2 Announcements

We are releasing HW1 today

- It is due in 2 weeks (4/16 at 23:59pm)
- The homework is long
 - Requires proving theorems as well as coding
- Please start early

Recitation sessions:

Spark Tutorial using Colab 0:
 Today, April 2, 1-3pm on Zoom

Frequent Itemset Mining & Association Rules

CS547 Machine Learning for Big Data
Tim Althoff
PAUL G. ALLEN SCHOOL

Association Rule Discovery

Supermarket shelf management – Market-basket model:

- Goal: Identify items that are bought together by sufficiently many customers
- Approach: Process the sales data collected with barcode scanners to find dependencies among items
- A "classic" rule:
 - If someone buys diaper and milk, then he/she is likely to buy beer
 - Don't be surprised if you find six-packs next to diapers!

The Market-Basket Model

- A large set of items
 - e.g., things sold in a supermarket
- A large set of baskets
 - Each basket is a small subset of items
 - e.g., the things one customer buys on one day (or "cart")

Basket	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Output:

Input:

Rules Discovered:

{Milk} --> {Coke} {Diaper, Milk} --> {Beer}

Discover association rules:

People who bought {x,y,z} tend to buy {v,w}

Example application: Amazon

More generally

- A general many-to-many mapping (association) between two kinds of things
 - But we ask about connections among "items", not "baskets"
- Items and baskets are abstract:
 - For example:
 - Items/baskets can be products/shopping basket
 - Items/baskets can be words/documents
 - Items/baskets can be basepairs/genes
 - Items/baskets can be drugs/patients

Applications – (1)

- Items = products; Baskets = sets of products someone bought in one trip to the store
- Real market baskets: Chain stores keep TBs of data about what customers buy together
 - Tells how typical customers navigate stores, lets them position tempting items:
 - Apocryphal story of "diapers and beer" discovery
 - Used to position potato chips between diapers and beer to enhance sales of potato chips
- Amazon's 'people who bought X also bought Y'

Applications – (2)

- Baskets = sentences; Items = documents in which those sentences appear
 - Items that appear together too often could represent plagiarism
 - Notice items do not have to be "in" baskets
- Baskets = patients; Items = drugs & side-effects
 - Has been used to detect combinations of drugs that result in particular side-effects
 - But requires extension: Absence of an item needs to be observed as well as presence

Outline

First: Define

Frequent itemsets

Association rules:

Confidence, Support, Interestingness

Then: Algorithms for finding frequent itemsets

Finding frequent pairs

A-Priori algorithm

PCY algorithm

Frequent Itemsets

- Simplest question: Find sets of items that appear together "frequently" in baskets
- Support for itemset I: Number of baskets containing all items in I
 - (Often expressed as a fraction of the total number of baskets)
- Given a support threshold s, then sets of items that appear in at least s baskets are called frequent itemsets

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Support of {Beer, Bread} = 2

Example: Frequent Itemsets

- Items = {milk, coke, pepsi, beer, juice}
- Support threshold = 3 baskets

$$B_1 = \{m, c, b\}$$
 $B_2 = \{m, p, j\}$
 $B_3 = \{m, b\}$ $B_4 = \{c, j\}$
 $B_5 = \{m, p, b\}$ $B_6 \neq \{m, c, b, j\}$
 $B_7 \neq \{c, b, j\}$ $B_8 = \{b, c\}$

Frequent itemsets: {m}, {c}, {b}, {j}, {m,b}, {b,c}, {c,j}.

Define: Association Rules

- Define: Association Rules:
 If-then rules about the contents of baskets
- $\{i_1, i_2, ..., i_k\} \rightarrow j$ means: "if a basket contains all of $i_1, ..., i_k$ then it is *likely* to contain j"
- In practice there are many rules, want to find significant/interesting ones!
- <u>Confidence</u> of association rule is the probability of j given $I = \{i_1, ..., i_k\}$

$$conf(I \to j) = \frac{support(I \cup j)}{support(I)}$$

Where confidence falls short

What if everyone buys milk?

```
conf({Beer} \rightarrow Milk) = 1

conf({Bread} \rightarrow Milk) = 1

...

conf({Beer,Bread,Diapers} \rightarrow Milk) = 1
```

Observations				
Bread, Coke, Milk				
Beer, Bread, Milk				
Beer, Coke, Diapers, Milk				
Beer, Bread, Diapers, Milk				
Coke, Diapers, Milk				

We have 100% confidence for $I \rightarrow \text{milk}$, no matter what I we choose!

Interesting Association Rules

- Not all high-confidence rules are interesting
 - The rule X → milk may have high confidence for many itemsets X, because milk is just purchased very often (independent of X) and the confidence will be high
- Interest of an association rule $I \rightarrow j$: abs. difference between its confidence and the fraction of baskets that contain j

$$Interest(I \rightarrow j) = |conf(I \rightarrow j) - Pr[j]|$$

 Interesting rules are those with high positive or negative interest values (usually above 0.5)

Example: Confidence and Interest

$$B_1 = \{m, c, b\}$$
 $B_2 = \{m, p, j\}$
 $B_3 = \{m, b\}$ $B_4 = \{c, j\}$
 $B_5 = \{m, p, b\}$ $B_6 = \{m, c, b, j\}$
 $B_7 = \{c, b, j\}$ $B_8 = \{b, c\}$

- Association rule: {m, b} →c
 - Support = 2
 - Confidence = 2/4 = 0.5
 - Interest = |0.5 5/8| = 1/8
 - Item c appears in 5/8 of the baskets
 - The rule is not very interesting!

Association Rule Mining

- Problem: Find all association rules with support $\geq s$ and confidence $\geq c$
 - Note: Support of an association rule is the support of the set of items in the rule (left and right side)
- Hard part: Finding the frequent itemsets!
 - If $\{i_1, i_2, ..., i_k\} \rightarrow j$ has high support and confidence, then both $\{i_1, i_2, ..., i_k\}$ and $\{i_1, i_2, ..., i_k, j\}$ will be "frequent"

$$conf(I \rightarrow j) = \frac{support(I \cup j)}{support(I)}$$

Mining Association Rules

- Step 1: Find all frequent itemsets $I \quad conf(I \rightarrow j) = \frac{support(I \cup j)}{support(I)}$
 - (we will explain this next)
- Step 2: Rule generation
 - For every subset A of I, generate a rule $A \rightarrow I \setminus A$
 - Since I is frequent, A is also frequent (monotonicity)
 - Variant 1: Single pass to compute the rule confidence
 - confidence($A,B \rightarrow C,D$) = support(A,B,C,D) / support(A,B)
 - Variant 2:
 - Observation: If A,B,C \rightarrow D is below confidence, so is A,B \rightarrow C,D
 - Can generate "bigger" rules from smaller ones!
 - Output the rules above the confidence threshold

Example

$$B_1 = \{m, c, b\}$$
 $B_2 = \{m, p, j\}$
 $B_3 = \{m, c, b, n\}$ $B_4 = \{c, j\}$
 $B_5 = \{m, p, b\}$ $B_6 = \{m, c, b, j\}$
 $B_7 = \{c, b, j\}$ $B_8 = \{b, c\}$

- Support threshold s = 3, confidence c = 0.75
- Step 1) Find frequent itemsets:
 - {b,m} {b,c} {c,m} {c,j} {m,c,b}
- Step 2) Generate rules:

Compacting the Output

- To reduce the number of rules, we can post-process them and only output:
 - Maximal frequent itemsets:
 No immediate superset is frequent
 - Gives more pruning

or

Closed itemsets:

No immediate superset has the same support (> 0)

Stores not only frequent information, but exact supports/counts

Example: Maximal/Closed

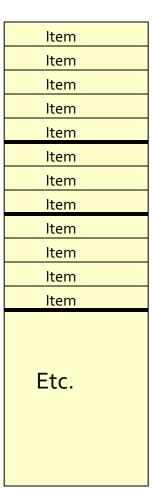
	Support	Frequent (s=3)	Maximal	Closed	Superset AB also frequent
A	4	Yes	No	No	Superset BC has same support ABC (only superset) not freq
В	5	Yes	No	Yes	
C	3	Yes	No	No 🐣	
AB	4	Yes	Yes -	Yes	
AC	2	No	No	No	
BC	3	Yes	Yes	Yes ←	——— ABC (only
ABC	2	No	No	Yes	superset) has smaller support

Step 1: Finding Frequent Itemsets

Itemsets: Computation Model

- Back to finding frequent itemsets
- Typically, data is kept in flat files rather than in a database system:
 - Stored on disk
 - Stored basket-by-basket
 - Baskets are small but we have many baskets and many items
 - Expand baskets into pairs, triples, etc. as you read baskets
 - Use k nested loops to generate all sets of size k

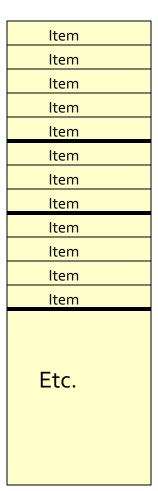
Note: We want to find frequent itemsets. To find them, we have to count them. To count them, we have to enumerate them.



Items are positive integers, and boundaries between baskets are -1.

Computation Model

- The true cost of mining diskresident data is usually the number of disk I/Os
- In practice, association-rule algorithms read the data in passes
 - all baskets read in turn
- We measure the cost by the number of passes an algorithm makes over the data



Items are positive integers, and boundaries between baskets are -1.

Main-Memory Bottleneck

- For many frequent-itemset algorithms,
 main-memory is the critical resource
 - As we read baskets, we need to count something, e.g., occurrences of pairs of items
 - The number of different things we can count is limited by main memory
 - Swapping counts in/out is a disaster

Finding Frequent Pairs

- The hardest problem often turns out to be finding the frequent pairs of items $\{i_1, i_2\}$
 - Why? Freq. pairs are common, freq. triples are rare
 - Why? Probability of being frequent drops exponentially with size; number of sets grows more slowly with size
- Let's first concentrate on pairs, then extend to larger sets
- The approach:
 - We always need to generate all the itemsets
 - But we would only like to count (keep track) of those itemsets that in the end turn out to be frequent

Naïve Algorithm

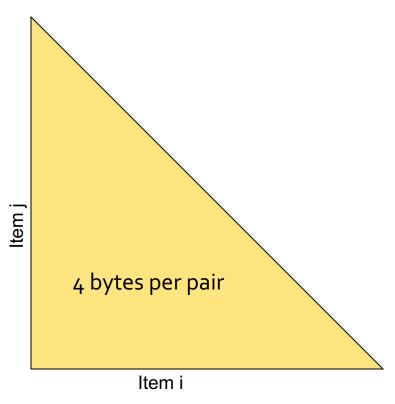
- Naïve approach to finding frequent pairs
- Read file once, counting in main memory the occurrences of each pair:
 - From each basket of n items, generate its n(n-1)/2 pairs by two nested loops
- Fails if (#items)² exceeds main memory
 - Remember: #items can be 100K (Wal-Mart) or 10B (Web pages)
 - Suppose 10⁵ items, counts are 4-byte integers
 - Number of pairs of items: $10^5(10^5-1)/2 \approx 5*10^9$
 - Therefore, 2*10¹⁰ (20 gigabytes) of memory is needed

Counting Pairs in Memory

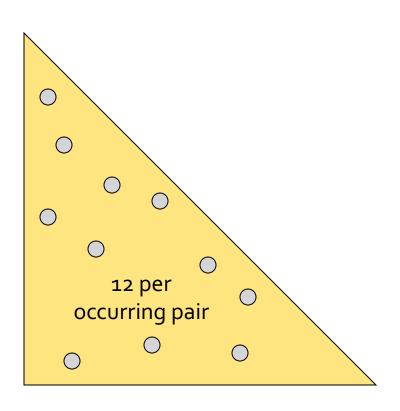
Goal: Count the number of occurrences of each pair of items (i,j):

- Approach 1: Count all pairs using a matrix
- Approach 2: Keep a table of triples [i, j, c] = "the count of the pair of items {i, j} is c."
 - If integers and item ids are 4 bytes, we need approximately 12 bytes for pairs with count > 0
 - Plus some additional overhead for the hashtable

Comparing the 2 Approaches



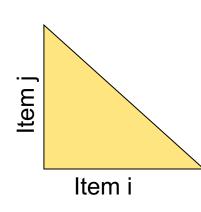
Triangular Matrix



Triples

Comparing the two approaches

- Approach 1: Triangular Matrix
 - n = total number items
 - Count pair of items {i, j} only if i<j</p>
 - Keep pair counts in lexicographic order:
 - $\{1,2\}, \{1,3\},..., \{1,n\}, \{2,3\}, \{2,4\},..., \{2,n\}, \{3,4\},...$
 - Pair {i, j} is at position: [n(n 1) (n i)(n i + 1)]/2 + (j i)
 - Total number of pairs n(n-1)/2; total bytes= $O(n^2)$
 - Triangular Matrix requires 4 bytes per pair
- Approach 2 uses 12 bytes per occurring pair (but only for pairs with count > 0)
- Approach 2 beats Approach 1 if less than 1/3 of possible pairs actually occur



Comparing the two approaches

- Approach 1: Triangular Matrix
 - **n** = total number items

 - Ke
 - Problem is if we have too
 - many items so the pairs
- (but
- do not fit into memory.
 - Can we do better?

Approach 2 beats Approach 1 if less than 1/3 of possible pairs actually occur

[)]/2 + (j - i)

A-Priori Algorithm

- Monotonicity of "Frequent"
- Notion of Candidate Pairs
- Extension to Larger Itemsets

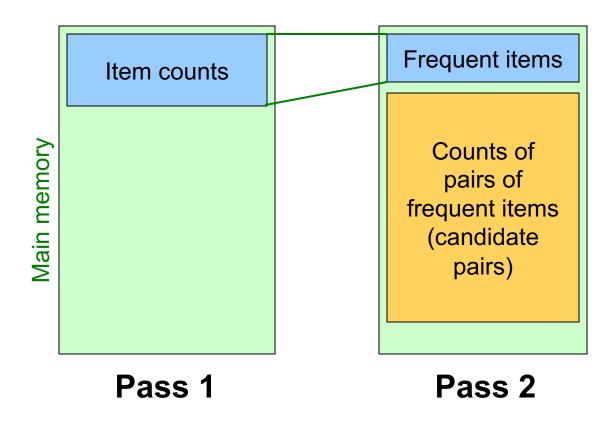
A-Priori Algorithm – (1)

- A two-pass approach called A-Priori limits the need for main memory
- Key idea: monotonicity
 - If a set of items I appears at least s times, so does every **subset** J of I
- Contrapositive for pairs:
 If item i does not appear in s baskets, then no pair including i can appear in s baskets
- So, how does A-Priori find freq. pairs?

A-Priori Algorithm – (2)

- Pass 1: Read baskets and count in main memory the # of occurrences of each individual item
 - Requires only memory proportional to #items
- Items that appear $\geq s$ times are the <u>frequent items</u>
- Pass 2: Read baskets again and keep track of the count of <u>only</u> those pairs where both elements are frequent (from Pass 1)
 - Requires memory proportional to square of frequent items only (for counts)
 - Plus a list of the frequent items (so you know what must be counted)

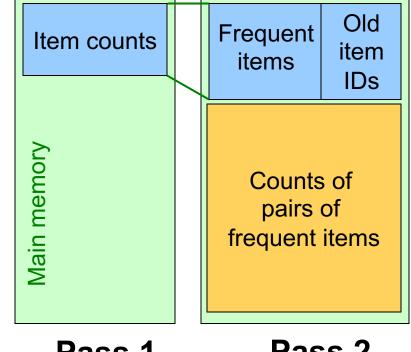
Main-Memory: Picture of A-Priori



Green box represents the amount of available main memory. Smaller boxes represent how the memory is used.

Detail for A-Priori

- You can use the triangular matrix method with n = numberof frequent items
 - May save space compared with storing triples
- Trick: re-number frequent items 1,2,... and keep a table relating new numbers to original item numbers

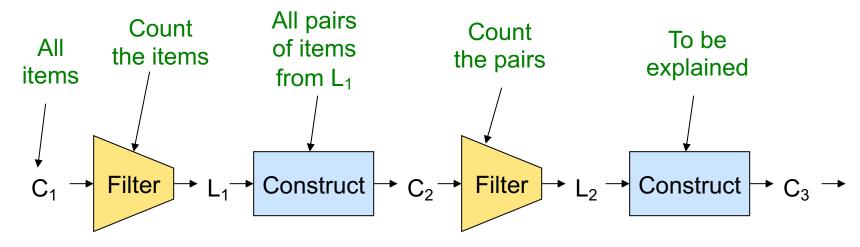


Pass 1

Pass 2

Frequent Triples, Etc.

- For each k, we construct two sets of k-tuples (sets of size k):
 - C_k = candidate k-tuples = those that might be frequent sets (support ≥ s) based on information from the pass for k-1
 - L_k = the set of truly frequent k-tuples



Example

$$C_1 = \{ \{b\}, \{c\}, \{j\}, \{m\}, \{n\}, \{p\} \} \}$$

baskets
{m, c, b}
{m, p, j}
{m, c, b, n}
{c, j}
{m, p, b}
{m, c, b, j}
{c, b, j}
{b, c}

$$s = 3$$

** In order for a triple

to be frequent, the

Supports: $\{b\} \rightarrow 6$, $\{c\} \rightarrow 6$, $\{j\} \rightarrow 4$, $\{m\} \rightarrow 5$, $\{n\} \rightarrow 1$, $\{p\} \rightarrow 2$ L_1 = $\{\{b\}, \{c\}, \{j\}, \{m\}\}\}$

 $\{c,j\} \rightarrow 3, \{c,m\} \rightarrow 3, \{j,m\} \rightarrow 2$

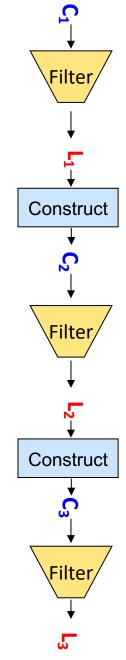
C_2 = { {b,c}, {b,j}, {b,m}, {c,j}, {c,m}, {j,m} } Supports: {b,c} \rightarrow 5, {b,j} \rightarrow 2, {b,m} \rightarrow 4

 $L_2 = \{ \{b,c\}, \{b,m\}, \{c,j\}, \{c,m\} \}$

C_3 = { {b,c,m}, {b,c,j}, {b,m,j}, {c,m,j} }

Supports: {b,c,m} \rightarrow 3

$$L_3 = \{ \{b,c,m\} \}$$



three pairs it contains must all be frequent.

A-Priori for All Frequent Itemsets

- One pass for each k (itemset size)
- Needs room in main memory to count each candidate k-tuple
- For typical market-basket data and reasonable support (e.g., 1%), k = 2 requires the most memory
- Many possible extensions:
 - Association rules with intervals:
 - For example: Men over 65 have 2 cars
 - Association rules when items are in a taxonomy
 - Bread, Butter → FruitJam
 - BakedGoods, MilkProduct → PreservedGoods
 - Lower the support s as itemset gets bigger

PCY (Park-Chen-Yu) Algorithm

- Improvement to A-Priori
- Exploits Empty Memory on First Pass
- Frequent Buckets

PCY (Park-Chen-Yu) Algorithm

- Observation:
 - In pass 1 of A-Priori, most memory is idle
 - We store only individual item counts
 - Can we use the idle memory to reduce memory required in pass 2?
- Pass 1 of PCY: In addition to item counts, maintain a hash table with as many buckets as fit in memory

Note: Bucket≠Basket

- Keep a count for each bucket into which pairs of items are hashed
 - For each bucket just keep the count, not the actual pairs that hash to the bucket!

PCY Algorithm – First Pass

```
FOR (each basket):

FOR (each item in the basket):

add 1 to item's count;

New in  
hash the pair to a bucket;

add 1 to the count for that bucket;
```

Few things to note:

- Pairs of items need to be generated from the input file; they are not present in the file
- We are not just interested in the presence of a pair, but we need to see whether it is present at least s (support) times

Observations about Buckets

- Observation: If a bucket contains a frequent pair, then the bucket is surely frequent
- However, even without any frequent pair, a bucket can still be frequent ⁽³⁾
 - So, we cannot use the hash to eliminate any member (pair) of a "frequent" bucket
- But, for a bucket with total count less than s, none of its pairs can be frequent ©
 - Pairs that hash to this bucket can be eliminated as candidates (even if the pair consists of 2 frequent items)
- Pass 2:
 Only count pairs that hash to frequent buckets

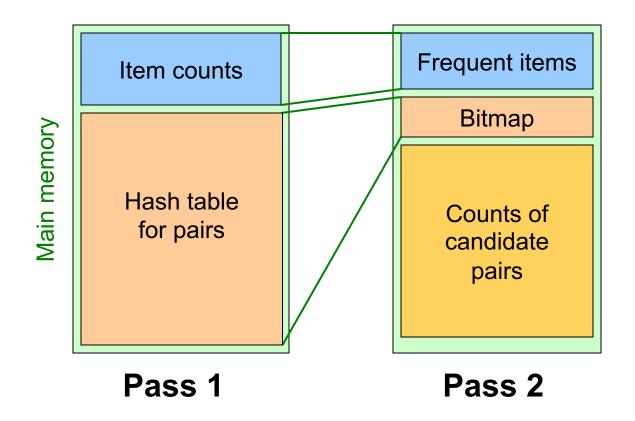
PCY Algorithm – Between Passes

- Replace the buckets by a bit-vector:
 - 1 means the bucket count exceeded the support s
 (call it a frequent bucket); 0 means it did not
- 4-byte integer counts are replaced by bits,
 so the bit-vector requires 1/32 of memory
- Also, decide which items are frequent and list them for the second pass

PCY Algorithm – Pass 2

- Count all pairs {i, j} that meet the conditions for being a candidate pair:
 - 1. A-priori: Both i and j are frequent items
 - 2. PCY: The pair $\{i, j\}$ hashes to a bucket whose bit in the bit vector is 1 (i.e., a frequent bucket)
 - Both conditions are necessary for the pair to have a chance of being frequent

Main-Memory: Picture of PCY



More Extensions to A-Priori

 The MMDS book covers several other extensions beyond the PCY idea: "Multistage" and "Multihash"

- For reading on your own, Sect. 6.4 of MMDS
- Recommended video (starting about 10:10):
 https://www.youtube.com/watch?v=AGAkNiQnbjY

Frequent Itemsets in < 2 Passes

- Simple Algorithm
- Savasere-Omiecinski- Navathe (SON) Algorithm
- Toivonen's Algorithm

Frequent Itemsets in ≤ 2 Passes

- A-Priori, PCY, etc., take k passes to find frequent itemsets of size k
- Can we use fewer passes?
- Use 2 or fewer passes for all sizes,
 but may miss some frequent itemsets
 - Random sampling
 - Do not sneer; "random sample" is often a cure for the problem of having too large a dataset.
 - SON (Savasere, Omiecinski, and Navathe)
 - Toivonen

Random Sampling (1)

- Take a random sample of the market baskets
- Run a-priori or one of its improvements in main memory
 - So we don't pay for disk I/O each time we increase the size of itemsets
 - Reduce support threshold proportionally to match the sample size
 - Example: if your sample is 1/100 of the baskets, use s/100 as your support threshold instead of s.

Copy of sample baskets

Main memory

Space for counts

Random Sampling (2)

- To avoid false positives: Optionally, verify that the candidate pairs are truly frequent in the entire data set by a second pass
- But you don't catch sets frequent in the whole but not in the sample (false negative)
 - Smaller threshold, e.g., s/125, helps catch more truly frequent itemsets
 - But requires more space

SON Algorithm – (1)

- SON Algorithm: Repeatedly read small subsets of the baskets into main memory and run an in-memory algorithm to find all frequent itemsets
 - Note: we are not sampling, but processing the entire file in memory-sized chunks
- An itemset becomes a candidate if it is found to be frequent in any one or more subsets of the baskets.

SON Algorithm – (2)

- On a second pass, count all the candidate itemsets and determine which are frequent in the entire set
- Key "monotonicity" idea: An itemset cannot be frequent in the entire set of baskets unless it is frequent in at least one subset

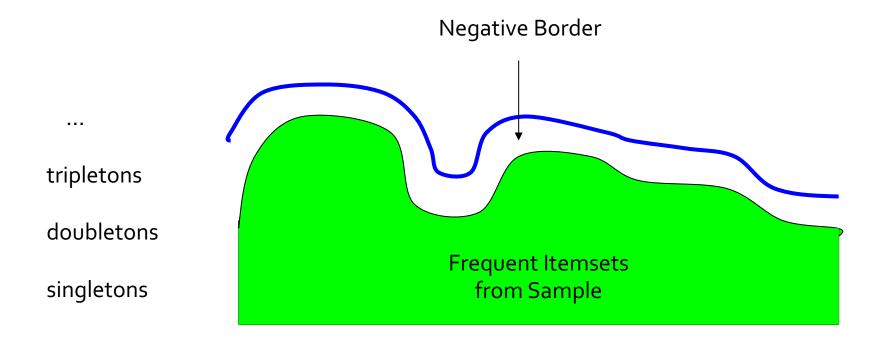
Toivonen's Algorithm: Intro

Pass 1:

- Start with a random sample, but lower the threshold slightly for the sample:
 - **Example:** if the sample is 1% of the baskets, use s/125 as the support threshold rather than s/100
- Find frequent itemsets in the sample
- Add to the itemsets that are frequent in the sample the negative border of these itemsets:
 - Negative border: An itemset is in the negative border if it is not frequent in the sample, but all its immediate subsets are
 - Immediate subset = "delete exactly one element"

Example: Negative Border

- {A,B,C,D} is in the negative border if and only if:
 - 1. It is not frequent in the sample, but
 - 2. All of {*A*,*B*,*C*}, {*B*,*C*,*D*}, {*A*,*C*,*D*}, and {*A*,*B*,*D*} are.



Toivonen's Algorithm

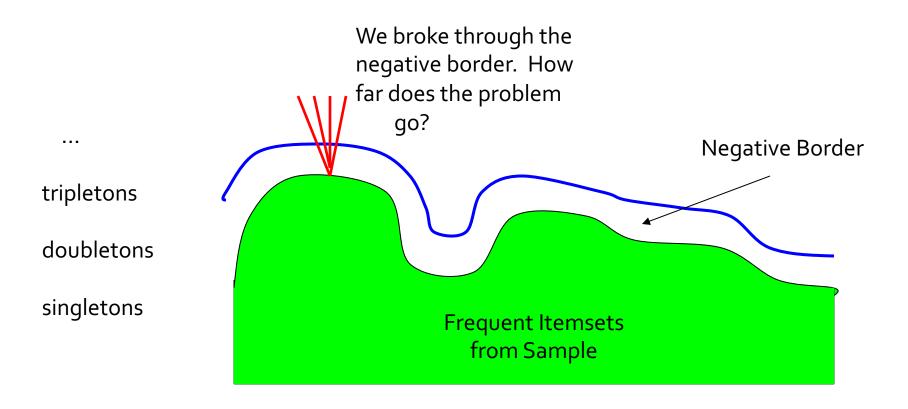
Pass 1:

- Start with the random sample, but lower the threshold slightly for the subset
- Add to the itemsets that are frequent in the sample the negative border of these itemsets

Pass 2:

- Count all candidate frequent itemsets from the first pass, and also count sets in their negative border
- Key: If no itemset from the negative border turns out to be frequent, then we found all the frequent itemsets.
 - What if we find that something in the negative border is frequent?
 - We must start over again with another sample!
 - Try to choose the support threshold so the probability of failure is low, while the number of itemsets checked on the second pass fits in main-memory.

If Something in the Negative Border Is Frequent . . .



Summary

- Frequent Itemset Mining
- Association Rules
- A Priori Algorithm: Dynamic Programming
- PCY: Improvement using Hashing
- Announcements:
 - Spark Tutorial Today!
 - HW1 posted today start early
 - Ed Search for Teammates!