Reinforcement Learning
Spring 2024

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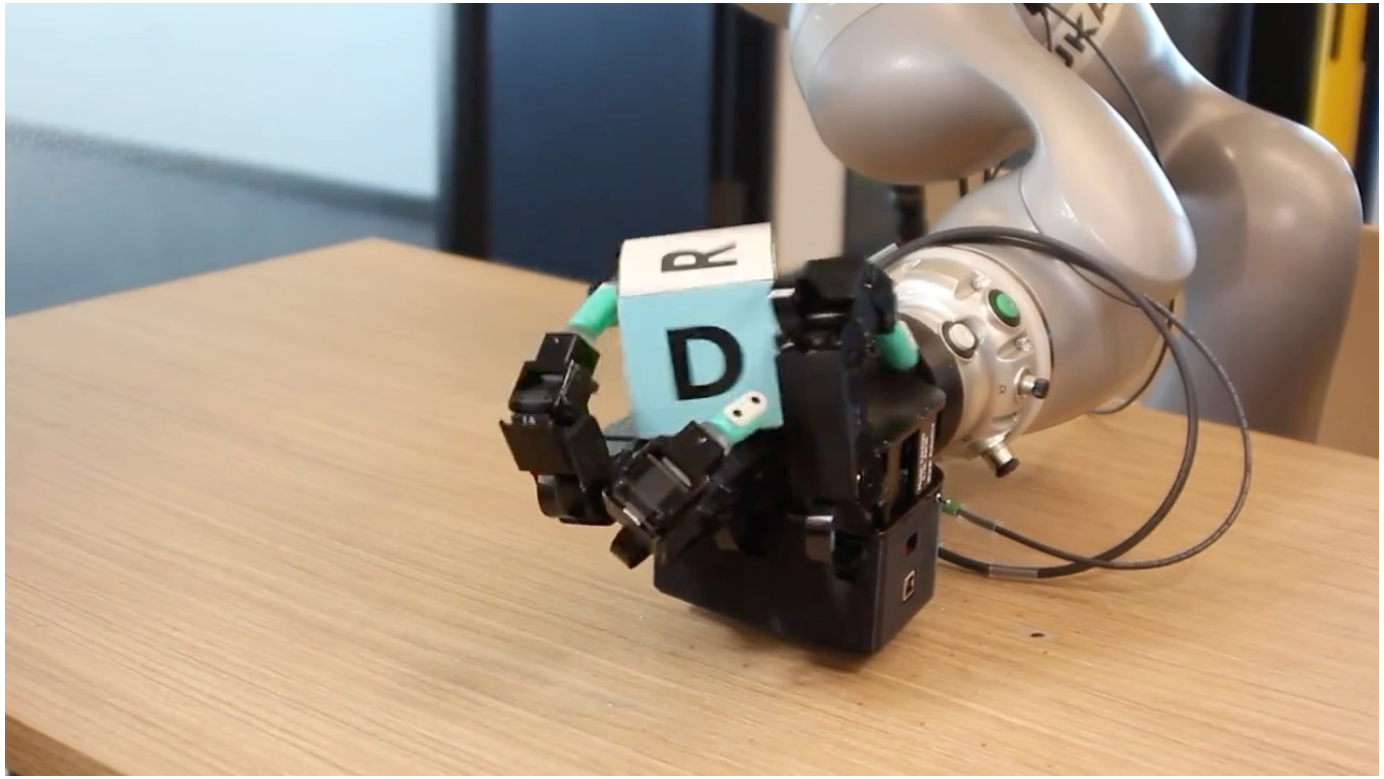
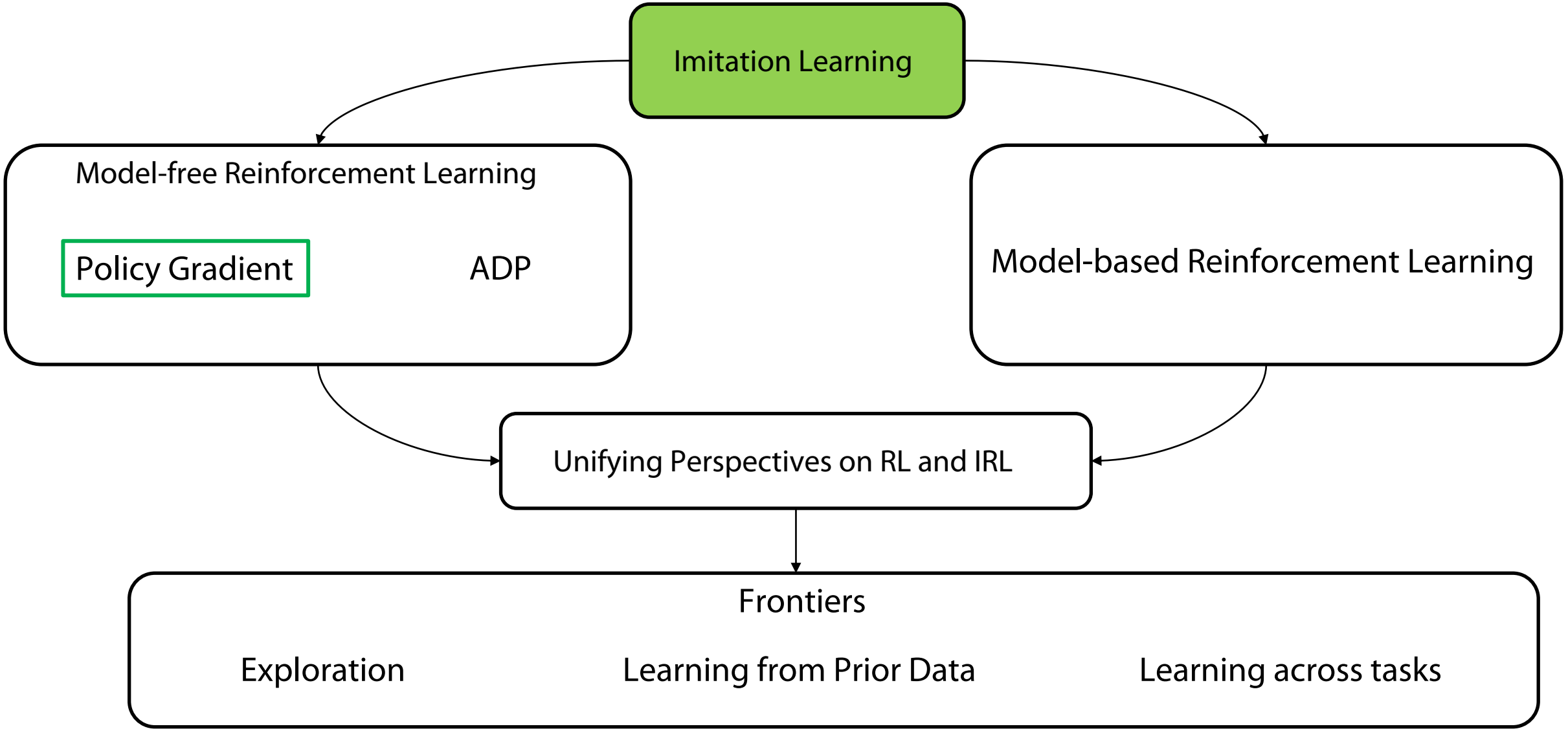
TAs: Patrick Yin, Qiuyu Chen



Slides inspired by Sergey Levine

Class Structure



Taking the gradient of return

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim p_\theta(\tau)} \left[\nabla_\theta \log p_\theta(\tau) \sum_{t=0}^{T} r(s_t, a_t) \right]$$

$$\nabla_\theta J(\theta) = \mathbb{E}_{\substack{s_0 \sim p(s_0) \\ s_{t+1} \sim p(s_{t+1}|s_t, a_t) \\ a_t \sim \pi(a_t|s_t)}} \left[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t) \sum_{t'=0}^{T} r(s_t, a_t) \right]$$

$$\approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i|s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$ (approximating using samples)

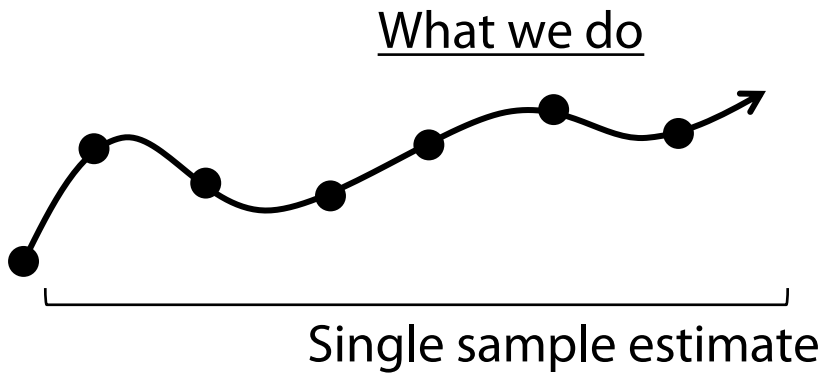
(Monte-Carlo approximation)

What makes policy gradient challenging?

Hard to tell what matters without many samples

$$\nabla_\theta J(\theta) = \int p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) d\tau$$

$$\approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i | s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$



For every (s, a) pair, weight by only the sum of rewards in the current trajectory

Couples together all actions

Susceptible to scale variations

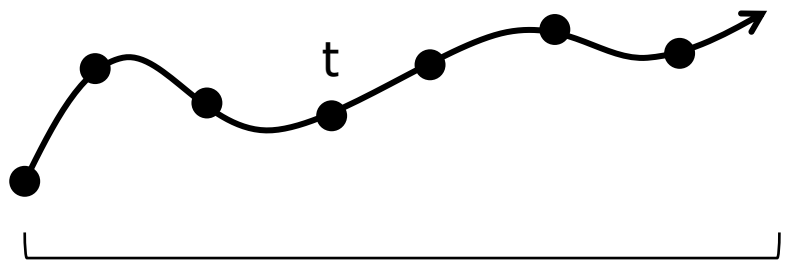
Susceptible to lucky samples

Makes policy gradient unstable, requires huge numbers of samples and huge batch size

Variance Reduction with Causality

Idea: Trajectory returns depend on past and future, but we only care about the future, since actions cannot affect the past. Instead, consider **"return-to-go"**

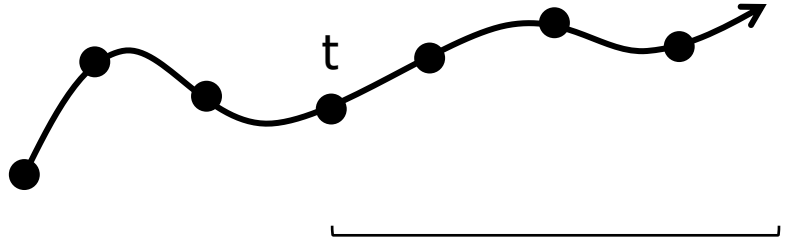
$$\approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i | s_t^i) \underbrace{\sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)}_{\text{Includes } t' < t}$$



Full trajectory return

Ignore past terms ⇩

$$\frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i | s_t^i) \sum_{t'=t}^{T} r(s_t^i, a_t^i)$$



Return to go

Variance Reduction with a Baseline

Idea: We can reduce variance by subtracting a current state dependent function from the policy gradient return

$$\frac{1}{N}\sum_{i=1}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\left[\sum_{t'=t}^{T} r(s_{t'}^i a_{t'}^i) - b(s_t)\right]$$

Baseline: Centers the returns, reduces variance

Does *not* increase bias

Take a deeper look at REINFORCE

$$\nabla_\theta J(\theta) = \int p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) d\tau \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i | s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$

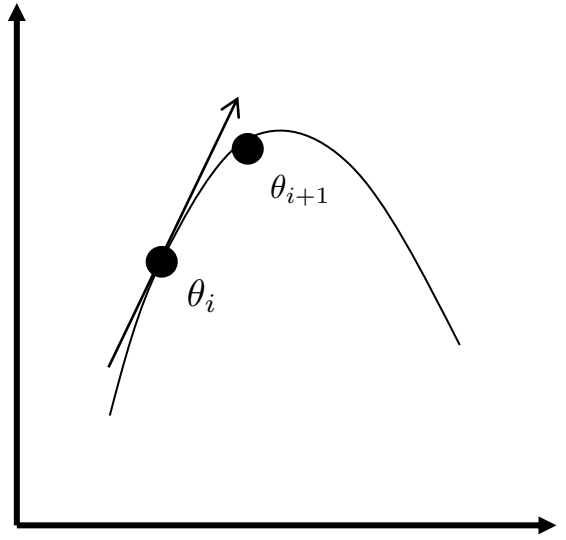
Gradient ascent is steepest ascent on linear approximation under the Euclidean metric!

$$\max \quad J(\theta_i) + \nabla_\theta J(\theta)|_{\theta=\theta_i}(\theta - \theta_i) \qquad \text{Linear approximation}$$

$$(\theta - \theta_i)^T(\theta - \theta_i) \leq \epsilon \qquad \text{Quadratic Constraint}$$

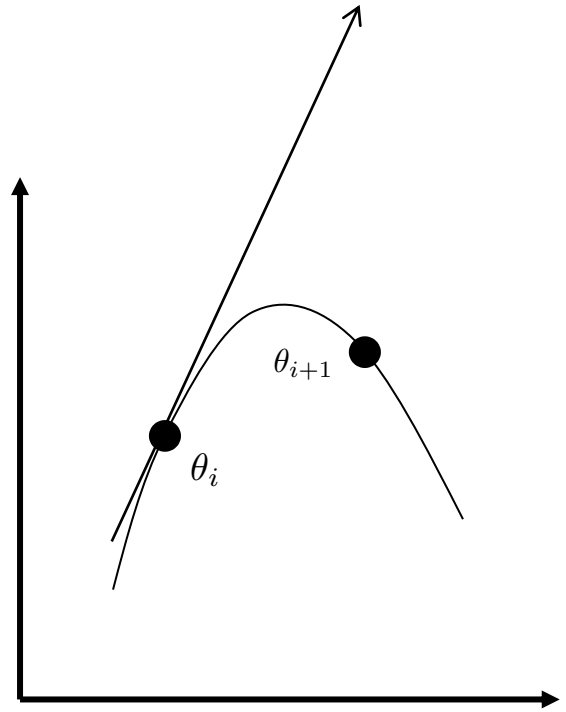
$$\downarrow$$

$$\theta = \theta_i + \alpha \nabla_\theta J(\theta)|_{\theta=\theta_i}$$



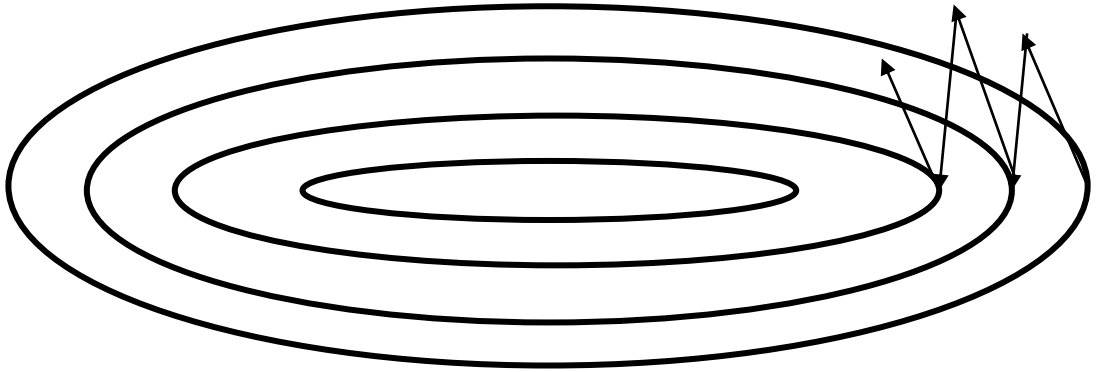
When might this fail?

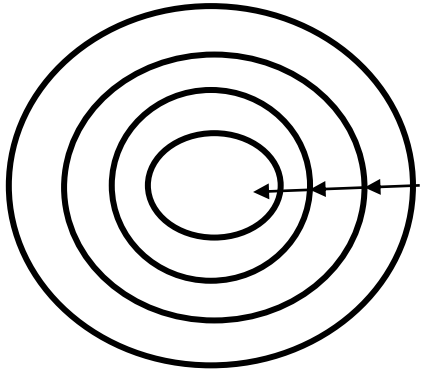
Large step sizes may cause collapse



Must use very small step sizes, slow!

Sensitive to Policy Parameterization





Very different!

Can struggle for a deep neural network!

Covariant Policy Gradient Updates

$$\max \; J(\theta_i) + \nabla_\theta J(\theta)|_{\theta=\theta_i}(\theta - \theta_i)$$
$$(\theta - \theta_i)^T G(\theta - \theta_i) \leq \epsilon$$

What should G be?

$$\max \; J(\theta_i) + \nabla_\theta J(\theta)|_{\theta=\theta_i}(\theta - \theta_i)$$
$$D_{\mathrm{KL}}(\pi_\theta || \pi_{\theta_i}) \leq \epsilon$$

Let us use the constraint as KL divergence on the policy (2^{nd} order Taylor expansion)

Measures functional distance, not parameter distance

Resulting "Natural" Policy Gradient

$$\max \quad J(\theta_i) + \nabla_\theta J(\theta)|_{\theta=\theta_i}(\theta - \theta_i)$$
$$D_{\mathrm{KL}}(\pi_\theta || \pi_{\theta_i}) \leq \epsilon$$

2nd order approximation of KL → Fisher Information Metric

$$F = \mathbb{E}_{\pi_\theta}\left[(\nabla_\theta \log \pi_\theta)(\nabla_\theta \log \pi_\theta)^T\right]$$

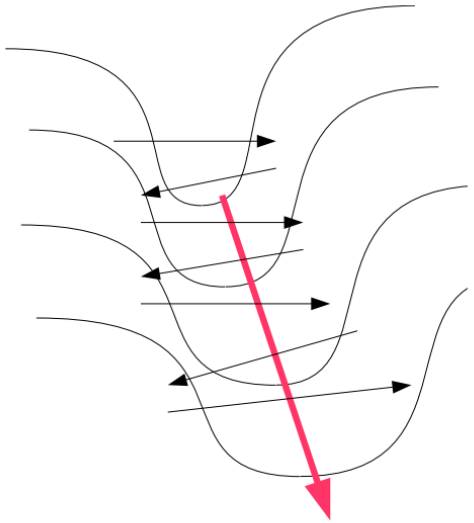
$$\max \quad J(\theta_i) + \nabla_\theta J(\theta)|_{\theta=\theta_i}(\theta - \theta_i)$$
$$(\theta - \theta_i)^T F(\theta - \theta_i) \leq \epsilon$$

Resulting update $\theta_{i+1} = \theta_i + \alpha F^{-1} \nabla_\theta J(\theta)|_{\theta=\theta_i}$ Covariant to parameterization

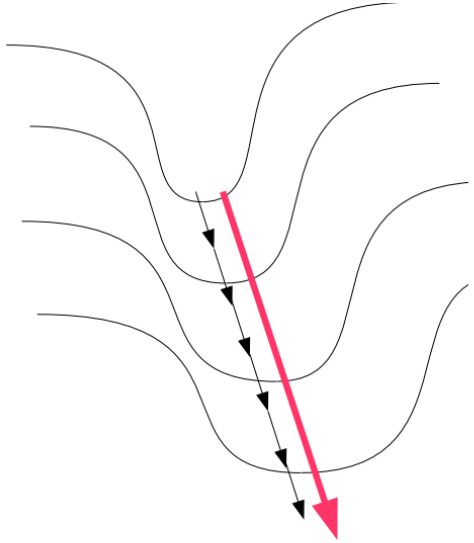
Fisher Information as a Gauss-Newton Approximation

Fisher Information is generalized Gauss Newton Approximation to the Hessian

SGD



Natural Gradient

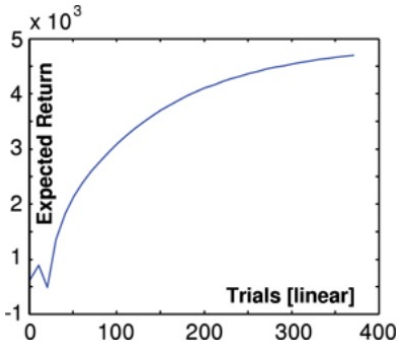


$$E(\boldsymbol{\theta}) = \mathbb{E}_{(\mathbf{x},\mathbf{y})}\left[-\log p(\mathbf{y}|\mathbf{x};\boldsymbol{\theta})\right]$$

$$\begin{aligned}
\mathbf{H}(E)(\boldsymbol{\theta}) &= \nabla^2 \mathop{\mathbb{E}}_{(\mathbf{x},\mathbf{y})}\left[-\log p(\mathbf{y}|\mathbf{x};\boldsymbol{\theta})\right] \\
&= -\mathop{\mathbb{E}}_{(\mathbf{x},\mathbf{y})}\left[\nabla^2 \log p(\mathbf{y}|\mathbf{x};\boldsymbol{\theta})\right] \\
&= -\mathop{\mathbb{E}}_{(\mathbf{x},\mathbf{y})}\left[-\frac{\nabla p(\mathbf{y}|\mathbf{x};\boldsymbol{\theta})\nabla p(\mathbf{y}|\mathbf{x};\boldsymbol{\theta})^{\mathrm{T}}}{p(\mathbf{y}|\mathbf{x};\boldsymbol{\theta})^2} + \frac{\nabla^2 p(\mathbf{y}|\mathbf{x};\boldsymbol{\theta})}{p(\mathbf{y}|\mathbf{x};\boldsymbol{\theta})}\right] \\
&= \mathbf{F}_{\boldsymbol{\theta}} - \mathop{\mathbb{E}}_{(\mathbf{x},\mathbf{y})}\left[\frac{\nabla^2 p(\mathbf{y}|\mathbf{x};\boldsymbol{\theta})}{p(\mathbf{y}|\mathbf{x};\boldsymbol{\theta})}\right]
\end{aligned}$$

https://towardsdatascience.com/introducing-k-fac-and-its-application-for-large-scale-deep-learning-4e3f9b443414

Natural Policy Gradient in Action

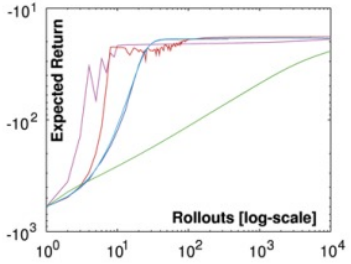


(a) Performance.

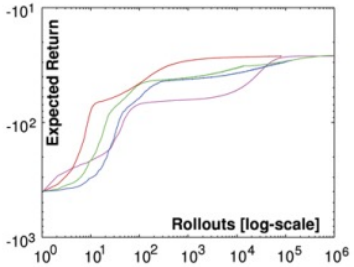

(b) Imitation learning.


(c) Initial reproduction.

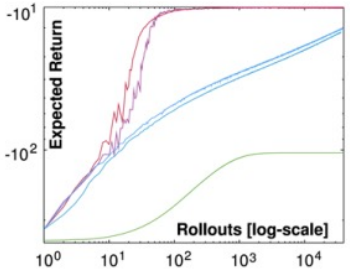

(d) After reinforcement learning.



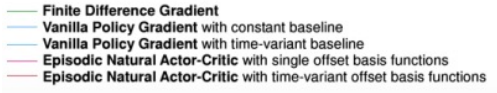
(b) Minimum motor command with motor primitives



(c) Passing through a point with splines



(d) Passing through a point with motor primitives



Peters, Schaal ‘08

Lecture outline

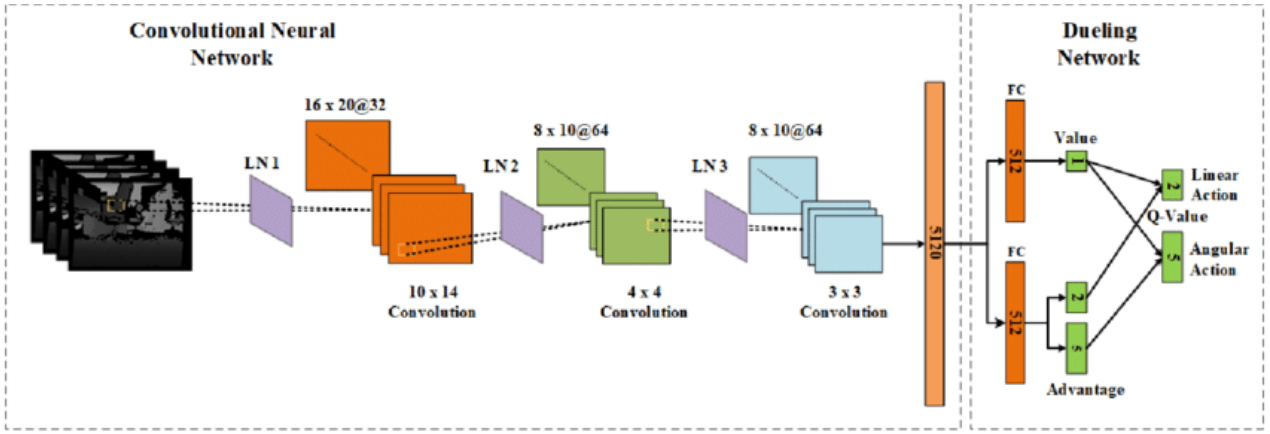
Making NPG Practical → Trust Region Policy Optimization

Making FIM Inversion Tractable → K-FAC

One Algorithm to Rule Them All - Proximal Policy Optimization

Natural Policy Gradient - is it enough?

Huge matrix inversion

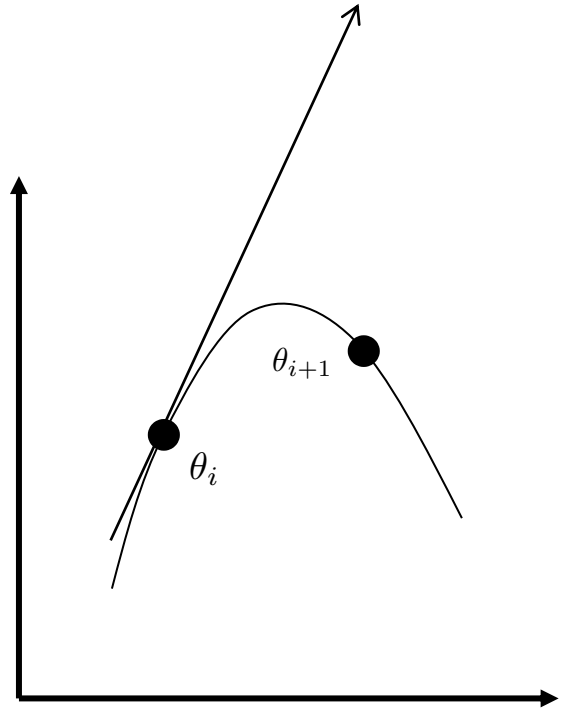


$F - R^{dxd}$

For a standard convnet – d is in the millions

Hessian is way out of memory / hard to invert!

Step-size?



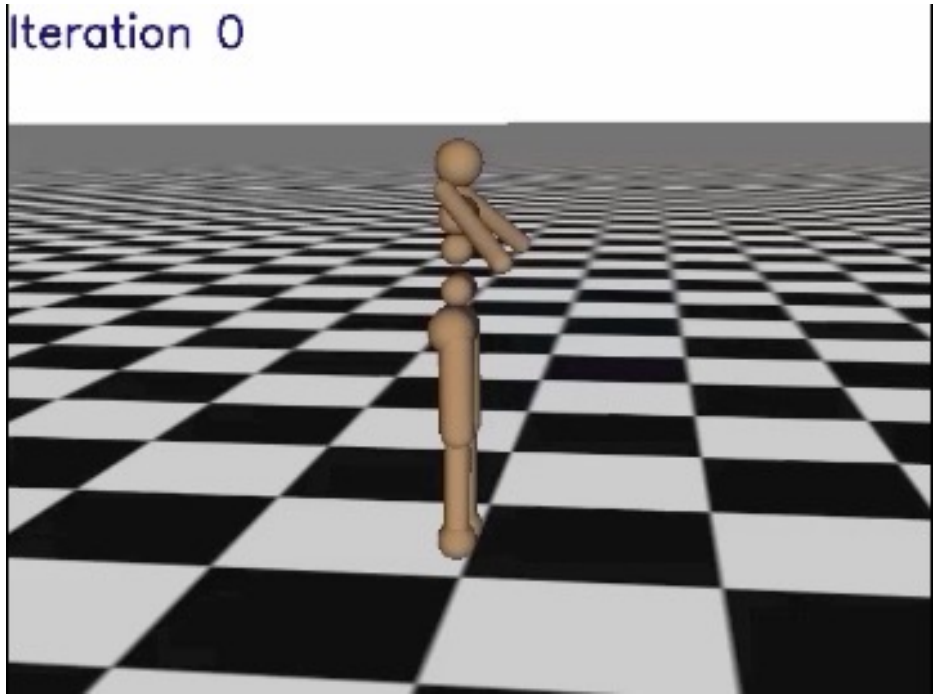
Can easily overstep and collapse performance

Also, only a single gradient step at a time before recollecting data!

Trust Region Policy Optimization

3 key ideas:

1. On-policy updates → importance sampled objective
2. Huge matrix inversion → conjugate gradient method
3. Step size may be too large → backtracking line search



TRPO, Schulman '15

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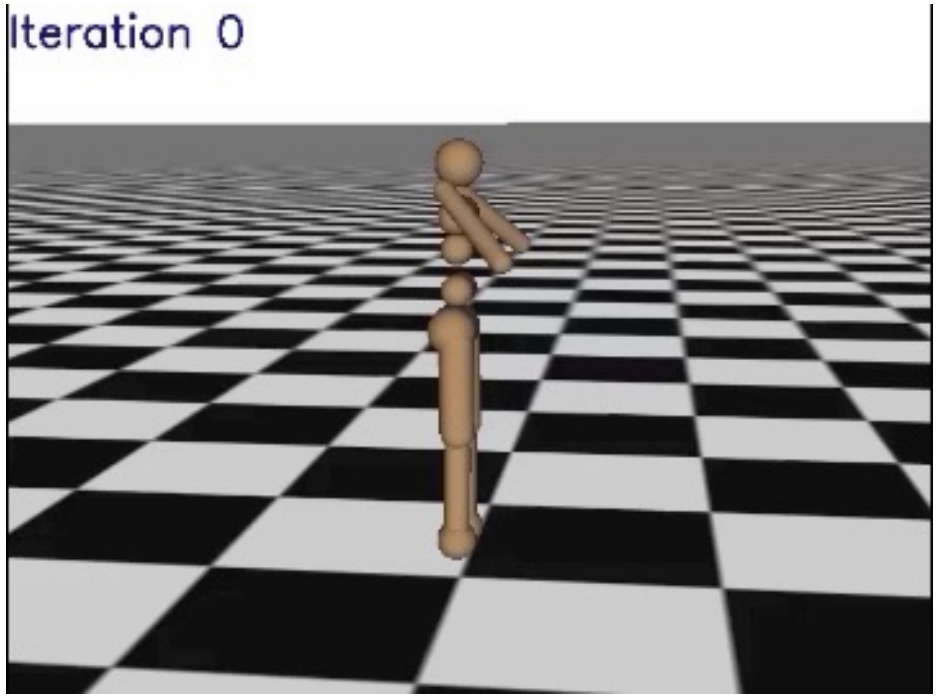

TRPO, Schulman '15

Off Policy Policy Gradient – Importance Sampling

Problem with original policy gradient objective – hard to evaluate without samples from θ

$$J(\theta) = \mathbb{E}_{\tau \sim p_\theta(\tau)}[R(\tau)] \quad \nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim p_\theta(\tau)}[\nabla_\theta \log \pi_\theta(\tau) R(\tau)]$$

Must sample to evaluate

Can we be off-policy?

$$J(\theta) = \int p_\theta(\tau) R(\tau)$$

Can rederive policy gradient from this perspective

$$J(\theta) = \int \frac{q(\tau)}{q(\tau)} p_\theta(\tau) R(\tau)$$

Has numerical challenges

$$J(\theta) = \mathbb{E}_{\tau \sim q(\tau)}\left[\frac{p_\theta(\tau)}{q(\tau)} R(\tau)\right]$$

Deriving Policy Gradient from Importance Sampling

$$J(\theta) = \int p_\theta(\tau) R(\tau)$$

$$J(\theta) = \int \frac{q(\tau)}{q(\tau)} p_\theta(\tau) R(\tau)$$

$$J(\theta) = \mathbb{E}_{\tau \sim q(\tau)} \left[\frac{p_\theta(\tau)}{q(\tau)} R(\tau) \right]$$

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim q(\tau)} \left[\frac{\nabla_\theta p_\theta(\tau)}{q(\tau)} R(\tau) \right]$$

Set $q(\tau) = p_{\theta_i}(\tau)$

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim p_{\theta_i}(\tau)} \left[\frac{\nabla_\theta p_\theta(\tau)}{p_{\theta_i}(\tau)} R(\tau) \right]$$

$$\nabla_\theta J(\theta)|_{\theta=\theta_i} = \mathbb{E}_{\tau \sim p_{\theta_i}(\tau)} \left[\nabla_\theta \log p_\theta(\tau)|_{\theta=\theta_i} R(\tau) \right]$$

Same as PG in first order!

But can do more than first order, and be off-policy

Expanding the Importance Sampling Objective

$$J(\theta) = \int p_\theta(\tau) R(\tau)$$

$$J(\theta) = \int \frac{q(\tau)}{q(\tau)} p_\theta(\tau) R(\tau)$$

$$J(\theta) = \mathbb{E}_{\tau \sim q(\tau)} \left[\frac{p_\theta(\tau)}{q(\tau)} R(\tau) \right]$$

$$J(\theta) = \mathbb{E}_{\tau \sim q(\tau)} \left[\left(\Pi_{t=1}^{T} \frac{\pi_\theta(a_t|s_t)}{q(a_t|s_t)} \right) \left(R(\tau) \right) \right]$$

Numerically unstable, high variance!

Trust Region Policy Optimization – Importance Sampling without the Pain

Cannot evaluate without resampling

Original Objective $J(\theta) = \mathbb{E}_{\tau \sim p_\theta(\tau)} [R(\tau)]$

$J(\theta) = \mathbb{E}_{s \sim p_\theta(s), a \sim \pi_\theta(a|s)} [Q(s,a)]$

Importance Sampling(ish) $J(\theta) = \mathbb{E}_{s \sim p_{\theta_i}(s), a \sim \pi_{\theta_i}(a|s)} \left[\frac{p_\theta(s)}{p_{\theta_i}(s)} \frac{\pi_\theta(a|s)}{\pi_\theta(a|s)} Q(s,a) \right]$

(Surrogate Objective) $\approx \mathbb{E}_{s \sim p_{\theta_i}(s), a \sim \pi_{\theta_i}(a|s)} \left[\frac{\pi_\theta(a|s)}{\pi_\theta(a|s)} Q(s,a) \right]$

If policies are close, we can show that this is not so bad!

Often replaced by A = Q - V

Trust Region Policy Optimization

3 key ideas:

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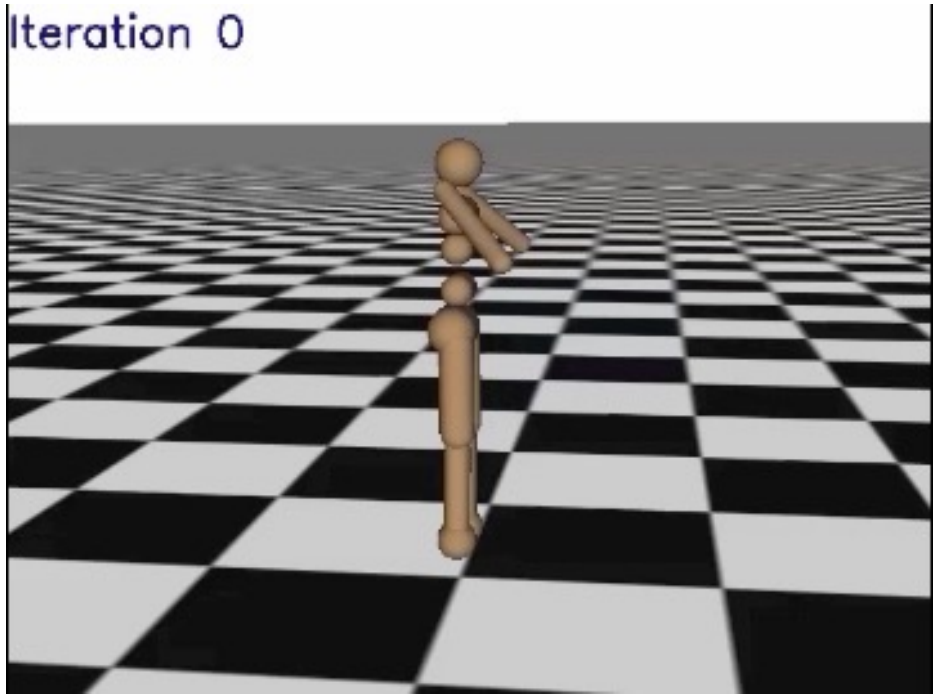


TRPO, Schulman '15

Trust Region Policy Optimization – Conjugate Gradient

Challenging to compute F^{-1} and then get $F^{-1}g$

⇩

Convert into an iterative minimization problem!

Solution to

$$Fx = g$$

same as

Solution to

$$\min_x \frac{1}{2} x^T F x - x^T g + c$$

Trust Region Policy Optimization – Conjugate Gradient

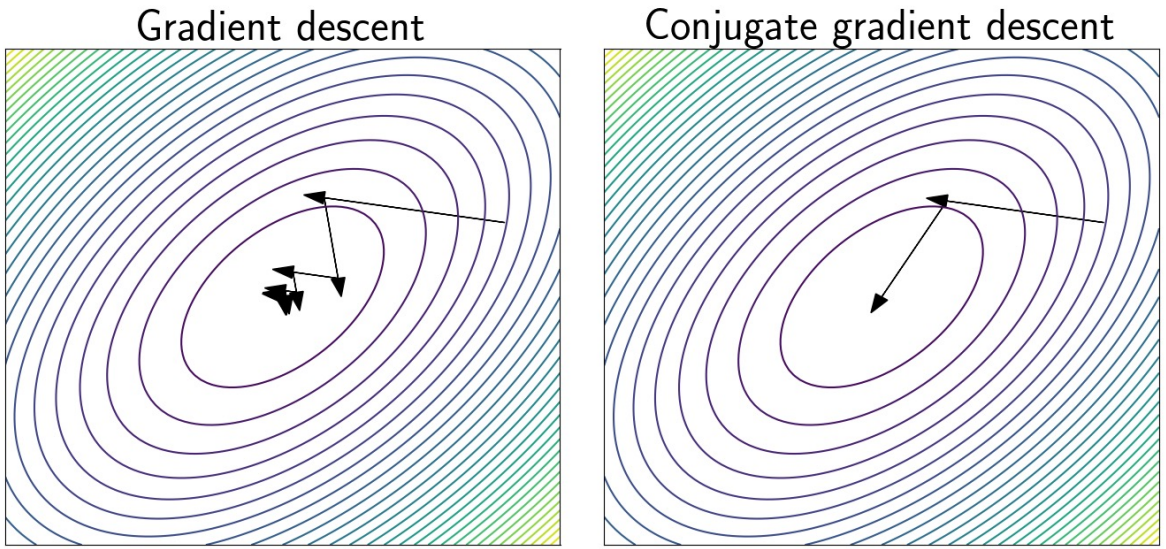
Challenging to compute F^{-1} and then get $F^{-1}g$

⇩

Convert into an iterative minimization problem!

⇩

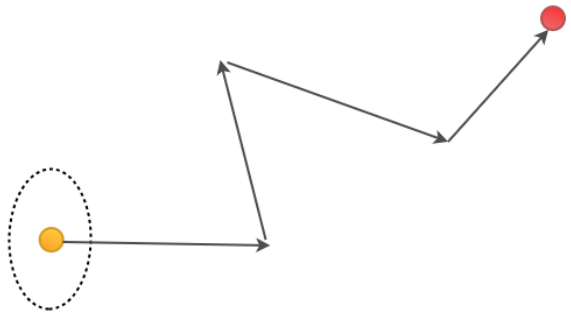
Solve with conjugate gradient



Do coordinate descent in geometry aligned orthogonal directions

https://www.cs.cmu.edu/~quake-papers/painless-conjugate-gradient.pdf

Trust Region Policy Optimization – Conjugate Gradient



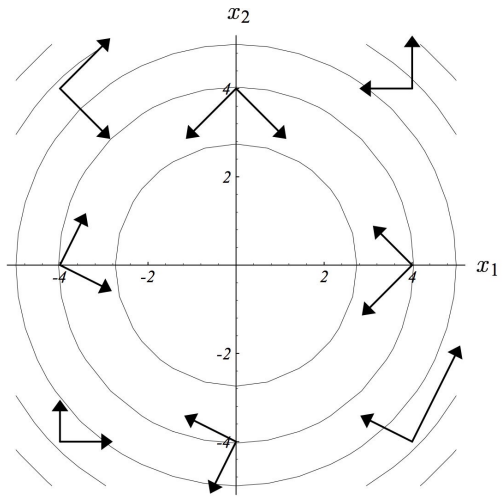
Gradient ascent

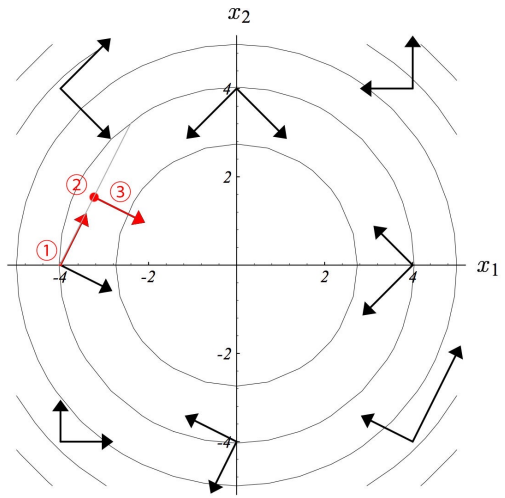
Conjugate gradient

$$\min_x \frac{1}{2} x^T F x - x^T g + c$$

Find search directions at ever step that are F-orthogonal with previous directions

$$d_{(i)}^T F d_{(i)} = 0$$





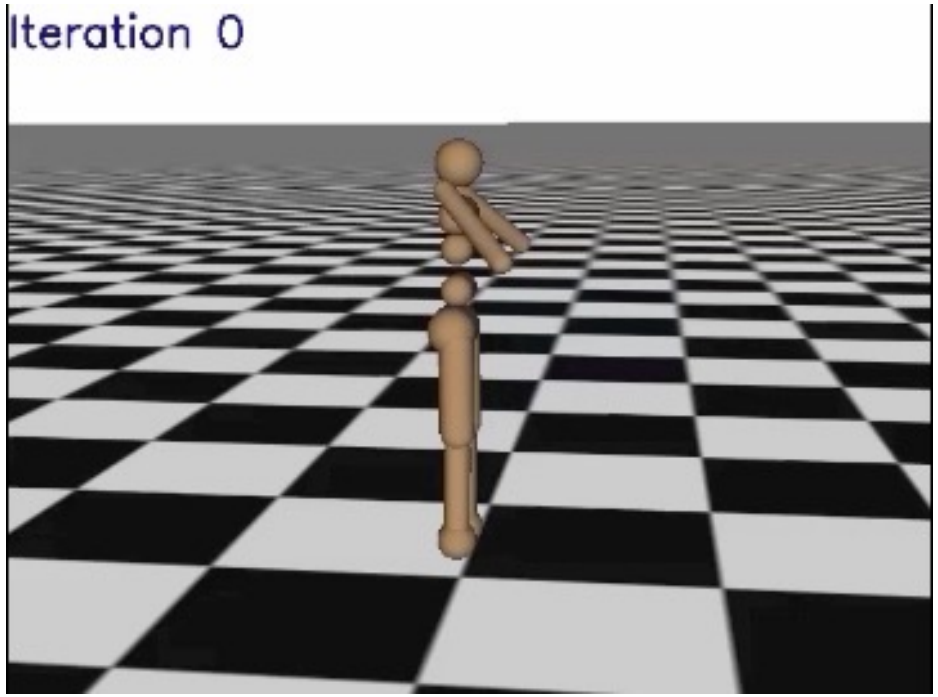
Converges in approx N steps!

Only requires matrix-vector product

Trust Region Policy Optimization

3 key ideas:

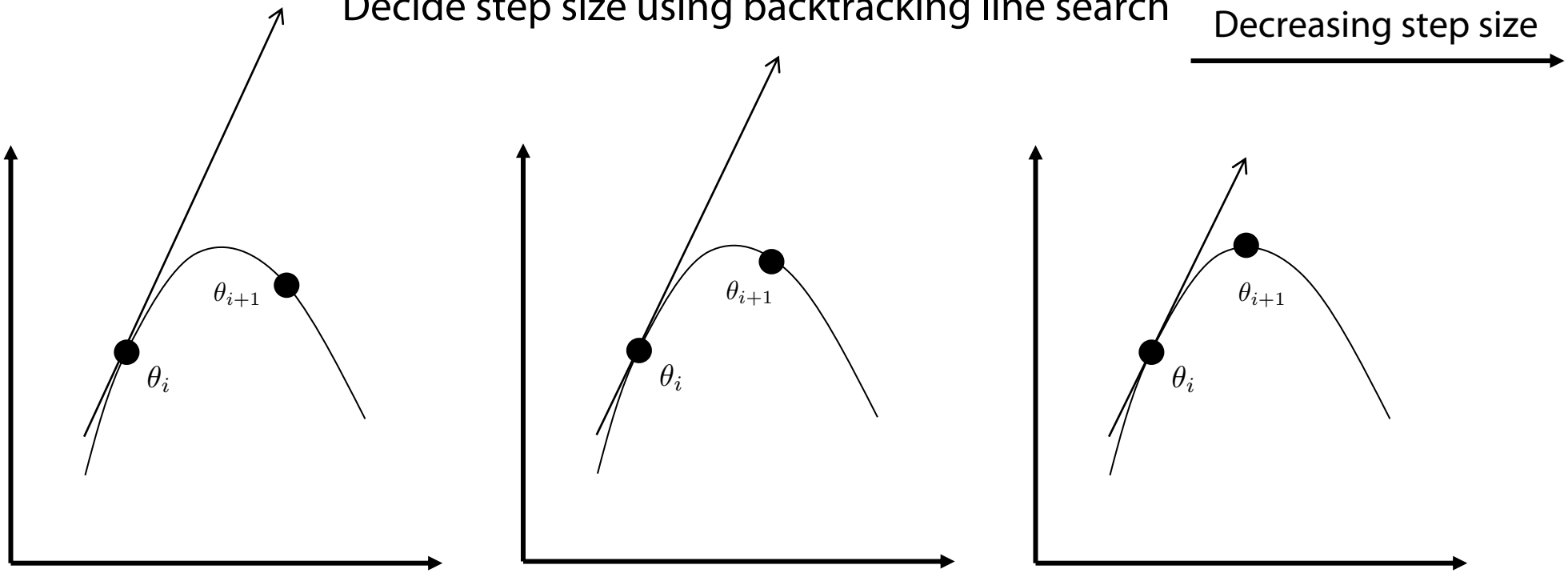
1. On-policy updates → importance sampled objective
2. Huge matrix inversion → conjugate gradient method
3. Step size may be too large → backtracking line search



TRPO, Schulman '15

Trust Region Policy Optimization – Backtracking line search

Decide step size using backtracking line search



1. Choose parameter $\beta \in (0, 1)$, given search direction s = $F^{-1}g$
2. Compute maximal step size such that constraint is satisfied - $\frac{1}{2}(ts)^T F(ts) = \epsilon \rightarrow t = \sqrt{\frac{2\epsilon}{s^T F s}}$
3. While $J(\theta_i + ts) < J(\theta_i)$, set $t = \beta t$

Backtracking

Trust Region Policy Optimization

3 key ideas:

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Algorithm 3 Trust Region Policy Optimization

Input: initial policy parameters θ_0

for $k = 0, 1, 2, \ldots$ **do**

Collect set of trajectories $\mathcal{D}_k$ on policy $\pi_k = \pi(\theta_k)$

Estimate advantages $\hat{A}_t^{\pi_k}$ using any advantage estimation algorithm

Form sample estimates for

- policy gradient $\hat{g}_k$ (using advantage estimates)
- and KL-divergence Hessian-vector product function $f(v) = \hat{H}_k v$

Use CG with n_{cg} iterations to obtain $x_k \approx \hat{H}_k^{-1} \hat{g}_k$

Estimate proposed step $\Delta_k \approx \sqrt{\frac{2\delta}{x_k^T \hat{H}_k x_k}} x_k$

Perform backtracking line search with exponential decay to obtain final update

$$\theta_{k+1} = \theta_k + \alpha^j \Delta_k$$

end for

TRPO, Schulman '15

Can we say anything formal about updates?

$$\eta(\tilde{\pi}) \geq L_{\pi}(\tilde{\pi}) - C D_{\mathrm{KL}}^{\max}(\pi, \tilde{\pi}),$$
$$\text{where } C = \frac{4\epsilon\gamma}{(1-\gamma)^2}.$$

Ensures that policies are non-decreasing in performance

Performance difference lemma

$$\eta(\tilde{\pi}) = \eta(\pi) + \mathbb{E}_{\tau \sim \tilde{\pi}} \left[\sum_{t=0}^{\infty} \gamma^t A_{\pi}(s_t, a_t) \right]$$

Express advantage in terms of TVD

Theorem 1. *Let* $\alpha = D_{\mathrm{TV}}^{\max}(\pi_{\mathrm{old}}, \pi_{\mathrm{new}})$. *Then the following bound holds:*

$$\eta(\pi_{\mathrm{new}}) \geq L_{\pi_{\mathrm{old}}}(\pi_{\mathrm{new}}) - \frac{4\epsilon\gamma}{(1-\gamma)^2}\alpha^2$$
$$\textit{where } \epsilon = \max_{s,a} |A_{\pi}(s,a)| \qquad (8)$$

Key idea: by bounding how different the policies are, we can bound how different returns are

TRPO, Schulman '15

TRPO in action

TRPO, Schulman ‘15

Why might TRPO not be enough?

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Form sample estimates for

- policy gradient $\hat{g}_k$ (using advantage estimates)
- and KL-divergence Hessian-vector product function $f(v) = \hat{H}_k v$

Use CG with n_{cg} iterations to obtain $x_k \approx \hat{H}_k^{-1} \hat{g}_k$

Estimate proposed step $\Delta_k \approx \sqrt{\frac{2\delta}{x_k^T \hat{H}_k x_k}} x_k$

Perform backtracking line search with exponential decay to obtain final update

$$\theta_{k+1} = \theta_k + \alpha^j \Delta_k$$

end for

Advantage estimation is too high variance

Optimization expensive/unstable

TRPO, Schulman '15

Better Advantage Estimation - Generalized Advantage Estimation

Advantage estimator $A_N^\theta(s_1, a_1) = r_1 + \gamma r_2 + \cdots + \gamma^{N-1} r_N - V(s_1)$

High variance!

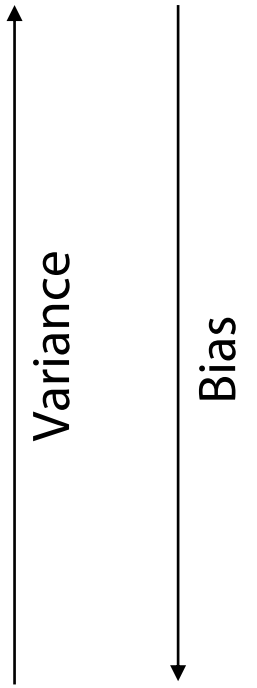
N step advantage estimator $A_N^\theta(s_1, a_1) = r_1 + \gamma r_2 + \cdots + \gamma^{N-1} r_N - V(s_1)$

N-1 step advantage estimator $A_{N-1}^\theta(s_1, a_1) = r_1 + \gamma r_2 + \cdots + \gamma^{N-2} V(s_{N-1}) - V(s_1)$

$\vdots$

2 step advantage estimator $A_2^\theta(s_1, a_1) = r_1 + \gamma r_2 + \cdots + \gamma^2 V(s_3) - V(s_1)$

1 step advantage estimator $A_1^\theta(s_1, a_1) = r_1 + \gamma V(s_2) - V(s_1)$



GAE, Schulman '15

Generalized Advantage Estimation

Sum up all the estimators in a geometric sum

$$A_N^\theta(s_1, a_1) = r_1 + \gamma r_2 + \cdots + \gamma^{N-1} r_N - V(s_1)$$

$$A_{N-1}^\theta(s_1, a_1) = r_1 + \gamma r_2 + \cdots + \gamma^{N-2} V(s_{N-1}) - V(s_1)$$

$$A_2^\theta(s_1, a_1) = r_1 + \gamma r_2 + \cdots + \gamma^2 V(s_3) - V(s_1)$$

$$A_1^\theta(s_1, a_1) = r_1 + \gamma V(s_2) - V(s_1)$$

Geometric sum

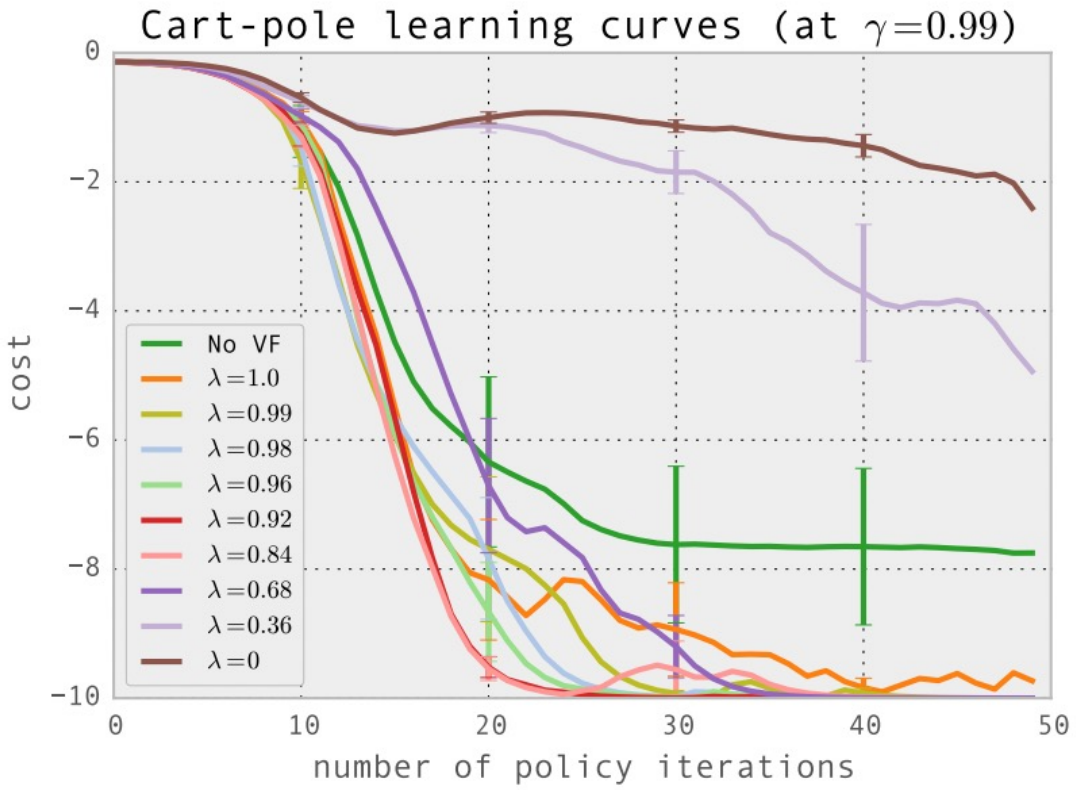
$$A_\lambda^\theta(s_1, a_1) = \sum_{j=1}^{N} \lambda^j A_j^\theta(s, a)$$

λ controls bias-variance tradeoff

Best of both worlds – very similar idea to eligibility traces

GAE, Schulman '15

Generalized Advantage Estimation in Action



GAE, Schulman ‘15

Lecture outline

Making NPG Practical → Trust Region Policy Optimization

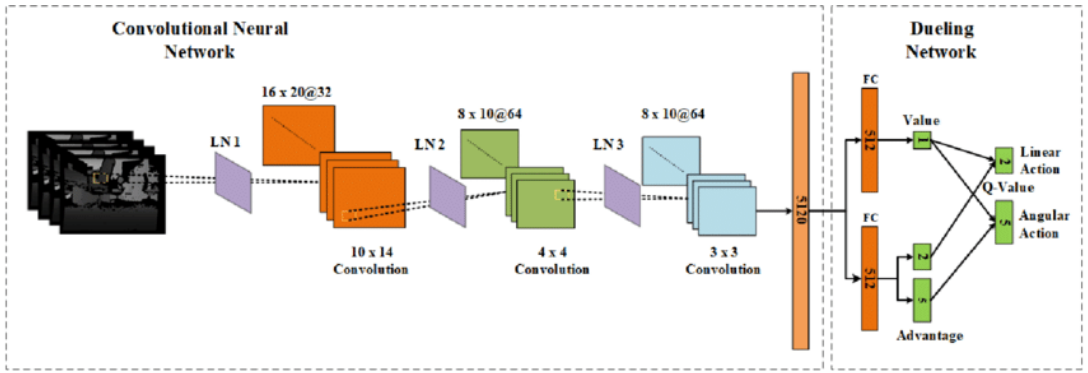
Making FIM Inversion Tractable → K-FAC

One Algorithm to Rule Them All - Proximal Policy Optimization

Approximations to effectively invert FIM

Instead of solving with conjugate gradient, what if we approximated FIM

Major issue with FIM is huge dimensionality



Dimensionality of FIM – huge!
Very challenging to invert

Kronecker factorization (K-FAC) makes it tractable with 2 approx:

1. Block diagonalize FIM by layer
2. Represent each block diagonal as a Kronecker product (easy inversion)

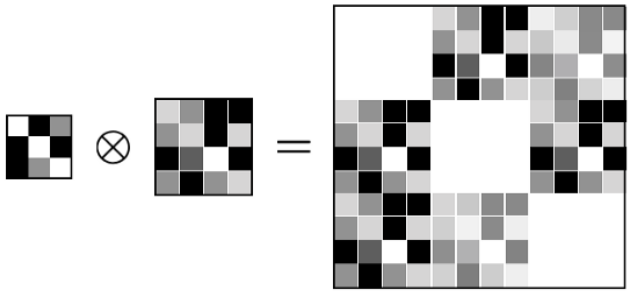
ACKTR, Wu '17

What is a Kronecker product?

Generalization of the tensor product

$$\mathbf{A} \otimes \mathbf{B} := \begin{pmatrix} [\mathbf{A}]_{1,1}\mathbf{B} & \cdots & [\mathbf{A}]_{1,n}\mathbf{B} \\ \vdots & \ddots & \vdots \\ [\mathbf{A}]_{m,1}\mathbf{B} & \cdots & [\mathbf{A}]_{m,n}\mathbf{B} \end{pmatrix} \in \mathbb{R}^{ma \times nb}$$

$\mathbf{A} \in \mathbb{R}^{m \times n}, \mathbf{B} \in \mathbb{R}^{a \times b}$: Kronecker factors

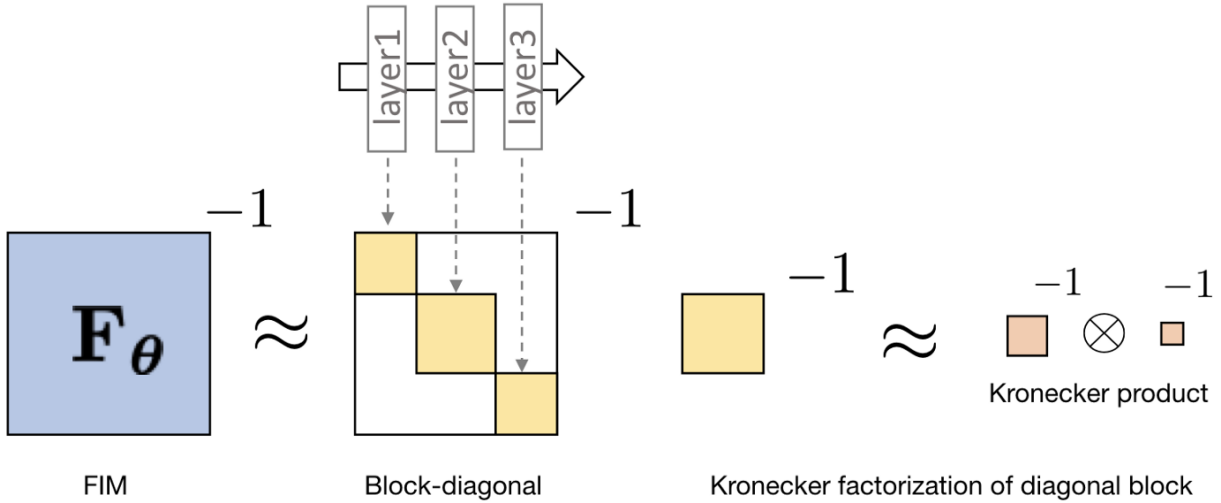


Identities

$$\text{vec}(uv^T) = u \otimes v$$

$$(a \otimes b)(c \otimes d) = (ac \otimes bd)$$

$$(\mathbf{A} \otimes \mathbf{B})^{-1} = \mathbf{A}^{-1} \otimes \mathbf{B}^{-1}$$



We will represent the (per layer) fisher information metric as a Kronecker product of two smaller matrices

ACKTR, Wu '17

K-FAC approximation for NPG

Kronecker factorization (K-FAC) makes it tractable with 2 approx:

1. Block diagonalize FIM by layer
2. Represent each block diagonal as a Kronecker product (easy inversion)

Weight defs

$$s = Wa$$
$$\nabla_W L = (\nabla_s L) a^\intercal$$

Matrix Identities

$$\text{vec}(uv^T) = u \otimes v$$
$$(a \otimes b)(c \otimes d) = (ac \otimes bd)$$

K-FAC reparam

$$F_\ell = \mathbb{E}[\text{vec}\{\nabla_W L\}\text{vec}\{\nabla_W L\}^\intercal] = \mathbb{E}[aa^\intercal \otimes \nabla_s L(\nabla_s L)^\intercal]$$
$$\approx \mathbb{E}[aa^\intercal] \otimes \mathbb{E}[\nabla_s L(\nabla_s L)^\intercal] := A \otimes S := \hat{F}_\ell,$$

Easy to compute and invert (order of magnitude smaller matrices)

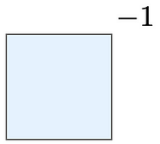
ACKTR, Wu '17

How much does this help?

All layers in AlexNet
60,000,000 parameters

Fisher information matrix

$$\mathbf{F}_{\boldsymbol{\theta}} \in \mathbb{R}^{60{,}000{,}000 \times 60{,}000{,}000}$$



Final layer of AlexNet
Input dimension: 4,096
Output dimension: 1,000
4,096,000 parameters

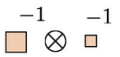
Fisher block

$$\mathbf{F}_i \in \mathbb{R}^{4{,}096{,}000 \times 4{,}096{,}000}$$

Kronecker factors

$$\mathbf{A}_{i-1} \in \mathbb{R}^{4{,}096 \times 4{,}096}$$

$$\mathbf{G}_i \in \mathbb{R}^{1{,}000 \times 1{,}000}$$



ACKTR, Wu ‘17

Lecture outline

Making NPG Practical → Trust Region Policy Optimization

Making FIM Inversion Tractable → K-FAC

One Algorithm to Rule Them All - Proximal Policy Optimization

Avoiding Second Order Optimization

Algorithm 3 Trust Region Policy Optimization

Input: initial policy parameters θ_0
for $k = 0, 1, 2, \ldots$ **do**
Collect set of trajectories $\mathcal{D}_k$ on policy $\pi_k = \pi(\theta_k)$
Estimate advantages $\hat{A}_t^{\pi_k}$ using any advantage estimation algorithm
Form sample estimates for

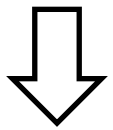
- policy gradient $\hat{g}_k$ (using advantage estimates)
- and KL-divergence Hessian-vector product function $f(v) = \hat{H}_k v$

Use CG with n_{cg} iterations to obtain $x_k \approx \hat{H}_k^{-1} \hat{g}_k$
Estimate proposed step $\Delta_k \approx \sqrt{\frac{2\delta}{x_k^T \hat{H}_k x_k}} x_k$
Perform backtracking line search with exponential decay to obtain final update

$$\theta_{k+1} = \theta_k + \alpha^j \Delta_k$$

end for

Expensive second order optimization, can we avoid?



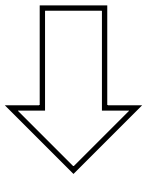
What if we just restricted how much the policy changes directly!

Proximal Policy Optimization Update

Trust Region Policy Optimization

$$\max \quad J(\theta_i) + \nabla_\theta J(\theta)|_{\theta=\theta_i}(\theta - \theta_i)$$
$$D_{\mathrm{KL}}(\pi_\theta || \pi_{\theta_i}) \leq \epsilon$$

Restrict the amount the policy moves



Proximal Policy Optimization

$$\mathcal{L}(s, a, \theta_i, \theta) = \min\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_i}(a|s)} A(s,a), \mathrm{clip}\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_i}(a|s)}, 1-\epsilon, 1+\epsilon\right) A(s,a)\right)$$

PPO, Schulman '17

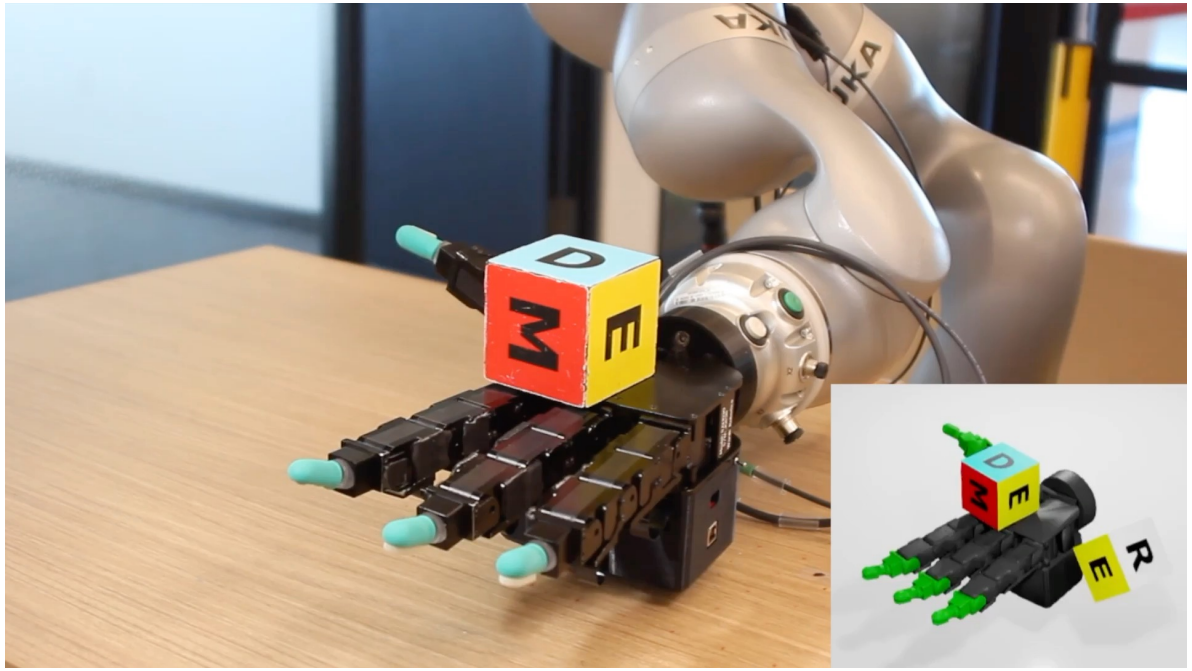
Proximal Policy Optimization Algorithm

$$\mathcal{L}(s, a, \theta_i, \theta) = \min\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_i}(a|s)} A(s,a), \text{clip}\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_i}(a|s)}, 1-\epsilon, 1+\epsilon\right) A(s,a)\right)$$

- ✓ Multiple minibatch gradient steps
- ✓ No second order optimization
- ✓ Simple and stable, without huge updates

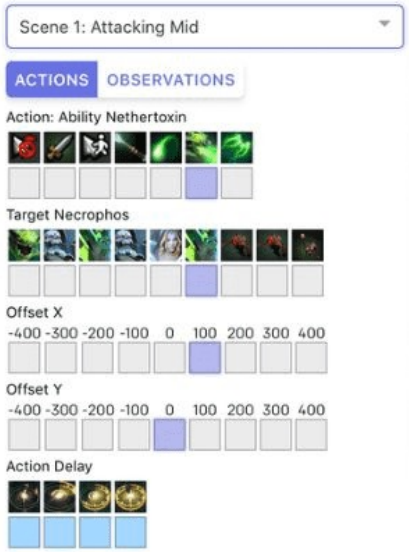
PPO, Schulman '17

PPO in Action



Dextreme, Handa ‘22
A system for general in-hand reorientation, Chen ‘21

PPO in Action



PPO, Schulman ‘17

So should we just use PPO for everything?

Implementation Matters in Deep Policy Gradients: A Case Study on PPO and TRPO

Logan Engstrom[1*], Andrew Ilyas[1*], Shibani Santurkar[1], Dimitris Tsipras[1],
Firdaus Janoos[2], Larry Rudolph[1,2], and Aleksander Mądry[1]

Open question!

	MuJoCo Task		
Step	Walker2d-v2	Hopper-v2	Humanoid-v2
PPO	3292 [3157, 3426]	2513 [2391, 2632]	806 [785, 827]
PPO-M	2735 [2602, 2866]	2142 [2008, 2279]	674 [656, 695]
TRPO	2791 [2709, 2873]	2043 [1948, 2136]	586 [576, 596]
TRPO+	3050 [2976, 3126]	2466 [2381, 2549]	1030 [979, 1083]
AAI	242	99	224
ACLI	557	421	444

Code optimizations may make a bigger impact than algorithmic ones
→ needs more investigation!

Engstrom et al, ‘20

Lecture outline

Making NPG Practical → Trust Region Policy Optimization

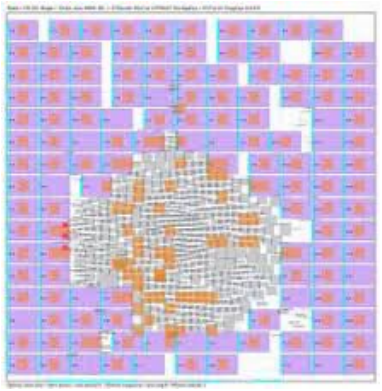
Making FIM Inversion Tractable → K-FAC

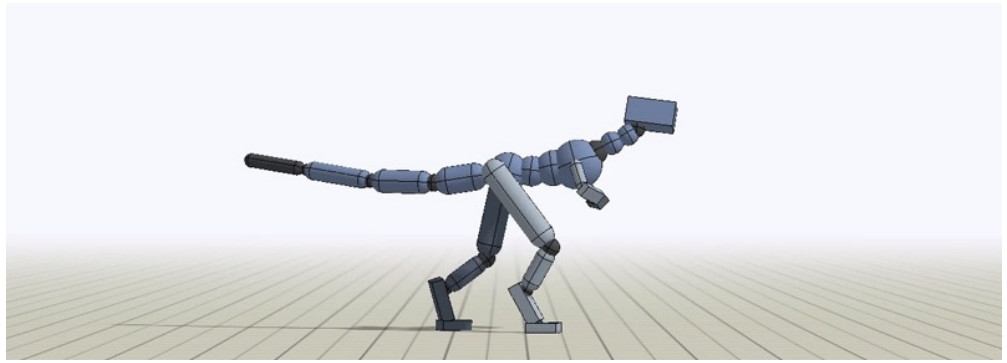
One Algorithm to Rule Them All - Proximal Policy Optimization

Pros/Cons of Policy Gradient Methods

Pros

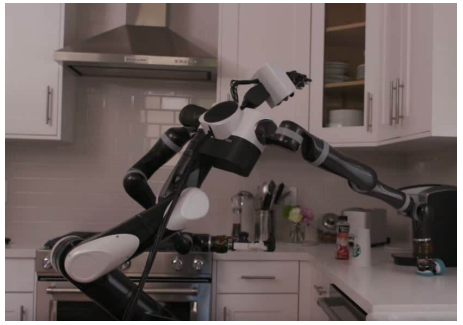
- Conceptually simple, easy to implement
- Stable, good asymptotic performance
- Compatible with deep models
- Require minimal modeling

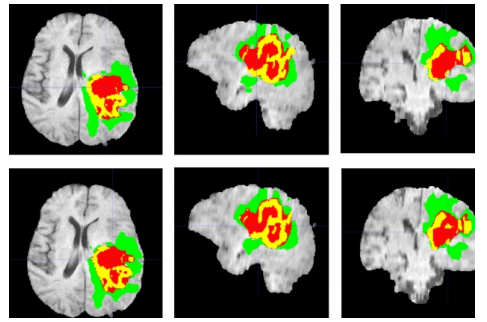




Cons

- Sample inefficient
- Unable to reuse prior data effectively → on-policy
- Blackbox, can be hard to debug





Frontiers of Policy Gradient Research

Major open challenges in policy gradient research:

- Convergence guarantees
- Asynchronous/Parallel Methods
- Better Variance Reduction
- Learning from high-dimensional inputs
- Bootstrapping from prior data
- Multi-agent Policy Gradient

Frontiers of Policy Gradient Research

Convergence guarantees and empirical investigations

Globally Convergent in LQR/LQG Case

- *Gradient descent case: For an appropriate (constant) setting of the stepsize η,*

$$\eta = \text{poly}\left(\frac{\mu\sigma_{min}(Q)}{C(K_0)}, \frac{1}{\|A\|}, \frac{1}{\|B\|}, \frac{1}{\|R\|}, \sigma_{min}(R)\right)$$

and for

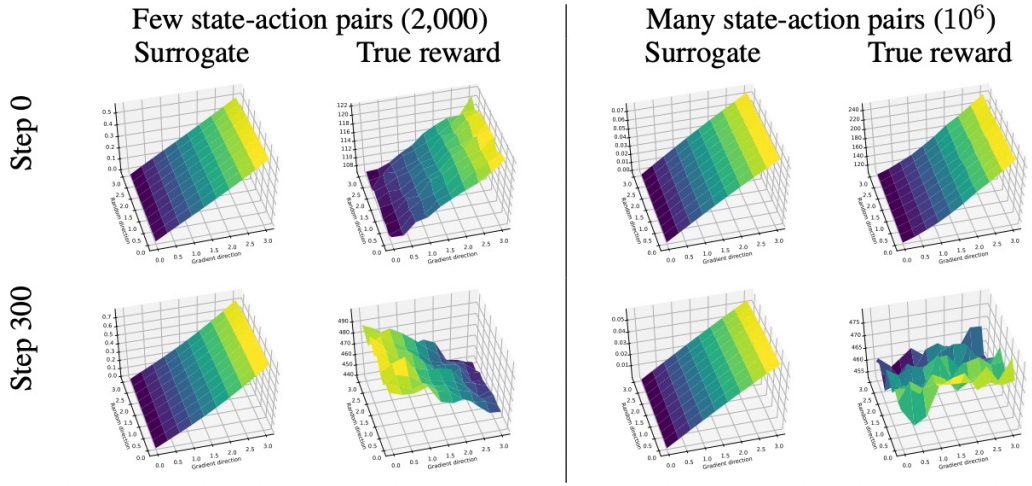
$$N \geq \frac{\|\Sigma_{K^*}\|}{\mu} \log \frac{C(K_0) - C(K^*)}{\varepsilon} \times \text{poly}\left(\frac{C(K_0)}{\mu\sigma_{min}(Q)}, \|A\|, \|B\|, \|R\|, \frac{1}{\sigma_{min}(R)}\right),$$

then, with high probability, gradient descent (Equation 8) enjoys the following performance bound:

$$C(K_N) - C(K^*) \leq \varepsilon\,.$$

Global Convergence of Policy Gradient Methods for the Linear Quadratic Regulator, Fazel et al '19
Global Convergence of Policy Gradient Methods to (Almost) Locally Optimal Policies, Zhang et al, '19
Globally convergent policy search over dynamic filters for output estimation, Umenberger '21

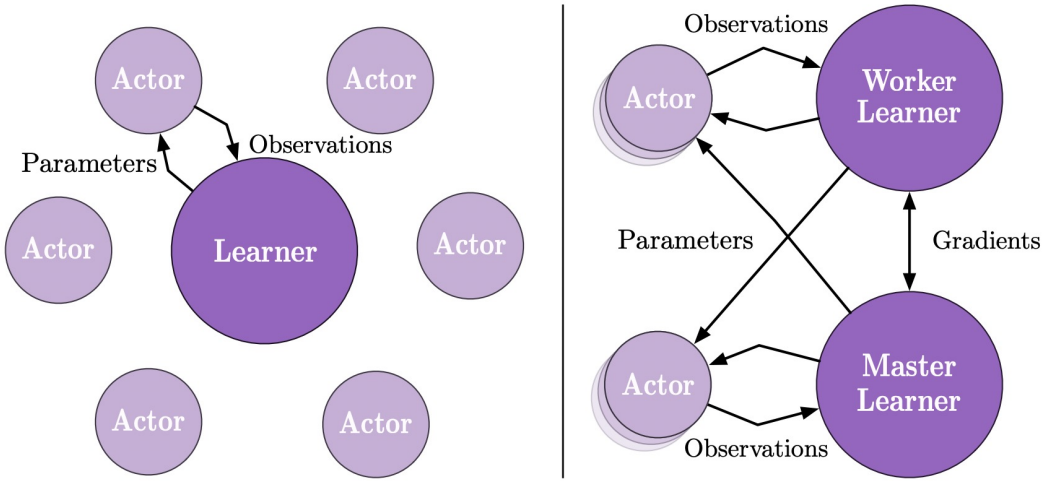
Practical Algorithms Deviate from Theory



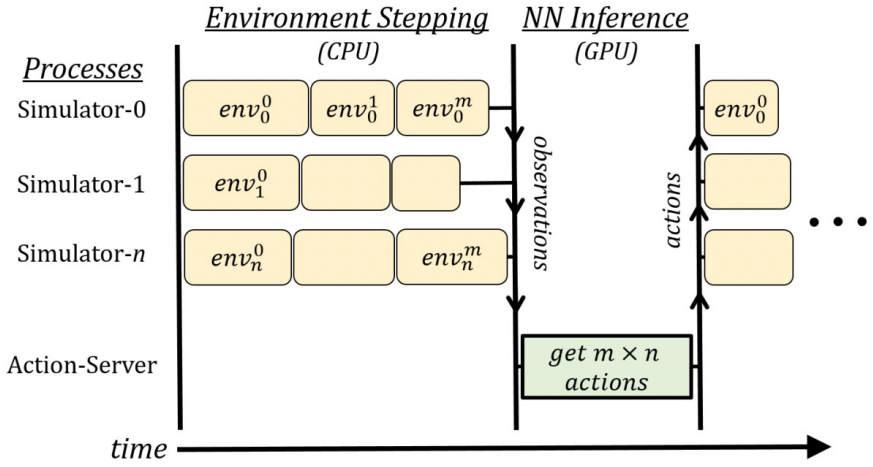
Is the Policy Gradient a Gradient?, Nota et al, '19
A Closer Look at Deep Policy Gradients, Ilyas et al '19
An Empirical Analysis of Proximal Policy Optimization with Kronecker-factored Natural Gradients, Song et al '18
What Matters In On-Policy Reinforcement Learning? A Large-Scale Empirical Study, Andrychowicz et al '20

Frontiers of Policy Gradient Research

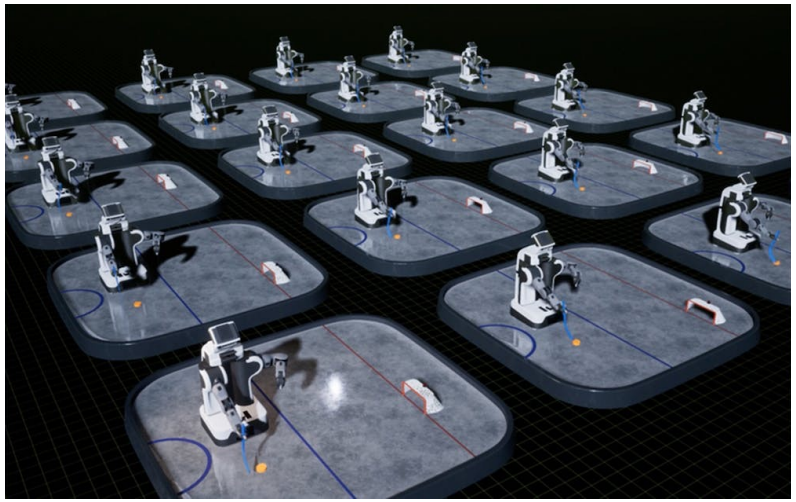
Asynchronous methods for large scale speedup



IMPALA: Scalable Distributed Deep-RL with Importance Weighted Actor-Learner Architectures, Espeholt '18



Accelerated Methods for Deep Reinforcement Learning, Stooke et al '19



Frontiers of Policy Gradient Research

Better Variance Reduction Methods

Action dependent baselines

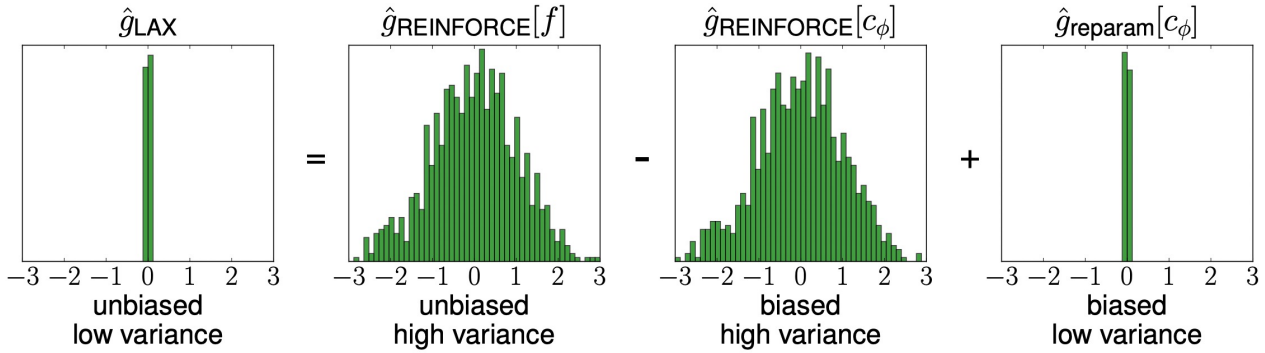
$$\pi_\theta(a_t|s_t) = \prod_{i=1}^{m} \pi_\theta(a_t^i|s_t)$$

$$\nabla_\theta \eta(\pi_\theta) = \mathbb{E}_{\rho_\pi,\pi}\left[\sum_{i=1}^{m} \nabla_\theta \log \pi_\theta(a_t^i|s_t)\left(\hat{Q}(s_t,a_t) - b_i(s_t,a_t^{-i})\right)\right]$$

For factorized spaces, baselines can depend on independent action factors

The Mirage of Action-Dependent Baselines in Reinforcement Learning, Tucker et al '18
Variance Reduction for Policy Gradient with Action-Dependent Factorized Baselines, Wu et al '18

Alternative Estimators

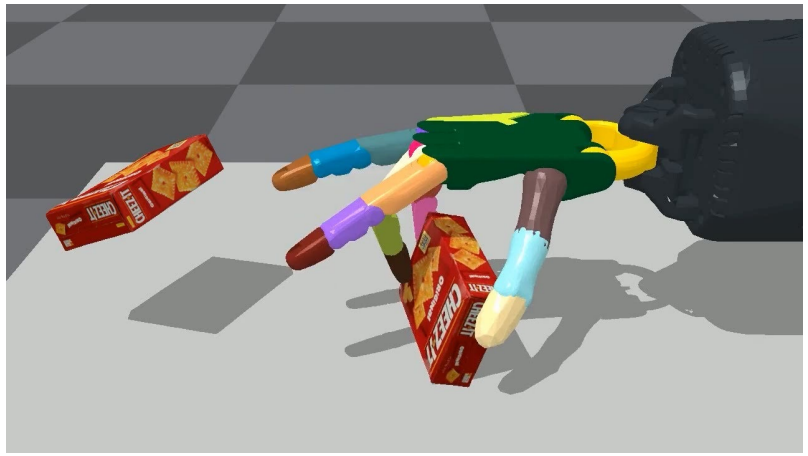


Q-Prop: Sample-Efficient Policy Gradient with An Off-Policy Critic, Gu et al '16
Backpropagation through the Void: Optimizing control variates for black-box gradient estimation, Grathwohl et al '17
Categorical Reparameterization with Gumbel-Softmax, Jang et al '16

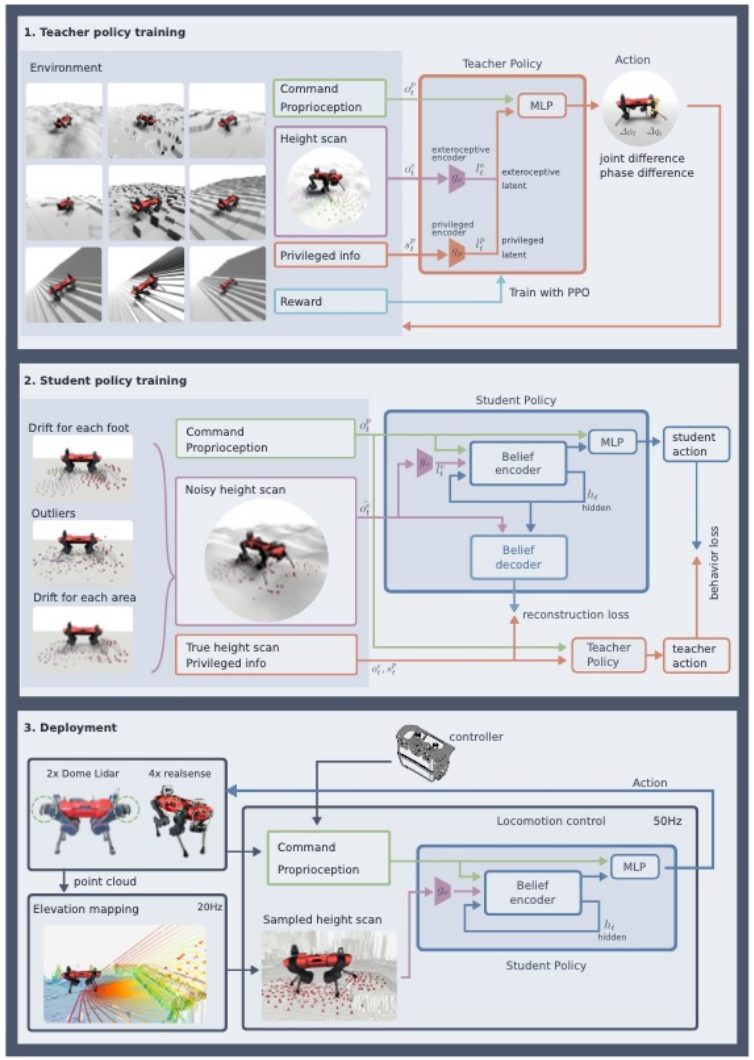
Frontiers of Policy Gradient Research

Learning from High Dimensional Observations

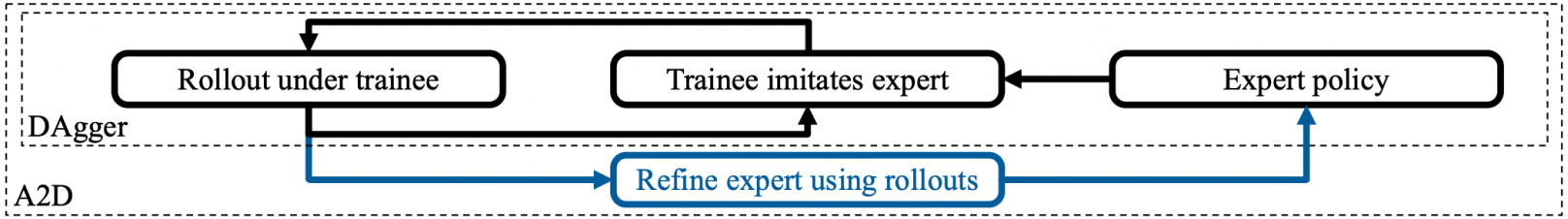
Learning Quadrupedal Locomotion over Challenging Terrain, Lee et al '20



A System for General In-Hand Reorientation, Chen et al '21



Challenging to provide guarantees in partially observed settings!



Robust Asymmetric Learning in POMDPs, Warrington et al '20

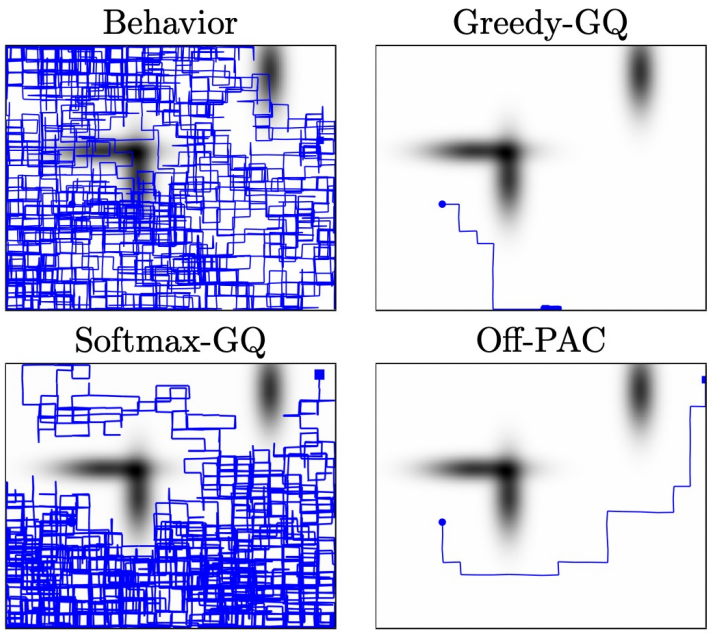
Learning robust perceptive locomotion for quadrupedal robots in the wild, Miki et al '22

Frontiers of Policy Gradient Research

Bootstrapping from Prior/Off-Policy Data

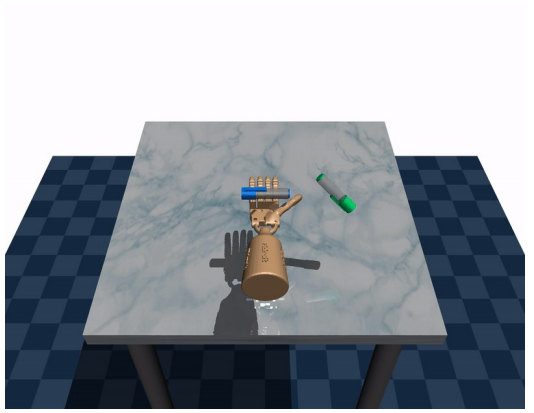
Off-policy policy gradient

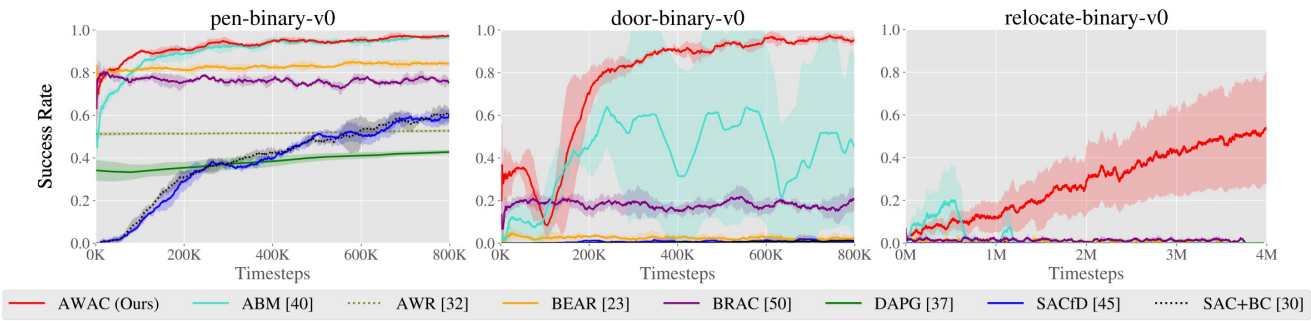
$$\mathbb{E}_\beta\left[\frac{\pi_\theta(a|s)}{\beta(a|s)}Q^\pi(s,a)\nabla_\theta \ln \pi_\theta(a|s)\right]$$



Off-Policy Actor-Critic, Degris et al '13

Learning from Prior Data

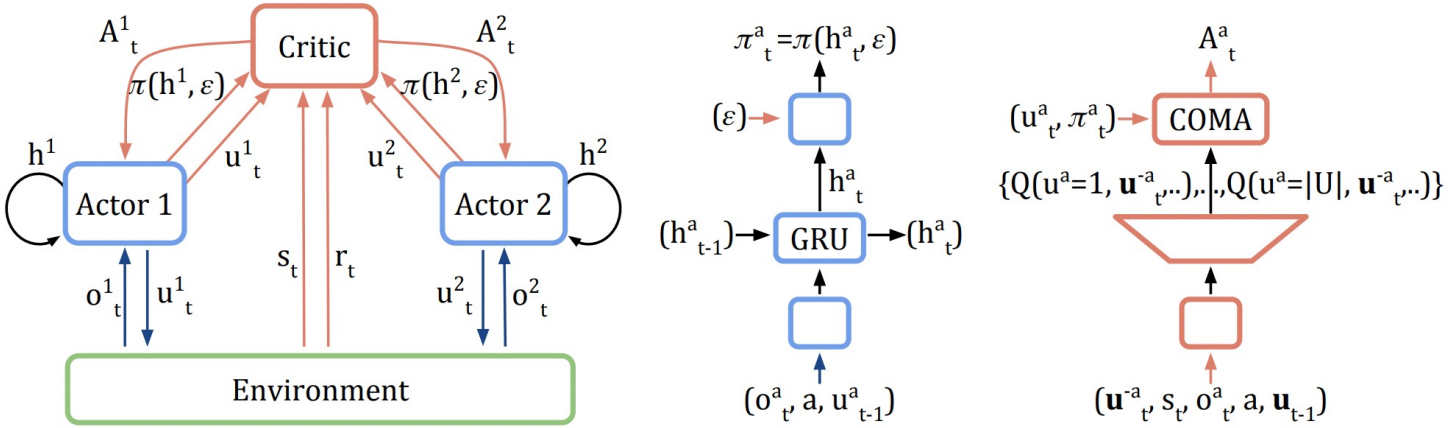




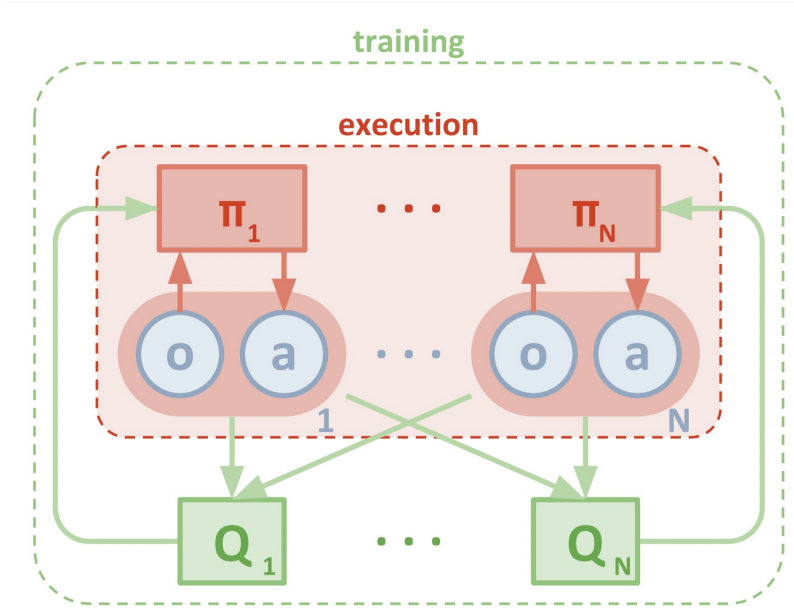
Advantage Weighted Actor Critic, Nair et al '20
DDPGfD, Vecerik '17
DAPG, Rajeswaran '17

Frontiers of Policy Gradient Research

Multi-agent policy gradient



Counterfactual Multi-Agent Policy Gradients, Foerster et al '17



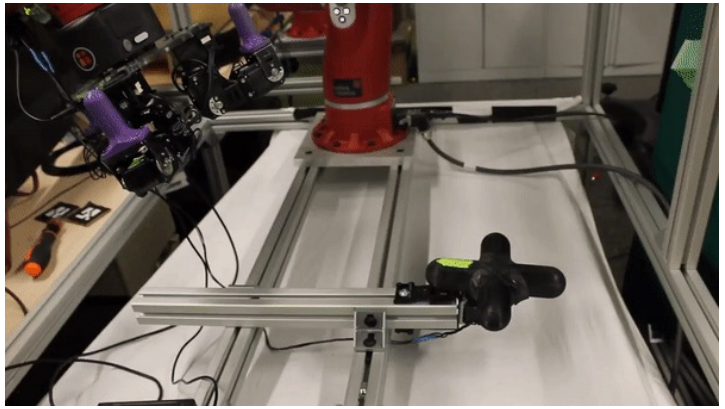
Multi-Agent Actor-Critic for Mixed Cooperative-Competitive Environments, Lowe et al '17

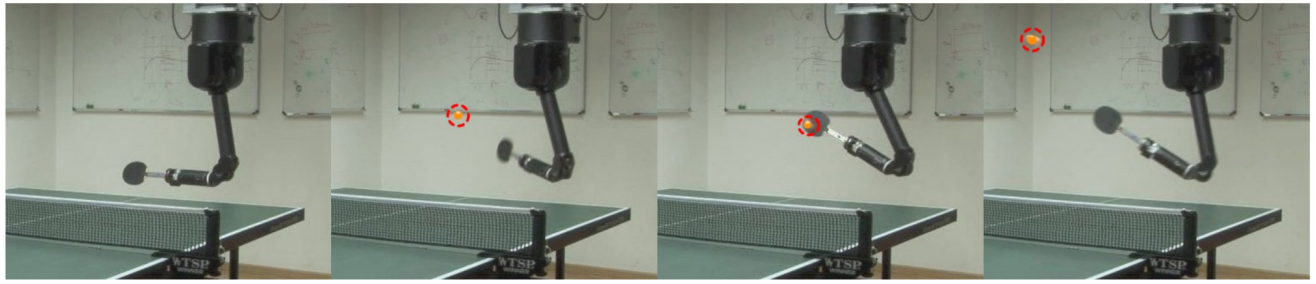
Primary challenges:

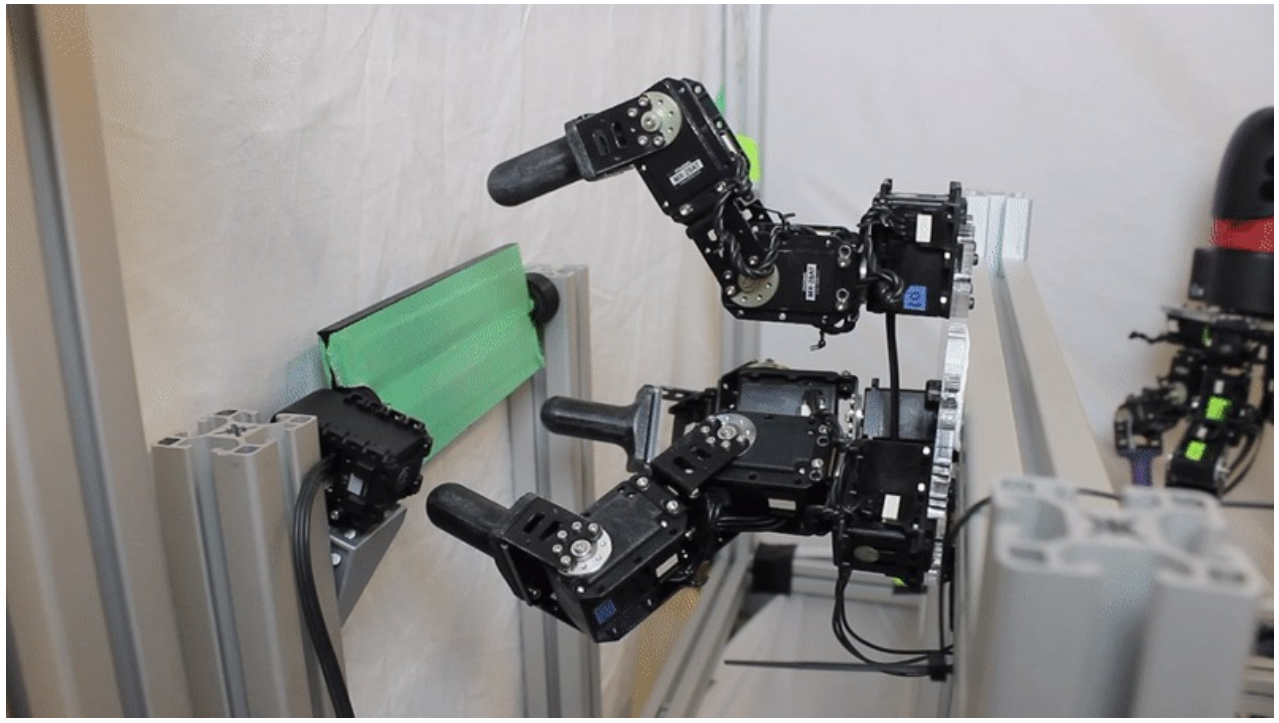
1. Non-stationarity
2. Data-efficiency
3. Communication

How is this useful for robotics?

Can be used to train robots in the real world but only in limited settings

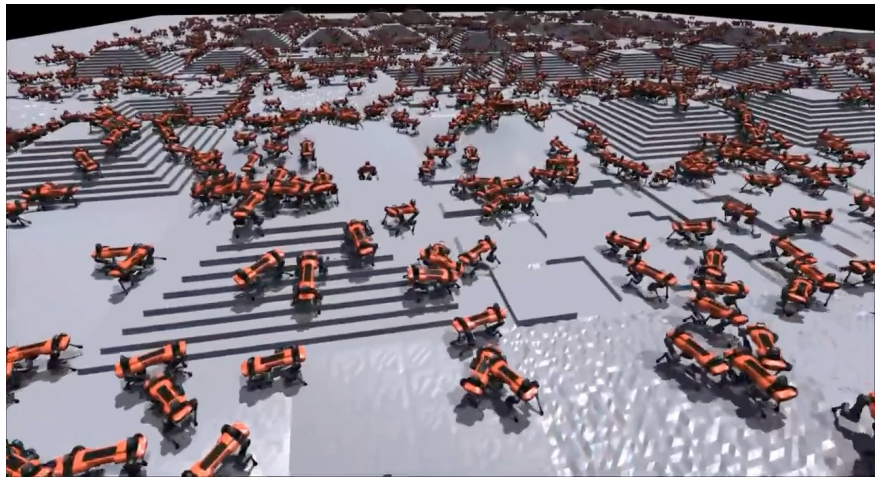


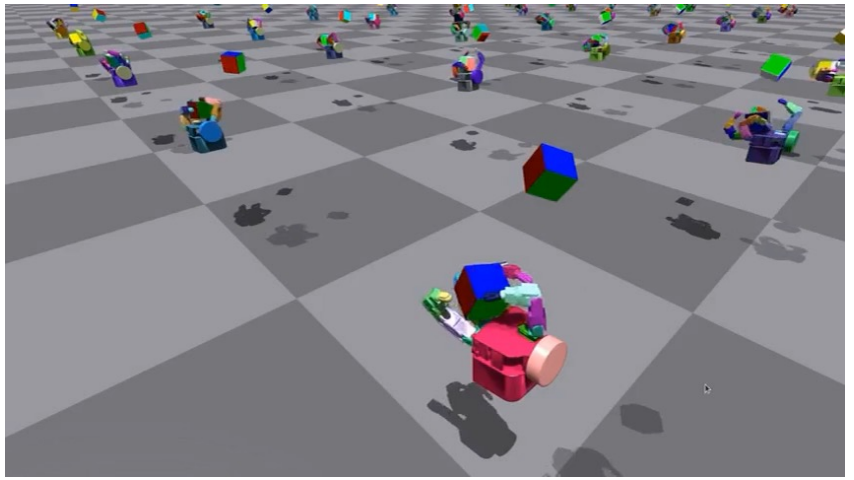


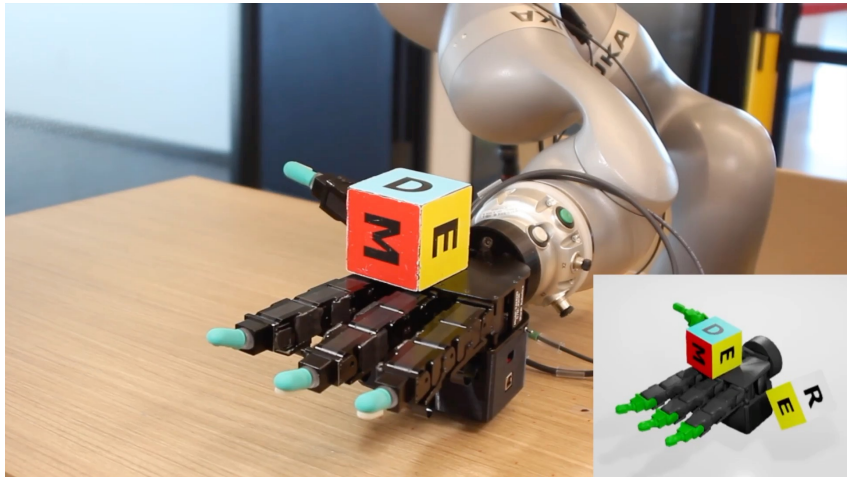


How is this useful for robotics?

Largely useful for pretraining in simulation







More in the sim2real lecture!

Summary

- Policy gradient methods form an effective solution technique to the RL problem
- Techniques range from vanilla policy gradient to NPG to PPO, each with it's own pros and cons
- PG can be very adept at solving black-box optimization asymptotically, but can be very slow
- Several open frontiers still exist for research into PG methods
- Most promising use of PG methods in robotics is through simulation to reality transfer

Fin.