

11/26/13 Recitation

## Naive Bayes and Bayesian Networks

# Naive Bayes Background

- Classification algorithm
- Requires discrete features
- Popular, fast, easy to implement
- Versatile: text classification, weather, computer vision
- Performs well even when main assumption is violated

# Naive Bayes Details

- Classification rule: choose most likely label given features:
  - Pick  $y$  to max  $P(y | x) = P(x | y) P(y) / P(x)$  [Bayes]
  - Equivalent to maximizing  $P(y, x) = P(x | y) P(y)$
- Assume conditional independence of features  $x$ 
  - $P(x_1, x_2, x_3 | y) = P(x_1 | y) P(x_2 | y) P(x_3 | y)$
  - If no conditional independence, given  $d$  binary features and binary  $y$ , what is the order of the number of parameters needed to know  $P(x | y)$  for all possible  $x$  and  $y$ ?
  - Order of the number of parameters with conditional independence?

# Prior and Likelihood

- Final classification rule:  $\max P(y) \text{Prod}[P(x_i | y)]$
- Prior  $P(y) = (\# y) / N$ 
  - Fraction of examples with label  $y$
- Likelihood  $P(x_i | y) = (\# y \text{ and } x_i) / (\# y)$ 
  - Among examples with label  $y$ , the fraction whose  $i$ -th feature equals the value  $x_i$

# Naive Bayes Example

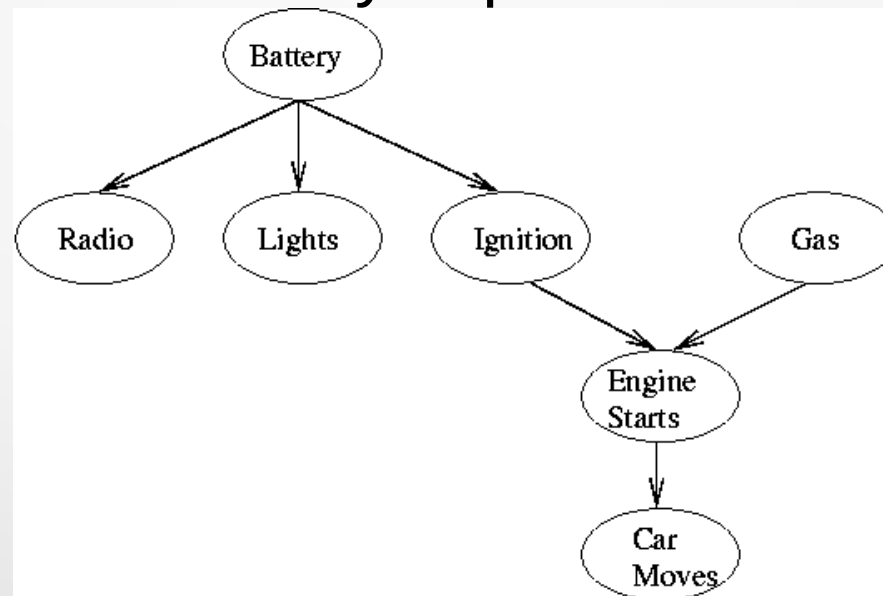
- Label  $y$ : does the patient have lung cancer?
  - 1 = yes, 0 = no
- Features  $x_1, x_2$ :
  - $x_1$  whether the patient smokes
  - $x_2$  whether any family member has had lung cancer
  - Are they conditionally independent realistically?

# NB Example

- 3 training examples
  - 1. has cancer, smokes, family does not have cancer
  - 2. has cancer, does not smoke, family has cancer
  - 3. no cancer, does not smoke, family does not have cancer
- Classify the following:
  - 4. smokes, family has cancer
  - 5. does not smoke, family has cancer
- Note “smoothing” issue

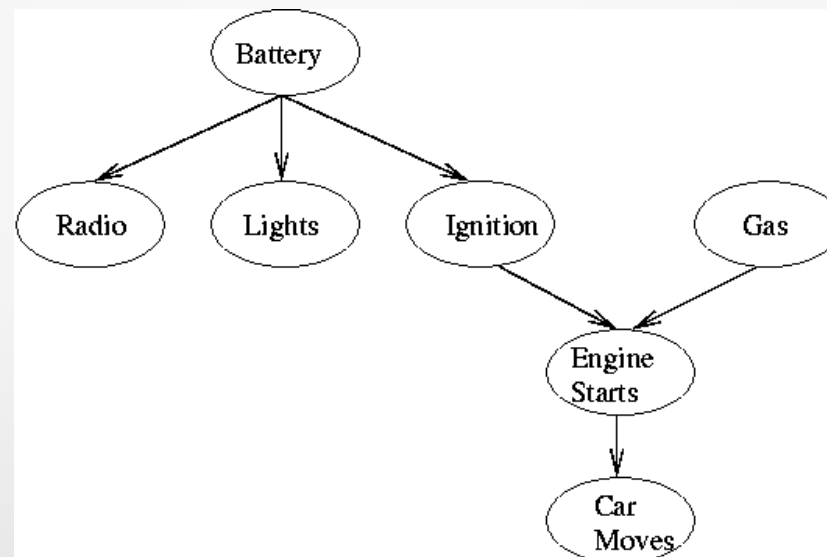
# Bayesian Network Background

- AKA Bayes Net
- Graphical model on a directed acyclic graph
- Generalizes Naive Bayes and logistic regression
- Fast
- For inferring the most likely explanation of an outcome



# Bayes Net Details

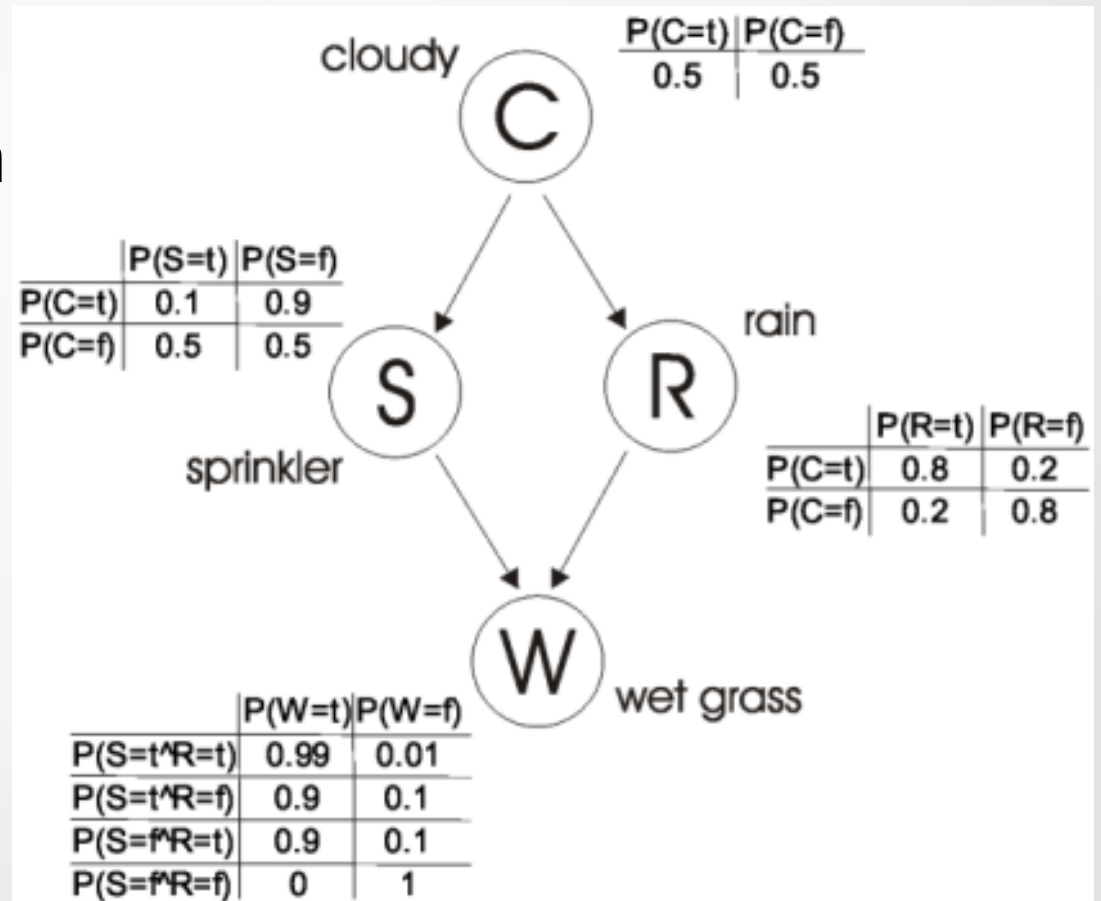
- Nodes = variables
- Edges = dependencies
- Conditional independence assumption: Given all parents of node  $v$ ,  $v$  is independent of non descendants
- Parameters are the conditional probability tables (CPTs) of a variable given all combinations of its parents



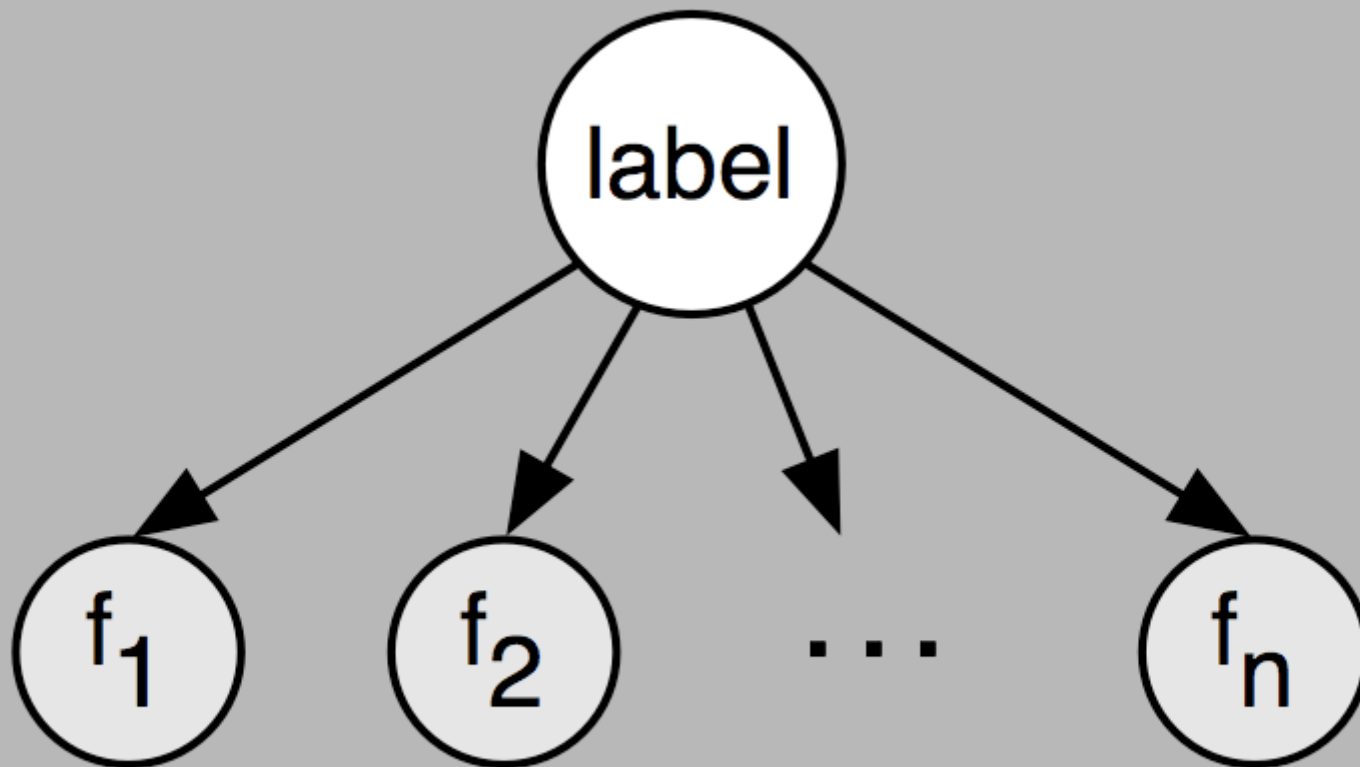


# Bayes Net Example

- Joint:  $P(C,S,R,W) = P(C) P(R|C) P(S|C) P(W|R,S)$
- $P(C=t \mid W=t, R=f, S=t) = ?$
- $P(C=t \mid W=t, R=f) = ?$ 
  - Inferring variable from evidence
  - Could also infer most likely situation



# Naive Bayes as Bayes Net



# Efficiency of Bayes Net

- Assume binary variables
- For a node with  $k$  parents, what is the order of the number of parameters for its CPT?
- If there are  $n$  nodes with at most  $k$  parents, what is the order of the total number of parameters?
- If there was no conditional independence, what would be the order of the total number of parameters?