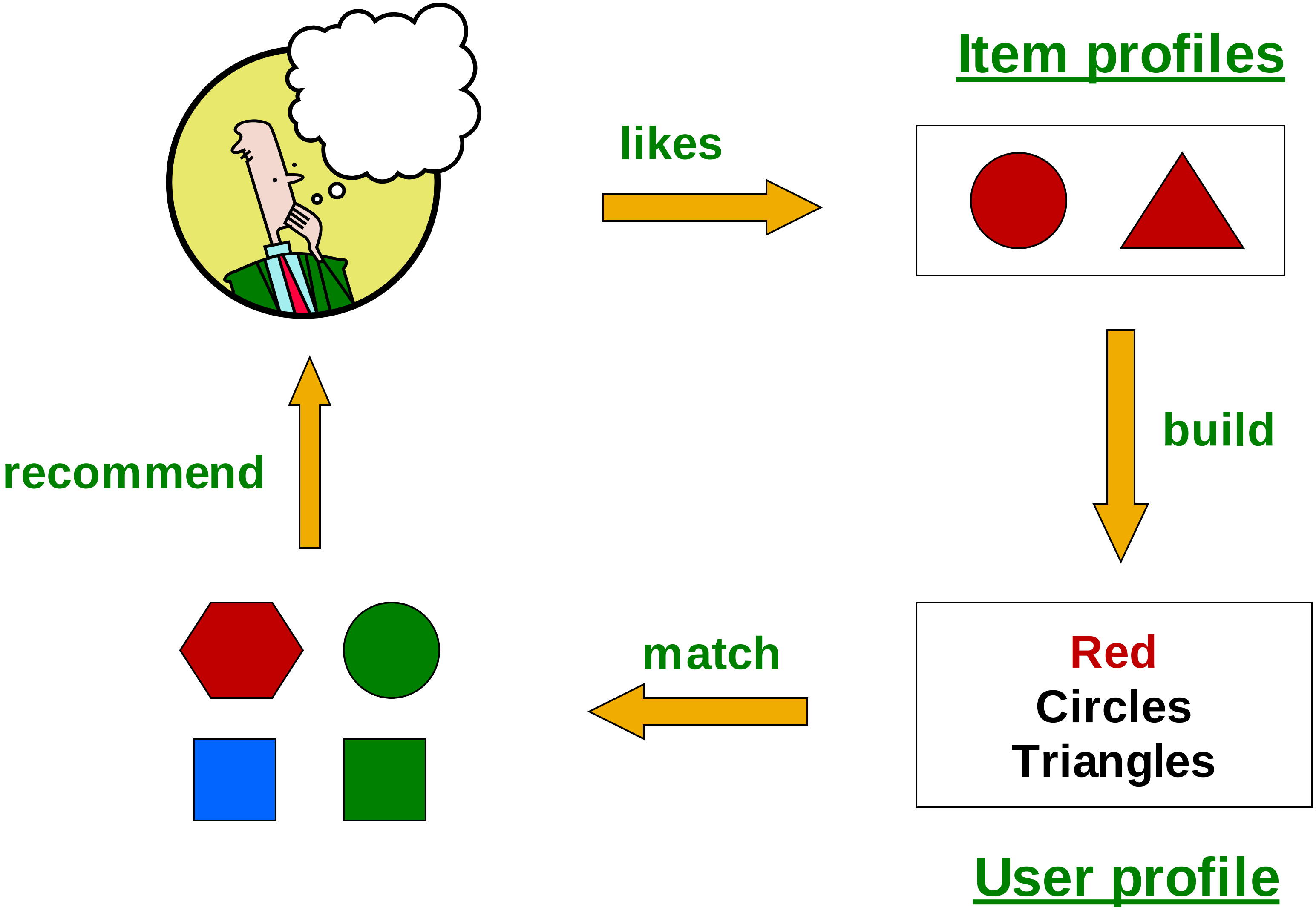
Content-based Recommendations

- **Main idea:** Recommend items to customer ***x*** similar to previous items rated highly by ***x***

Example:

- **Movie recommendations**
  - Recommend movies with same actor(s), director, genre, …
- **Websites, blogs, news**
  - Recommend other sites with "similar" content

Plan of Action



likes

Item profiles

build

Red
Circles
Triangles

User profile

match

recommend

Item Profiles

- For each item, create an **item profile**
- **Profile is a set (vector) of features**
  - **Movies:** author, title, actor, director,…
  - **Text:** Set of "important" words in document
- **How to pick important features?**
  - Usual heuristic from text mining is **TF-IDF** (Term frequency * Inverse Doc Frequency)
    - **Term … Feature**
    - **Document … Item**

Sidenote: TF-IDF

f_{ij} = frequency of term (feature) ***i*** in doc (item) ***j***

$$TF_{ij} = \frac{f_{ij}}{\max_k f_{kj}}$$

Large when term ***i*** appears often in doc ***j***

Note: we normalize TF to discount for "longer" documents

n_i = number of docs that mention term ***i***

N = total number of docs

$$IDF_i = \log \frac{N}{n_i}$$

Large when term ***i*** appears in very few documents

TF-IDF score: $w_{ij} = TF_{ij} \times IDF_i$

Doc profile = set of words with highest **TF-IDF** scores, together with their scores

User Profiles and Prediction

- **User profile possibilities:**
  - Weighted average of rated item profiles
  - **Variation:** weight by difference from average rating for item
- **Prediction heuristic: Cosine similarity of user and item profiles)**
  - Given user profile ***x*** and item profile ***i***, estimate
    $$u(\boldsymbol{x}, \boldsymbol{i}) = \cos(\boldsymbol{x}, \boldsymbol{i}) = \frac{\boldsymbol{x} \cdot \boldsymbol{i}}{||\boldsymbol{x}|| \cdot ||\boldsymbol{i}||}$$
- **How do you quickly find items closest to x?**
  - Job for LSH!

Pros: Content-based Approach

- **+: No need for data on other users**
  - No cold-start or sparsity problems
- **+: Able to recommend to users with unique tastes**
- **+: Able to recommend new & unpopular items**
  - No first-rater problem
- **+: Able to provide explanations**
  - Can provide explanations of recommended items by listing content-features that caused an item to be recommended

Cons: Content-based Approach

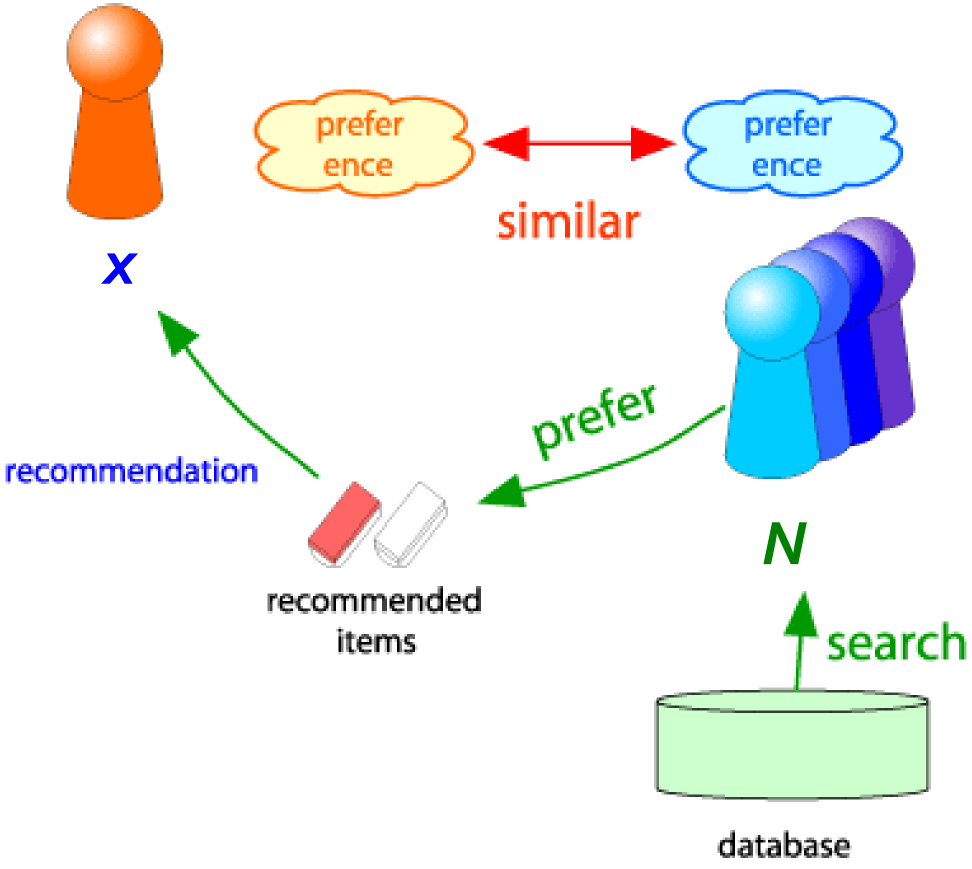
- **–: Finding the appropriate features is hard**
  - E.g., images, movies, music
- **–: Recommendations for new users**
  - **How to build a user profile?**
- **–: Overspecialization**
  - Never recommends items outside user's content profile
  - People might have multiple interests
  - **! Unable to exploit quality judgments of other users!**

Collaborative Filtering

Harnessing quality judgments of other users

Collaborative Filtering

- Consider user ***x***
- Find set ***N*** of other users whose ratings are "**similar**" to ***x***'s ratings
- Estimate ***x***'s ratings based on ratings of users in ***N***



Finding "Similar" Users

r_x = [*, _, _, *, ***]
r_y = [*, _, **, **, _]

- Let r_x be the vector of user x's ratings
- **Jaccard similarity metric**
  - **Problem:** Ignores the value of the rating
- **Cosine similarity metric**
  - sim(x, y) = cos(r_x, r_y) = $\frac{r_x \cdot r_y}{||r_x|| \cdot ||r_y||}$
  - **Problem:** Treats some missing ratings as "negative"
- **Better: Pearson correlation coefficient**
  - S_{xy} = items rated by both users x and y

$$sim(x, y) = \frac{\sum_{s \in S_{xy}} (r_{xs} - \overline{r_x})(r_{ys} - \overline{r_y})}{\sqrt{\sum_{s \in S_{xy}} (r_{xs} - \overline{r_x})^2} \sqrt{\sum_{s \in S_{xy}} (r_{ys} - \overline{r_y})^2}}$$

r_x, r_y as sets:
r_x = {1, 4, 5}
r_y = {1, 3, 4}

r_x, r_y as points:
r_x = {1, 0, 0, 1, 3}
r_y = {1, 0, 2, 2, 0}

$\overline{r_x}$, $\overline{r_y}$ … avg. rating of **x**, **y**

Similarity Metric

Cosine sim:
$$sim(x,y) = \frac{\sum_i r_{xi} \cdot r_{yi}}{\sqrt{\sum_i r_{xi}^2} \cdot \sqrt{\sum_i r_{yi}^2}}$$

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					3

- **Intuitively we want: sim(*A*, *B*) > sim(*A*, *C*)**
- **Jaccard similarity:** 1/5 < 2/4
- **Cosine similarity:** 0.380 > 0.322
  - Considers missing ratings as “negative”
  - **Solution: subtract the (row) mean**

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	$2/3$			$5/3$	$-7/3$		
B	$1/3$	$1/3$	$-2/3$				
C				$-5/3$	$1/3$	$4/3$	
D		0					0

sim A,B vs. A,C:
0.092 > -0.559

Notice cosine sim. is correlation when data is centered at 0

Rating Predictions

From similarity metric to recommendations:

- Let ***r*$_x$** be the vector of user ***x***'s ratings
- Let ***N*** be the set of ***k*** users most similar to ***x*** who have rated item ***i***
- **Prediction for item *i* of *user x*:**
  - $r_{xi} = \frac{1}{k} \sum_{y \in N} r_{yi}$
  - Or even better: $r_{xi} = \frac{\sum_{y \in N} s_{xy} \cdot r_{yi}}{\sum_{y \in N} s_{xy}}$

Shorthand:
$\boldsymbol{s_{xy} = sim(x, y)}$

- **Many other tricks possible…**

Item-Item Collaborative Filtering

- **So far: User-user collaborative filtering**
- **Another view: Item-item**
  - For item ***i***, find other similar items
  - Estimate rating for item ***i*** based on ratings for similar items
  - Can use same similarity metrics and prediction functions as in user-user model

$$r_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}$$

s_{ij}… similarity of items ***i*** and ***j***
r_{xj}…rating of user ***x*** on item ***j***
N(i;x)… set items which were rated by ***x*** and similar to ***i***

Item-Item CF (|N|=2)

users

movies

	1	2	3	4	5	6	7	8	9	10	11	12
1	1		3			5			5		4	
2			5	4			4			2	1	3
3	2	4		1	2		3		4	3	5	
4		2	4		5			4			2	
5			4	3	4	2					2	5
6	1		3		3			2			4	

☐ - unknown rating 🟨 - rating between 1 to 5

Item-Item CF (|N|=2)

users

movies

	1	2	3	4	**5**	6	7	8	9	10	11	12
1	1		3		?	5			5		4	
2			5	4			4			2	1	3
3	2	4		1	2		3		4	3	5	
4		2	4		5			4			2	
5			4	3	4	2					2	5
6	1		3		3			2			4	

🟥 - estimate rating of movie **1** by user **5**

Item-Item CF (|N|=2)

users

movies

	1	2	3	4	**5**	6	7	8	9	10	11	12	$s_{1,m}$
1	1		3		?	5			5		4		**1.00**
2			5	4			4			2	1	3	**-0.18**
3	2	4		1	2		3		4	3	5		**0.41**
4		2	4		5			4			2		**-0.10**
5			4	3	4	2					2	5	**-0.31**
6	1		3		3			2			4		**0.59**

Neighbor selection:
Identify movies similar to movie **1**, **rated by user 5**

Here we use Pearson correlation as similarity:

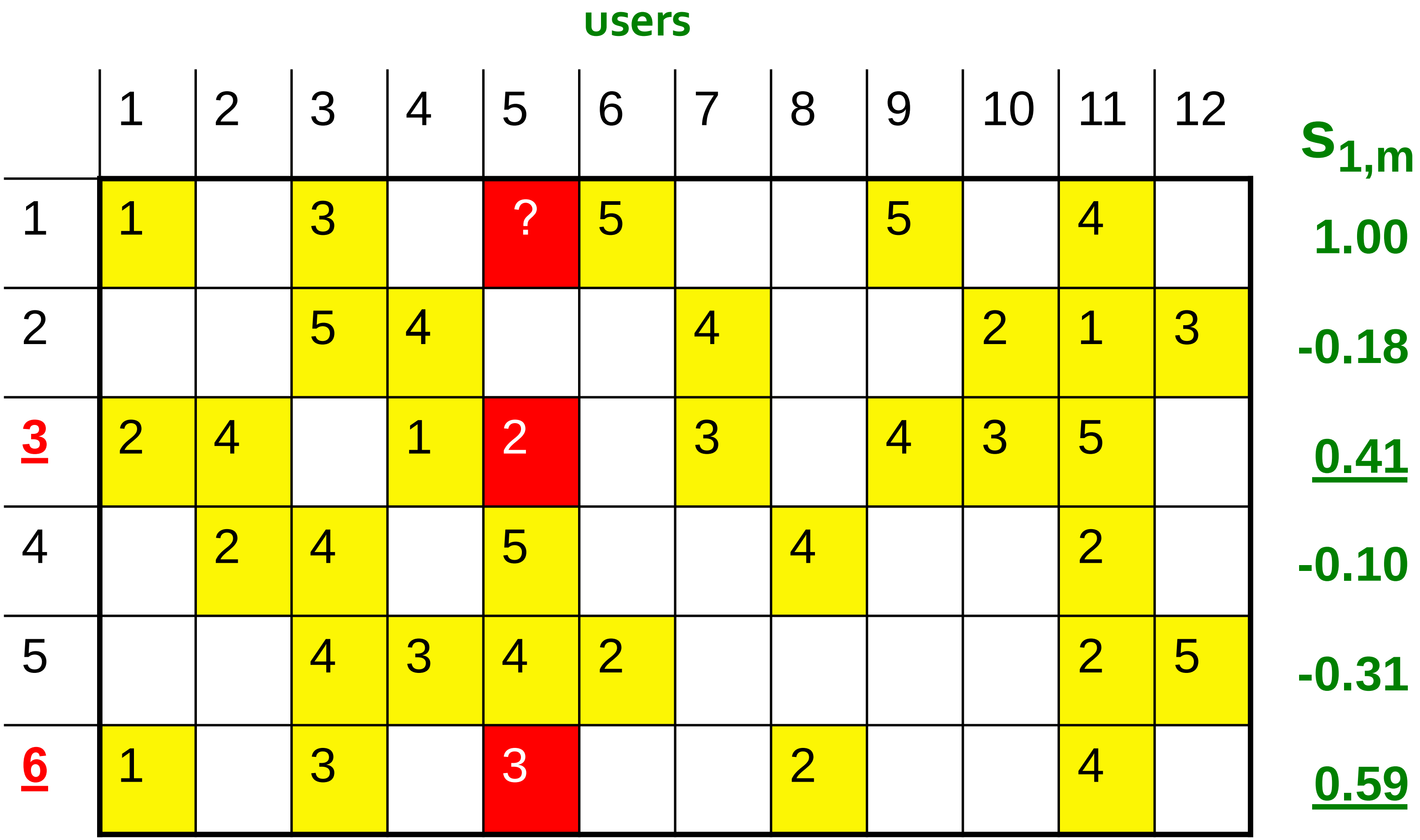
1) Subtract mean rating m_i from each movie i

$m_1 = (1+3+5+5+4)/5 = \mathbf{3.6}$

row 1: $[-2.6, 0, -0.6, 0, 0, 1.4, 0, 0, 1.4, 0, 0.4, 0]$

2) Compute dot products between rows

Item-Item CF (|N|=2)



users

movies

	1	2	3	4	5	6	7	8	9	10	11	12	$s_{1,m}$
1	1		3		?	5			5		4		1.00
2			5	4			4			2	1	3	-0.18
3	2	4		1	2		3		4	3	5		0.41
4		2	4		5			4			2		-0.10
5			4	3	4	2					2	5	-0.31
6	1		3		3			2			4		0.59

Compute similarity weights:

$s_{1,3}$=0.41, $s_{1,6}$=0.59

Item-Item CF (|N|=2)

users

movies

	1	2	3	4	5	6	7	8	9	10	11	12
1	1		3		**2.6**	5			5		4	
2			5	4			4			2	1	3
3	2	4		1	2		3		4	3	5	
4		2	4		5			4			2	
5			4	3	4	2					2	5
6	1		3		3			2			4	

Predict by taking weighted average:

$$r_{ix} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{jx}}{\sum s_{ij}}$$

$r_{1.5}$ = (0.41*2 + 0.59*3) / (0.41+0.59) = 2.6

CF: Common Practice

Before:

$$r_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}$$

- Define **similarity** ***s_{ij}*** of items ***i*** and ***j***
- Select ***k*** nearest neighbors ***N(i; x)***
  - Items most similar to ***i***, that were rated by ***x***
- Estimate rating ***r_{xi}*** as the weighted average:

$$r_{xi} = b_{xi} + \frac{\sum_{j \in N(i;x)} s_{ij} \cdot (r_{xj} - b_{xj})}{\sum_{j \in N(i;x)} s_{ij}}$$

baseline estimate for r_{xi}

$$\boldsymbol{b_{xi} = \mu + b_x + b_i}$$

- ***μ*** = overall mean movie rating
- ***b_x*** = rating deviation of user ***x***
  = *(avg. rating of user **x**)* **– *μ***
- ***b_i*** = rating deviation of movie ***i***

Item-Item vs. User-User

	Avatar	LOTR	Matrix	Pirates
Alice	1		0.8	
Bob		0.5		0.3
Carol	0.9		1	0.8
David			1	0.4

- **In practice, it has been observed that item-item often works better than user-user**
- **Why?** Items are simpler, users have multiple tastes

Pros/Cons of Collaborative Filtering

- **+ Works for any kind of item**
  - No feature selection needed
- **- Cold Start:**
  - Need enough users in the system to find a match
- **- Sparsity:**
  - The user/ratings matrix is sparse
  - Hard to find users that have rated the same items
- **- First rater:**
  - Cannot recommend an item that has not been previously rated
  - New items, Esoteric items
- **- Popularity bias:**
  - Cannot recommend items to someone with unique taste
  - Tends to recommend popular items

Hybrid Methods

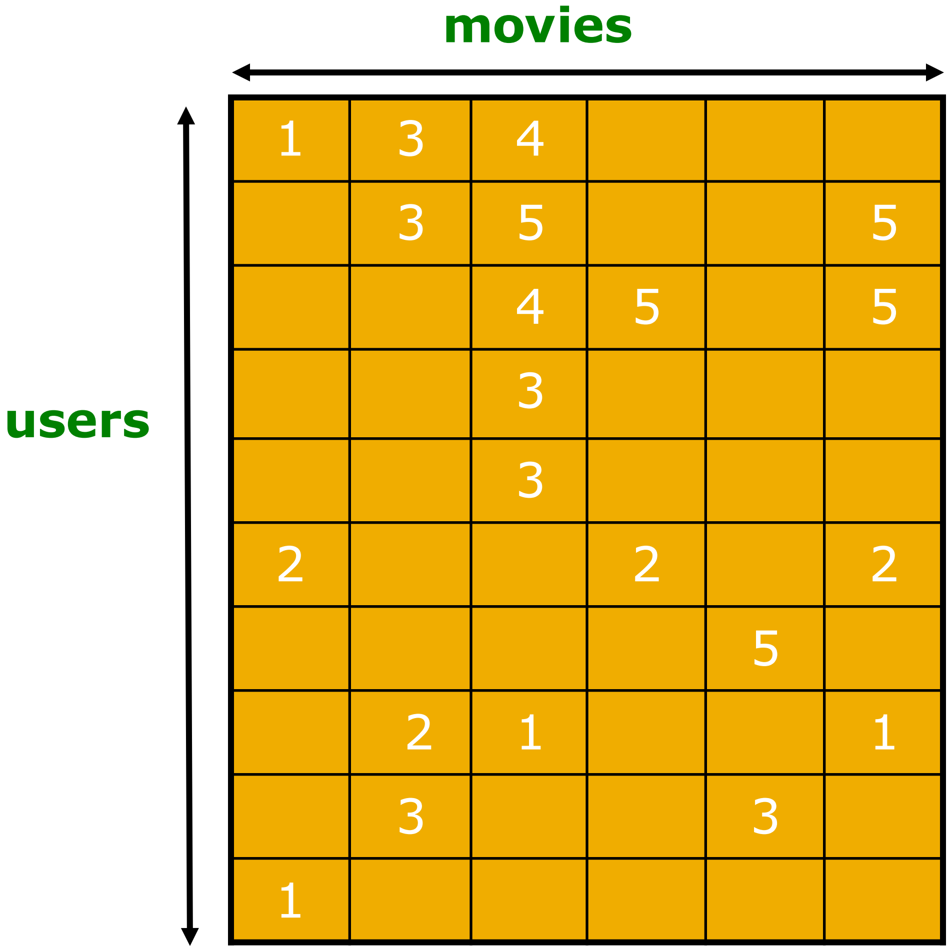
- **Implement two or more different recommenders and combine predictions**
  - Perhaps using a linear model

- **Add content-based methods to collaborative filtering**
  - Item profiles for new item problem
  - Demographics to deal with new user problem

Remarks & Practical Tips

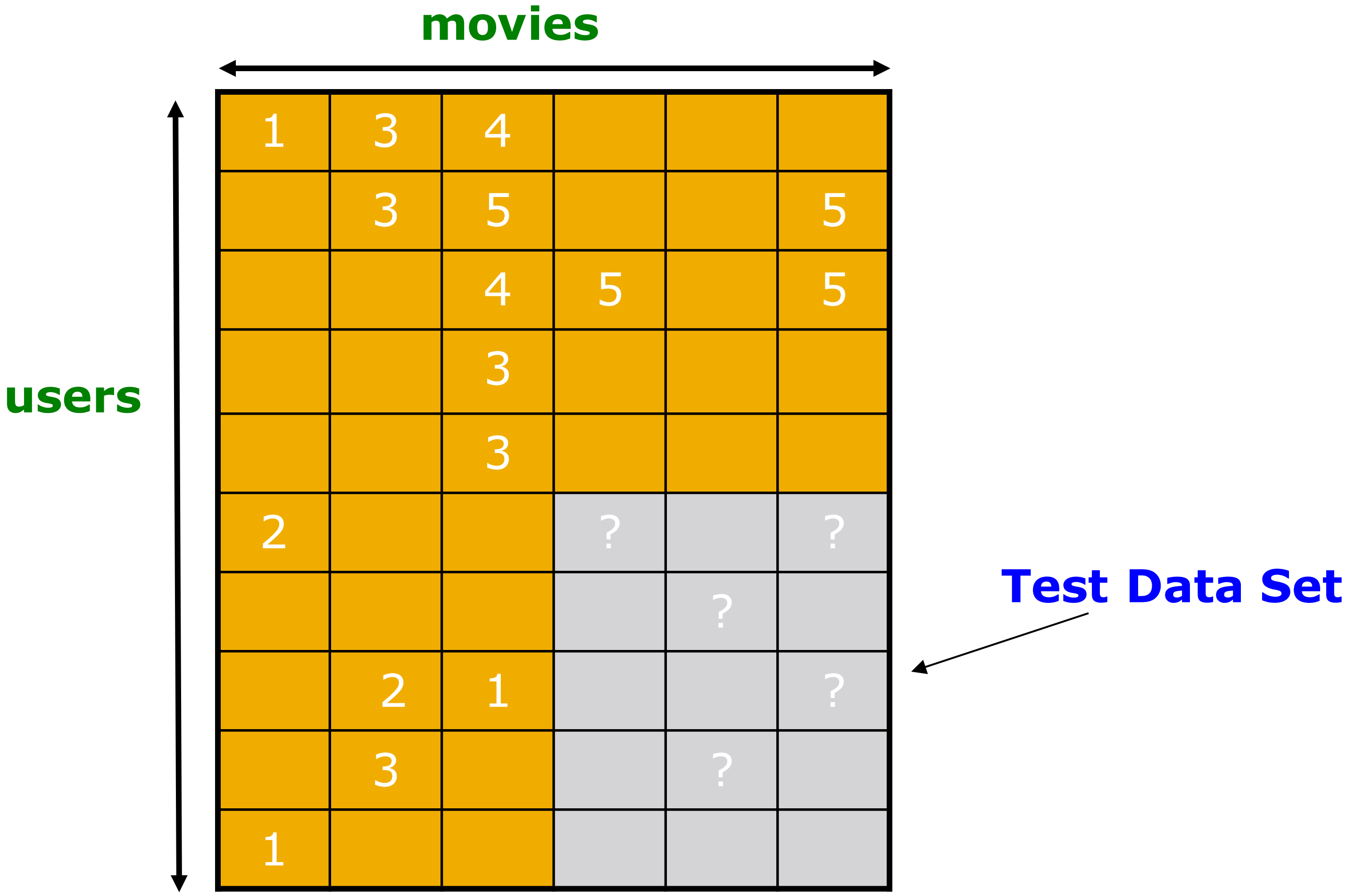
- Evaluation
- Error metrics
- Complexity / Speed

Evaluation



1	3	4			
	3	5			5
		4	5		5
		3			
		3			
2			2		2
				5	
	2	1			1
	3			3	
1					

Evaluation



movies

users

Test Data Set

1	3	4			
	3	5			5
		4	5		5
		3			
		3			
2			?		?
				?	
	2	1			?
	3			?	
1					

Evaluating Predictions

- **Compare predictions with known ratings**
  - **Root-mean-square error** (RMSE)
    - $\sqrt{\frac{1}{N}\sum_{xi}\left(r_{xi}-r_{xi}^{*}\right)^{2}}$ where $\boldsymbol{r}_{\boldsymbol{xi}}$ is predicted, $\boldsymbol{r}_{\boldsymbol{xi}}^{*}$ is the true rating of ***x*** on ***i***
      - ***N is the number of points we are making comparisons on***
  - **Rank Correlation**:
    - Spearman's *correlation* between system's and user's complete rankings
  - **Precision at top 10 (or k)**:
    - % of those in top 10 (or k)

Idea: ignore lowly-ranked items

- **Another approach: 0/1 model**
  - **Coverage:**
    - Number of items/users for which the system can make predictions
  - **Precision:**
    - Accuracy of predictions
  - **Receiver operating characteristic** (ROC)
    - Tradeoff curve between false positives and false negatives

Problems with Error Metrics

- **Narrow focus on accuracy sometimes misses the point**
  - Prediction Diversity
  - Prediction Context
  - Order of predictions
- **In practice, we care only to predict high ratings:**
  - RMSE might penalize a method that does well for high ratings and badly for others

Collaborative Filtering: Complexity

- Expensive step is finding ***k*** most similar customers: **O(|X|)**
- **Too expensive to do at runtime**
  - Could pre-compute
- Pre-computation takes time **O(k ·|X|)**
    - X … set of customers
- **We already know how to do this!**
  - Near-neighbor search in high dimensions (**LSH**)
  - Clustering
  - Dimensionality reduction

Tip: Add Data

- **Leverage all the data**
  - Don’t try to reduce data size in an effort to make fancy algorithms work
  - Simple methods on large data do best
- **Add more data**
  - e.g., add IMDB data on genres
- **More data beats better algorithms**

http://anand.typepad.com/datawocky/2008/03/more-data-usual.html

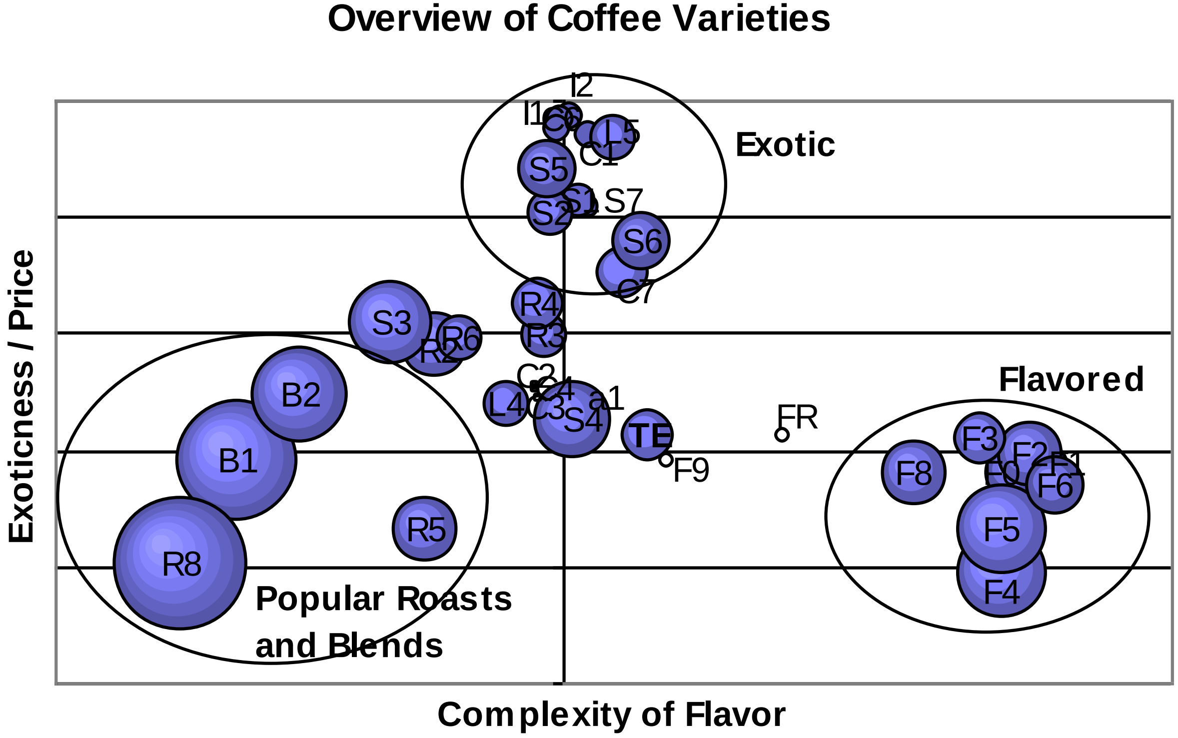
On Thursday:
The Netflix prize and the Latent Factor Models

On Thursday: The Netflix Prize

- **Training data**
  - 100 million ratings, 480,000 users, 17,770 movies
    - Lots of ratings – still 99% sparsity!
  - 6 years of data: 2000-2005
- **Test data (private)**
  - Last few ratings of each user (2.8 million)
  - Evaluation criterion: root mean squared error (RMSE)
  - Netflix Cinematch RMSE (production): 0.9514
- **Competition**
  - 2700+ teams
  - $1 million prize for 10% improvement on Cinematch

On Thursday: Latent Factor Models

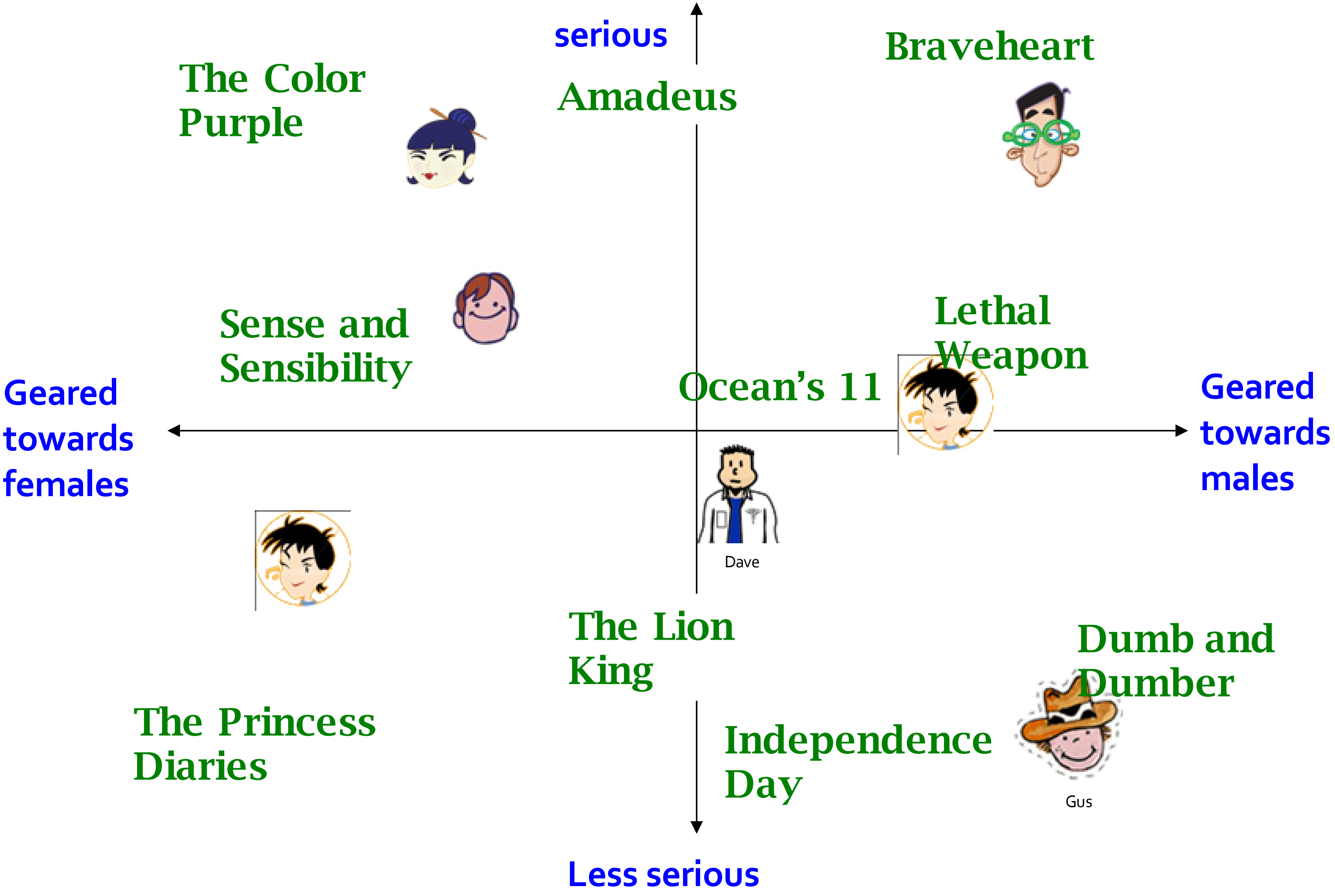
- **Next topic: Recommendations via Latent Factor models**

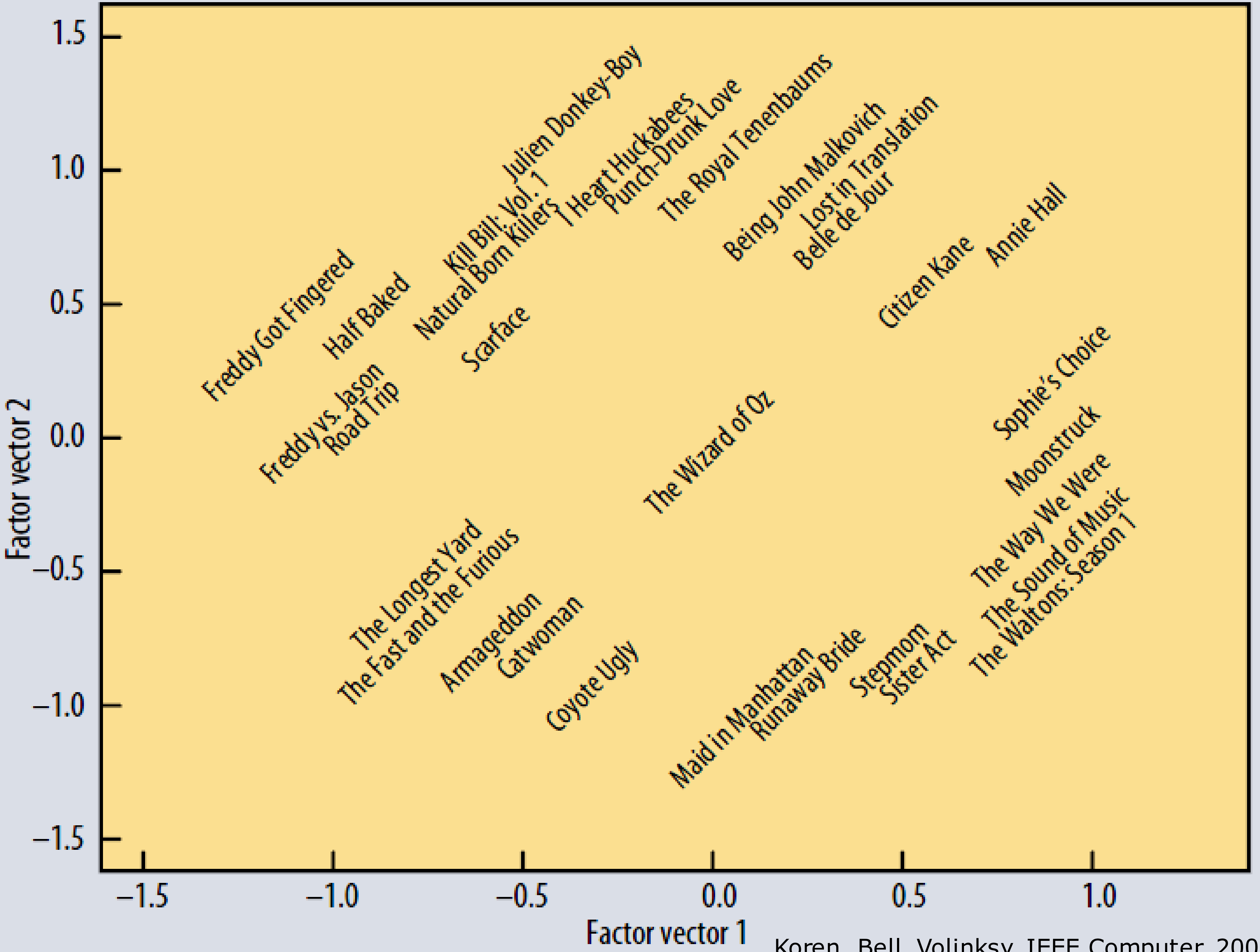


The bubbles above represent products sized by sales volume.
Products close to each other are recommended to each other.

[slide from winning Bellkor Team]

Latent Factor Models (i.e., SVD++)





Koren, Bell, Volinksy, IEEE Computer, 2009