Boosting

Machine Learning – CSE546
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Fighting the bias-variance tradeoff

- **Simple (a.k.a. weak) learners are good**
  - e.g., naïve Bayes, logistic regression, decision stumps (or shallow decision trees)
  - Low variance, don't usually overfit too badly
- **Simple (a.k.a. weak) learners are bad**
  - High bias, can't solve hard learning problems
- Can we make weak learners always good???
  - **No!!!**
  - **But often yes…**

Voting (Ensemble Methods)

- Instead of learning a single (weak) classifier, learn **many weak classifiers** that are **good at different parts of the input space**
- **Output class:** (Weighted) vote of each classifier
  - Classifiers that are most "sure" will vote with more conviction
  - Classifiers will be most "sure" about a particular part of the space
  - On average, do better than single classifier!
- **But how do you ???**
  - force classifiers to learn about different parts of the input space?
  - weigh the votes of different classifiers?

Boosting [Schapire, 1989]

- Idea: given a weak learner, run it multiple times on (reweighted) training data, then let learned classifiers vote
- On each iteration *t*:
  - weight each training example by how incorrectly it was classified
  - Learn a hypothesis – h_t
  - A strength for this hypothesis – α_t
- Final classifier:
- **Practically useful**
- **Theoretically interesting**

Learning from weighted data

- **Sometimes not all data points are equal**
  - Some data points are more equal than others
- **Consider a weighted dataset**
  - D(j) – weight of *j* th training example ($\mathbf{x}^j, y^j$)
  - Interpretations:
    - *j* th training example counts as D(j) examples
    - If I were to "resample" data, I would get more samples of "heavier" data points
- **Now, in all calculations, whenever used, *j* th training example counts as D(j) "examples"**

AdaBoost

- Initialize weights to uniform dist: $D_1(j) = 1/N$
- For t = 1…T
  - Train weak learner h_t on distribution D_t over the data
  - Choose weight α_t
  - Update weights:

$$D_{t+1}(j) = \frac{D_t(j)\exp(-\alpha_t y^j h_t(x^j))}{Z_t}$$

    - Where Z_t is normalizer:

$$Z_t = \sum_{j=1}^{N} D_t(j)\exp(-\alpha_t y^j h_t(x^j))$$

- Output final classifier:

Picking Weight of Weak Learner

- Weigh h_t higher if it did well on training data (weighted by D_t):

$$\alpha_t = \frac{1}{2} \ln \left(\frac{1 - \epsilon_t}{\epsilon_t} \right)$$

  - Where ε_t is the weighted training error:

$$\epsilon_t = \sum_{j=1}^{N} D_t(j) \mathbb{1}[h_t(x^j) \neq y^j]$$

Why choose α_t for hypothesis h_t this way?

[Schapire, 1989]

Training error of final classifier is bounded by:

$$\frac{1}{N} \sum_{j=1}^{N} \mathbb{1}[H(x^j) \neq y^j] \leq \frac{1}{N} \sum_{j=1}^{N} \exp(-y^j f(x^j))$$

Where $f(x) = \sum_t \alpha_t h_t(x); H(x) = sign(f(x))$

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Where $f(x) = \sum_t \alpha_t h_t(x); H(x) = sign(f(x))$

If we minimize $\prod_t Z_t$, we minimize our training error

We can tighten this bound greedily, by choosing α_t and h_t on each iteration to minimize Z_t.

$$Z_t = \sum_{j=1}^{N} D_t(j) \exp(-\alpha_t y^j h_t(x^j))$$

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For boolean target function, this is accomplished by [Freund & Schapire '97]:

$$\alpha_t = \frac{1}{2} \ln \left(\frac{1 - \epsilon_t}{\epsilon_t} \right)$$

You'll prove this in your homework! ☺

Strong, weak classifiers

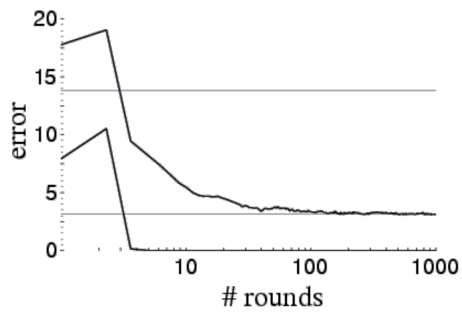
- If each classifier is (at least slightly) better than random
  - $\varepsilon_t < 0.5$
- AdaBoost will achieve zero *training error* (exponentially fast):

$$\frac{1}{N} \sum_{j=1}^{N} \mathbb{1}[H(x^j) \neq y^j] \leq \prod_{t=1}^{T} Z_t \leq \exp\left(-2 \sum_{t=1}^{T} (1/2 - \epsilon_t)^2 \right)$$

- Is it hard to achieve better than random training error?

Boosting results – Digit recognition

[Schapire, 1989]

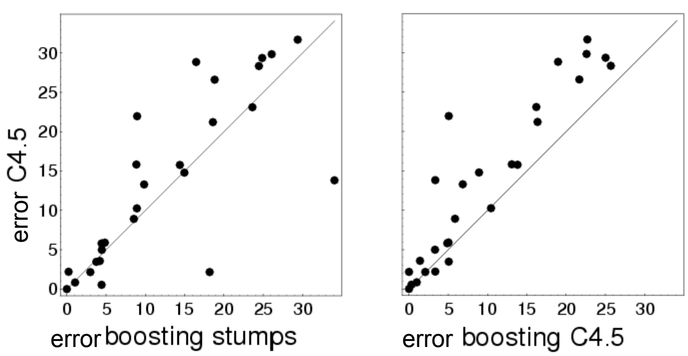


- Boosting often
  - Robust to overfitting
  - Test set error decreases even after training error is zero

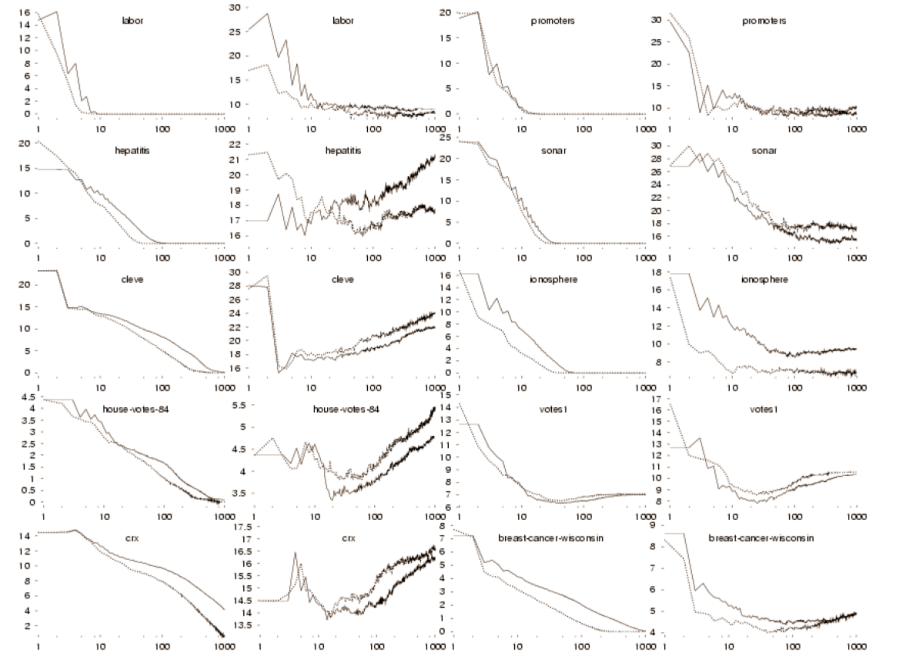
Boosting: Experimental Results

[Freund & Schapire, 1996]

Comparison of C4.5, Boosting C4.5, Boosting decision stumps (depth 1 trees), 27 benchmark datasets



AdaBoost and AdaBoost.MH on Train (left) and Test (right) data from Irvine repository. [Schapire and Singer, ML 1999]



Boosting and Logistic Regression

Logistic regression assumes:

$$P(Y=1|X) = \frac{1}{1+\exp(f(x))}$$

And tries to maximize data likelihood:

$$P(\mathcal{D}|H) = \prod_{j=1}^{N} \frac{1}{1+\exp(-y^j f(x^j))}$$

Equivalent to minimizing log loss

$$\sum_{j=1}^{N} \ln(1+\exp(-y^j f(x^j)))$$

Boosting and Logistic Regression

Logistic regression equivalent to minimizing log loss

$$\sum_{j=1}^{N} \ln(1+\exp(-y^j f(x^j)))$$

Boosting minimizes similar loss function!!

$$\frac{1}{N}\sum_{j=1}^{N} \exp(-y^j f(x^j)) = \prod_{t=1}^{T} Z_t$$

Both smooth approximations of 0/1 loss!

Logistic regression and Boosting

Logistic regression:

- Minimize loss fn

$$\sum_{j=1}^{N} \ln(1+\exp(-y^j f(x^j)))$$

- Define

$$f(x) = w_0 + \sum_i w_i x_i$$

where features x_i are predefined

- Weights w_i are learned in joint optimization

Boosting:

- Minimize loss fn

$$\sum_{j=1}^{N} \exp(-y^j f(x^j))$$

- Define

$$f(x) = \sum_t \alpha_t h_t(x)$$

where $h_t(x)$ defined dynamically to fit data (not a linear classifier)

- Weights α_t learned incrementally

What you need to know about Boosting

- Combine weak classifiers to obtain very strong classifier
  - Weak classifier – slightly better than random on training data
  - Resulting very strong classifier – can eventually provide zero training error
- AdaBoost algorithm
- Boosting v. Logistic Regression
  - Similar loss functions
  - Single optimization (LR) v. Incrementally improving classification (B)
- Most popular application of Boosting:
  - Boosted decision stumps!
  - Very simple to implement, very effective classifier