Case Study 1: Estimating Click Probabilities

Tackling an Unknown Number of Features with Sketching

Machine Learning for Big Data

CSE547/STAT548, University of Washington

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Problem 1: Complexity of Update Rules for Logistic Regression

- Logistic regression update: stoch. grad. asc.

$$w_i^{(t+1)} \leftarrow w_i^{(t)} + \eta_t \left\{ -\lambda w_i^{(t)} + x_i^{(t)} [y^{(t)} - P(Y = 1 | \mathbf{x}^{(t)}, \mathbf{w}^{(t)})] \right\}$$

- Complexity of updates:
  - Constant in number of data points ✓
  - In number of features? O(d)
    - Problem both in terms of computational complexity and sample complexity

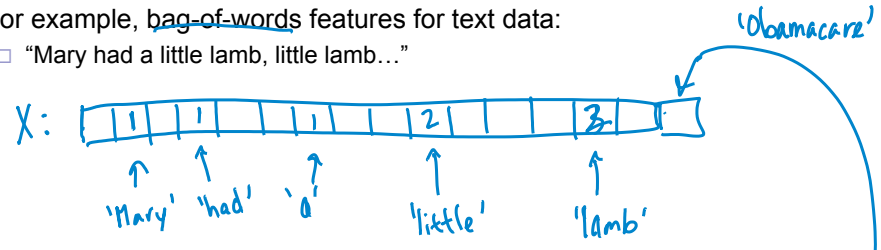
d very large

What if we have 1B features??

- What can we with very high dimensional feature spaces?
  - Kernels not always appropriate, or scalable ← "kernel trick"
  - What else?

Problem 2: Unknown Number of Features

- For example, bag-of-words features for text data:
  - "Mary had a little lamb, little lamb…"



- What's the dimensionality of **x**? size of vocabulary … millions
- What if we see new word that was not in our vocabulary?
  - Obamacare
  - Theoretically, just keep going in your learning, and initialize $\mathbf{w}_{\text{Obamacare}} = 0$
  - In practice, need to re-allocate memory, fix indices,… A big problem for Big Data

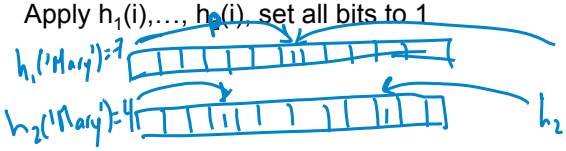
d can change

Bloom Filter: Multiple Hash Tables

- Single hash table -> Many false positives

just kept track of binary bit vector of length m (m << d)

- Multiple hash tables with independent hash functions
  - Apply $h_1(i), \ldots, h_k(i)$, set all bits to 1



- Query Q(i)?

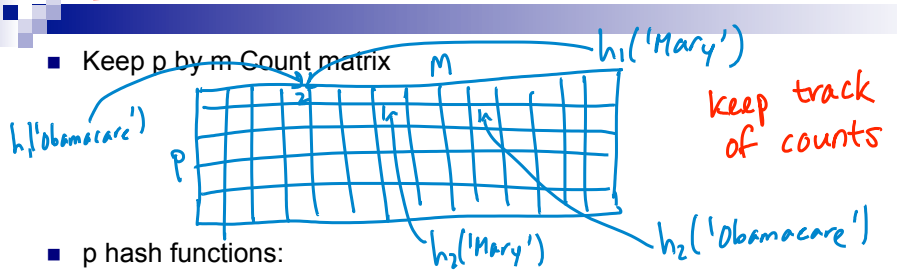
if $\forall j \; h_j(i) = 1$
  Q(i) = very probably yes
else Q(i) = no

'Mary' + 'Obamacare' collide in h_1 but not h_2

- Significantly decrease probability of false positives

Count-Min Sketch: general case

- Keep p by ~~m~~ Count matrix



- p hash functions:
  - Just like in Bloom Filter, decrease errors with multiple hashes
  - Every time see string *i*:

row, j^{th} hash fcn, input

$$\forall j \in \{1, \ldots, p\} : Count[j, h_j(i)] \leftarrow Count[j, h_j(i)] + 1$$

Finally, Sketching for LR

$$w_i^{(t+1)} \leftarrow w_i^{(t)} + \eta_t \left\{ -\lambda w_i^{(t)} + x_i^{(t)} [y^{(t)} - P(Y = 1 | \mathbf{x}^{(t)}, \mathbf{w}^{(t)})] \right\}$$

- Never need to know size of vocabulary!
- At every iteration, update Count-Min matrix:

$\forall j,k \quad Count[j,k] = (1 - \eta_t \lambda)\, Count[j,k]$

"soft counts" instead

$\forall x_i^{(t)} \neq 0$

$\forall j \quad Count[j, h_j(i)] \mathrel{+}= x_i^{(t)} \cdot const$

$const = \eta_t (y^{(t)} - \hat{P}(Y=1 | \ldots))$

- Making a prediction:

Remember our est. of $w_i^{(t)}$: $\underset{j}{\text{median}}\ Count[j, h_j(i)]$

Make pred: $\text{log odds} = w_0^{(t)} + \sum_{i: x_i \neq 0} \underset{j}{\text{median}}\ Count[j, h_j(i)]\, x_i^{(t)}$

- Scales to huge problems, great practical implications…

Hash Kernels

- Count-Min sketch not designed for negative updates
- Biased estimates of dot products
- **Hash Kernels**: Very simple, but powerful idea to remove bias
- Pick 2 hash functions:
  - *h* : Just like in Count-Min hashing → $h: X \to \{1, \ldots, m\}$
  - ξ : Sign hash function → $\xi: X \to \{+1, -1\}$
    - Removes the bias found in Count-Min hashing (see homework)

can think of as random projection of X

- Define a "kernel", a projection ϕ for **x**:

$\phi(x) =$ 

For each non-zero element of X, add to bin h(j): $\xi(j)x_j$

$$\phi_i(x) = \sum_{j:h(j)=i} \xi(j) x_j$$

If $x_j = 7$, $h(j) = 4$, $\xi(j) = -1$, add -7 to bin 4

Hash Kernels Preserve Dot Products

$$\phi_i(\mathbf{x}) = \sum_{j:h(j)=i} \xi(j)\mathbf{x}_j$$

- Hash kernels provide unbiased estimate of dot-products!

$E_{h,\xi}[\phi(x) \cdot \phi(y)] = x \cdot y$ pf: by homework

- Variance decreases as O(1/*m*) ← gets better w/ more dims
- Choosing *m*? For ε>0, if

$$m = \mathcal{O}\left(\frac{\log \frac{N}{\delta}}{\epsilon^2}\right)$$

log in data size

  - Under certain conditions…
  - Then, with probability at least 1-δ:

$$(1-\epsilon)||\mathbf{x}-\mathbf{x}'||_2^2 \leq ||\phi(\mathbf{x})-\phi(\mathbf{x}')||_2^2 \leq (1+\epsilon)||\mathbf{x}-\mathbf{x}'||_2^2$$

Learning With Hash Kernels

- Given hash kernel of dimension *m*, specified by *h* and ξ
  - Learn *m* dimensional weight vector
- Observe data point **x**
  - Dimension does not need to be specified a priori!
- Compute ϕ(**x**):
  - Initialize ϕ(**x**) = 0
  - For non-zero entries *j* of $\mathbf{x}_j$:

$$\phi_{h(j)} \mathrel{+}= \xi(j)\, x_j$$

e.g. j = 'Mary' $h(\text{'Mary'}) = 7$ $\xi(\text{'Mary'}) = -1$

$\phi_7 \mathrel{+}= -x_{\text{'Mary'}}$

- Use normal update as if observation were ϕ(**x**), e.g., for LR using SGD:

$$w_i^{(t+1)} \leftarrow w_i^{(t)} + \eta_t \left\{ -\lambda w_i^{(t)} + \phi_i(\mathbf{x}^{(t)})[y^{(t)} - P(Y = 1 | \phi(\mathbf{x}^{(t)}), \mathbf{w}^{(t)})] \right\}$$

w length m, $i = 0, 1, \ldots, m-1$

$$P(Y=1 \mid \phi(x^{(t)}), w^{(t)}) = \frac{\exp(\phi(x^{(t)}) \cdot w^{(t)})}{1 + \exp(\phi(x^{(t)}) \cdot w^{(t)})}$$

Interesting Application of Hash Kernels: Multi-Task Learning

- Personalized click estimation for many users:
  - One global click prediction vector **w**:
    - But… people are unique
  - A click prediction vector $\mathbf{w}_u$ per user *u*:
    - But… people don't each provide much data (label)

predict using $w \cdot x$ ← $\frac{\exp(w \cdot x)}{1 + \exp(w \cdot x)}$

predict with $w_u \cdot x$

- Multi-task learning: Simultaneously solve multiple learning related problems:
  - Use information from one learning problem to inform the others
- In our simple example, learn both a global **w** and one $\mathbf{w}_u$ per user:
  - Prediction for user *u*: $(w + w_u) \cdot x = w \cdot x + w_u \cdot x$
  - If we know little about user *u*: basically $w \cdot x$
  - After a lot of data from user *u*: using $w + w_u$ as your vector

Problems with Simple Multi-Task Learning

- Dealing with new user is annoying, just like dealing with new words in vocabulary
- Dimensionality of joint parameter space is HUGE, e.g. personalized email spam classification from Weinberger et al.:
  - 3.2M emails
  - 40M unique tokens in vocabulary
  - 430K users
  - 16T parameters needed for personalized classification!

Hash Kernels for Multi-Task Learning

- Simple, pretty solution with hash kernels:
  - Very multi-task learning as (sparse) learning problem with (huge) joint data point **z** for point **x** and user *u*:

$$z_{(x,u)} = (x_1, \ldots, x_d, 0, \ldots, 0, x_1, \ldots, x_d, 0, \ldots, 0)$$

(global; length d; # of users; user u)

- Estimating click probability as desired:

$$w = (w_0, w_1, \ldots, w_u, \ldots, w_{\#\text{users}}) \quad (w_0 \text{ global}) \quad \rightarrow z_{(x,u)} \cdot w = w \cdot x + w_u \cdot x$$

- Address huge dimensionality, new words, and new users using hash kernels:

$$\phi(z_{(x,u)}) \quad \text{just like w/ hash kernels} \quad \phi_i = \sum_{j: h(j)=i} \xi(j)\, z_{(x,u),j} \quad = (w + w_u) \cdot x$$

  - Desired effect achieved if *j* includes both
    - just word (for global **w**)
    - word,user (for personalized $\mathbf{w}_u$)

Simple Trick for Forming Projection $\phi(\mathbf{x},u)$

- Observe data point $\mathbf{x}$ for user u
  - Dimension does not need to be specified a priori and user can be ~~unknown~~ new!
- Compute $\phi(\mathbf{x},u)$:
  - Initialize $\phi(\mathbf{x},u) = 0$
  - For non-zero entries j of $\mathbf{x}_j$:
    - E.g., j='Obamacare'
    - Need two contributions to ϕ:
      - Global contribution
      - Personalized Contribution
    - Simply: $u = 17$

$$\phi_h(\text{'Obamacare\_user17'}) \mathrel{+}= g(\text{'Obamacare\_user17'})\, x_j$$

- Learn as usual using $\phi(\mathbf{x},u)$ instead of $\phi(\mathbf{x})$ in update function

Results from Weinberger et al. on Spam Classification: Effect of *m*

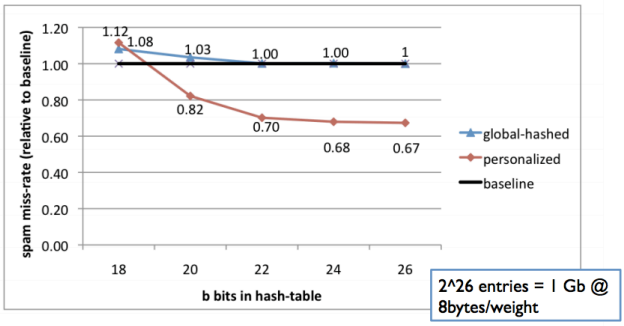


Figure 2. The decrease of uncaught spam over the baseline classifier averaged over all users. The classification threshold was chosen to keep the not-spam misclassification fixed at 1%. The hashed global classifier (*global-hashed*) converges relatively soon, showing that the distortion error ϵ_d vanishes. The personalized classifier results in an average improvement of up to 30%.

Results from Weinberger et al. on Spam Classification: Illustrating Multi-Task Effect

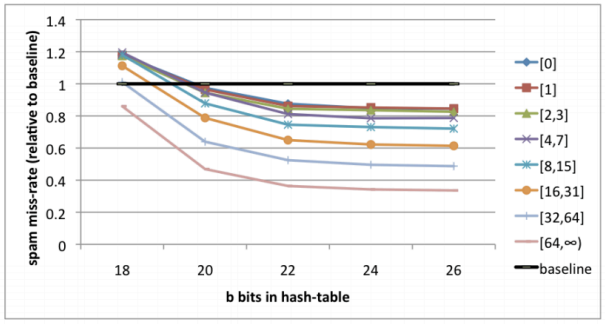


Figure 3. Results for users clustered by training emails. For example, the bucket $[8, 15]$ consists of all users with eight to fifteen training emails. Although users in buckets with large amounts of training data do benefit more from the personalized classifier (up-to 65% reduction in spam), even users that did not contribute to the training corpus at all obtain almost 20% spam-reduction.

What you need to know

- Hash functions
- Bloom filter
  - Test membership with some false positives, but very small number of bits per element
- Count-Min sketch
  - Positive counts: upper bound with nice rates of convergence
  - General case
- Application to logistic regression
- Hash kernels:
  - Sparse representation for feature vectors
  - Very simple, use two hash function (Can use one hash function…take least significant bit to define ξ)
  - Quickly generate projection $\phi(\mathbf{x})$
  - Learn in projected space
- Multi-task learning:
  - Solve many related learning problems simultaneously
  - Very easy to implement with hash kernels
  - Significantly improve accuracy in some problems (if there is enough data from individual users)

Case Study 2: Document Retrieval

Task Description: Finding Similar Documents

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Document Retrieval

- **Goal:** Retrieve documents of interest
- **Challenges:**
  - Tons of articles out there
  - How should we measure similarity?

Task 1: Find Similar Documents

- **To begin…**
  - **Input:** Query article
  - **Output:** Set of k similar articles

X

k related articles

Document Representation

- Bag of words model

ignore order of the words

word count

$$x = \begin{bmatrix} wc_1 \\ wc_2 \\ \vdots \\ wc_d \end{bmatrix} \qquad d = |V|$$

size of vocabulary

1-Nearest Neighbor

- Articles $X = \{x^1, \ldots, x^N\}$ $x^i \in \mathbb{R}^d$ (N articles)
- Query: x
- 1-NN
  - Goal: find article in X "closest" to x
  - Formulation: ★ need distance metric $d(u,v)$ ★

$$x^{NN} = \arg\min_{x^i \in X} d(x^i, x)$$

(x: query article)

k-Nearest Neighbor

- Articles $X = \{x^1, \ldots, x^N\}, \quad x^i \in \mathbb{R}^d$
- Query: $x \in \mathbb{R}^d$
- k-NN
  - Goal: find k articles in X closest to x
  - Formulation:

$$X^{NN} = \{x^{NN_1}, \ldots, x^{NN_k}\} \subseteq X$$

s.t. $\forall\, x^i \in X \setminus X^{NN}$ ← "not in nearest neighbor set"

$$d(x^i, x) \geq \max_{x^{NN_i} \in X^{NN}} d(x^{NN_i}, x)$$

Distance Metrics – Euclidean

$$d(u,v) = \sqrt{\sum_{i=1}^{d}(u_i - v_i)^2} = \|u-v\|_2$$

Or, more generally, $d(u,v) = \sqrt{\sum_{i=1}^{d}\sigma_i^2(u_i - v_i)^2}$ "scaled L_2" ↑ weight for dim i

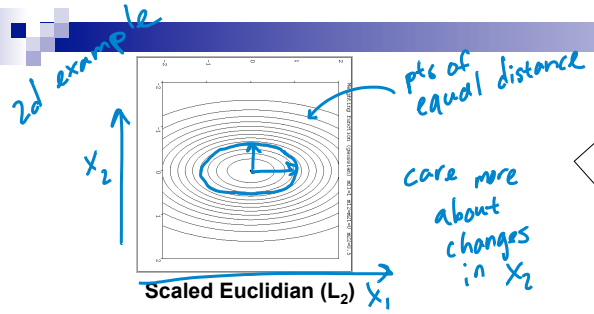
Equivalently,

$$d(u,v) = \sqrt{(u-v)'\Sigma(u-v)}$$ where $\Sigma = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \sigma_d^2 \end{bmatrix}$

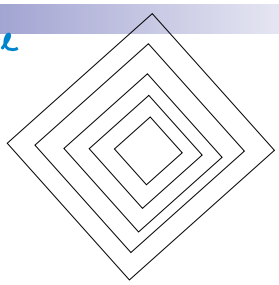
Other Metrics…

- Mahalanobis, Rank-based, Correlation-based, cosine similarity…

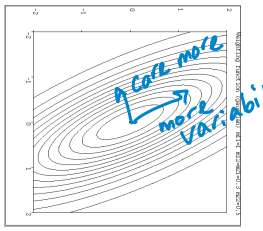
Notable Distance Metrics (and their level sets)



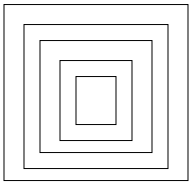
Scaled Euclidian (L_2)



L_1 norm (absolute)



Mahalanobis
(Σ is general sym pos def matrix, on previous slide = diagonal)



L_∞ (max) norm

Euclidean Distance + Document Retrieval

- Recall distance metric

$$d(u,v) = \sqrt{\sum_{i=1}^{d}(u_i - v_i)^2}$$

- What if each document were α times longer?
  - Scale word count vectors

$u \leftarrow \alpha u$
$v \leftarrow \alpha v$

  - What happens to measure of similarity?

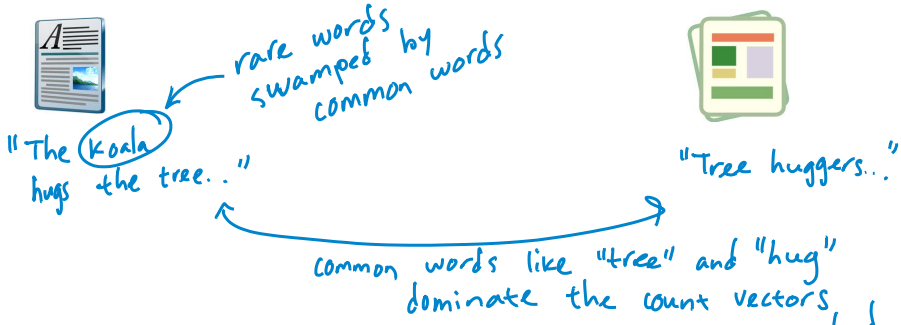
$\|\alpha u - \alpha v\|_2 = \alpha \|u - v\|_2 > \|u - v\|_2$
$\alpha > 1$
Now less similar

- Good to normalize vectors

$\|u\|_2 = \|v\|_2 = 1$

Issues with Document Representation

- Words counts are ***bad*** for standard similarity metrics



- Term Frequency – Inverse Document Frequency (tf-idf)
  - Increase importance of rare words

TF-IDF

- Term frequency:

$$\mathrm{tf}(t,d) =$$

  - Could also use $\{0,1\}, 1+\log f(t,d), \ldots$

- Inverse document frequency:

$$\mathrm{idf}(t,\mathcal{D}) = \log \frac{|\mathcal{X}|}{1+|\{d \in \mathcal{X} : t \in d\}|}$$

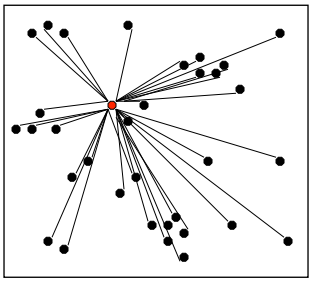
- tf-idf:

$$\mathrm{tfidf}(t,d,\mathcal{D}) =$$

  - High for document d with high frequency of term t (high "term frequency") and few documents containing term t in the corpus (high "inverse doc frequency")

Issues with Search Techniques

- Naïve approach:
  Brute force search
  - Given a query point x
  - Scan through each point x^i
  - O(*N*) distance computations per 1-NN query!
  - O(*N*log*k*) per k-NN query!

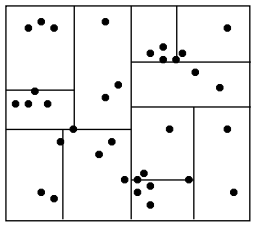


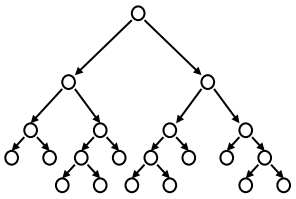
33 Distance Computations

- What if *N* is huge???
  (and many queries)

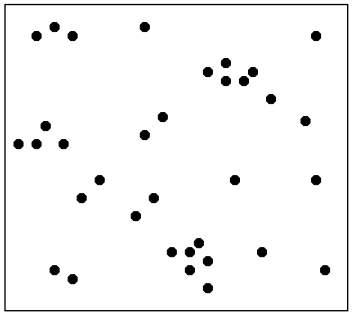
KD-Trees

- Smarter approach: ***kd-trees***
  - Structured organization of documents
    - Recursively partitions points into axis aligned boxes.
  - Enables more efficient pruning of search space
    - Examine nearby points first.
    - Ignore any points that are further than the nearest point found so far.
- ***kd-trees*** work "well" in "low-medium" dimensions
  - We'll get back to this…





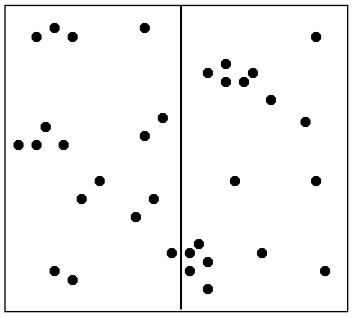
KD-Tree Construction

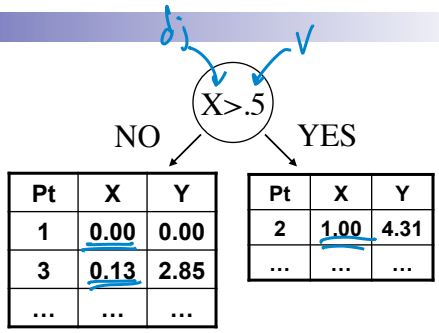


Pt	X	Y
1	0.00	0.00
2	1.00	4.31
3	0.13	2.85
…	…	…

- Start with a list of *d*-dimensional points.

KD-Tree Construction



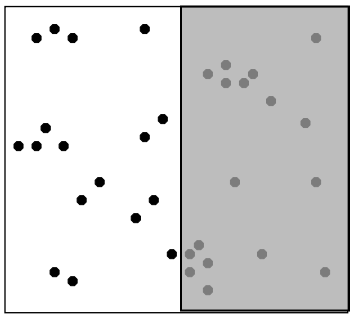


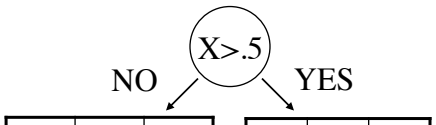
Pt	X	Y
1	0.00	0.00
3	0.13	2.85
…	…	…

Pt	X	Y
2	1.00	4.31
…	…	…

- Split the points into 2 groups by:
  1. Choosing dimension d_j and value V (methods to be discussed…)
  2. Separating the points into $x^i_{d_j}$ > V and $x^i_{d_j}$ <= V.

KD-Tree Construction





Pt	X	Y
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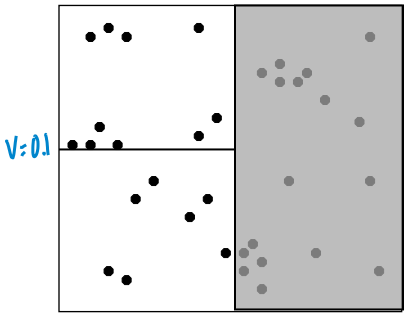
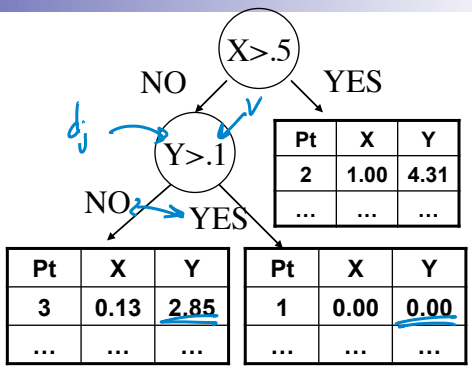
3	0.13	2.85
…	…	…

Pt	X	Y
2	1.00	4.31
…	…	…

- Consider each group separately and possibly split again (along same/different dimension).
  - Stopping criterion to be discussed…
  - how to choose dimension

KD-Tree Construction





Pt	X	Y
2	1.00	4.31
…	…	…

Pt	X	Y
3	0.13	2.85
…	…	…

Pt	X	Y
1	0.00	0.00
…	…	…

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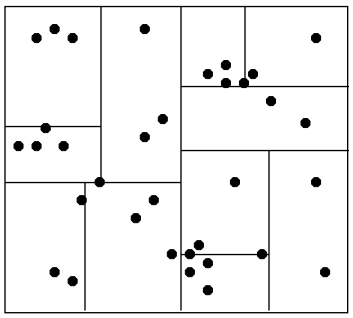
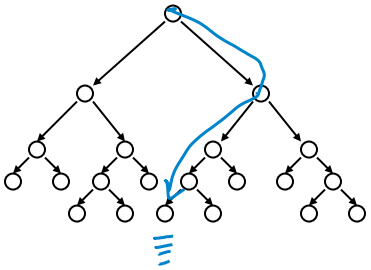
KD-Tree Construction

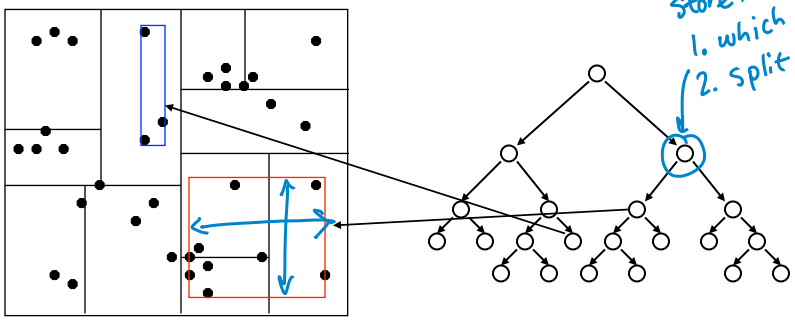




- Continue splitting points in each set
  - creates a binary tree structure
- Each leaf node contains a list of points

satisfying all conditions down the tree to that point

KD-Tree Construction



3.

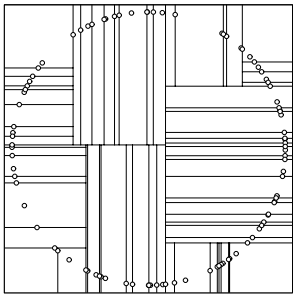
- Keep one additional piece of information at each node:
  - The (tight) bounds of the points at or below this node.

KD-Tree Construction

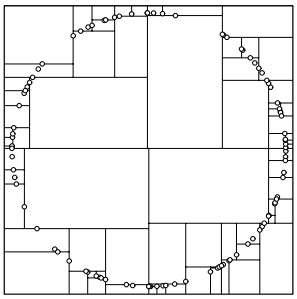
Use heuristics to make splitting decisions:

- Which dimension do we split along?
  widest dim (or alternate...)
- Which value do we split at?
  median of chose split dim (or center)
- When do we stop?
  fewer than m pts left
  or
  box hits minimum width

Many heuristics…

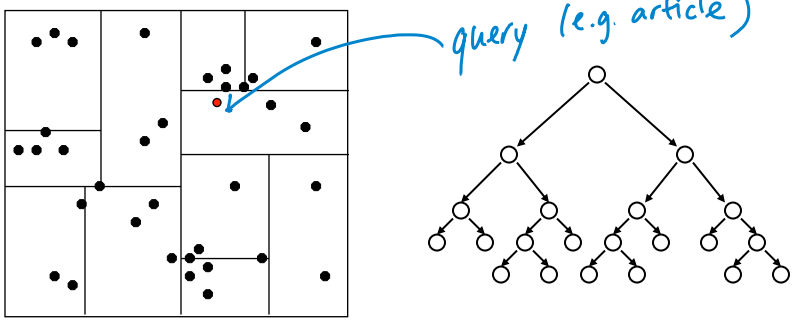


median heuristic



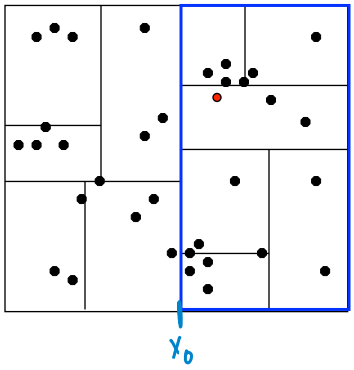
center-of-range heuristic

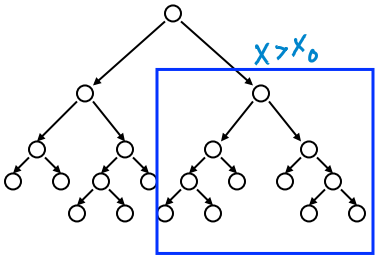
Nearest Neighbor with KD Trees



- Traverse the tree looking for the nearest neighbor of the query point.

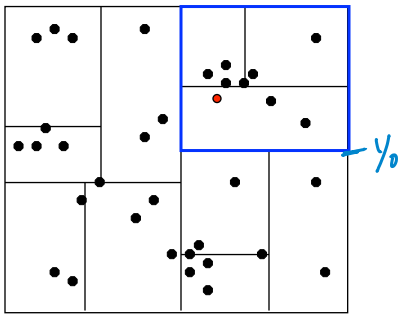
Nearest Neighbor with KD Trees

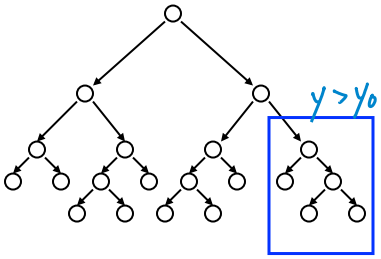




- Examine nearby points first:
  - Explore branch of tree closest to the query point first.

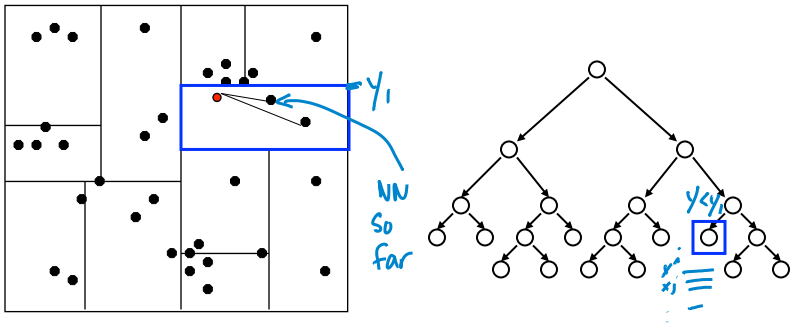
Nearest Neighbor with KD Trees





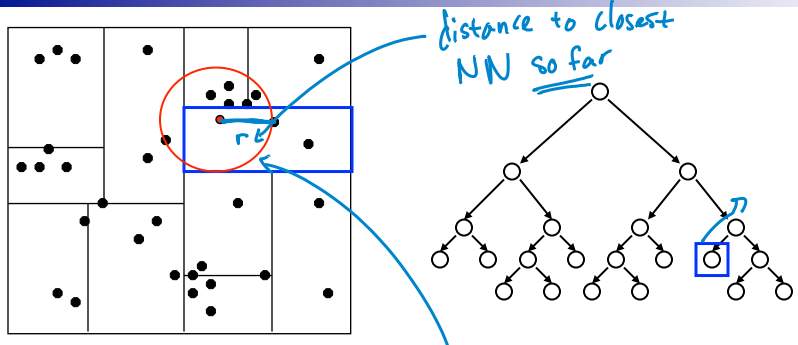
- Examine nearby points first:
  - Explore branch of tree closest to the query point first.

Nearest Neighbor with KD Trees



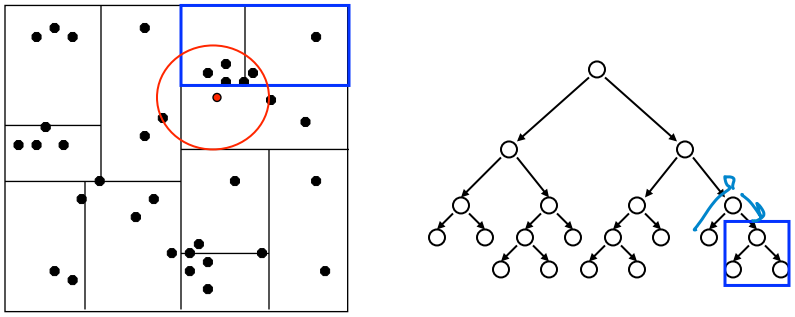
- When we reach a leaf node:
  - Compute the distance to each point in the node.

Nearest Neighbor with KD Trees



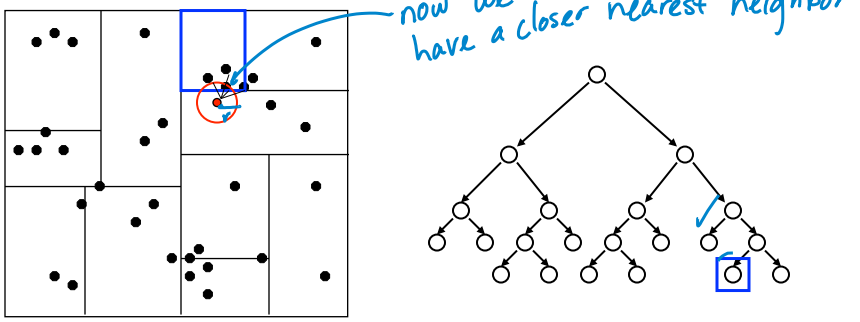
- When we reach a leaf node:
  - Compute the distance to each point in the node.

Nearest Neighbor with KD Trees



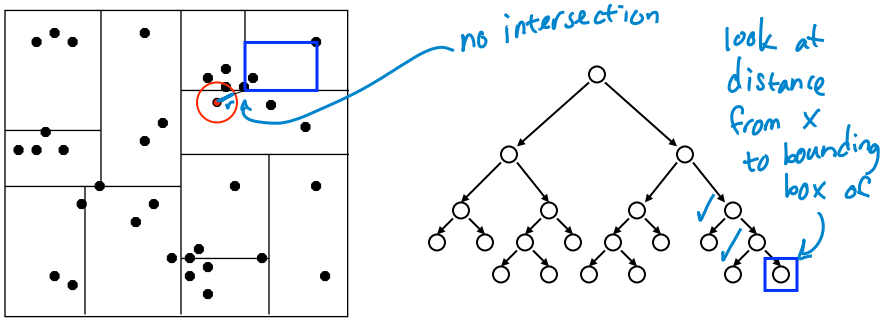
- Then backtrack and try the other branch at each node visited

Nearest Neighbor with KD Trees



- Each time a new closest node is found, update the distance bound

Nearest Neighbor with KD Trees



- Using the distance bound and bounding box of each node:
  - Prune parts of the tree that could NOT include the nearest neighbor

No article in this box could be the NN.