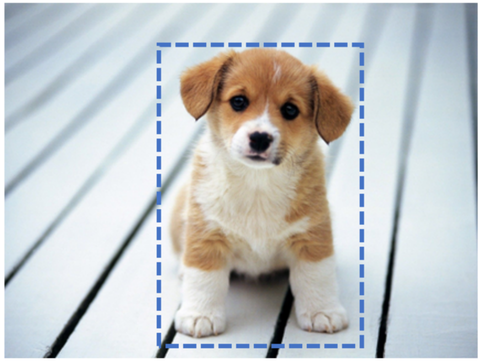
Deep Reinforcement Learning

Supervised Learning

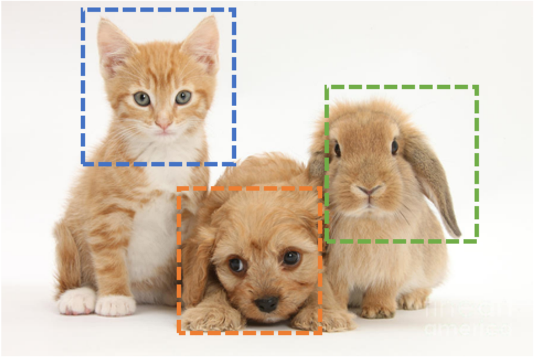
- Data: (x, y)
- Goal: Learn a function f(x)=y
- Examples: Classification, Regression, …

Single-label classification



Dog

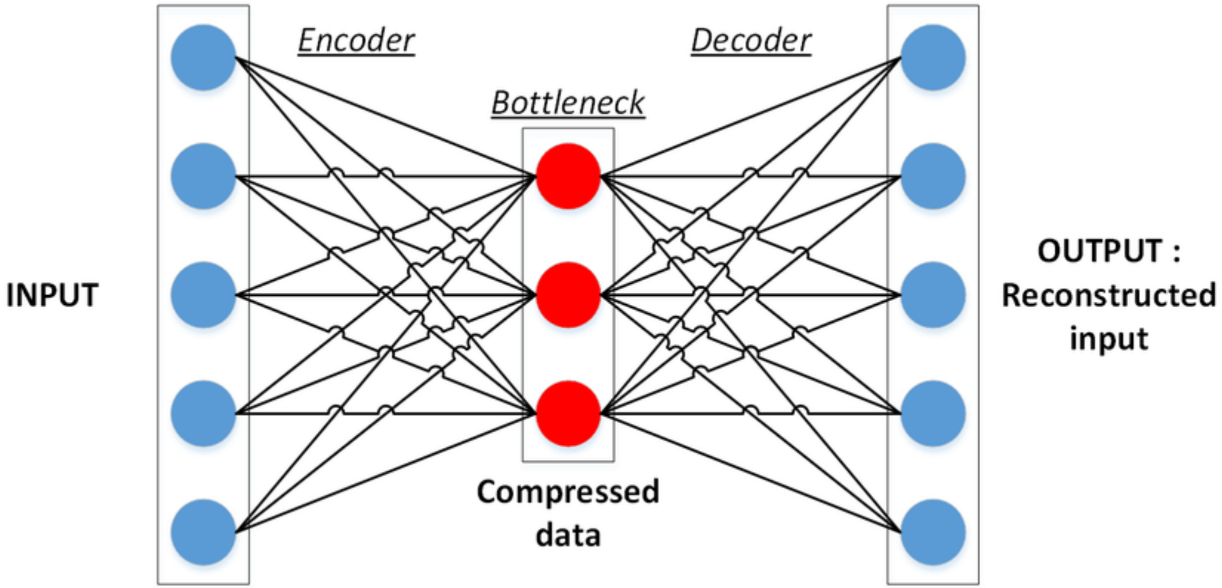
Multi-label classification



Cat, Dog, Rabbit

Self-supervised Learning

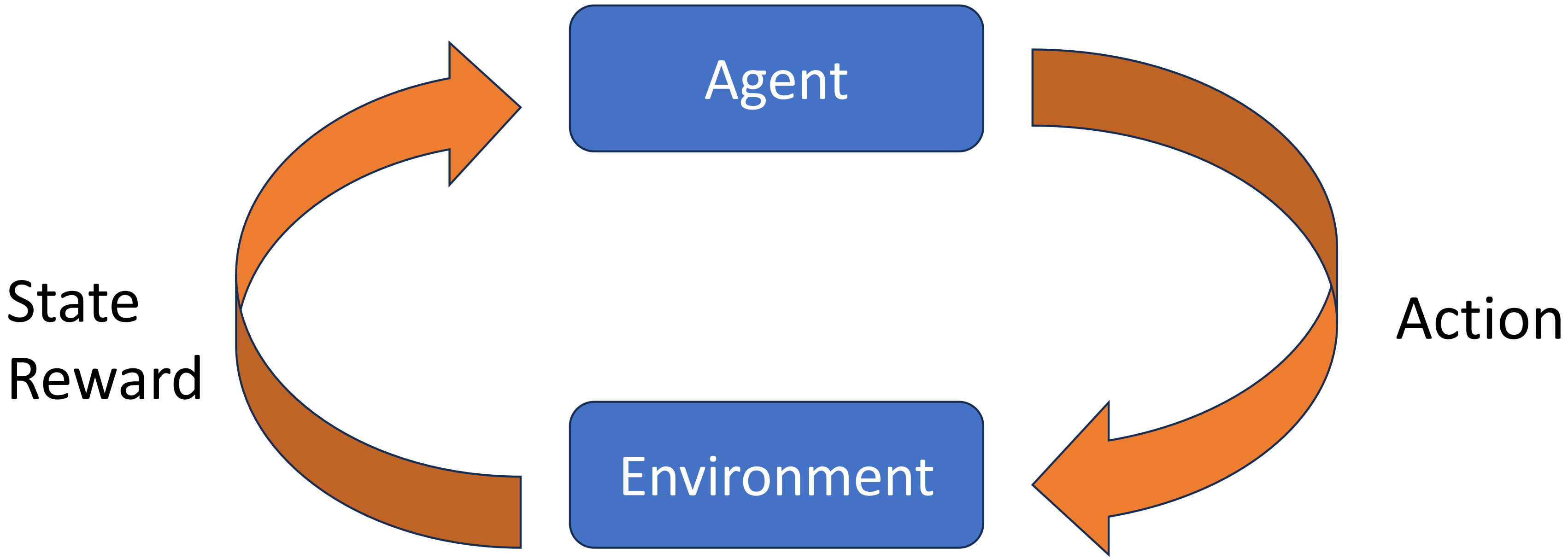
- Data: x
- Goal: Learn underlying structure of the data
- Examples: Representation Learning, Contrastive Learning, Autoregressive Pretraining



Reinforcement Learning

- Goal: Learn a policy to maximize reward
- Examples: Chess, Go, Poker, Self-driving

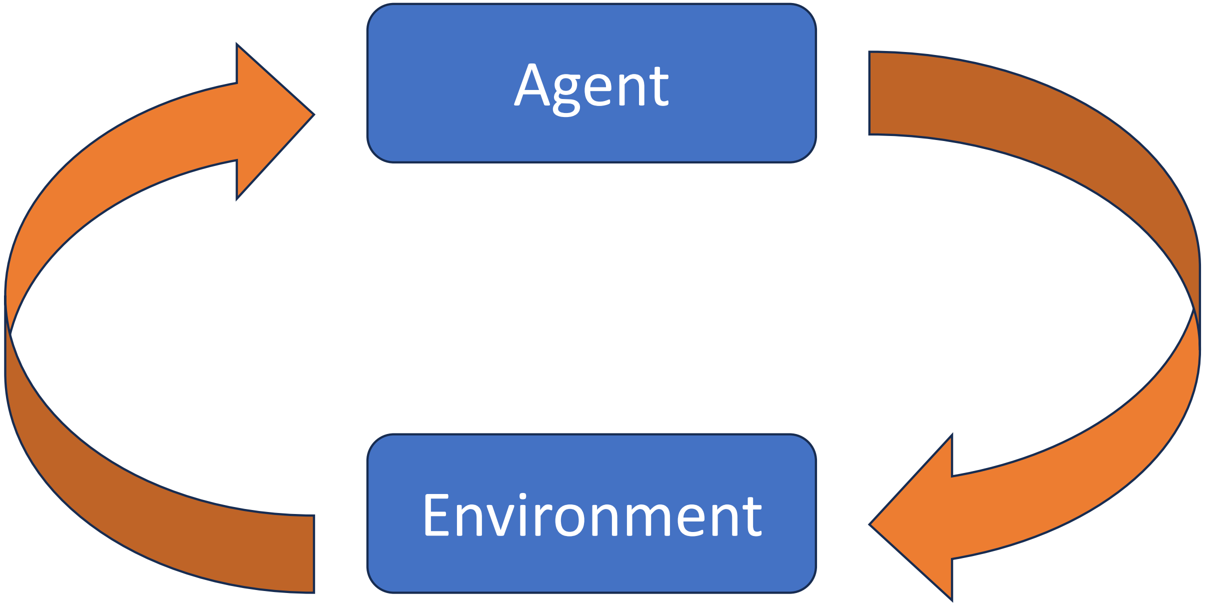
Markov Decision Process



- Goal: Collect as much reward as possible.

Markov Decision Process

State:
$s_{t+1} \sim P(\cdot \mid s_t, a_t)$
Reward:
$r_{t+1} = r(s_t, a_t)$



Action $a_t = \pi(s_t)$

Maximize total discounted reward $\sum \gamma^t r_t$.

$\gamma = 0.99, 0.9$

Markov Decision Process

- Policy: $\pi(s) = a$.
- Discount factor: $\gamma \in (0,1)$.
- Value function: $V^{\pi}(s_0) = \mathbb{E}_{\pi}[\sum_t \gamma^t r_t]$, where $s_0, a_0, r_0, s_1, a_1, r_1, \ldots$ is a trajectory sampled by using policy π.
- Q function: $Q^{\pi}(s_0, a_0) = \mathbb{E}_{\pi}[\sum_t \gamma^t r(s_t, a_t)]$. /
- Optimal policy: $\pi^* = \mathrm{argmax}_{\pi} \mathrm{V}^{\pi}(\mathrm{s})$.
- There exists an optimal policy that achieves the argmax for all s **simultaneously**!

Optimal Q Function

- Optimal Q function: $Q^{\pi^*}(s_0, a_0) = \mathbb{E}_{\pi^*}[\sum_t \gamma^t r(s_t, a_t)]$.
- Property: $\pi^*(s) = \text{argmax}_a Q^{\pi^*}(s, a)$.
- If we know Q^*, we know π^*.

Reinforcement Learning

- If we know $r(s, a)$ and $P(s' \mid s, a)$, we can use dynamic programming to solve the optimal policy.
- How to learn the optimal policy **without** the knowledge of $r(s, a)$ and $P(s' \mid s, a)$?

- Collect **samples**!

$$r(s, a) = \frac{\sum r(\quad)}{\#\text{ of } (s, a) \text{ in data}}$$

$$P(s' \mid s, a) = \frac{\#\,(s, a, s')}{\#\,(s, a)}$$

Challenge: Large State Space

- 3^{361} possible board configurations in Go.
- Impossible to enumerate.

- Theorem: $\Omega(SA)$ samples are necessary for learning MDP without structures, where S is # of states and A is # of actions.

Function Approximation

$\{(s, a, s', r)\}_{i=1}^{N}$

- Challenge in RL: large state and action space.
- Many states and actions are similar and have similar Q^{π^*}.
- Use a function class $\mathcal{F} = \{f_\theta\}$ to approximate Q function.

- Suppose we have a dataset $\mathcal{D} = \{Q^{\pi^*}(s, a)\}$, then we can fit a f_θ to approximate Q^{π^*}:

$$\theta^* = \operatorname{argmin}_\theta \sum_{(s,a)\in\mathcal{D}} \left(f_\theta(s, a) - Q^{\pi^*}(s, a)\right)^2.$$

Offline Reinforcement Learning

- Dataset: trajectories $s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_T$ sampled from some behavior policy π_b.

- Challenge: unknown $Q^{\pi^*}(s, a)$.

Q-learning

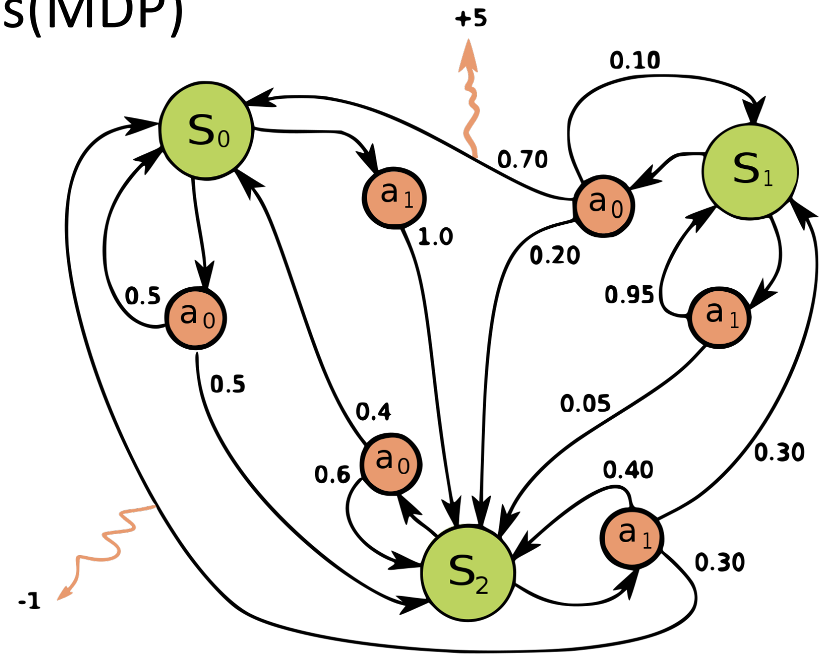
- Reminder: Markov-Decision Process(MDP)

State:

$$s_{h+1} = P(\cdot|\, s_h, a_h)$$

Reward:

$$r_{h+1} = r(s_h, a_h)$$



Q-learning

- Value-based method:
  - Evaluate all the states, then find the action leading to the best state.
- Reminder: Value function and Q function:

  - $V_\pi(s) = E_\pi[\sum_h \gamma^h r_h \mid s]$

- We need to know which action leads to the given reward:

  - $Q_\pi(s, a) = E_\pi[\sum_h \gamma^h r_h \mid s, a]$

Q-learning

Q-function:

$$Q_{\pi}(s,a) = E_{\pi}[\sum_{h} \gamma^h r_h \mid s,a]$$

- Target: derive the Q function for the optimal policy π^*, Q^*
- How to solve this system?

- Of course, we can use Monte Carlo's Method to estimate Q function.
- But it takes $\Omega(SA^h)$ sample trajectories.

- Can we do better?

Q-learning: Tabular learning

Q-function:

$$\mathrm{Q}_{\pi}(s,a) = E_{\pi}[\sum_{h} \gamma^{h} r_{h} \mid s,a]$$

- Notice that Q function should satisfy the successor relationship;
- Bellman's Equation:
  - $\mathrm{Q}^{*}(s,a) = r(s,a) + \gamma E_{\pi^{*}}[V^{*}(s') \mid s,a]$
  - $V^{*}(s) = \max_{a} Q^{*}(s,a)$

- Then we can solve it with polynomial samples!

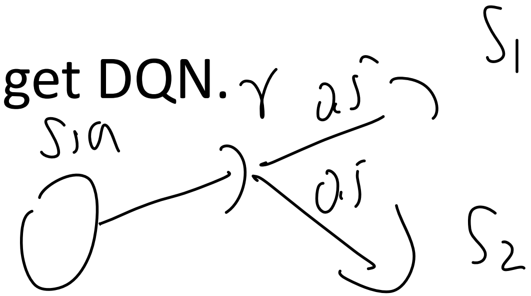
Q-learning: Tabular learning

- First, initialize $Q(\cdot) = 0$;
- Then we do iterative DP:
  - Until convergency, do:
    - For $(s, a) \in S \times A$:
      - Update Q: $Q(s, a) \leftarrow \frac{1}{N_{s,a}} \sum_{s_i = s, a_i = a} \left(r_i + \gamma V(s_{i+1}) \right)$
    - For $s \in S$:
      - Update V: $V(s) \leftarrow \max_a Q(s, a)$

Here $N_{s,a}$ is the counter of (s,a) in dataset.

Deep Q Network (DQN)

- When we combine Deep Learning with Q-learning, we get DQN.



- Reminder: function approximation
  - Structure/function class: MLP, CNN, Transformer, etc.
  - Solve the Bellman's Equation with gradient descent!
    - $Q^*(s,a) = r(s,a) + \gamma E_{\pi^*}[V^*(s') \mid s,a]$
    - $V^*(s) = \max_a Q^*(s,a)$
  - Loss function:

$$L(\theta) = \mathbf{E}_\theta\left[(Q_\theta(s,a) - r(s,a) - \gamma \mathbf{E}[V_\theta(s') \mid s,a])^2\right]$$

(s, a, s', r)

$\downarrow \; Q_{\theta_t}$

$\rightarrow \; (s, a, s', r, V_{\theta_t}(s'))$

Deep Q Network (DQN)

- Loss function:

$$L(\theta) = \mathbf{E}_\theta\Big[(Q_\theta(s,a) - r(s,a) - \gamma \mathbf{E}[V_\theta(s') \mid s,a])^2\Big]$$

- Estimated loss:

$$\mathscr{L}(\theta) = \frac{1}{N}\sum_{i=1}^{N}[Q_\theta(s_i,a_i) - r_i - \gamma \max_{a'} Q_\theta(s_i',a')]^2$$

- Other tricks:
  1. Double network trick for stronger stability;
  2. Replay buffer for higher sample efficiency.

DQN: double network structure

$$\mathcal{L}(\theta) = \frac{1}{N} \sum_{i=1}^{N} [Q_\theta(s_i, a_i) - r_i - \gamma \max_{a'} Q_{\theta'}(s_i', a')]^2$$

- Evaluate network: trained network θ
  - Updated in each iteration
  - The first Q is the evaluate network
- Target network: temporal copy of evaluate network θ'
  - Updated at regular intervals
  - The second Q is fixed to be target network

- Avoid overfitting problem;
- Don’t need to solve a max problem in each iteration;
- Stabilize the training process.

DQN: experience replay

$$\mathcal{L}(\theta) = \frac{1}{N}\sum_{i=1}^{N}[Q_\theta(s_i, a_i) - r_i - \gamma \max_{a'} Q_{\theta'}(s_i', a')]^2$$

- Problem: batch size is very small compared with the dataset
  - Each batch may only contain the transitions from a single trajectory
  - Not mutually independent!

- Notice that we only need transitions $\{s_i, a_i, r_i, s_i{}'\}$,instead of complete trajectories.
- Solution: In each iteration, we randomly sample data from the replay buffer to form the training batch.
- The replay buffer can be the offline dataset, or the data collected with latest policy, which gives better sample efficiency.

Policy-Gradient

$\pi(S) \to a$

- Sometimes we don't want to estimate the Value function!
  - Value function approximation can be extremely tricky;
    - Empirical experiments tell us simpler algorithm leads to better performance;
  - We need to solve an argmax/max problem for each update, which can be very expensive.

$$\pi(s) \leftarrow \arg\max_{a} \{\mathbf{E}_{\pi}[\sum \gamma^{h} r_{h} \mid s, a]\}$$

- Policy-Gradient(PG) directly optimize the policy!
- Directly approximate $\pi^{*}(\cdot)$ with DNN.
  - Now we use π_{θ} to denote the policy learnt.

Policy-Gradient

- Denote the probability of getting a certain trajectory τ as $P(\tau,\theta)$, and the corresponding reward as $R(\tau)$.

$$P(\tau,\theta) = \prod_h \pi_\theta(\, a_h \mid s_h \,)$$
$$R(\tau) = \sum_h \gamma^h r_h$$

- Target: maximize $J(\theta) = E_{\pi_\theta}[\sum_h \gamma^h r_h] = \sum_h P(\tau,\theta)R(\tau)$
- Gradient ascent: $\theta \leftarrow \theta + \eta\nabla_\theta J(\theta)$
- Great so far!
- The problem lies in the estimation of $\nabla_\theta J(\theta)$.

Policy-Gradient

- Target: maximize $J(\theta) = E_{\pi_\theta}[\sum_h \gamma^h r_h] = \sum_h P(\tau, \theta) R(\tau)$
- Gradient ascent: $\theta \leftarrow \theta + \eta \nabla_\theta J(\theta)$
- Directly calculation of the gradient of empirical reward gives:

$$J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} R(\tau_i),$$

$$\nabla_\theta J(\theta) \approx \nabla_\theta [\frac{1}{N} \sum_{i=1}^{N} R(\tau_i)]?$$

- Remember that $R(\tau)$ doesn't depend on θ directly:
  - $P(\tau, \theta) = \prod_h \pi_\theta(a_h \mid s_h)$
  - $R(\tau) = \sum_h \gamma^h r_h$

Policy-Gradient

- Target: maximize $J(\theta) = E_{\pi_\theta}[\sum_h \gamma^h r_h] = \sum_\tau P(\tau, \theta) R(\tau)$
- Directly calculation of the gradient of empirical reward gives:

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$$\nabla_\theta J(\theta) \approx \nabla_\theta [\frac{1}{N} \sum_{i=1}^{N} R(\tau_i)]$$

- Problem: We are not calculating the exact reward with probability, but with sampling!
  - Therefore, we cannot backpropagate the gradient to DNN;
  - (Sad news, can’t leave differential to loss.backword() this time)

Policy-Gradient — Theorem

- Target: maximize $J(\theta) = E_{\pi_\theta}[\sum_h \gamma^h r_h] = \sum_h P(\tau,\theta)R(\tau)$

$$\begin{aligned}
\nabla_\theta J(\theta) &= \sum_\tau \nabla_\theta P(\tau,\theta)R(\tau) \\
&= \sum_\tau \frac{P(\tau,\theta)}{P(\tau,\theta)} \nabla_\theta P(\tau,\theta)R(\tau) \\
&= \sum_\tau P(\tau,\theta) \frac{\nabla_\theta P(\tau,\theta)}{P(\tau,\theta)} R(\tau) \\
&= \sum_\tau P(\tau,\theta) \nabla_\theta \log P(\tau,\theta) R(\tau) \\
&= \mathbf{E}_{\pi_\theta}[R(\tau)\nabla_\theta \log P(\tau,\theta)]
\end{aligned}$$

- Good! The gradient can be also understood as an expectation!
- Therefore, the empirical update function is:

$$\theta \leftarrow \theta + \frac{\eta}{N} \sum_{i=1}^{N} R(\tau_i) \nabla_\theta \log P(\tau_i, \theta)$$

Decision Transformers

Decision making

Architecture in language modeling

- Language modeling: autoregressive conditional sequence modeling
  - Predict next **token** (≈word) with some probability

$$P(\text{“you”}|[\text{“How”}, \text{“ ”}, \text{“are”}, \text{“ ”}])$$

  - **Autoregressive:** sample, and predict next

$$P(\text{“?”}|[\text{“How”}, \text{“ ”}, \text{“are”}, \text{“ ”}, \text{“you”}])$$

- Just like **policy** in RL!

$$\pi(a_t|s_1, a_1, r_1, \dots, s_t)$$

Decision Transformers for Offline RL [🔗]

- Offline dataset:
  - Consider **deterministic reward, finite horizon H, and discount $\gamma = 1$**

$$D = \left\{\tau^i = (s_0^i, a_0^i, r_0^i; s_1^i, a_1^i, r_1^i; \cdots; s_H^i, a_H^i, r_H^i)\right\}_{i=1}^N$$

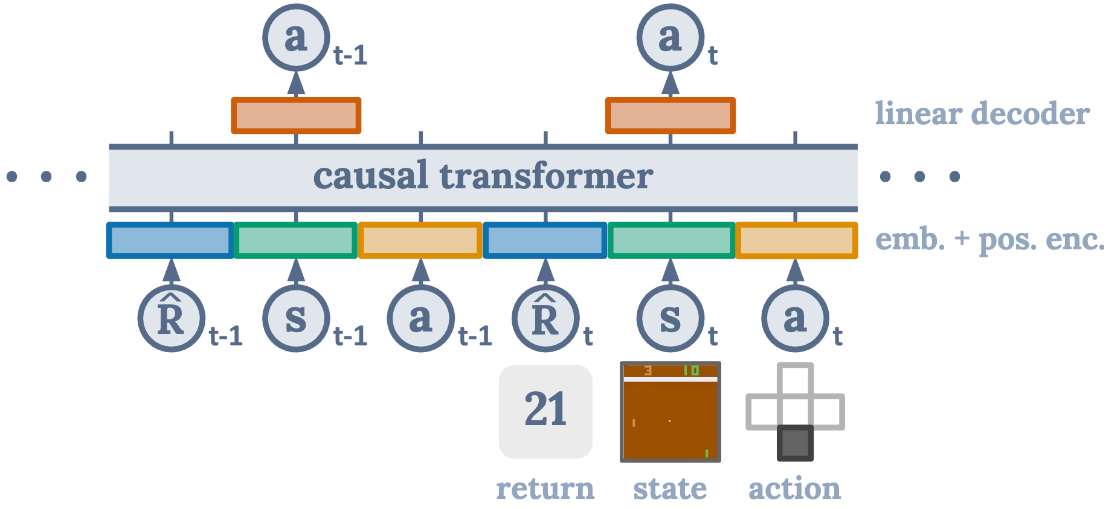
- Decision Transformers:
  - **Return-to-go:** $\hat{R}_t = \sum_{h=t}^H r_h$

$$D = \left\{\tau^i = (\hat{R}_0^i, s_0^i, a_0^i; \hat{R}_1^i, s_1^i, a_1^i; \cdots; \hat{R}_H^i, s_H^i, a_H^i)\right\}_{i=1}^N$$

Decision Transformers for Offline RL

- Decision Transformers:
  - **Return-to-go (RTG):** $\hat{R}_t = \sum_{h=t}^{H} r_h$

$$D = \left\{\tau^i = \left(\hat{R}_0^i, s_0^i, a_0^i; \hat{R}_1^i, s_1^i, a_1^i; \cdots; \hat{R}_H^i, s_H^i, a_H^i\right)\right\}_{i=1}^{N}$$



Decision Transformers for Offline RL

- Decision Transformers:
  - **Return-to-go (RTG):** $\hat{R}_t = \sum_{h=t}^{H} r_h$

$$D = \left\{\tau^i = \left(\hat{R}_0^i, s_0^i, a_0^i; \hat{R}_1^i, s_1^i, a_1^i; \cdots; \hat{R}_H^i, s_H^i, a_H^i\right)\right\}_{i=1}^{N}$$

```
# main model
def DecisionTransformer(R, s, a, t):
    # compute embeddings for tokens
    pos_embedding = embed_t(t)  # per-timestep (note: not per-token)
    s_embedding = embed_s(s) + pos_embedding
    a_embedding = embed_a(a) + pos_embedding
    R_embedding = embed_R(R) + pos_embedding

    # interleave tokens as (R_1, s_1, a_1, ..., R_K, s_K)
    input_embeds = stack(R_embedding, s_embedding, a_embedding)

    # use transformer to get hidden states
    hidden_states = transformer(input_embeds=input_embeds)

    # select hidden states for action prediction tokens
    a_hidden = unstack(hidden_states).actions

    # predict action
    return pred_a(a_hidden)
```

```
self.embed_timestep = nn.Embedding(max_ep_len, hidden_size)
self.embed_return = torch.nn.Linear(1, hidden_size)
self.embed_state = torch.nn.Linear(self.state_dim, hidden_size)
self.embed_action = torch.nn.Linear(self.act_dim, hidden_size)
```

Training

- Minibatch of sequence with length K
  - Context length K: use previous K steps to predict next action
  - Slice τ^i into $\tau^i_{[\max\{j-K+1,1\}:j]}$ for $j = 1,2,\ldots,H$

$$\tau^i_{[l:r]} = \left(\hat{R}^i_l, s^i_l, a^i_l; \ldots; \hat{R}^i_r, s^i_r, a^i_r\right)$$
$$\check{\tau}^i_{[l:r]} = \left(\hat{R}^i_l, s^i_l, a^i_l; \ldots; \hat{R}^i_r, s^i_r\right)$$

Training

- Loss function
  - **Cross-entropy loss** for discrete action space

$$\mathcal{L}_{\text{decision}} = \sum_{i=1}^{N} \sum_{j=1}^{H} -\log \pi\left(a_j^i \,\middle|\, \check{\tau}^i_{[\max\{j-K+1,1\}:j]}\right)$$

  - ***L*2 loss** for continuous action space

$$\mathcal{L}_{\text{decision}} = \sum_{i=1}^{N} \sum_{j=1}^{H} \mathbb{E}_{a\sim\pi(\cdot|\check{\tau}^i_{[\max\{j-K+1,1\}:j]})}\left(a_j^i - a\right)^2$$

```
# training loop
for (R, s, a, t) in dataloader:  # dims: (batch_size, K, dim)
    a_preds = DecisionTransformer(R, s, a, t)
    loss = mean((a_preds - a)**2)  # L2 loss for continuous actions
    optimizer.zero_grad(); loss.backward(); optimizer.step()
```

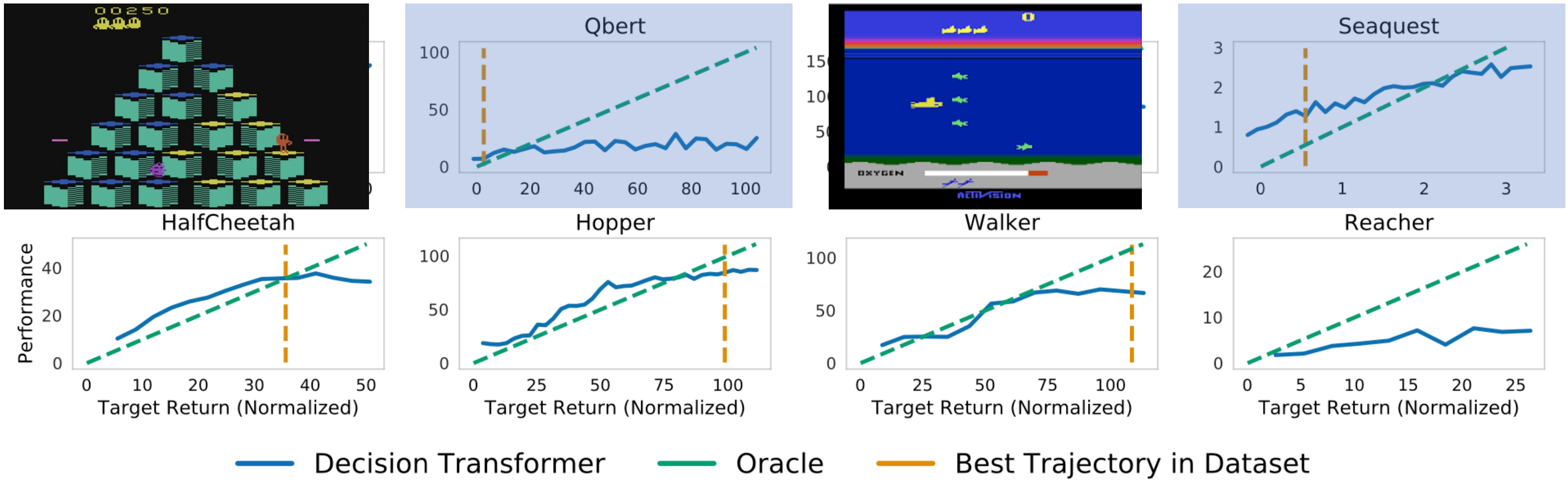
Evaluation

- Set an **initial RTG** (large enough)
- Run the DT and subtract the current return-to-go with the observed reward
- Crop the sequence to length K

```
# evaluation loop
target_return = 1  # for instance, expert-level return
R, s, a, t, done = [target_return], [env.reset()], [], [1], False
while not done:  # autoregressive generation/sampling
    # sample next action
    action = DecisionTransformer(R, s, a, t)[-1]  # for cts actions
    new_s, r, done, _ = env.step(action)

    # append new tokens to sequence
    R = R + [R[-1] - r]  # decrement returns-to-go with reward
    s, a, t = s + [new_s], a + [action], t + [len(R)]
    R, s, a, t = R[-K:], ...  # only keep context length of K
```

Results



- **Possible** to outperform the best trajectory in dataset

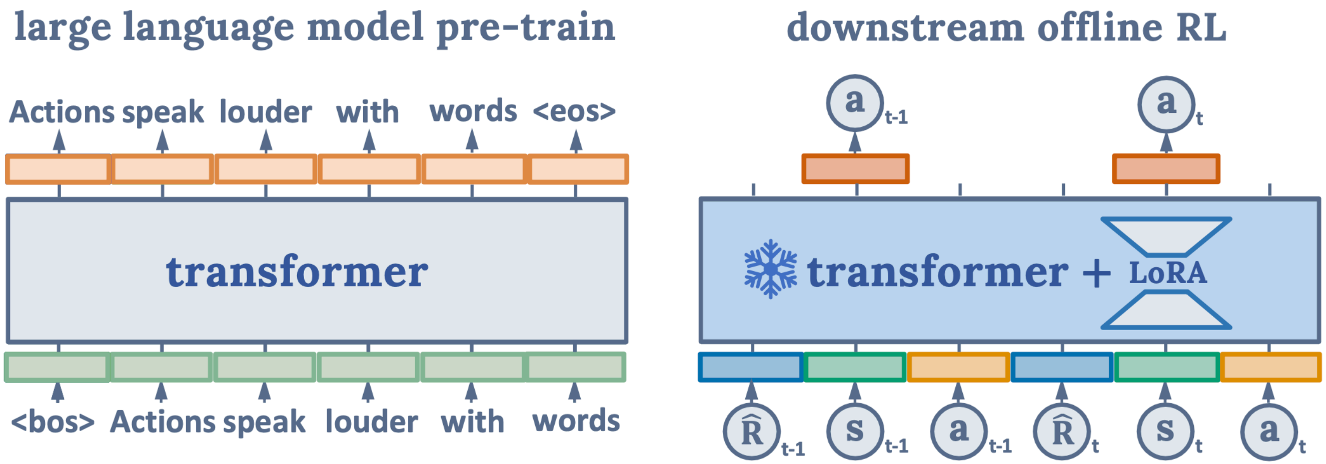
Results

Dataset	Environment	DT (Ours)	CQL	BEAR	BRAC-v	AWR	BC
Medium-Expert	HalfCheetah	**86.8 ± 1.3**	62.4	53.4	41.9	52.7	59.9
Medium-Expert	Hopper	107.6 ± 1.8	**111.0**	96.3	0.8	27.1	79.6
Medium-Expert	Walker	**108.1 ± 0.2**	98.7	40.1	81.6	53.8	36.6
Medium-Expert	Reacher	**89.1 ± 1.3**	30.6	-	-	-	73.3
Medium	HalfCheetah	42.6 ± 0.1	44.4	41.7	**46.3**	37.4	43.1
Medium	Hopper	**67.6 ± 1.0**	58.0	52.1	31.1	35.9	63.9
Medium	Walker	74.0 ± 1.4	79.2	59.1	**81.1**	17.4	77.3
Medium	Reacher	**51.2 ± 3.4**	26.0	-	-	-	**48.9**
Medium-Replay	HalfCheetah	36.6 ± 0.8	46.2	38.6	**47.7**	40.3	4.3
Medium-Replay	Hopper	**82.7 ± 7.0**	48.6	33.7	0.6	28.4	27.6
Medium-Replay	Walker	**66.6 ± 3.0**	26.7	19.2	0.9	15.5	36.9
Medium-Replay	Reacher	**18.0 ± 2.4**	**19.0**	-	-	-	5.4
Average (Without Reacher)		**74.7**	63.9	48.2	36.9	34.3	46.4
Average (All Settings)		**69.2**	54.2	-	-	-	47.7

- CQL: conservative Q-learning
- BEAR: off-policy Q-learning
- BRAC-v: behavior regularized offline RL
- AWR: advantage-weighted regression
- BC: behavior cloning

Pretraining DTs on Language Tasks[🔗]

- Use a pretrained language model (GPT2) as initialization



Pretraining DTs on Language Tasks

- Language prediction as an auxiliary objective
  - WikiText dataset
  - $\mathcal{L}_{\text{language}} = \sum_i -\log T(w_{i+1}|w_1, \ldots, w_i)$

$$\mathcal{L} = \mathcal{L}_{\text{decision}} + \lambda \mathcal{L}_{\text{language}}$$