Case Study 1: Estimating Click Probabilities

Tackling an Unknown Number of Features with Sketching

Machine Learning for Big Data
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What you should know about Logistic Regression (LR) and Click Prediction

- Click prediction problem:
  - Estimate probability of clicking
  - Can be modeled as logistic regression
- Logistic regression model: Linear model
- Gradient ascent to optimize conditional likelihood
- Overfitting + regularization
- Regularized optimization
  - Convergence rates and stopping criterion
- Stochastic gradient ascent for large/streaming data
  - Convergence rates of SGD
- AdaGrad motivation, derivation, and algorithm

Problem 1: Complexity of LR Updates

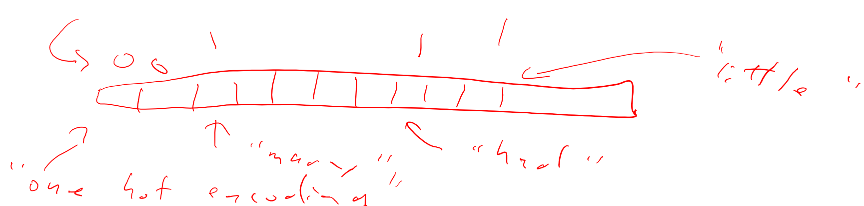
- Logistic regression update:

$$w_i^{(t+1)} \leftarrow w_i^{(t)} + \eta_t \left\{ -\lambda w_i^{(t)} + x_i^{(t)} [y^{(t)} - P(Y = 1 | \mathbf{x}^{(t)}, \mathbf{w}^{(t)})] \right\}$$

- Complexity of updates:
  - Constant in number of data points
  - In number of features?
    - Problem both in terms of computational complexity and sample complexity
- What can we with very high dimensional feature spaces?
  - Kernels not always appropriate, or scalable
  - What else?

Problem 2: Unknown Number of Features

- For example, bag-of-words features for text data: "obamacare"
  - "Mary had a little lamb, little lamb…"



- What's the dimensionality of **x**? ← size of vocab.
- What if we see new word that was not in our vocabulary?
  - Obamacare
  - Theoretically, just keep going in your learning, and initialize $\mathbf{w}_{\text{Obamacare}} = 0$
  - In practice, need to re-allocate memory, fix indices,… A big problem for Big Data

What Next?

- Hashing & Sketching!
  - Addresses both dimensionality issues and new features in one approach!
- Let's start with a much simpler problem: Is a string in our vocabulary?
  - Membership query
- How do we keep track?
  - Explicit list of strings
    - Very slow

{ "many", "hard", "a", "little", ... }

  - Fancy Trees and Tries
    - Hard to implement and maintain
  - Hash tables?

h('many') ⟶

h('obamacare') ⟶

Hash Functions and Hash Tables

values

$h: X \longrightarrow \{1, \dots, m\}$

string

- Hash functions map **keys** to integers (bins):
  - Keys can be integers, strings, objects,…

key is an integer

- Simple example: **mod**
  - *h(i)* = (*a.i* + *b*) % *m*

$a = 7, \quad b = 11, \quad m = 32$

$i = 4 \quad h(i) = 39 \% 32 = 7$

  - Random choice of (*a,b*) (usually primes)
  - If inputs are uniform, bins are uniformly used
  - From two results can recover (*a,b*), so not pairwise independent -> Typically use fancier hash functions
- Hash table:
  - Store list of objects in each bin
  - Exact, but storage still linear in size of object ids, which can be very long
    - E.g., hashing very long strings, entire documents

Hash Bit-Vector Table-Based Membership Query

- Approximate queries with one-sided error: Accept false positives only
  - If we say no, element is not in set
  - If we say yes, element is very to be likely in set

h('had') h('mary')

- Given hash function, keep binary bit vector **v** of length *m*:

$V = [1\,1\,1\,1\,1\,0\,1\,1\,1\,0\,1]$ m buckets

h('Affordable') h('obamacare')

- Query *Q(i)*: Element *i* in set?
  - $V(h(i)) = 0 \Rightarrow Q(i) = 0$
- Collisions: $V(h(i)) = 1 \Rightarrow Q(i)$ probably yes

example 'had' & 'obamacare' collide

- Guarantee: One-sided errors, but may make many mistakes
  - How can we improve probability of correct answer?

Bloom Filter: Multiple Hash Tables

have p hash functions

- Single hash table → Many false positives
- Multiple hash tables with independent hash functions
  - Apply $h_1(i),\dots, h_p(i)$, set all bits to 1

V_1 h_1('mary') h_1('obamacare')

⋮

V_p h_p('mary') h_p('obamacare')

- Query *Q(i)*?

if $\forall j$, $V_j(h_j(i)) = 1 \Longrightarrow$ "yes" (very likely yes)

else no!

- Significantly decrease probability of false positives

Analysis of Bloom Filter

- Want to keep track of n elements with false positive probability of δ>0… how large m & p?

- Simple analysis yields:

$$m = \frac{n \log_2 \frac{1}{\delta}}{\ln 2} \approx 1.5 n \log_2 \frac{1}{\delta}$$

$$p = \log_2 \frac{1}{\delta}$$

$m = O(n)$

$p = O(\log \frac{1}{\delta})$

$\Rightarrow$ Prob (false positive) $\leq \delta$

Sketching Counts

- Bloom Filter is super cool, but not what we need…
  - We don't just care about whether a feature existed before, but to keep track of counts of occurrences of features! (assuming x_i integer)
- Recall the LR update:

$$w_i^{(t+1)} \leftarrow w_i^{(t)} + \eta_t \left\{ -\lambda w_i^{(t)} + x_i^{(t)} [y^{(t)} - P(Y = 1 | \mathbf{x}^{(t)}, \mathbf{w}^{(t)})] \right\}$$

- Must keep track of (weighted) counts of each feature:
  - E.g., with sparse data, for each non-zero dimension i in $\mathbf{x}^{(t)}$:

- Can we generalize the Bloom Filter?

Count-Min Sketch: single vector

- Simpler problem: Count how many times you see each string
- Single hash function:
  - Keep *Count* vector of length *m*
  - every time see string *i*:

$$Count[h(i)] \leftarrow Count[h(i)] + 1$$

  - Again, collisions could be a problem:
    - a_i is the count of element *i*:

Count-Min Sketch: general case

- Keep *p* by *m* Count matrix

- *p* hash functions:
  - Just like in Bloom Filter, decrease errors with multiple hashes
  - Every time see string *i*:

$$\forall j \in \{1, \ldots, p\} : Count[j, h_j(i)] \leftarrow Count[j, h_j(i)] + 1$$

Querying the Count-Min Sketch

$$\forall j \in \{1, \ldots, p\} : Count[j, h_j(i)] \leftarrow Count[j, h_j(i)] + 1$$

- Query Q(i)?
  - What is in *Count*[*j*,*k*]?
  - Thus:
  - Return:

Analysis of Count-Min Sketch

$$\hat{a}_i = \min_j Count[j, h(i)] \geq a_i$$

- Set:

$$m = \left\lceil \frac{e}{\epsilon} \right\rceil \qquad p = \left\lceil \ln \frac{1}{\delta} \right\rceil$$

- Then, after seeing n elements:

$$\hat{a}_i \leq a_i + \epsilon n$$

- With probability at least 1-δ

Proof of Count-Min for Point Query with Positive Counts: Part 1 – Expected Bound

- $I_{i,j,k}$ = indicator that i & k collide on hash j:
- Bounding expected value:
- $X_{i,j}$ = total colliding mass on estimate of count of i in hash j:
- Bounding colliding mass:
- Thus, estimate from each hash function is close in expectation

Proof of Count-Min for Point Query with Positive Counts: Part 2 – High Probability Bounds

- What we know: $Count[j, h_j(i)] = a_i + X_{i,j} \qquad E[X_{i,j}] \leq \frac{\epsilon}{e} n$
- Markov inequality: For $z_1,\ldots,z_k$ positive iid random variables

$$P(\forall z_i : z_i > \alpha E[z_i]) < \alpha^{-k}$$

- Applying to the Count-Min sketch:

But updates may be positive or negative

$$w_i^{(t+1)} \leftarrow w_i^{(t)} + \eta_t \left\{ -\lambda w_i^{(t)} + x_i^{(t)} [y^{(t)} - P(Y = 1 | \mathbf{x}^{(t)}, \mathbf{w}^{(t)})] \right\}$$

- Count-Min sketch for positive & negative case
  - a_i no longer necessarily positive
- Update the same: Observe change Δ_i to element *i*:

$$\forall j \in \{1, \ldots, p\} : Count[j, h_j(i)] \leftarrow Count[j, h_j(i)] + \Delta_i$$

  - Each Count[*j*,*h(i)*] no longer an upper bound on a_i
- How do we make a prediction?
- Bound: $|\hat{a}_i - a_i| \leq 3\epsilon ||\mathbf{a}||_1$
  - With probability at least $1-\delta^{1/4}$, where $||\mathbf{a}|| = \Sigma_i |a_i|$

Finally, Sketching for LR

$$w_i^{(t+1)} \leftarrow w_i^{(t)} + \eta_t \left\{ -\lambda w_i^{(t)} + x_i^{(t)} [y^{(t)} - P(Y = 1 | \mathbf{x}^{(t)}, \mathbf{w}^{(t)})] \right\}$$

- Never need to know size of vocabulary!
- At every iteration, update Count-Min matrix:
- Making a prediction:
- Scales to huge problems, great practical implications…

Hash Kernels

- Count-Min sketch not designed for negative updates
- Biased estimates of dot products

- **Hash Kernels**: Very simple, but powerful idea to remove bias
- Pick 2 hash functions:
  - *h* : Just like in Count-Min hashing
  - ξ : Sign hash function
    - Removes the bias found in Count-Min hashing (see homework)
- Define a "kernel", a projection ϕ for **x**:

Hash Kernels Preserve Dot Products

$$\phi_i(\mathbf{x}) = \sum_{j:h(j)=i} \xi(j)\mathbf{x}_j$$

- Hash kernels provide unbiased estimate of dot-products!

- Variance decreases as O(1/*m)*

- Choosing *m*? For ε>0, if

$$m = \mathcal{O}\left(\frac{\log \frac{N}{\delta}}{\epsilon^2}\right)$$

  - Under certain conditions…
  - Then, with probability at least 1-δ:

$$(1-\epsilon)||\mathbf{x}-\mathbf{x}'||_2^2 \leq ||\phi(\mathbf{x})-\phi(\mathbf{x}')||_2^2 \leq (1+\epsilon)||\mathbf{x}-\mathbf{x}'||_2^2$$

Learning With Hash Kernels

- Given hash kernel of dimension m, specified by h and ξ
  - Learn m dimensional weight vector
- Observe data point **x**
  - Dimension does not need to be specified a priori!
- Compute $\phi(\mathbf{x})$:
  - Initialize $\phi(\mathbf{x})$
  - For non-zero entries j of $\mathbf{x}_j$:
- Use normal update as if observation were $\phi(\mathbf{x})$, e.g., for LR using SGD:

$$w_i^{(t+1)} \leftarrow w_i^{(t)} + \eta_t \left\{ -\lambda w_i^{(t)} + \phi_i(\mathbf{x}^{(t)})[y^{(t)} - P(Y=1|\phi(\mathbf{x}^{(t)}), \mathbf{w}^{(t)})] \right\}$$

Interesting Application of Hash Kernels: Multi-Task Learning

- Personalized click estimation for many users:
  - One global click prediction vector **w**:
    - But…
  - A click prediction vector $\mathbf{w}_u$ per user u:
    - But…
- Multi-task learning: Simultaneously solve multiple learning related problems:
  - Use information from one learning problem to inform the others
- In our simple example, learn both a global **w** and one $\mathbf{w}_u$ per user:
  - Prediction for user u:
  - If we know little about user u:
  - After a lot of data from user u:

Problems with Simple Multi-Task Learning

- Dealing with new user is annoying, just like dealing with new words in vocabulary

- Dimensionality of joint parameter space is HUGE, e.g. personalized email spam classification from Weinberger et al.:
  - 3.2M emails
  - 40M unique tokens in vocabulary
  - 430K users
  - 16T parameters needed for personalized classification!

Hash Kernels for Multi-Task Learning

- Simple, pretty solution with hash kernels:
  - Very multi-task learning as (sparse) learning problem with (huge) joint data point **z** for point **x** and user *u*:

- Estimating click probability as desired:

- Address huge dimensionality, new words, and new users using hash kernels:

Simple Trick for Forming Projection $\phi(\mathbf{x},u)$

- Observe data point **x** for user *u*
  - Dimension does not need to be specified a priori and user can be new!
- Compute $\phi(\mathbf{x},u)$:
  - Initialize $\phi(\mathbf{x},u)$
  - For non-zero entries *j* of $\mathbf{x}_j$:
    - E.g., j='Obamacare'
    - Need two contributions to ϕ:
      - Global contribution
      - Personalized Contribution
    - Simply:
- Learn as usual using $\phi(\mathbf{x},u)$ instead of $\phi(\mathbf{x})$ in update function

Results from Weinberger et al. on Spam Classification: Effect of *m*

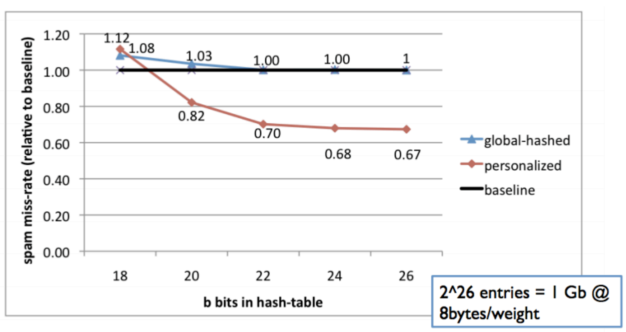


Figure 2. The decrease of uncaught spam over the baseline classifier averaged over all users. The classification threshold was chosen to keep the not-spam misclassification fixed at 1%. The hashed global classifier (*global-hashed*) converges relatively soon, showing that the distortion error ϵ_d vanishes. The personalized classifier results in an average improvement of up to 30%.

Results from Weinberger et al. on Spam Classification: Multi-Task Effect

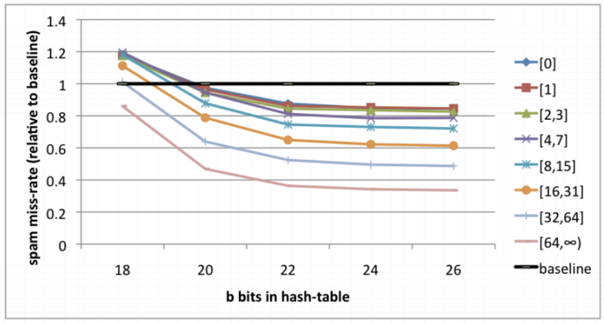


Figure 3. Results for users clustered by training emails. For example, the bucket $[8, 15]$ consists of all users with eight to fifteen training emails. Although users in buckets with large amounts of training data do benefit more from the personalized classifier (up-to 65% reduction in spam), even users that did not contribute to the training corpus at all obtain almost 20% spam-reduction.

What you need to know

- Hash functions
- Bloom filter
  - Test membership with some false positives, but very small number of bits per element
- Count-Min sketch
  - Positive counts: upper bound with nice rates of convergence
  - General case
- Application to logistic regression
- Hash kernels:
  - Sparse representation for feature vectors
  - Very simple, use two hash function (Can use one hash function...take least significant bit to define ξ)
  - Quickly generate projection φ(**x**)
  - Learn in projected space
- Multi-task learning:
  - Solve many related learning problems simultaneously
  - Very easy to implement with hash kernels
  - Significantly improve accuracy in some problems (if there is enough data from individual users)

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