# CSE 544 Principles of Database Management Systems 

Fall 2016
Lecture 9 - Structural query optimization

## Conjunctive Queries

- Definition:
Q(X) :- R1(X1), R2(X2), ..., Rm(Xm)
- Same as a single datalog rule
- Terminology:
- Atoms
- Head variables
- Existential variables
- $C Q=$ denotes the set of conjunctive queries


## Examples

- Example of CQ

$$
\begin{aligned}
& q(x, y)=\exists z \cdot(R(x, z) \wedge \exists u \cdot(R(z, u) \wedge R(u, y))) \\
& q(x)=\exists z \cdot \exists u \cdot(R(x, z) \wedge R(z, u) \wedge R(u, y))
\end{aligned}
$$

- Examples of non-CQ:

$$
\begin{aligned}
& q(x, y)=S(x, y) \wedge \forall z \cdot(R(x, z) \rightarrow R(y, z)) \\
& q(x)=T(x) \vee \exists z \cdot S(x, z)
\end{aligned}
$$

## Types of CQ

- Full CQ: head variables are all variables

$$
\mathrm{Q}(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{u}):-\mathrm{R}(\mathrm{x}, \mathrm{y}), \mathrm{S}(\mathrm{y}, \mathrm{z}), \mathrm{T}(\mathrm{z}, \mathrm{u})
$$

- Boolean CQ: no head variables

$$
Q():-R(x, y), S(y, z), T(z, u)
$$

- With or without self-joins:

$$
\begin{aligned}
& Q(x, u):-R(x, y), S(y, z), R(z, u) \\
& Q(x, u):-R(x, y), S(y, z), T(z, u)
\end{aligned}
$$

## Extensions

- With inequalities $\mathrm{CQ}^{<}$:

$$
Q(x):-R(x, y), S(y, z), T(z, u), y<u
$$

- With disequalities $\mathrm{CQ}^{\neq}$:

$$
Q(x):-R(x, y), S(y, z), T(z, u), y \neq u
$$

- With aggregates:

$$
\begin{aligned}
& Q(x, \operatorname{count}(*)):-R(x, y), S(y, z), T(z, u) \\
& Q(x, \operatorname{sum}(u)):-R(x, y), S(y, z), T(z, u)
\end{aligned}
$$

## Complexity of Query Evaluation

- The query evaluation problem is this: given a query $Q$ and a database $D$, compute $Q(D)$
- Three complexity measures:
- Data complexity. Fix $Q$. The complexity is $f(|D|)$ Variation: f(|Input|, |Output|)
- Query (or expression) complexity. Fix D. The complexity is $f(|Q|)$
- Combined complexity. The complexity if $f(|\mathrm{D}|,|\mathrm{Q}|)$
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- Discuss more about complexity in class...


## Question in Class

- $Q(x, w):-R(x, y), S(y, z), T(z, u), K(u, v), L(v, w)$
- Assume $|\mathrm{R}|=|\mathrm{S}|=|\mathrm{T}|=|\mathrm{K}|=|\mathrm{L}|=\mathrm{N}$
- What is the complexity of Q ?


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- Assume $|\mathrm{R}|=|\mathrm{S}|=|\mathrm{T}|=|\mathrm{K}|=|\mathrm{L}|=\mathrm{N}$
- What is the complexity of Q ?
- What is the complexity of this plan?



## Question in Class

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- Assume $|\mathrm{R}|=|\mathrm{S}|=|\mathrm{T}|=|\mathrm{K}|=|\mathrm{L}|=\mathrm{N}$
- What is the complexity of Q ?
- What is the complexity of this plan?
- Can you find a more efficient plan?


## Question in Class

- Push projections down: What about this complexity?
- Can we still improve?



## Law of Semijoins

- Input: R(A1,...An), S(B1,...,Bm)
- Output: T(A1,...,An)

Definition: the semi-join operation is

$$
R \ltimes S=\Pi_{A 1, \ldots, A_{n}}(R \bowtie S)
$$

- Data complexity: $\mathrm{O}(|\mathrm{R}|+|\mathrm{S}|)$ ignoring log-factors
- The law of semijoins is:

$$
R \bowtie S=(R \ltimes S) \bowtie S
$$

## Laws with Semijoins

- Very important in parallel databases
- Often combined with Bloom Filters (my plan is to discuss them in the next lecture)
- Read pp. 747 in the textbook
- Also used in query optimization, sometimes called "magic sets" (see Chaudhuri's paper)
- Historical note: magic sets were invented after semi-join reductions, and the connection became clear only later


## Semijoin Reducer

- Given a query:

$$
Q=R_{1} \bowtie R_{2} \bowtie \ldots \bowtie R_{n}
$$

- A semijoin reducer for $Q$ is

$$
\begin{aligned}
& R_{i 1}=R_{i 1} \ltimes R_{j 1} \\
& R_{i 2}=R_{i 2} \ltimes R_{j 2} \\
& \ldots \ldots=R_{i p} \ltimes R_{i p} \\
& R_{i p}
\end{aligned}
$$

such that the query is equivalent to.

$$
Q=R_{k 1} \bowtie R_{k 2} \bowtie \ldots \bowtie R_{k n}
$$

- A full reducer is such that no dangling tuples remain


## Example

- Example:

$$
Q=R(A, B) \bowtie S(B, C)
$$

- A semijoin reducer is:

$$
R_{1}(A, B)=R(A, B) \ltimes S(B, C)
$$

- The rewritten query is:

$$
Q=R_{1}(A, B) \bowtie S(B, C)
$$

## Semijoin Reducer

- More complex example:

$$
Q=R(A, B) \bowtie S(B, C) \bowtie T(C, D, E)
$$

- What is a full reducer?


## Semijoin Reducer

- More complex example:

$$
Q=R(A, B) \bowtie S(B, C) \bowtie T(C, D, E)
$$

- A full reducer is:

$$
\begin{aligned}
& S^{\prime}(B, C):=S(B, C) \ltimes R(A, B) \\
& T^{\prime}(C, D, E):=T(C, D, E) \ltimes S^{\prime}(B, C) \\
& S^{\prime \prime}(B, C):=S^{\prime}(B, C) \ltimes T^{\prime}(C, D, E) \\
& R^{\prime}(A, B):=R(A, B) \ltimes S^{\prime \prime}(B, C)
\end{aligned}
$$

$$
Q=R^{\prime}(A, B) \bowtie S^{\prime \prime}(B, C) \bowtie T^{\prime}(C, D, E)
$$

## Practice at Home...

- Find semi-join reducer for $R(x, y), S(y, z), T(z, u), K(u, v), L(v, w)$


## Not All Queries Have Full Reducers

- Example:

$$
Q=R(A, B) \bowtie S(B, C) \bowtie T(A, C)
$$

- Can write many different semi-join reducers
- But no full reducer of length $\mathrm{O}(1)$ exists


## Acyclic Queries

- Fix a Conjunctive Query without self-joins
- $Q$ is acyclic if its atoms can be organized in a tree such that for every variable the set of nodes that contain that variable form a connected component

$R(x, y), S(y, z), T(z, x)$ is cyclic


## Yannakakis Algorithm

- Given: acyclic query Q
- Compute Q on any database in time $O(|\ln n u t|+|O u t p u t|)$
- Step 1: semi-join reduction
- Pick any root node $x$ in the tree decomposition of $Q$
- Do a semi-join reduction sweep from the leaves to $x$
- Do a semi-join reduction sweep from $x$ to the leaves
- Step 2: compute the joins bottom up, with early projections


## Examples in Class

- Boolean query: Q() :- ...
- Non-boolean: $\mathrm{Q}(\mathrm{x}, \mathrm{m})$ :- ...
- With aggregate: $Q(x, s u m(m))$ :- ...
- And also: $\mathrm{Q}(\mathrm{x}, \operatorname{count(*))~:-~...~}$


In all cases: runtime $=\mathrm{O}(|\mathrm{R}|+|\mathrm{S}|+\ldots+\mid$ ㄴ| $+\mid$ Output $\mid)$

## Testing if $Q$ is Acyclic

- An ear of $Q$ is an atom $R(X)$ with the following property:
- Let $X^{\prime} \subseteq X$ be the set of join variables (meaning: they occur in at least one other atom)
- There exists some other atom $S(Y)$ such that $X^{\prime} \subseteq Y$
- The GYO algorithm (Graham,Yu,Özsoyoğlu) for testing if $Q$ is acyclic:
- While $Q$ has an ear $R(X)$, remove the atom $R(X)$ from the query
- If all atoms were removed, then $Q$ is acyclic
- If atoms remain but there is no ear, then $Q$ is cyclic
- Show example in class


## Computing Cyclic Queries



Next lecture...

