Reinforcement Learning
Spring 2024

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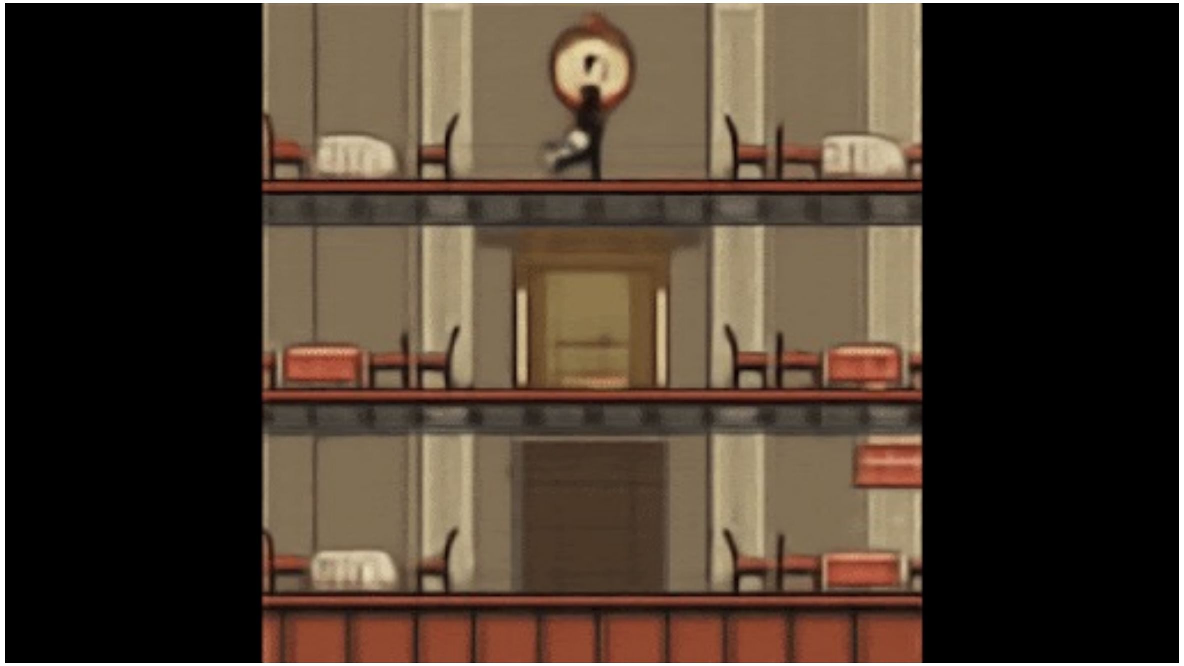
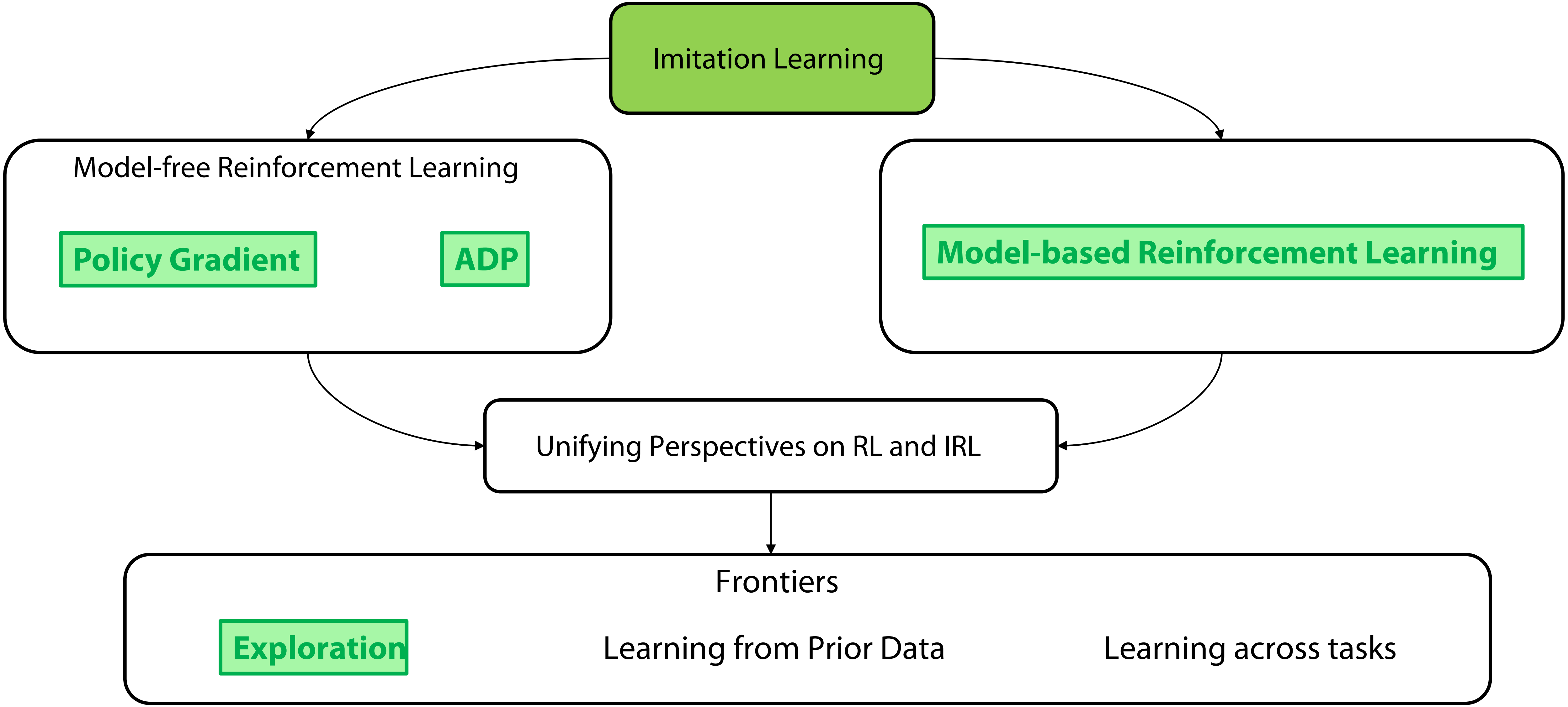


Class Structure



Imitation Learning

Model-free Reinforcement Learning

Policy Gradient

ADP

Model-based Reinforcement Learning

Unifying Perspectives on RL and IRL

Frontiers

Exploration

Learning from Prior Data

Learning across tasks

Past lecture outline

Control as Inference - Formulation

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Variational Inference

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Control as Inference to Derive Policy Gradient

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Control as Inference to Derive Q-learning

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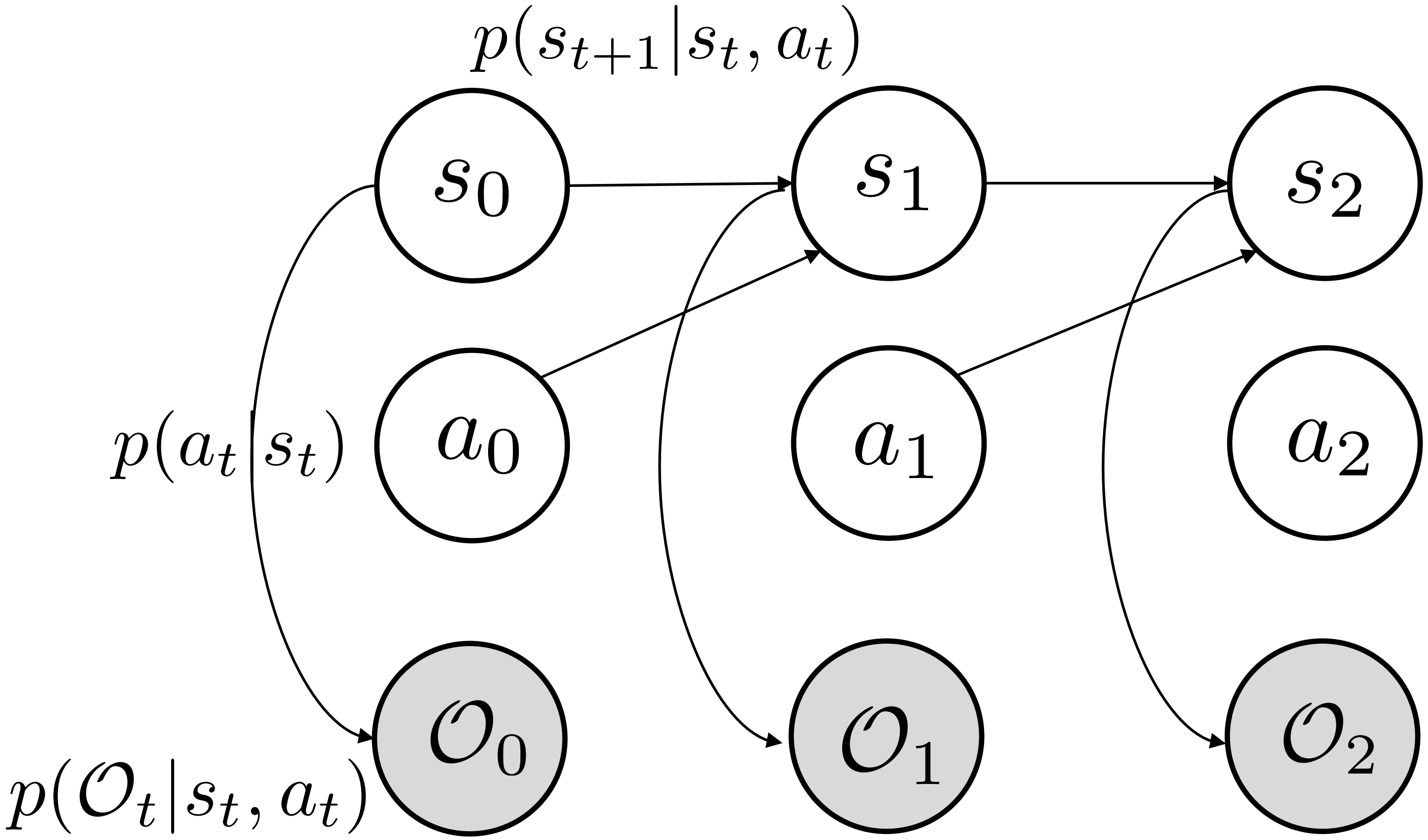
Control as Inference to Derive Model-Based RL

Using Probabilistic Graphical Models for Decision Making



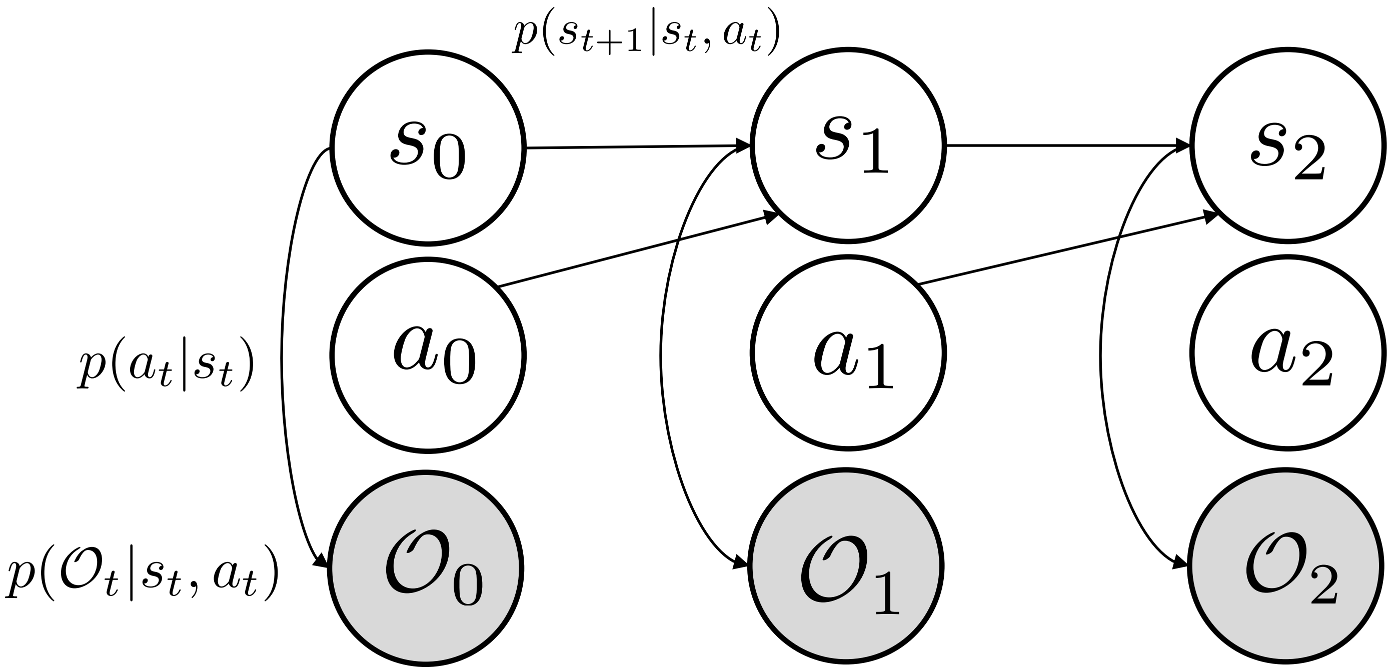
$$p(\mathcal{O}_t|s_t, a_t) = \exp(r(s_t, a_t))$$

Rewards must be negative
(subtract max reward WLOG)

Introduce binary "optimality" variables – optimal if O=1, suboptimal if O=0

Agents are observed to be **optimal**

So what are we doing inference over?



$$p(\mathcal{O}_t|s_t, a_t) = \exp(r(s_t, a_t))$$

$$p(\tau|\mathcal{O}_{0:T} = 1) \propto p(\tau) \exp(\sum_{t=0}^{T} r(s_t, a_t))$$

Use case 1:

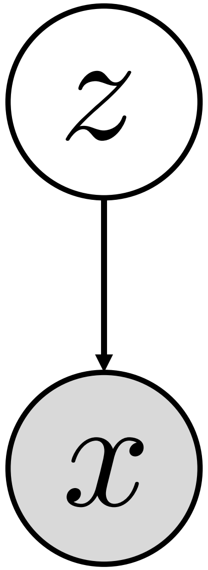
Derive soft RL algorithms

Insight: Computing optimal policy → posterior inference

$$p(a_t|s_t, \mathcal{O}_{t:T} = 1)$$

“Given that you are acting optimally, what is the likelihood of a particular action at a state”

Evidence Lower Bound: Best Posterior



$$p(z|x) = \frac{p(x|z)p(z)}{p(x)}$$

Intractable!

View 1: Find best posterior

$$D_{KL}(q_\phi(z|x)||p(z|x))$$

$$= D_{KL}(q(z|x)||p(z)) - \mathbb{E}_{z\sim q(z|x)}\left[\log p(x|z)\right] + \log p(x)$$

Likelihood/prior known – posterior hard to compute

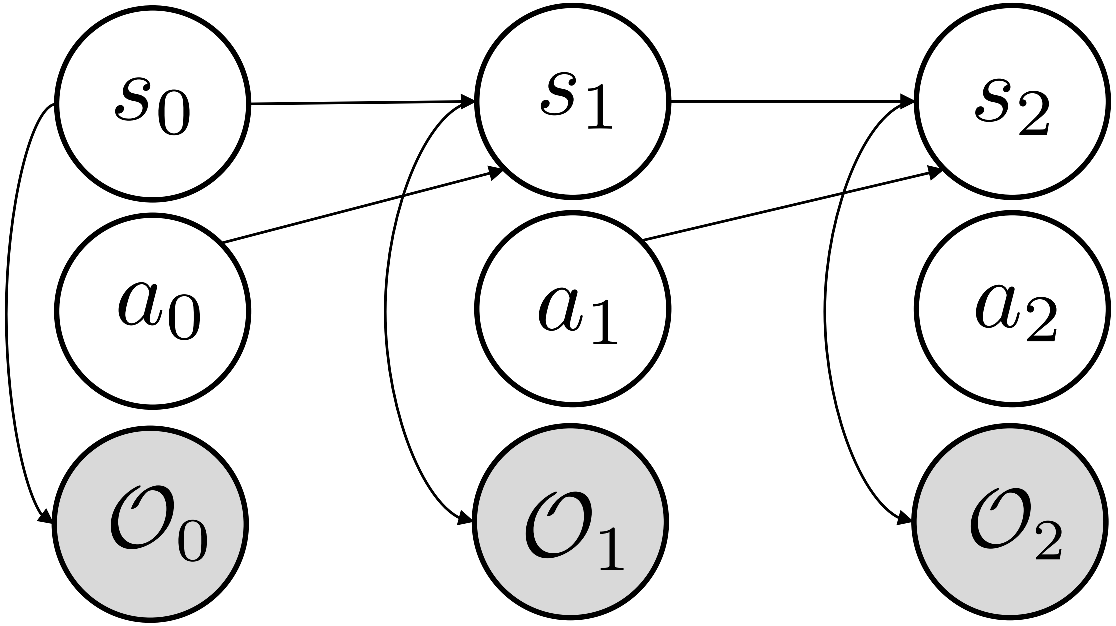
Maximum likelihood

Stay close to the prior

$$\max_q \mathbb{E}_{x\sim p(x)}\left[\mathbb{E}_{z\sim q(z|x)}\left[\log p(x|z)\right] - D_{KL}(q(z|x)||p(z))\right]$$

Learn a tractable posterior q(z|x) with known likelihood and sampling

Lets revisit our original inference problem in control



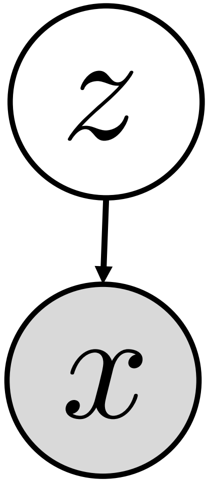
Optimal Policy → Posterior Inference

$$p(a_t|s_t, \mathcal{O}_{t:T} = 1)$$

$$= \frac{p(a_t, \mathcal{O}_{t:T} = 1|s_t)}{p(\mathcal{O}_{t:T} = 1|s_t)}$$

$$= \frac{\int\int\cdots\int p(a_{t:T}, \mathcal{O}_{t:T} = 1, s_{t:T})ds_{t+1:T}da_{t+1:T}}{\int\int\cdots\int p(a_{t:T}, \mathcal{O}_{t:T} = 1, s_{t:T})ds_{t+1:T}da_{t:T}}$$

Variational Inference



Approximate $p(a_t|s_t, \mathcal{O}_{t:T} = 1)$ by $q(a_t|s_t, \mathcal{O}_{t:T} = 1)$

$$\max_q \mathbb{E}_{x\sim p(x)} \left[\mathbb{E}_{z\sim q(z|x)} \left[\log p(x|z)\right] - D_{KL}(q(z|x)||p(z))\right]$$

$$x \updownarrow (\mathcal{O}_0, \mathcal{O}_1, \ldots, \mathcal{O}_T)$$

Tractable techniques for posterior policy computation

$$z \updownarrow (s_0, a_0, s_1, a_1, \ldots, s_T, a_T)$$

Computing Evidence Lower Bound

$$x \quad (\mathcal{O}_0, \mathcal{O}_1, \ldots, \mathcal{O}_T) \qquad z\ (s_0, a_0, s_1, a_1, \ldots, s_T, a_T)$$

$$\max_q \mathbb{E}_{x\sim p(x)}\left[\mathbb{E}_{z\sim q(z|x)}[\log p(x|z)] - D_{KL}(q(z|x)||p(z))\right]$$

$$\boxed{q(s_0, a_0, \ldots, s_T, a_T|\mathcal{O}_0, \ldots, \mathcal{O}_T) = p(s_0)\Pi_{t=0}^T p(s_{t+1}|s_t, a_t)q(a_t|s_t)}$$

1)

$$\mathbb{E}_{x\sim p(x), z\sim q(z|x)}\left[\log p(x, z) - \log q(z|x)\right]$$

2)

$$\mathbb{E}_{\substack{s_0\sim p(s_0)\\ a_t\sim q(a_t|s_t)\\ s_{t+1}\sim p(s_{t+1}|s_t,a_t)}}\Big[\log p(s_0, \ldots, s_T, a_0, \ldots, a_T, \mathcal{O}_0, \ldots, \mathcal{O}_T) - \\ \log q(s_0, a_0, \ldots, s_T, a_T|\mathcal{O}_0, \ldots, \mathcal{O}_T)\Big]$$

3)

$$\mathbb{E}_{\substack{s_0\sim p(s_0)\\ a_t\sim q(a_t|s_t)\\ s_{t+1}\sim p(s_{t+1}|s_t,a_t)}}\left[\left[\log p(s_0) + \sum_t [\log p(a_t|s_t) + \log p(s_{t+1}|s_t, a_t) + \log p(\mathcal{O}_t|s_t, a_t)]\right] - \\ \left[\log p(s_0) + \sum_t [\log q(a_t|s_t) + \log p(s_{t+1}|s_t, a_t)]\right]\right]$$

Computing Evidence Lower Bound

$$\max_q \mathbb{E}_{x\sim p(x)}\left[\mathbb{E}_{z\sim q(z|x)}\left[\log p(x|z)\right] - D_{KL}(q(z|x)||p(z))\right]$$

3) $$\mathbb{E}_{\substack{s_0\sim p(s_0)\\ a_t\sim q(a_t|s_t)\\ s_{t+1}\sim p(s_{t+1}|s_t,a_t)}}\left[\left[\cancel{\log p(s_0)} + \sum_t \left[\overset{\text{Set to uniform}}{\log p(a_t|s_t)} + \cancel{\log p(s_{t+1}|s_t,a_t)} + \log p(\mathcal{O}_t|s_t,a_t)\right]\right] - \left[\cancel{\log p(s_0)} + \sum_t \left[\log q(a_t|s_t) + \cancel{\log p(s_{t+1}|s_t,a_t)}\right]\right]\right]$$

4) $$\mathbb{E}_{\substack{s_0\sim p(s_0)\\ a_t\sim q(a_t|s_t)\\ s_{t+1}\sim p(s_{t+1}|s_t,a_t)}}\left[\sum_t \log p(\mathcal{O}_t|s_t,a_t) - \log q(a_t|s_t)\right]$$

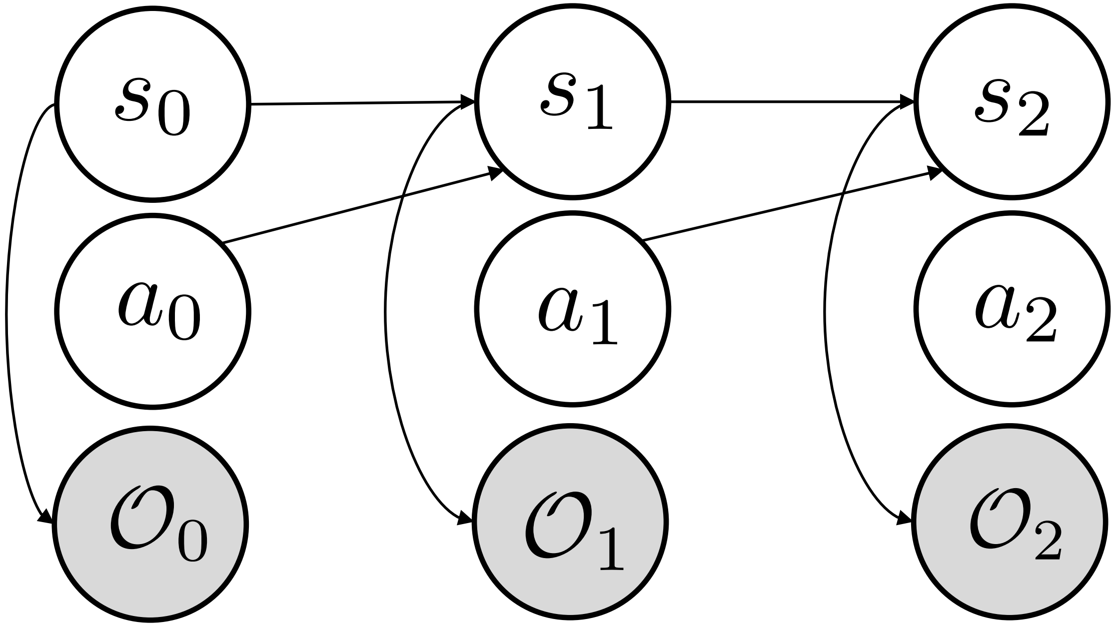
$$\boxed{p(\mathcal{O}_t|s_t,a_t) = \exp(r(s_t,a_t))}$$

5) $$\mathbb{E}_{\substack{s_0\sim p(s_0)\\ a_t\sim q(a_t|s_t)\\ s_{t+1}\sim p(s_{t+1}|s_t,a_t)}}\left[\sum_t r(s_t,a_t) + \mathcal{H}(q(\cdot|s_t))\right]$$

Maximum entropy RL

Gradient ascent = PG!

Ok so what did we show?



Find approximate posterior q(z|x) by optimizing the ELBO

$$\max_q \mathbb{E}_{x\sim p(x)}\left[\mathbb{E}_{z\sim q(z|x)}\left[\log p(x|z)\right] - D_{KL}(q(z|x)||p(z))\right]$$

$$\begin{array}{ccc} x & & z \\ \updownarrow & & \updownarrow \\ (\mathcal{O}_0, \mathcal{O}_1, \ldots, \mathcal{O}_T) & & (s_0, a_0, s_1, a_1, \ldots, s_T, a_T) \end{array}$$

$$\boxed{q(s_0, a_0, \ldots, s_T, a_T | \mathcal{O}_0, \ldots, \mathcal{O}_T) = p(s_0)\Pi_{t=0}^{T} p(s_{t+1}|s_t, a_t) q(a_t|s_t)}$$

$$\mathbb{E}_{\substack{s_0\sim p(s_0)\\ a_t\sim q(a_t|s_t)\\ s_{t+1}\sim p(s_{t+1}|s_t,a_t)}}\left[\sum_t \log p(\mathcal{O}_t|s_t, a_t) - \log q(a_t|s_t)\right] = \mathbb{E}_{\substack{s_0\sim p(s_0)\\ a_t\sim q(a_t|s_t)\\ s_{t+1}\sim p(s_{t+1}|s_t,a_t)}}\left[\sum_t r(s_t, a_t) + \mathcal{H}(q(\cdot|s_t))\right]$$

Maximize ELBO with SGD = policy gradient!

Lecture outline

Control as Inference to Derive Q-learning

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Control as Inference to Derive Model-Based RL

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Why inverse RL? + Problem formulation

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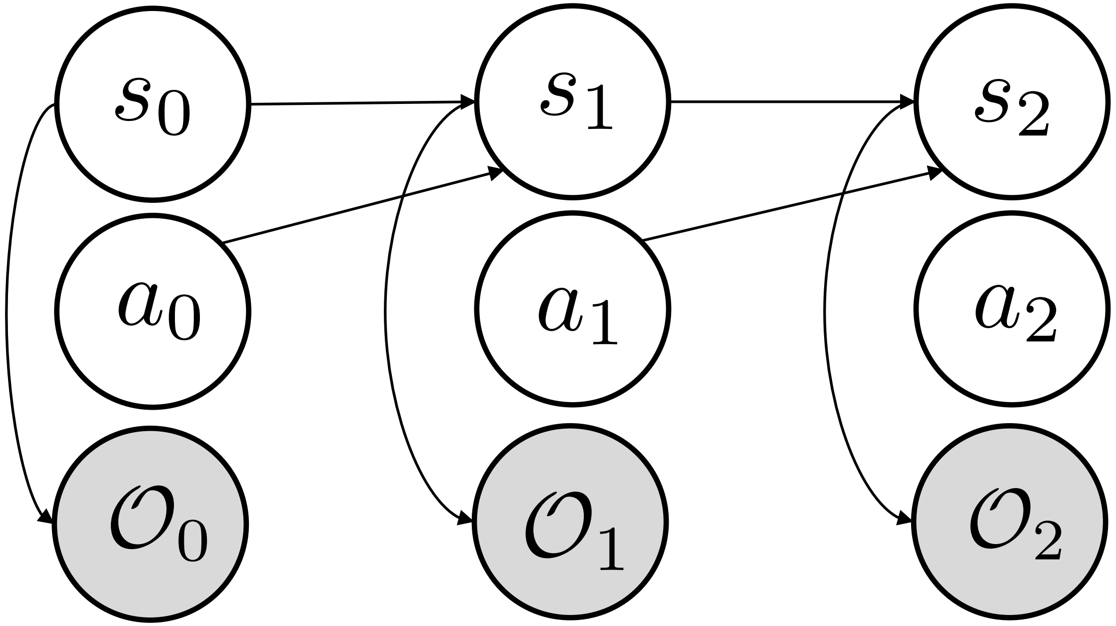
IRLv1 – max margin planning

Can we derive (soft) Q-learning from the ELBO?



Find approximate posterior q(z|x) by optimizing the ELBO

$$\max_q \mathbb{E}_{x\sim p(x)}\left[\mathbb{E}_{z\sim q(z|x)}\left[\log p(x|z)\right] - D_{KL}(q(z|x)||p(z))\right]$$

$$x \quad\quad z$$
$$\Updownarrow \quad\quad \Updownarrow$$
$$(\mathcal{O}_0, \mathcal{O}_1, \ldots, \mathcal{O}_T) \quad\quad (s_0, a_0, s_1, a_1, \ldots, s_T, a_T)$$

$$\boxed{q(s_0, a_0, \ldots, s_T, a_T|\mathcal{O}_0, \ldots, \mathcal{O}_T) = p(s_0)\Pi_{t=0}^{T} p(s_{t+1}|s_t, a_t)q(a_t|s_t)}$$

$$\mathbb{E}_{\substack{s_0\sim p(s_0)\\ a_t\sim q(a_t|s_t)\\ s_{t+1}\sim p(s_{t+1}|s_t,a_t)}}\left[\sum_t \log p(\mathcal{O}_t|s_t, a_t) - \log q(a_t|s_t)\right] = \mathbb{E}_{\substack{s_0\sim p(s_0)\\ a_t\sim q(a_t|s_t)\\ s_{t+1}\sim p(s_{t+1}|s_t,a_t)}}\left[\sum_t r(s_t, a_t) + \mathcal{H}(q(\cdot|s_t))\right]$$

Maximize ELBO with DP = Soft Q learning!

Let's optimize the last step of the ELBO

$$\mathbb{E}_{\substack{s_0 \sim p(s_0) \\ a_t \sim q(a_t|s_t) \\ s_{t+1} \sim p(s_{t+1}|s_t, a_t)}} \left[\sum_t r(s_t, a_t) + \mathcal{H}(q(\cdot|s_t)) \right]$$

Consider the last time step

$$\max \; \mathbb{E}_{s_T \sim q(s_T)} \left[\mathbb{E}_{a_T \sim q(a_T|s_T)} \left[r(s_T, a_T) - \log q(a_T|s_T) \right] \right]$$

(log-exp)

$$= \mathbb{E}_{s_T \sim q(s_T)} \left[\mathbb{E}_{a_T \sim q(a_T|s_T)} \left[\log \exp(r(s_T, a_T)) - \log q(a_T|s_T) \right] \right]$$

$$= \mathbb{E}_{s_T \sim q(s_T)} \left[\mathbb{E}_{a_T \sim q(a_T|s_T)} \left[\log \exp(r(s_T, a_T)) - \log \int \exp(r(s_T, a_T)) da_T - \log q(a_T|s_T) \right] + \log \int \exp(r(s_T, a_T)) da_T \right]$$

(Add subtract to normalize)

$$= \mathbb{E}_{s_T \sim q(s_T)} \left[\mathbb{E}_{a_T \sim q(a_T|s_T)} \left[\log \frac{\exp(r(s_T, a_T))}{\int \exp(r(s_T, a_T)) da_T} - \log q(a_T|s_T) \right] \right]$$

$$= \mathbb{E}_{s_T \sim q(s_T)} \left[\mathbb{E}_{a_T \sim q(a_T|s_T)} \left[- \log \frac{q(a_T|s_T)}{\frac{\exp(r(s_T, a_T))}{\int \exp(r(s_T, a_T)) da_T}} \right] \right] \quad = \mathbb{E}_{s_T \sim q(s_T)} \left[-D_{KL}(q(a_T|s_T) || \frac{\exp(r(s_T, a_T))}{\int \exp(r(s_T, a_T)) da_T}) \right]$$

(Definition of KL divergence)

Let’s optimize the last step of the ELBO

$$\mathbb{E}_{\substack{s_0\sim p(s_0)\\ a_t\sim q(a_t|s_t)\\ s_{t+1}\sim p(s_{t+1}|s_t,a_t)}}\left[\sum_t r(s_t,a_t)+\mathcal{H}(q(\cdot|s_t))\right]$$

Consider the last time step

$$\max\ \mathbb{E}_{s_T\sim q(s_T)}\left[\mathbb{E}_{a_T\sim q(a_T|s_T)}\left[r(s_T,a_T)-\log q(a_T|s_T)\right]\right]$$

$$=\mathbb{E}_{s_T\sim q(s_T)}\left[-D_{KL}(q(a_T|s_T)||\frac{\exp(r(s_T,a_T))}{\int\exp(r(s_T,a_T))da_T})\right]$$

KL-divergence D(p, q) is always non negative and is minimized when p == q

$$\boxed{q(a_T|s_T)=\frac{\exp(r(s_T,a_T))}{\int\exp(r(s_T,a_T))da_T}}$$

Ok let’s simplify

$$Q(s_T,a_T)=r(s_T,a_T)$$

$$V(s_T)=\log\int\exp(r(s_T,a_T))da_T$$

⇒

$$q(a_T|s_T)=\exp(Q(s_T,a_T)-V(s_T))$$

Optimal policy is proportional to exponential advantage (soft-max)

Let's optimize the last step of the ELBO

$$\mathbb{E}_{\substack{s_0 \sim p(s_0) \\ a_t \sim q(a_t|s_t) \\ s_{t+1} \sim p(s_{t+1}|s_t, a_t)}} \left[\sum_t r(s_t, a_t) + \mathcal{H}(q(\cdot|s_t)) \right]$$

Consider the last time step

$$\max \; \mathbb{E}_{s_T \sim q(s_T)} \left[\mathbb{E}_{a_T \sim q(a_T|s_T)} \left[r(s_T, a_T) - \log q(a_T|s_T) \right] \right]$$

$$= \mathbb{E}_{s_T \sim q(s_T)} \left[-D_{KL}\left(q(a_T|s_T) \,||\, \frac{\exp(r(s_T, a_T))}{\underset{\uparrow}{\int} \exp(r(s_T, a_T)) da_T}\right) + \log \underset{\uparrow}{\int \exp(r(s_T, a_T)) da_T} \right] \leftarrow$$

(added back)

Zero on optimal q

Simply the value $V(s_T)$

$$= \mathbb{E}_{s_T \sim q(s_T)} \left[V(s_T) \right]$$

Simply the expected **value**

$$q(a_T|s_T) = \exp(Q(s_T, a_T) - V(s_T))$$

Optimal policy is proportional to exponential advantage (soft-max)

Let’s optimize the step before

$$\mathbb{E}_{\substack{s_0 \sim p(s_0) \\ a_t \sim q(a_t|s_t) \\ s_{t+1} \sim p(s_{t+1}|s_t, a_t)}} \left[\sum_t r(s_t, a_t) + \mathcal{H}(q(\cdot|s_t)) \right]$$

Consider the second last time step

$$\arg\max_q \mathbb{E}_{s_{T-1} \sim q(s_{T-1})} \left[\mathbb{E}_{a_{T-1} \sim q(a_{T-1}|s_{T-1})} \left[r(s_{T-1}, a_{T-1}) - \log q(a_{T-1}|s_{T-1}) + \mathbb{E}_{\substack{s_T \sim p(s_T|s_{T-1}, a_{T-1}) \\ a_T \sim q(a_T|s_T)}} [r(s_T, a_T) - \log q(a_T|s_T)] \right] \right]$$

Exactly what we computed in the last step

$$\arg\max_q \mathbb{E}_{s_{T-1} \sim q(s_{T-1})} \left[\mathbb{E}_{a_{T-1} \sim q(a_{T-1}|s_{T-1})} \left[r(s_{T-1}, a_{T-1}) - \log q(a_{T-1}|s_{T-1}) + \mathbb{E}_{s_T \sim p(s_T|s_{T-1}, a_{T-1})} [V(s_T)] \right] \right]$$

From the last slide

Let us call this $Q(s_{T-1}, a_{T-1})$

$$Q(s_{T-1}, a_{T-1}) = r(s_{T-1}, a_{T-1}) + \mathbb{E}_{s_T \sim p(s_T|s_{T-1}, a_{T-1})} [V(s_T)]$$

(Looks like Bellman!)

$$\arg\max_q \mathbb{E}_{s_{T-1} \sim q(s_{T-1})} \left[\mathbb{E}_{a_{T-1} \sim q(a_{T-1}|s_{T-1})} \left[Q(s_{T-1}, a_{T-1}) - \log q(a_{T-1}|s_{T-1}) \right] \right]$$

Looks a lot like the previous time-step

Let’s optimize the step before

$$\mathbb{E}_{\substack{s_0\sim p(s_0)\\ a_t\sim q(a_t|s_t)\\ s_{t+1}\sim p(s_{t+1}|s_t,a_t)}}\left[\sum_t r(s_t,a_t)+\mathcal{H}(q(\cdot|s_t))\right]$$

Consider the second last time step

$$\arg\max_q \mathbb{E}_{s_{T-1}\sim q(s_{T-1})}\left[\mathbb{E}_{a_{T-1}\sim q(a_{T-1}|s_{T-1})}\left[Q(s_{T-1},a_{T-1})-\log q(a_{T-1}|s_{T-1})\right]\right]$$

$$Q(s_{T-1},a_{T-1})=r(s_{T-1},a_{T-1})+\mathbb{E}_{s_T\sim p(s_T|s_{T-1},a_{T-1})}[V(s_T)]$$

Referring back to the last time step math and pattern matching

$$V(s_{T-1})=\log\int \exp(Q(s_{T-1},a_{T-1}))da_{T-1}$$

$$Q(s_{T-1},a_{T-1})=r(s_{T-1},a_{T-1})+\mathbb{E}_{s_T\sim p(s_T|s_{T-1},a_{T-1})}[V(s_T)]$$

$$q(a_{T-1}|s_{T-1})=\exp(Q(s_{T-1},a_{T-1})-V(s_{T-1}))$$

Optimal policy is proportional to exponential advantage (soft-max)

Let’s make it recursive

This suggests a recursive dynamic programming algorithm!

$$Q(s_T, a_T) = r(s_T, a_T)$$

$$V(s_T) = \log \int \exp(r(s_T, a_T)) da_T$$

For t = T-1 to 1:

$$Q_t(s_t, a_t) = r(s_t, a_t) + \mathbb{E}_{s_{t+1} \sim p(s_{t+1}|s_t, a_t)} [V_{t+1}(s_{t+1})]$$

(Bellman update)

$$V_t(s_t) = \log \int \exp(Q(s_t, a_t)) da_t$$

(Soft-max)

$$q(a_t|s_t) = \exp(Q_t(s_t, a_t) - V_t(s_t))$$

(Soft-max)

Very similar to the “soft” (entropy) Q-learning procedure from earlier lectures!

What does this suggest as an algorithm?

Optimize a "soft" Bellman equation

$$Q(s_t, a_t) \leftarrow r_t + \gamma \mathbb{E}_{s_{t+1} \sim p_s} \left[V(s_{t+1}) \right]$$

$$V(s_t) \leftarrow \max_a Q(s_t, a)$$

$$\pi(a|s_t) \leftarrow \arg\max_a Q(s_t, a)$$

$$Q_{\text{soft}}(s_t, a_t) \leftarrow r_t + \gamma \mathbb{E}_{s_{t+1} \sim p_s} \left[V_{\text{soft}}(s_{t+1}) \right]$$

$$V_{\text{soft}}(s_t) \leftarrow \alpha \log \int_{\mathcal{A}} \exp\left(\frac{1}{\alpha} Q_{\text{soft}}(s_t, a') \right) da'$$

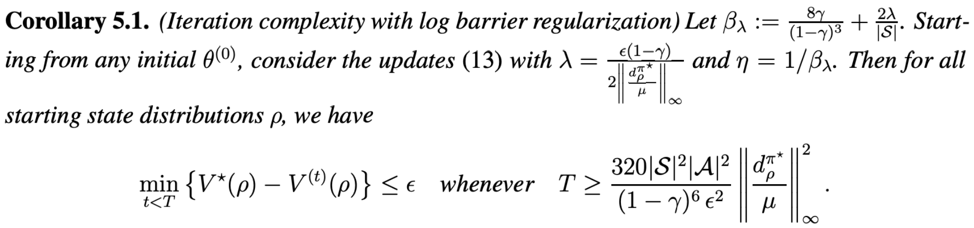
$$\pi_{\text{soft}}(a|s_t) = \exp\left(\frac{1}{\alpha} (Q_{\text{soft}}(s_t, a) - V_{\text{soft}}(s_t)) \right)$$

Go from max to "softmax" (imagine if α goes to 0, it becomes a max)

Prevents premature collapse of exploration while smoothing out optimization landscape!

Why should we ever do soft-Q learning?

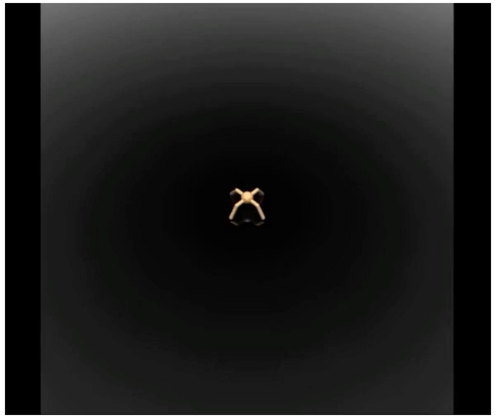
Optimization benefits



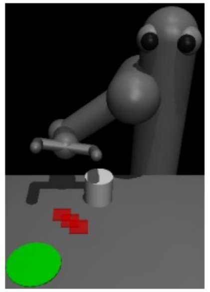
Corollary 5.1. *(Iteration complexity with log barrier regularization) Let* $\beta_\lambda := \frac{8\gamma}{(1-\gamma)^3} + \frac{2\lambda}{|\mathcal{S}|}$*. Starting from any initial* $\theta^{(0)}$*, consider the updates* (13) *with* $\lambda = \frac{\epsilon(1-\gamma)}{2\left\|\frac{d_\rho^{\pi^\star}}{\mu}\right\|_\infty}$ *and* $\eta = 1/\beta_\lambda$*. Then for all starting state distributions* ρ*, we have*

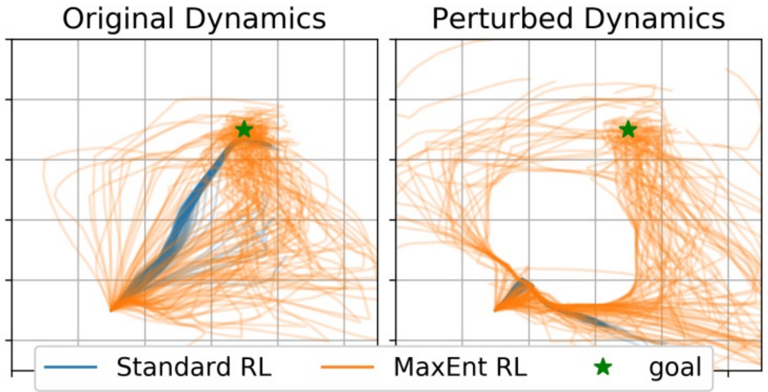
$$\min_{t<T}\left\{V^\star(\rho) - V^{(t)}(\rho)\right\} \le \epsilon \quad \textit{whenever} \quad T \ge \frac{320|\mathcal{S}|^2|\mathcal{A}|^2}{(1-\gamma)^6\,\epsilon^2}\left\|\frac{d_\rho^{\pi^\star}}{\mu}\right\|_\infty^2.$$

Transfer



Deals better with misspecification





Ok so what did we show?

Find approximate posterior q(z|x) by optimizing the ELBO using dynamic programming

$$\max_q \mathbb{E}_{x \sim p(x)} \left[\mathbb{E}_{z \sim q(z|x)} \left[\log p(x|z) \right] - D_{KL}(q(z|x) || p(z)) \right]$$

$$\mathbb{E}_{\substack{s_0 \sim p(s_0) \\ a_t \sim q(a_t|s_t) \\ s_{t+1} \sim p(s_{t+1}|s_t, a_t)}} \left[\sum_t \log p(\mathcal{O}_t | s_t, a_t) - \log q(a_t|s_t) \right] = \mathbb{E}_{\substack{s_0 \sim p(s_0) \\ a_t \sim q(a_t|s_t) \\ s_{t+1} \sim p(s_{t+1}|s_t, a_t)}} \left[\sum_t r(s_t, a_t) + \mathcal{H}(q(\cdot|s_t)) \right]$$

Can derive a “soft” dynamic programming Q-learning update

For t = T-1 to 1:

$$Q_t(s_t, a_t) = r(s_t, a_t) + \mathbb{E}_{s_{t+1} \sim p(s_{t+1}|s_t, a_t)} \left[V_{t+1}(s_{t+1}) \right]$$ (Bellman update)

$$V_t(s_t) = \log \int \exp(Q(s_t, a_t)) da_t$$ (Soft-max)

$$q(a_t|s_t) = \exp(Q_t(s_t, a_t) - V_t(s_t))$$ (Soft-max)

Lecture outline

Control as Inference to Derive Q-learning

↓

Control as Inference to Derive Model-Based RL

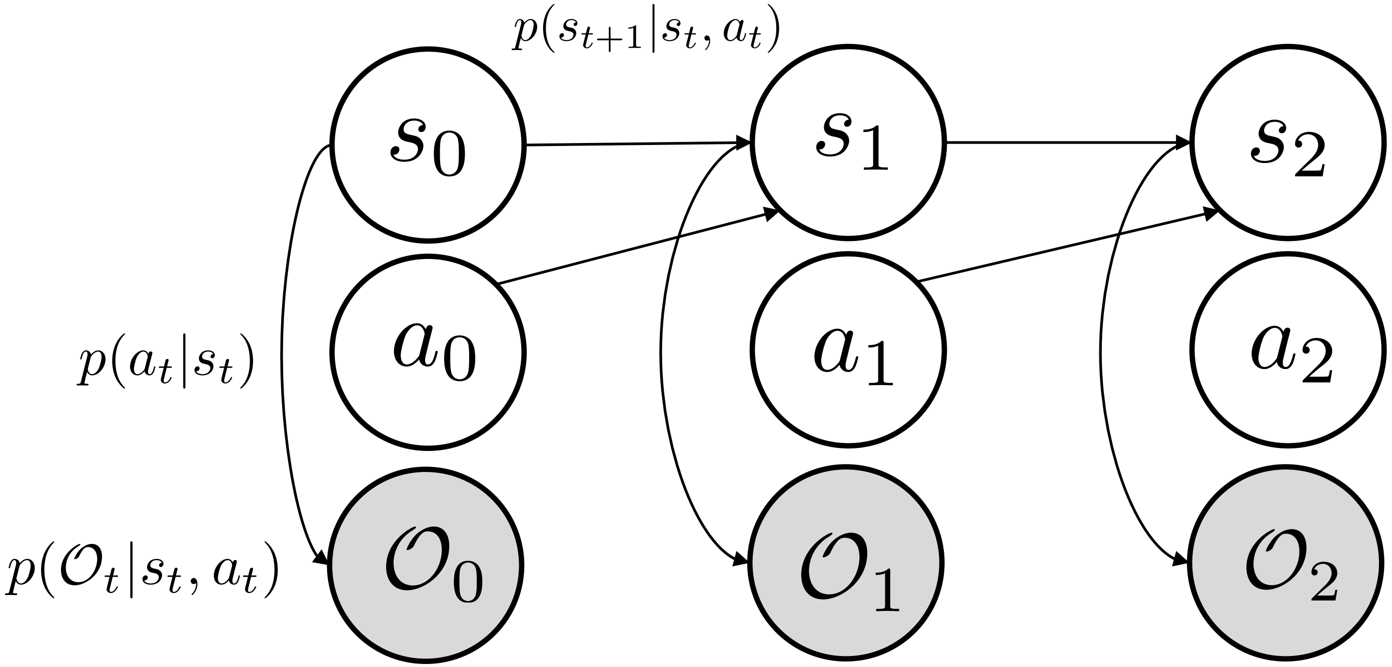
↓

Why inverse RL? + Problem formulation

↓

IRLv1 – max margin planning

Let’s back up from VI to max likelihood



$$p(\mathcal{O}_t|s_t, a_t) = \exp(r(s_t, a_t))$$

$$p(\tau|\mathcal{O}_{0:T} = 1) \propto p(\tau)\exp(\sum_{t=0}^{T} r(s_t, a_t))$$

Let us assume we get a bunch of data of (s, a, s′, r) from the true system p

We will try to learn a surrogate model $\widehat{p}$ to approximate p, use it for posterior inference

Model Learning via Maximum Likelihood

$$\min_{\hat{p}} D_{KL}(p(s_0, \ldots, s_T, a_0, \ldots, a_T, \mathcal{O}_0, \ldots, \mathcal{O}_T) || \hat{p}(s_0, \ldots, s_T, a_0, \ldots, a_T, \mathcal{O}_0, \ldots, \mathcal{O}_T)$$

$$\Big\downarrow$$

Definition of KLD

$$\max_{\hat{p}} \mathbb{E}_{p(s_0, \ldots, s_T, a_0, \ldots, a_T, \mathcal{O}_0, \ldots, \mathcal{O}_T)} [\log \hat{p}(s_0, \ldots, s_T, a_0, \ldots, a_T, \mathcal{O}_0, \ldots, \mathcal{O}_T)]$$

$$\Big\downarrow$$

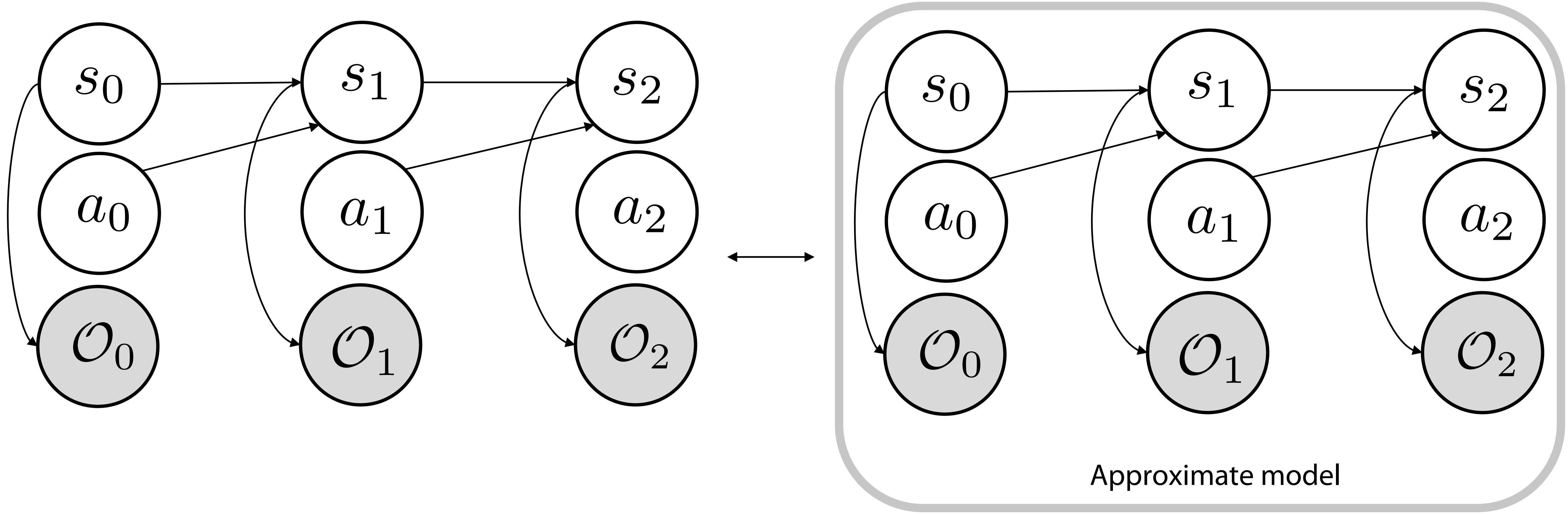
Expansion of joint

$$\max_{\hat{p}} \mathbb{E}_{p(s_0, \ldots, s_T, a_0, \ldots, a_T, \mathcal{O}_0, \ldots, \mathcal{O}_T)} \left[\log \hat{p}(s_0) + \sum_t [\log \hat{p}(s_{t+1}|s_t, a_t) + \log \hat{p}(\mathcal{O}_t|s_t, a_t)]\right]$$

Model learning   Reward learning

Fitting $\widehat{p}$ amounts to supervised learning on dynamics and rewards

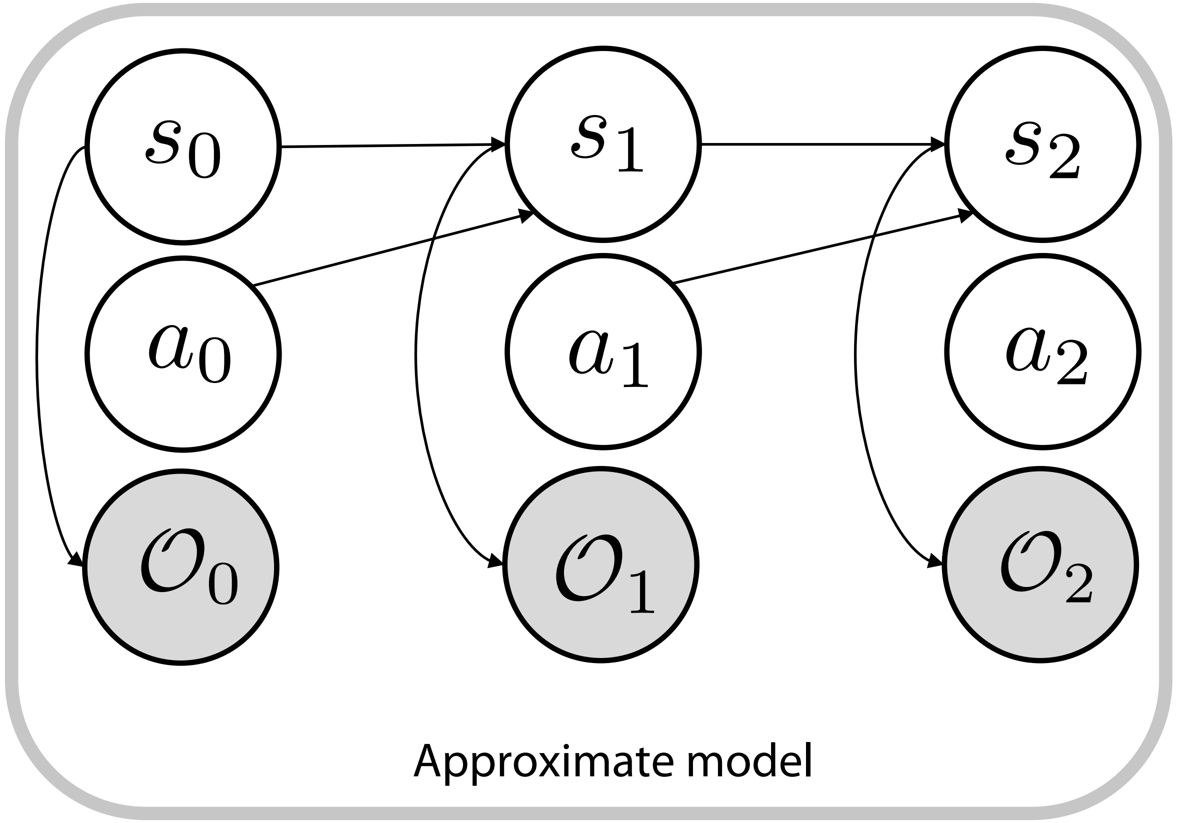
Model Learning via Maximum Likelihood



Fitting $\widehat{p}$ amounts to supervised learning on dynamics and rewards

How do we actually use this approximate model to obtain optimal actions?

Policy Extraction via Posterior Inference



Key idea: pretend that approximate model $\widehat{p}$ is the true model

↓

$$\hat{p}(a_t|s_t, \mathcal{O}_{t:T} = 1)$$

Just like in MFRL →
perform posterior inference

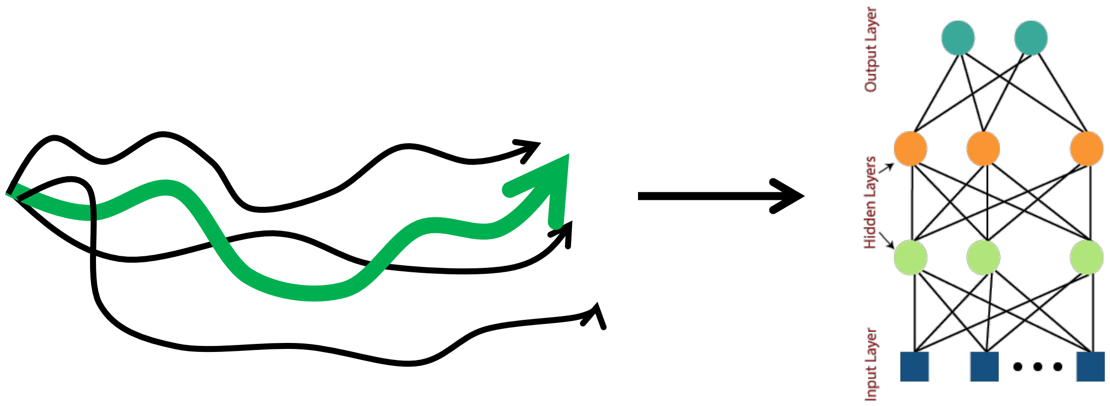
Certainty equivalence ← But pretend that the model were true

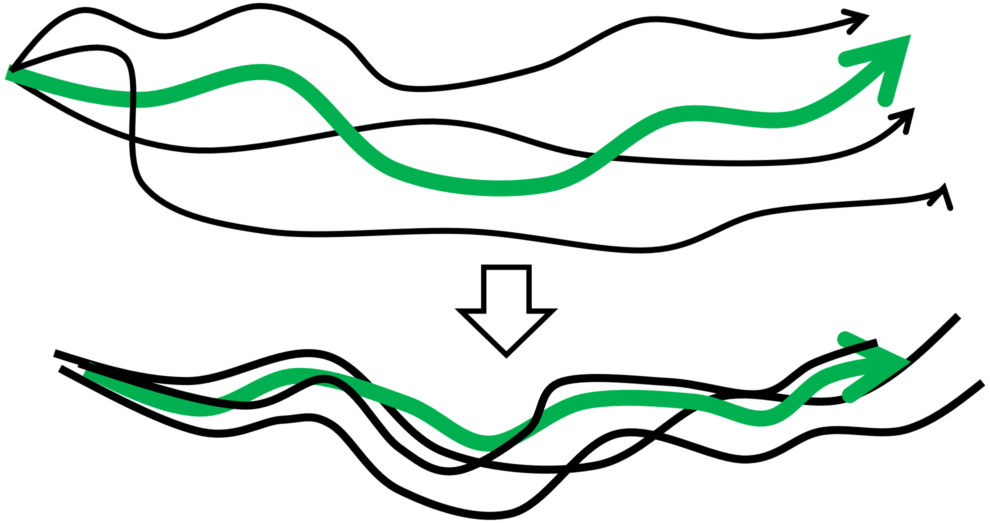
Ok so how we do perform this inference?

$$\hat{p}(a_t | s_t, \mathcal{O}_{t:T} = 1)$$

Idea 1: Variational inference in $\hat{p}$

Idea 2: Use Monte-Carlo Sampling for Inference





Model-based policy optimization methods (Dyna ++)

MPPI-style planning methods

Equivalence between posterior inference and MPPI

$$\hat{p}(a_t | s_t, \mathcal{O}_{t:T} = 1)$$

Let’s expand out the nasty integrals with Bayes rule

$$= \frac{\hat{p}(a_t, s_t, \mathcal{O}_{t:T} = 1)}{\hat{p}(s_t, \mathcal{O}_{t:T} = 1)}$$

$$= \frac{\int \int \cdots \int \hat{p}(a_t, s_t, a_{t+1}, s_{t+1}, \ldots, a_T, s_T, \mathcal{O}_{t:T} = 1) ds_{t+1} da_{t+1} \ldots ds_T da_T}{\hat{p}(s_t, \mathcal{O}_{t:T} = 1)}$$

$$\propto \int \int \cdots \int \hat{p}(a_t, s_t, a_{t+1}, s_{t+1}, \ldots, a_T, s_T, \mathcal{O}_{t:T} = 1) ds_{t+1} da_{t+1} \ldots ds_T da_T$$

Equivalence between posterior inference and MPPI

$$\hat{p}(a_t | s_t, \mathcal{O}_{t:T} = 1)$$

$$\propto \int \int \cdots \int \hat{p}(a_t, s_t, a_{t+1}, s_{t+1}, \ldots, a_T, s_T, \mathcal{O}_{t:T} = 1) ds_{t+1} da_{t+1} \ldots ds_T da_T$$

$$\propto \int \int \cdots \int \hat{p}(s_0) \Pi_t \left[\overset{\text{Dynamics}}{\hat{p}(s_{t+1} | s_t, a_t)} \overset{\text{Action prior}}{p(a_t | s_t)} \overset{\text{Optimality}}{p(\mathcal{O}_t | s_t, a_t)} \right] ds_{t+1} da_{t+1} \ldots ds_T da_T$$

$$\propto \int \int \cdots \int \hat{p}(s_0) \Pi_t \left[\hat{p}(s_{t+1} | s_t, a_t) p(a_t | s_t) \right] \exp \left[\sum_t r(s_t, a_t) \right] ds_{t+1} da_{t+1} \ldots ds_T da_T$$

Substituting optimality definition $p(\mathcal{O}_t | s_t, a_t) = \exp(r(s_t, a_t))$

$$\propto \mathbb{E}_{\substack{s_0 \sim \hat{p}(s_0) \\ a_t \sim \hat{p}(a_t | s_t) \\ s_{t+1} \sim \hat{p}(s_{t+1} | s_t, a_t)}} \left[\exp \left[\sum_t r(s_t, a_t) \right] \right]$$

Just using definition of expectation

Equivalence between posterior inference and MPPI

$$\hat{p}(a_t|s_t, \mathcal{O}_{t:T} = 1)$$

$$\propto \mathbb{E}_{\substack{s_0\sim\hat{p}(s_0)\\ a_t\sim\hat{p}(a_t|s_t)\\ s_{t+1}\sim\hat{p}(s_{t+1}|s_t,a_t)}}\left[\exp\left[\sum_t r(s_t, a_t)\right]\right]$$

Taking a bunch of samples through model →

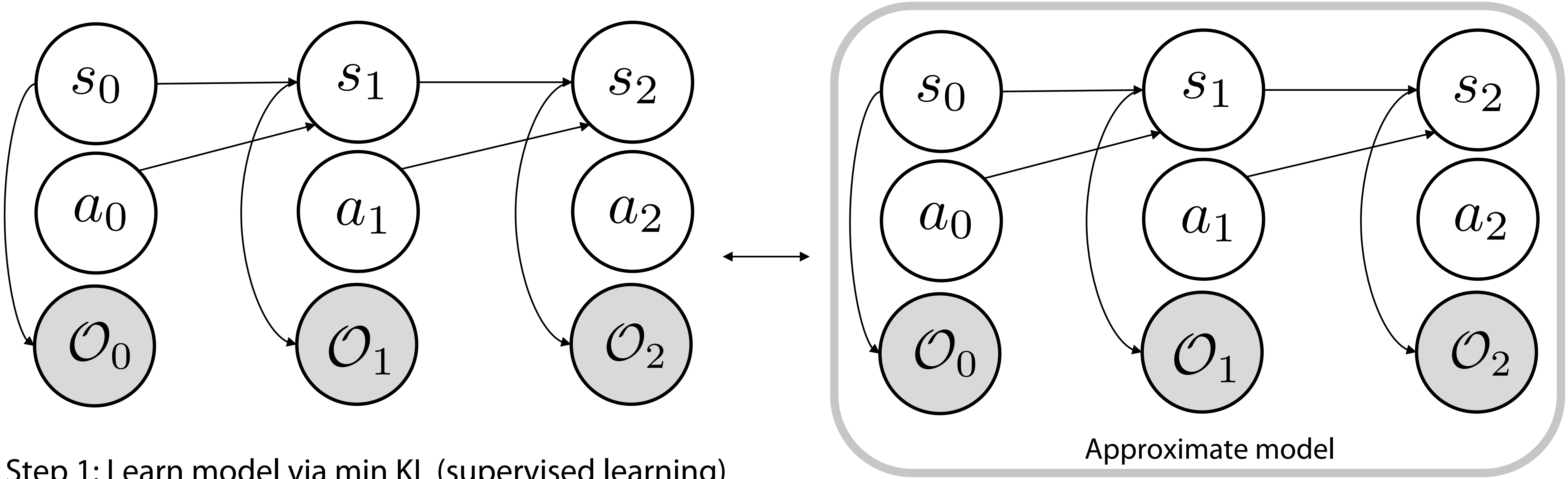
choose actions proportional to the expected sum of rewards

Can keep repeating with updated action prior

$$\propto \mathbb{E}_{\substack{s_0\sim\hat{p}(s_0)\\ a_t\sim\hat{p}(a_t|s_t)\\ s_{t+1}\sim\hat{p}(s_{t+1}|s_t,a_t)}}\left[\exp\left[\sum_t r(s_t, a_t)\right]\right]$$

Can be thought of as a sampling-based Monte-Carlo approximation to posterior

Ok so what did we show?



Step 1: Learn model via min KL (supervised learning)

$$\max_{\hat{p}} \mathbb{E}_{p(s_0,\ldots,s_T,a_0,\ldots,a_T,\mathcal{O}_0,\ldots,\mathcal{O}_T)}\left[\log \hat{p}(s_0) + \sum_t [\log \hat{p}(s_{t+1}|s_t,a_t) + \log \hat{p}(\mathcal{O}_t|s_t,a_t)]\right]$$

Step 2: Obtain posterior actions via Monte-Carlo approximation (approx MPPI)

$$\propto \mathbb{E}_{\substack{s_0\sim\hat{p}(s_0)\\ a_t\sim\hat{p}(a_t|s_t)\\ s_{t+1}\sim\hat{p}(s_{t+1}|s_t,a_t)}}\left[\exp\left[\sum_t r(s_t,a_t)\right]\right]$$

Lecture outline

Control as Inference to Derive Q-learning

↓

Control as Inference to Derive Model-Based RL

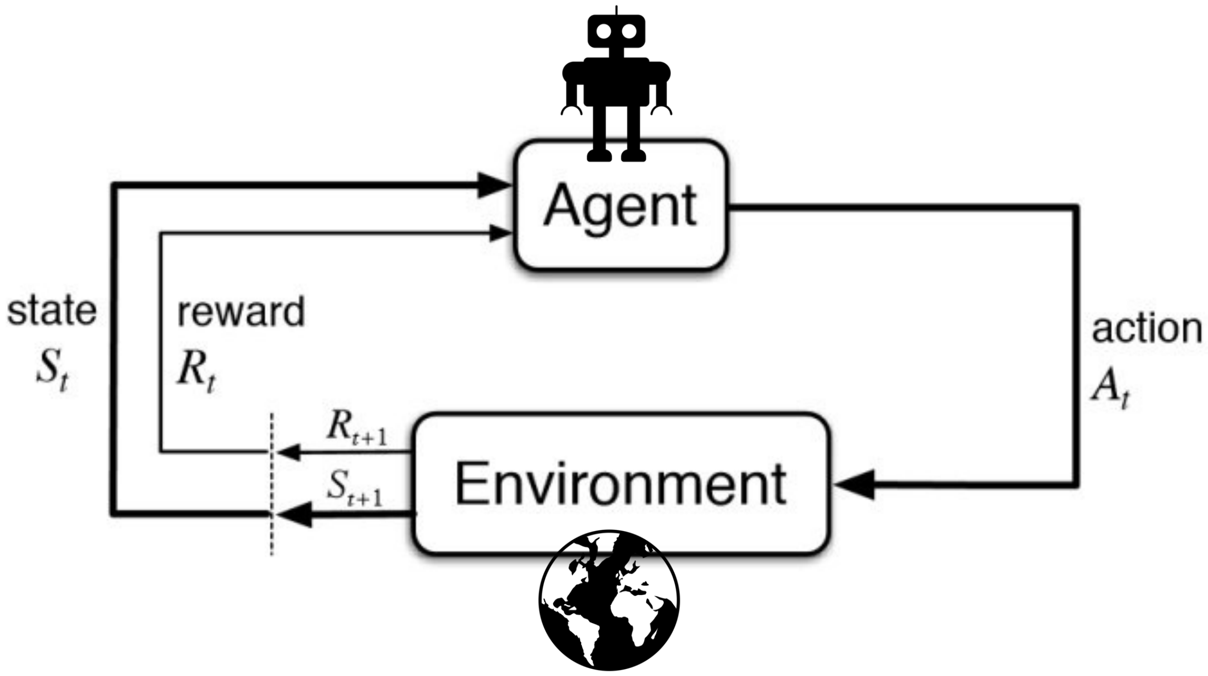
↓

Why inverse RL? + Problem formulation

↓

IRLv1 – max margin planning

Let's revisit the premise of reinforcement learning



$$\max_\theta \mathbb{E}_{\tau \sim \pi_\theta}\left[\sum_{t=0}^{T} r(s_t, a_t)\right]$$

or

$$\mathbb{E}_{\substack{s_0 \sim p(s_0) \\ a_t \sim q(a_t|s_t) \\ s_{t+1} \sim p(s_{t+1}|s_t, a_t)}}\left[\sum_t r(s_t, a_t) + \mathcal{H}(q(\cdot|s_t))\right]$$

We studied a bunch of different algorithms to solve this

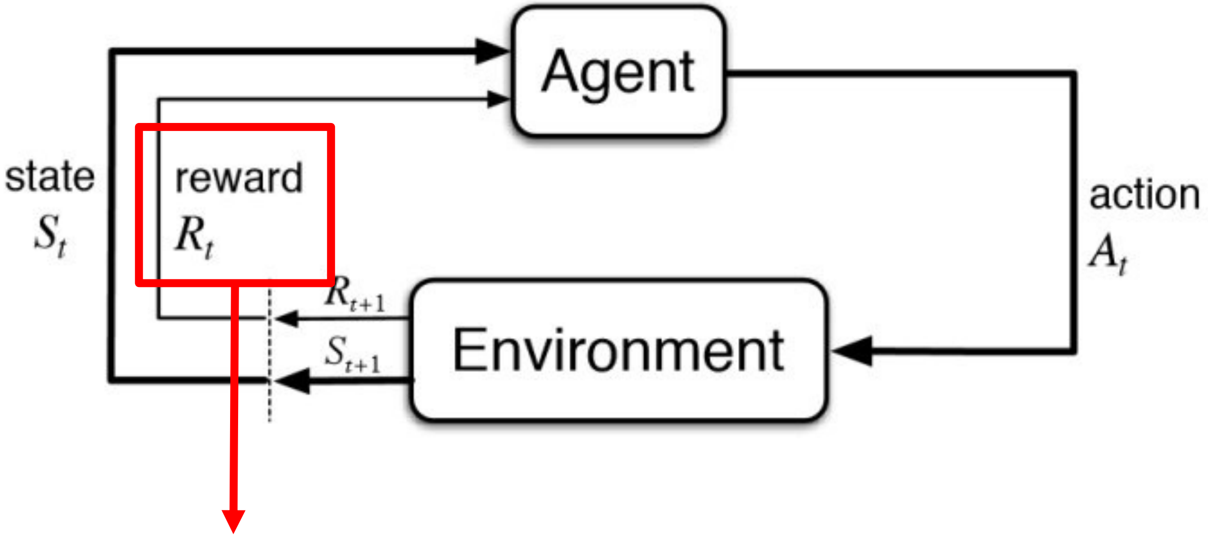
Model-based RL

Policy gradients

Actor-critic

But they all operate under the same assumption: **reward is known!**

Reinforcement Learning requires Task Specification

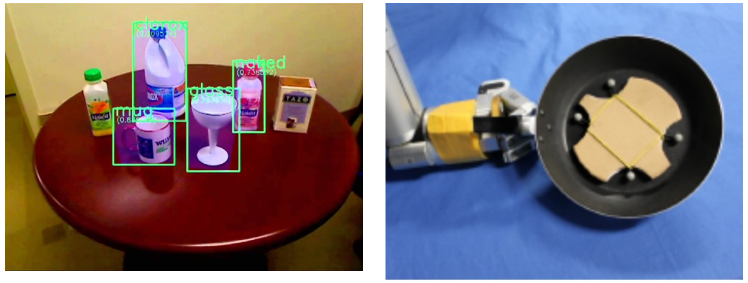


Does not magically appear in most settings

Has to be manually specified

→ can we do better?

Manual state estimation/perception



Complex reward specification

Name	Reward	Heroes	Description
Win	5	Team	
Hero Death	-1	Solo	
Courier Death	-2	Team	
XP Gained	0.002	Solo	
Gold Gained	0.006	Solo	For each unit of gold gained. Reward is not lost when the gold is spent or lost.
Gold Spent	0.0006	Solo	Per unit of gold spent on items without using courier.
Health Changed	2	Solo	Measured as a fraction of hero's max health.[‡]
Mana Changed	0.75	Solo	Measured as a fraction of hero's max mana.
Killed Hero	-0.6	Solo	For killing an enemy hero. The gold and experience reward is very high, so this reduces the total reward for killing enemies.
Last Hit	-0.16	Solo	The gold and experience reward is very high, so this reduces the total reward for last hit to ~ 0.4.
Deny	0.15	Solo	
Gained Aegis	5	Team	
Ancient HP Change	5	Team	Measured as a fraction of ancient's max health.
Megas Unlocked	4	Team	
T1 Tower[*]	2.25	Team	
T2 Tower[*]	3	Team	
T3 Tower[*]	4.5	Team	
T4 Tower[*]	2.25	Team	
Shrine[*]	2.25	Team	
Barracks[*]	6	Team	
Lane Assign[†]	-0.15	Solo	Per second in wrong lane.

[*] For buildings, two-thirds of the reward is earned linearly as the building loses health, and one-third is earned as a lump sum when it dies.

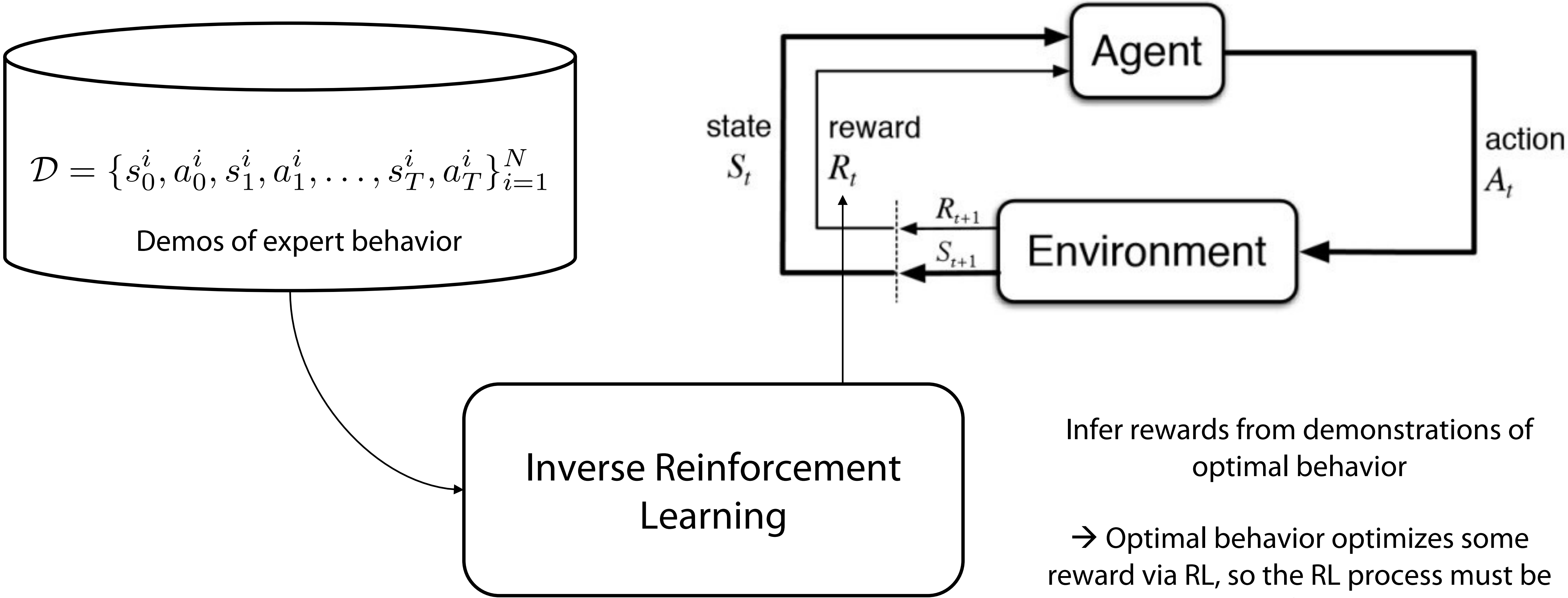
[†] See item O.2.

[‡] Hero's health is quartically interpolated between 0 (dead) and 1 (full health); health at fraction x of full health is worth $\left(x + 1 - (1 - x)^4\right)/2$. This function was not tuned; it was set once and then untouched for the duration of the project.

Table 6: Shaped Reward Weights

Learning from Demonstrations

Avoid manual reward specification by learning from demos of optimal behavior



$$\mathcal{D} = \{s_0^i, a_0^i, s_1^i, a_1^i, \ldots, s_T^i, a_T^i\}_{i=1}^N$$

Demos of expert behavior

Inverse Reinforcement Learning

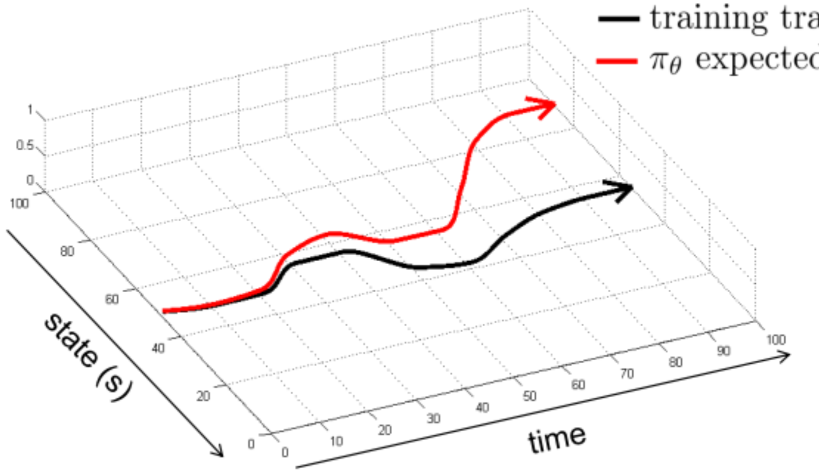
Infer rewards from demonstrations of optimal behavior

→ Optimal behavior optimizes some reward via RL, so the RL process must be "inverted" to find the reward

But haven't we already learned from demonstrations?

Imitation learning via Behavior Cloning (L2)

$$\arg\max_{\theta} \mathbb{E}_{(s^*,a^*)\sim\mathcal{D}} \left[\log \pi_\theta(a^*|s^*)\right]$$



Main difference between BC and IRL:

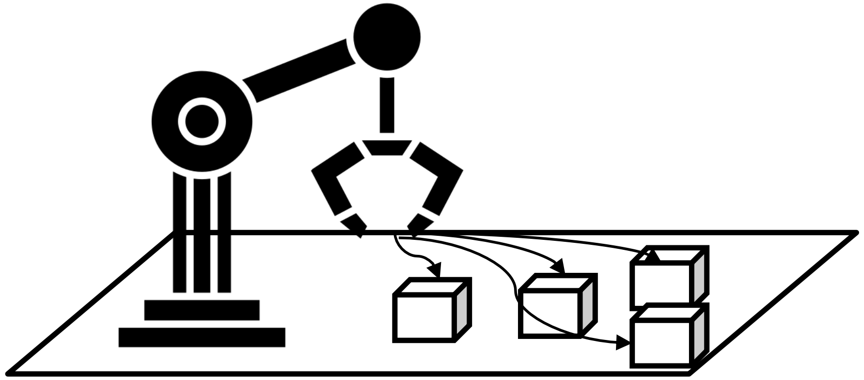
1. BC learns policies, IRL learns rewards
2. BC assumes no environment access, IRL typically assumes either known model or sampling access

Why does this matter?

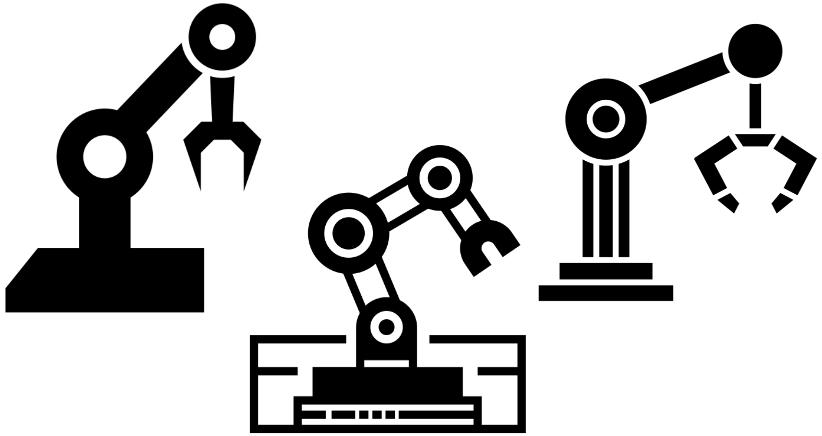
Zooming out – why do we care about imitation?

Imitation learning is all about generalization

Generalization across states



Generalization across dynamics



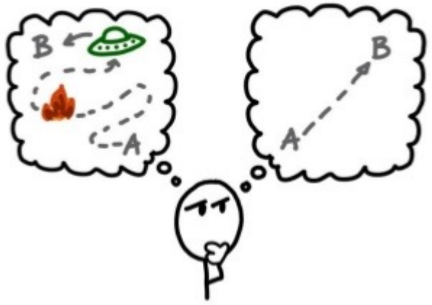
Covariate shift is just a manifestation of generalization

What if learning something else generalized better than policies?

Zooming out – why do we care about imitation?

Rewards may be simpler → better generalization

Occam's Razor

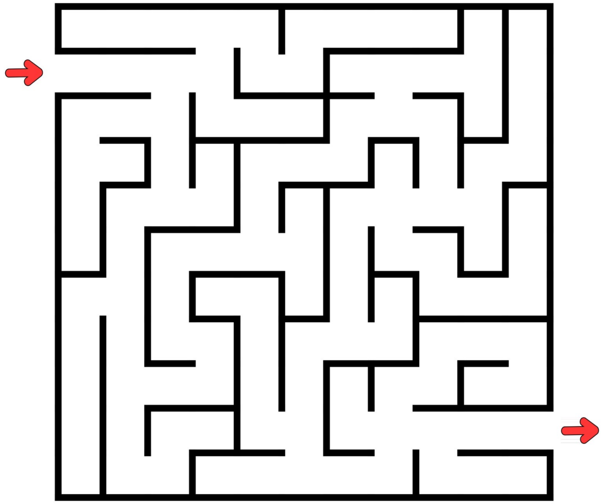


"When faced with two equally good hypotheses, always choose the simpler."

PAC-Bayes Bounds

$$R(h_S) \leq \frac{1}{m}\left(\log|\mathcal{H}| + \log\frac{1}{\delta}\right).$$

Smaller (yet sufficient) hypothesis class, better generalization

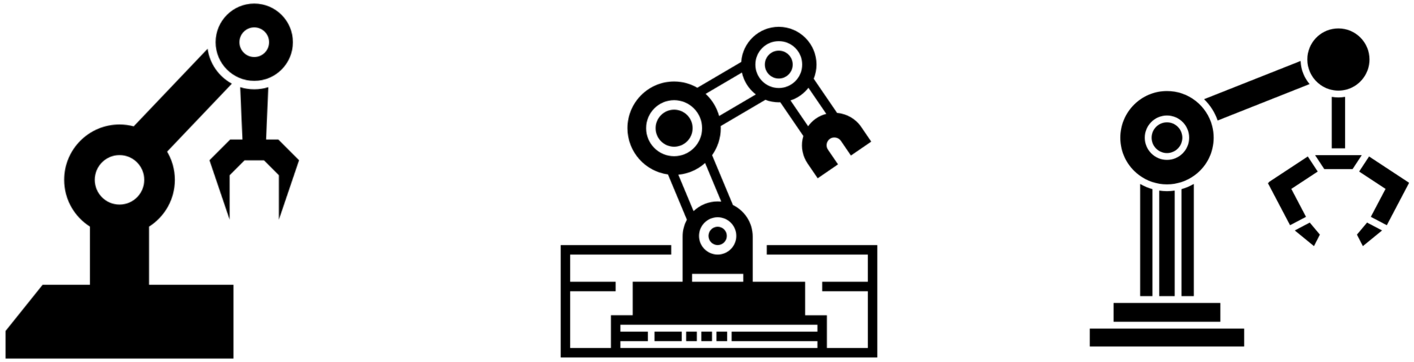


Policy – fairly complex
Reward – 1 when goal is reached, 0 otherwise

Reward <u>**can**</u> be much simpler

Cross-Embodiment/Dynamics Transfer

Rewards may allow for cross dynamics transfer



Can all share the same reward, even with different dynamics!

⇩

Policies and Q/V functions entangle dynamics, rewards do not

Addressing Compounding Error

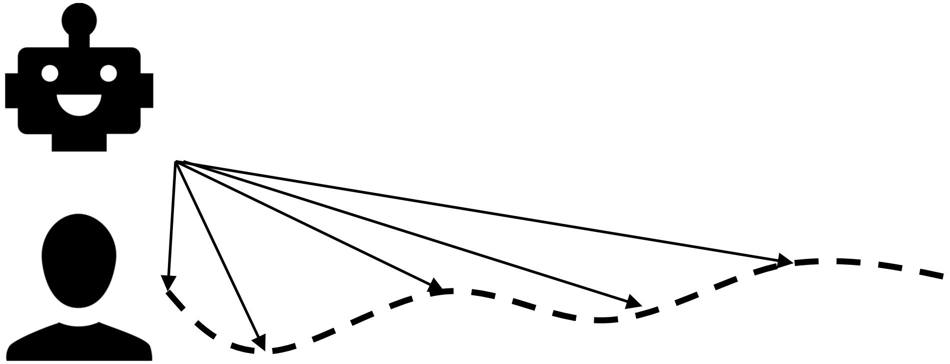
Reward can avoid covariate shift issues with forward KL

Imitation Learning via BC

$$\max_\theta \mathbb{E}_{(x,y)\sim\mathcal{D}} \left[\log \hat{p}_\theta(y|x)\right]$$

Sampling from expert

$$D_{\mathrm{KL}}(p^* || p_\theta)$$

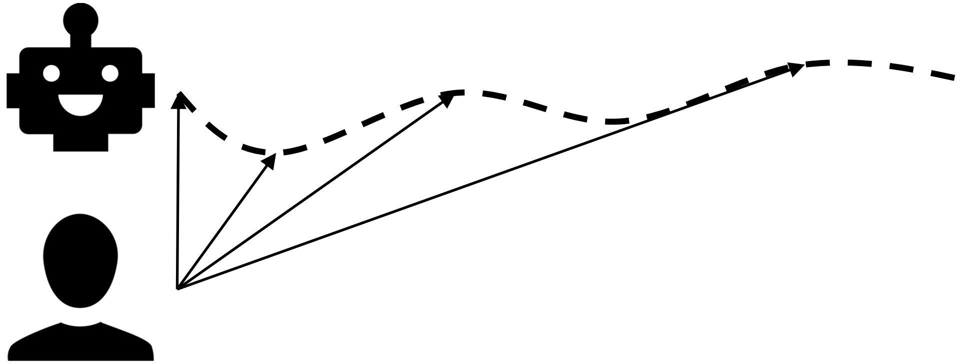


Reinforcement Learning with Inferred Reward

$$\max_\theta \mathbb{E}_{\tau\sim\pi_\theta} \left[\sum_{t=0}^{T} r(s_t, a_t)\right]$$

Sampling from policy

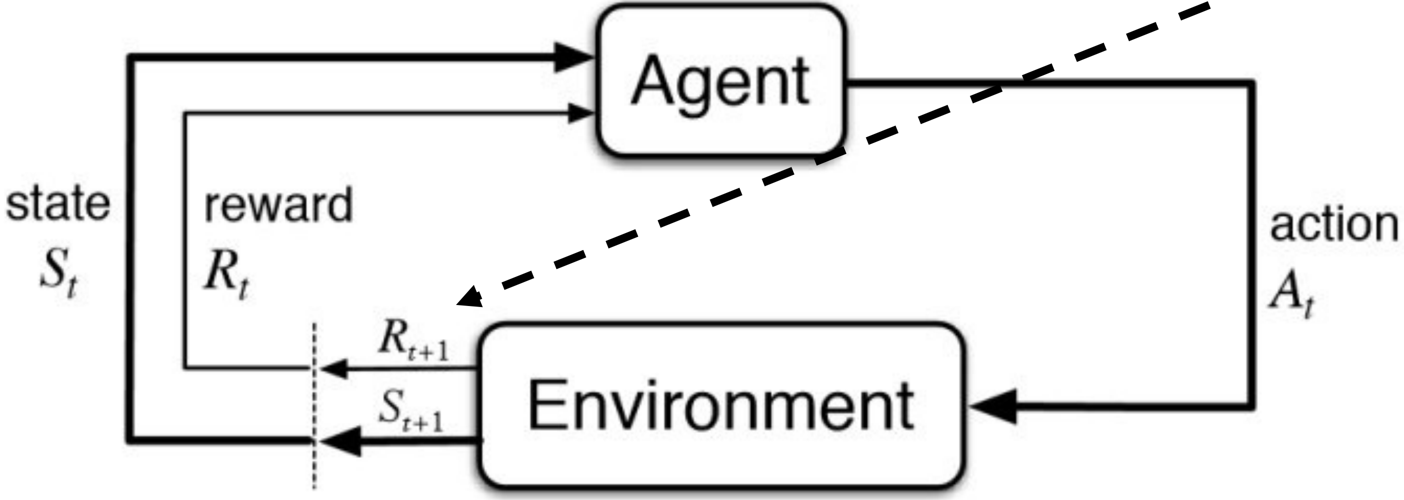
What we care about → $D_{\mathrm{KL}}(p_\theta || p^*)$



Learning Rewards from Human Data

Imitation Learning π

Inverse RL Reward

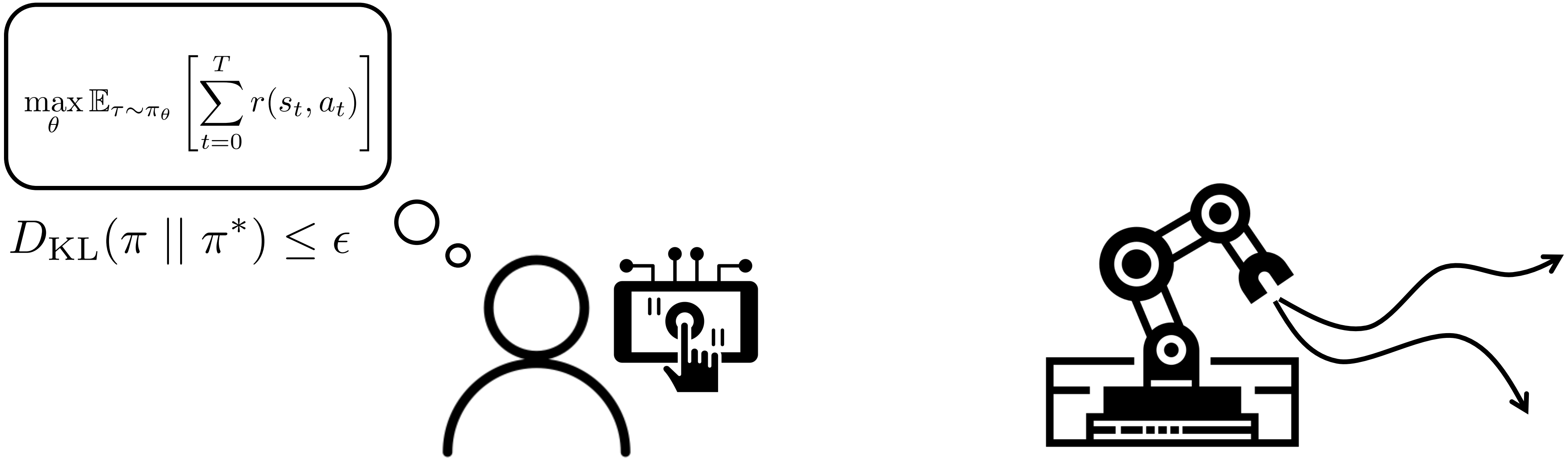


$$\max_{\theta} \mathbb{E}_{\tau \sim \pi_\theta} \left[\sum_{t=0}^{T} r(s_t, a_t) \right]$$

Is this even a well-defined problem?

How can we learn rewards?

We must make more assumptions on the expert provided data

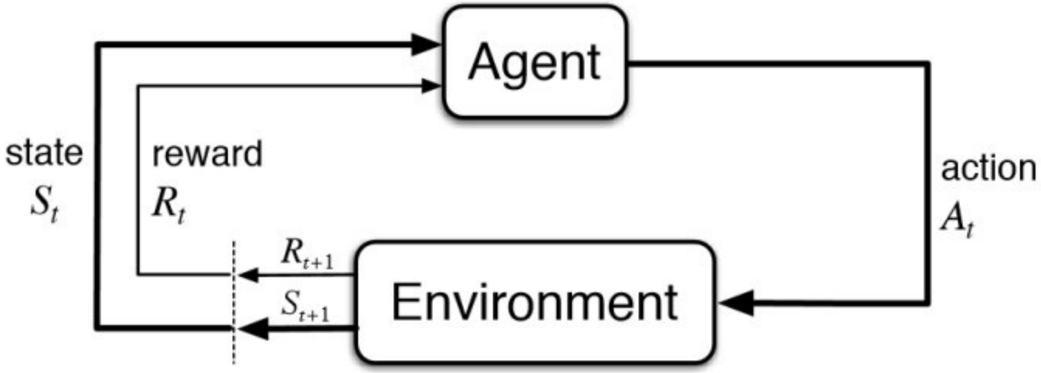


$$\max_{\theta} \mathbb{E}_{\tau \sim \pi_\theta} \left[\sum_{t=0}^{T} r(s_t, a_t) \right]$$

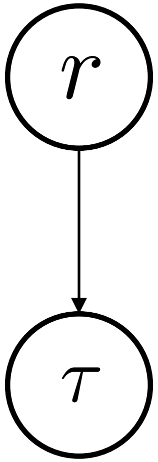
$$D_{\mathrm{KL}}(\pi \mid\mid \pi^*) \leq \epsilon$$

Experts are assumed to be “noisily” optimal

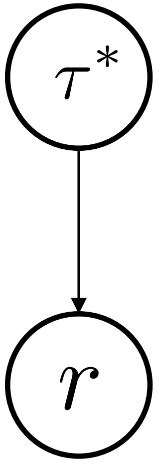
Why is this “inverse” reinforcement learning?



RL: Rewards generate trajectories



IRL: Expert trajectories generate rewards



Is this well defined?

IRL problem statement + assumptions

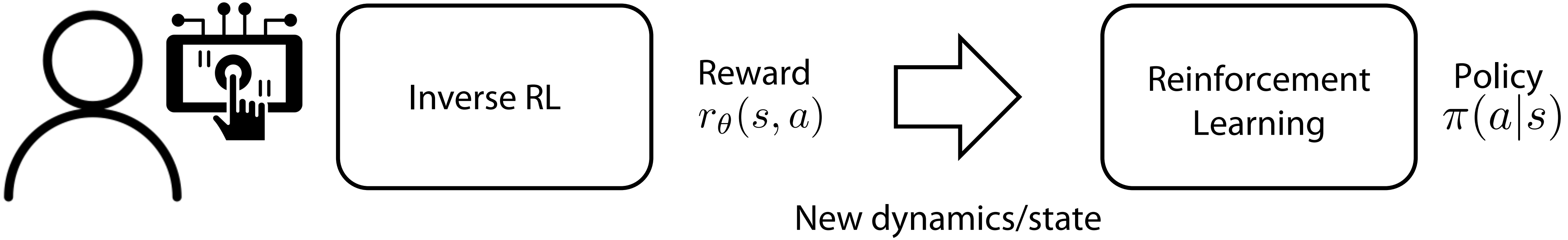
Reinforcement Learning

State: Known
Action: Known
Transition Dynamics: Unknown but can sample
Reward: **Known**
Expert policy: Unknown
Expert traces: **Unknown**

Inverse Reinforcement Learning

State: Known
Action: Known
Transition Dynamics: Unknown but can sample
Reward: **Unknown**
Expert policy: Unknown
Expert traces: **Known**

Find r that **explains** the demonstrator behavior as noisily optimal



Inverse RL

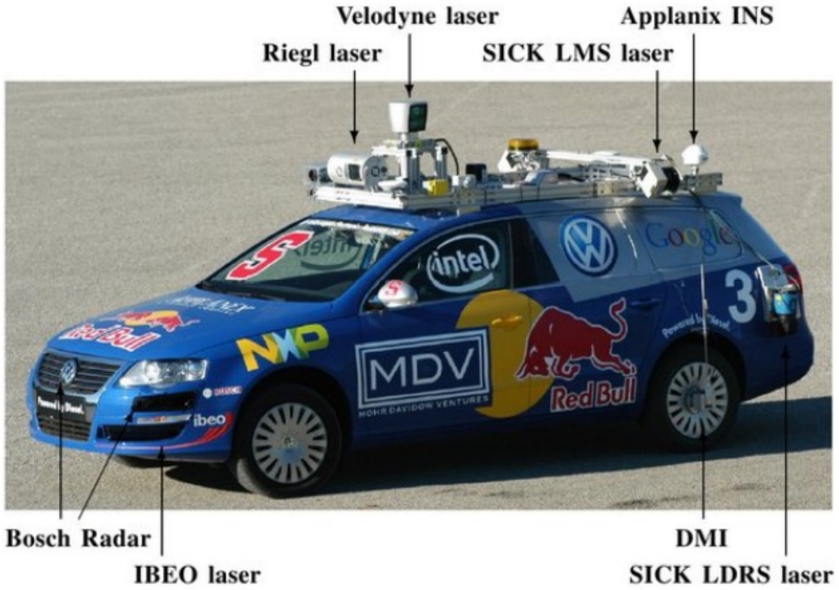
Reward
$r_\theta(s,a)$

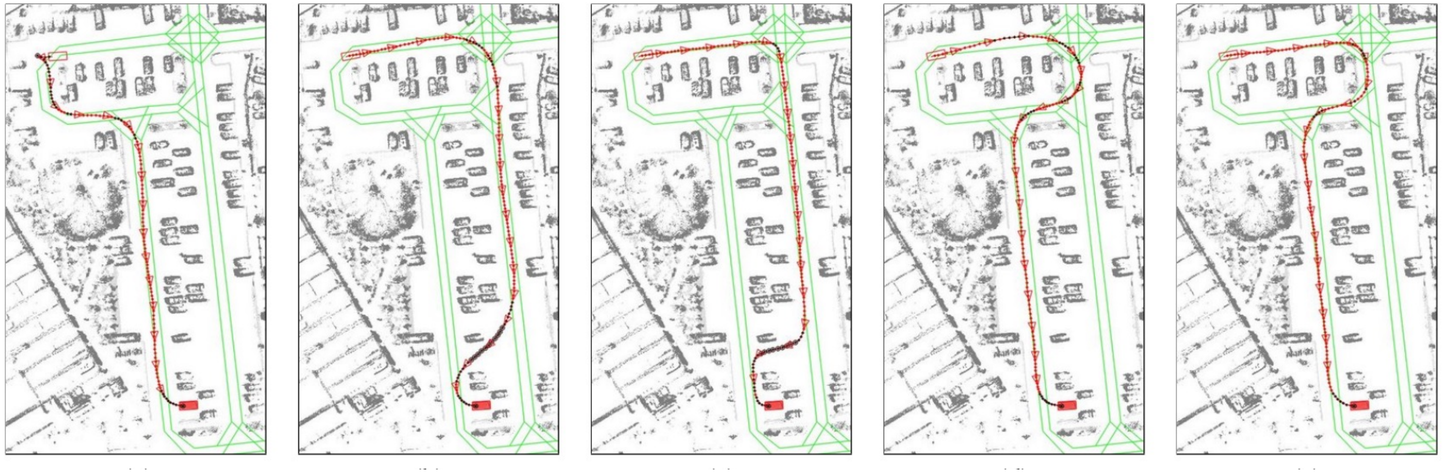
Reinforcement Learning

Policy
$\pi(a|s)$

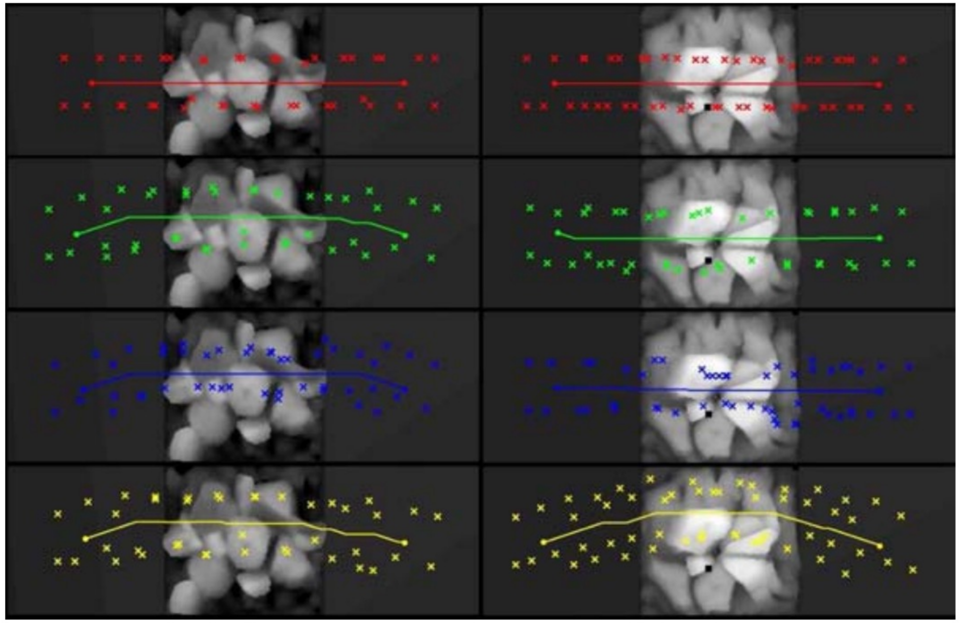
New dynamics/state

Inverse RL Applications



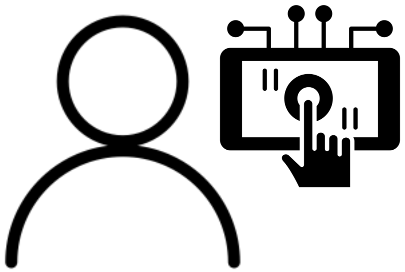


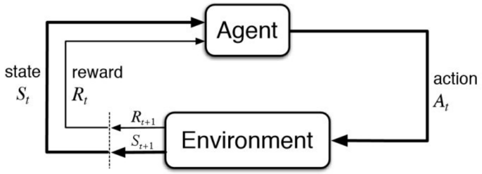
Inverse RL Applications



Why is this hard?

Find r that **explains** the demonstrator behavior as noisily optimal





Inverse RL

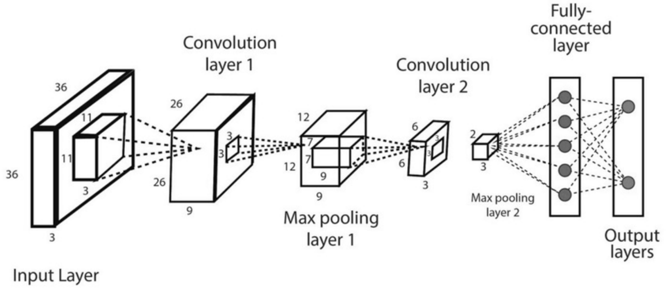
Challenging for a variety of reasons:

1. Inherently underspecified
2. R and π both unknown
3. Difficult optimization with T unknown.
4. Distributions/comparison metrics unknown

Reward Function

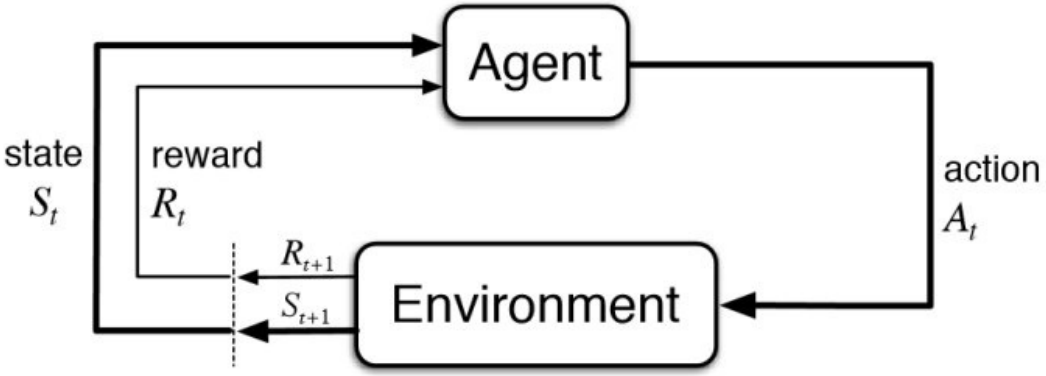
$$r_\theta(s, a)$$

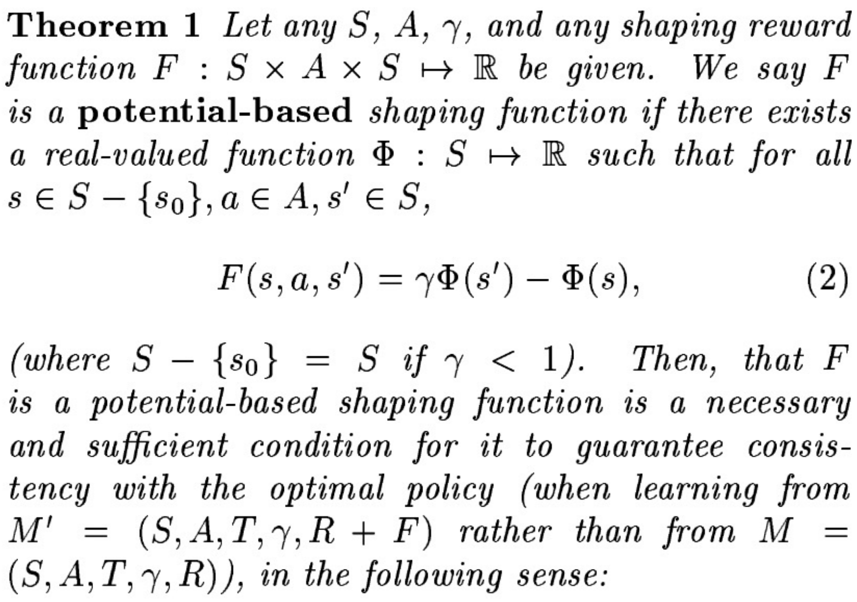
Can be parameterized by arbitrary function approximator



Underspecification in Reward Functions

Rewards are inherently underspecified → many rewards can give you the same optimal policy





Theorem 1 *Let any S, A, γ, and any shaping reward function $F : S \times A \times S \mapsto \mathbb{R}$ be given. We say F is a* **potential-based** *shaping function if there exists a real-valued function $\Phi : S \mapsto \mathbb{R}$ such that for all $s \in S - \{s_0\}, a \in A, s' \in S$,*

$$F(s,a,s') = \gamma\Phi(s') - \Phi(s), \qquad (2)$$

(where $S - \{s_0\} = S$ if $\gamma < 1$). Then, that F is a potential-based shaping function is a necessary and sufficient condition for it to guarantee consistency with the optimal policy (when learning from $M' = (S, A, T, \gamma, R + F)$ rather than from $M = (S, A, T, \gamma, R)$), in the following sense:

Original reward

$$r(s, a, s')$$

Reshaped reward

$$r_{\text{shaped}}(s, a, s') = r(s, a, s') + \gamma\phi(s') - \phi(s)$$

$$Q = r(s,a,s') + \gamma r(s',a',s'') + \gamma^2 r(s'',a'',s''') + \dots$$

$$\begin{aligned} Q = {} & r(s,a,s') + \gamma\cancel{\phi}(s') - \phi(s) + \\ & \gamma(r(s',a',s'') + \gamma\cancel{\phi}(s'') - \cancel{\phi}(s')) + \\ & \gamma^2(r(s'',a'',s''') + \gamma\phi(s''') - \cancel{\phi}(s'')) + \dots \end{aligned}$$

Unbiased policy optimization!

Lecture outline

Control as Inference to Derive Q-learning

↓

Control as Inference to Derive Model-Based RL

↓

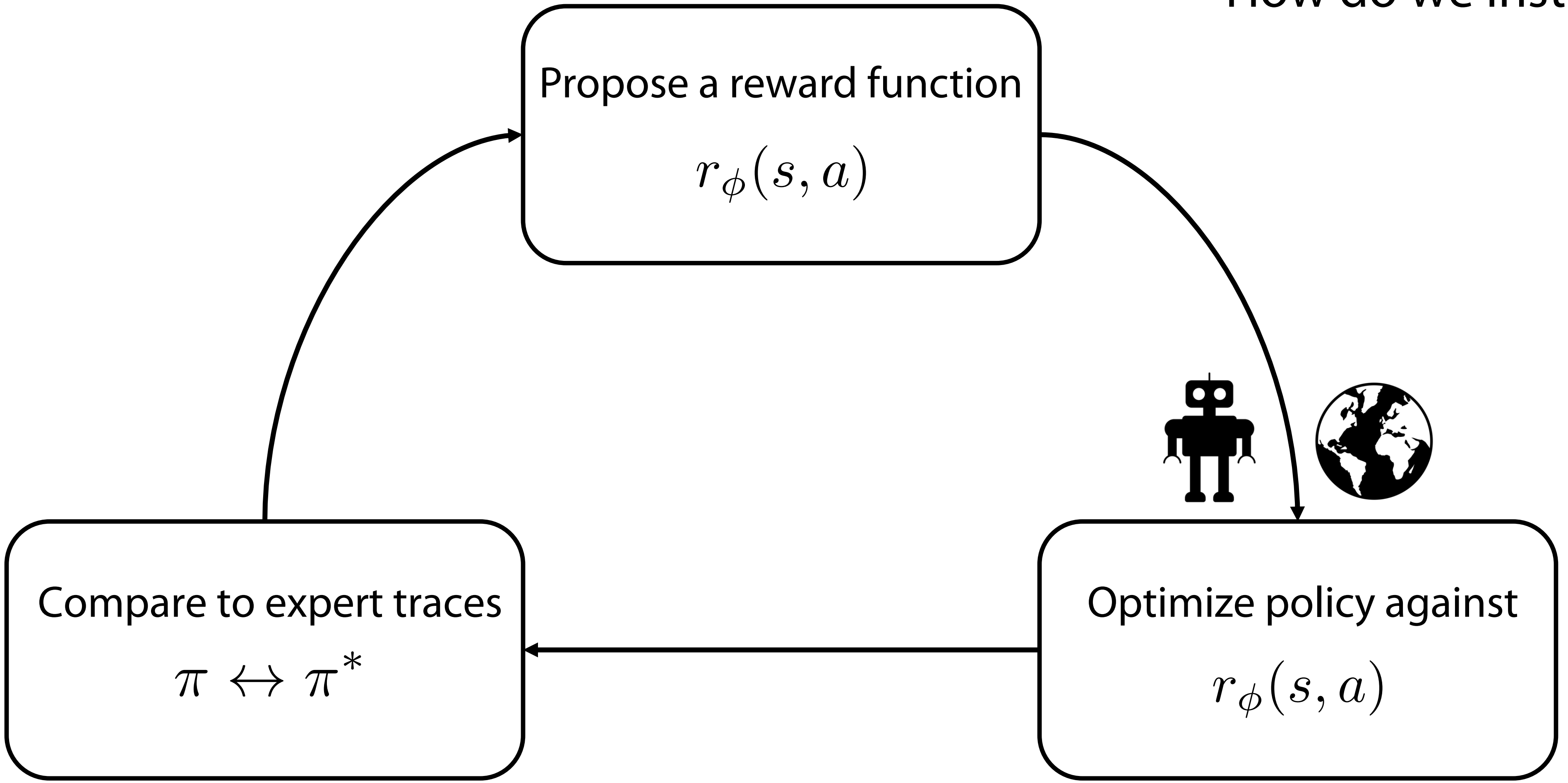
Why inverse RL? + Problem formulation

↓

IRLv1 – max margin planning

A Formula for Inverse Reinforcement Learning

How do we instantiate?



Propose a reward function

$r_\phi(s, a)$

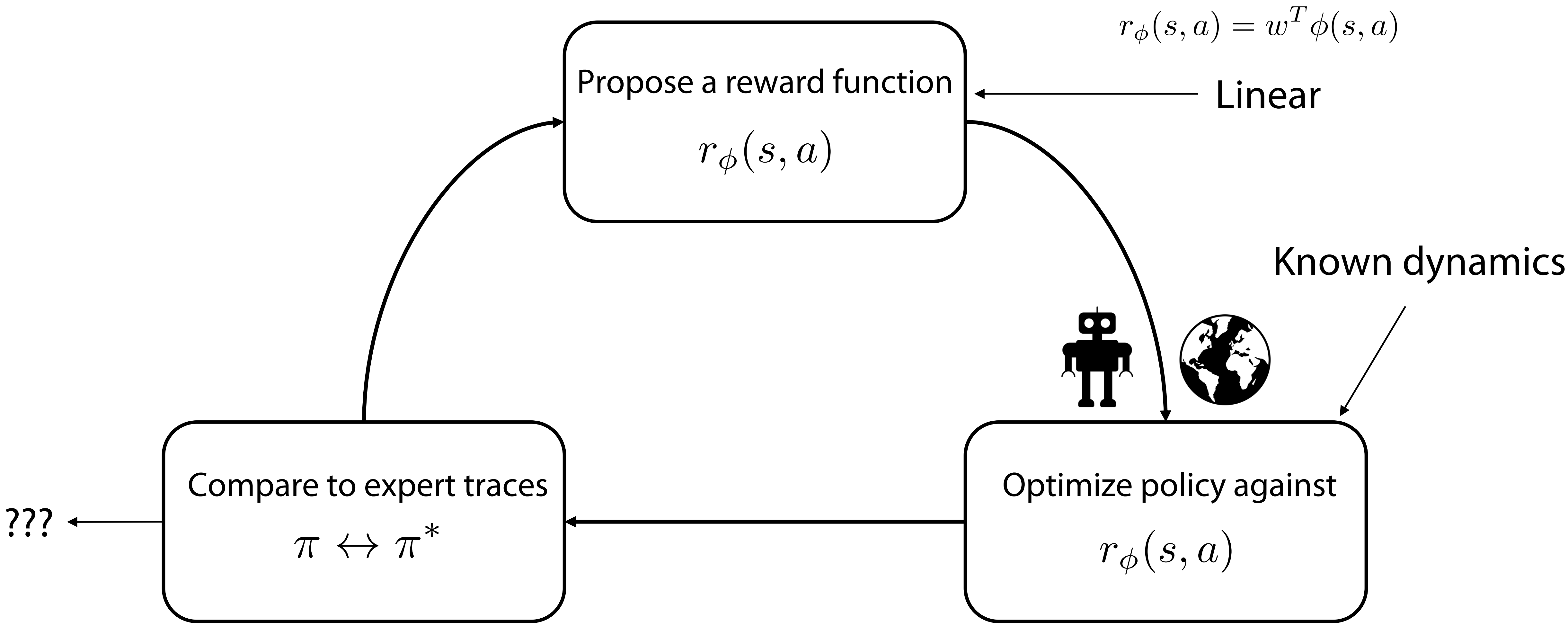
Optimize policy against

$r_\phi(s, a)$

Compare to expert traces

$\pi \leftrightarrow \pi^*$

IRL v0 – Assumptions



$r_\phi(s,a) = w^T\phi(s,a)$

Linear

Propose a reward function

$r_\phi(s,a)$

Known dynamics

Optimize policy against

$r_\phi(s,a)$

Compare to expert traces

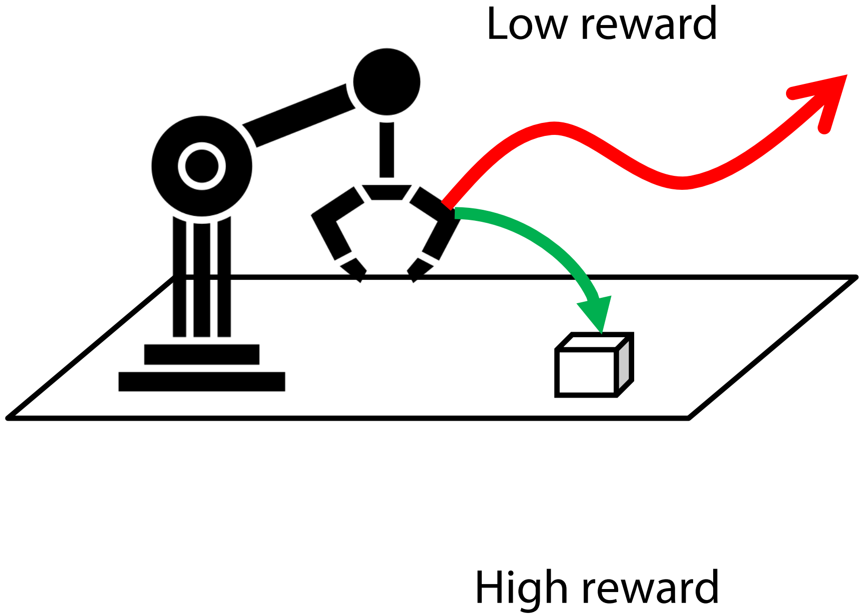
$\pi \leftrightarrow \pi^*$

???

IRL v0 – What is a good reward function?

A good reward would evaluate optimal data higher than all other data

$$V_r^{\pi^*}(s) \geq V_r^{\pi}(s) \ \ \forall \pi, \forall s$$



Find w* such that $r(s,a) = {w^*}^T \phi(s,a)$

$$\mathbb{E}_{\pi^*}\left[\sum_t \gamma^t r(s_t,a_t)\right] \geq \mathbb{E}_{\pi}\left[\sum_t \gamma^t r(s_t,a_t)\right], \quad \forall \pi$$

$$\mathbb{E}_{\pi^*}\left[\sum_t \gamma^t {w^*}^T \phi(s_t,a_t)\right] \geq \mathbb{E}_{\pi}\left[\sum_t \gamma^t {w^*}^T \phi(s_t,a_t)\right], \quad \forall \pi$$

$${w^*}^T \underbrace{\mathbb{E}_{\pi^*}\left[\sum_t \gamma^t \phi(s_t,a_t)\right]}_{\mu(\pi^*,\phi)} \geq {w^*}^T \underbrace{\mathbb{E}_{\pi}\left[\sum_t \gamma^t \phi(s_t,a_t)\right]}_{\mu(\pi,\phi)}, \quad \forall \pi$$

Underdefined, w* = 0 trivially satisfies!

IRL v0 – What is a good reward function?

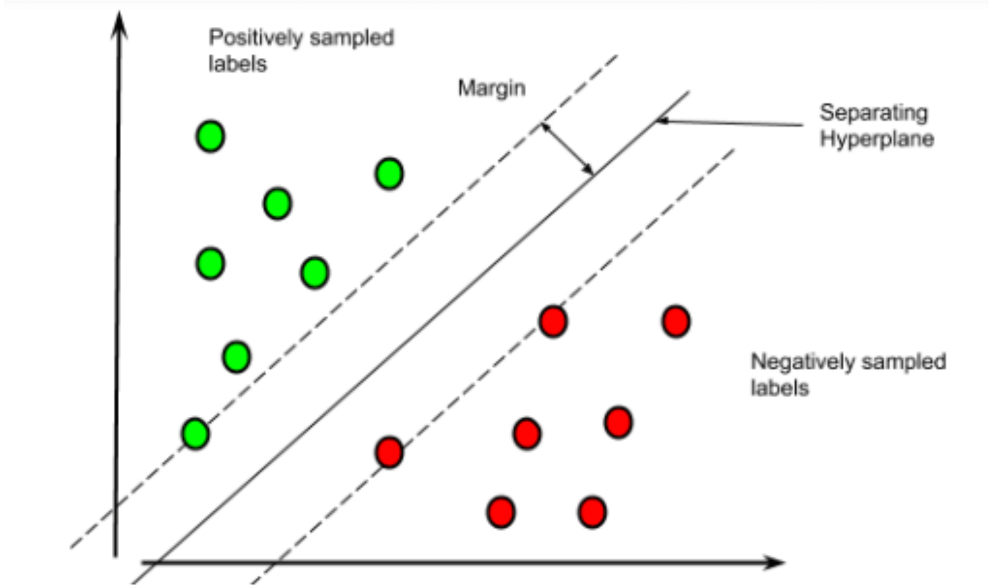
How do we tackle ambiguity?

$$w^{*T}\mathbb{E}_{\pi *}\left[\phi(s,a)\right] \geq w^{*T}\mathbb{E}_{\pi *}\left[\phi(s,a)\right] \quad \forall \pi, \forall s$$

$$\Downarrow$$

$$\max_{w,m} \quad m$$

$$\text{s.t} \quad w^T \mu^{\pi^*} \geq w^T \mu^{\pi} + m, \forall \pi \in \Pi$$



Find rewards which maximize the gap between the expert and all other policies

IRL v1 – Max Margin Feature Matching

Choose w such that “margin” is maximized

$$\max m$$

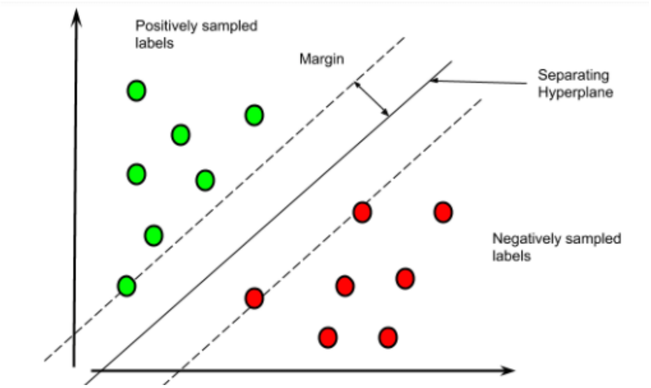
$$\text{s.t } \; w^T \mu^{\pi^*} \geq w^T \mu^{\pi} + m, \forall \pi \in \Pi$$

$$\Downarrow$$

$$\min \|w\|_2$$

$$\text{s.t } \; w^T \mu^{\pi^*} \geq w^T \mu^{\pi} + 1, \forall \pi \in \Pi$$

Looks a lot like an SVM!



What might the issues be →

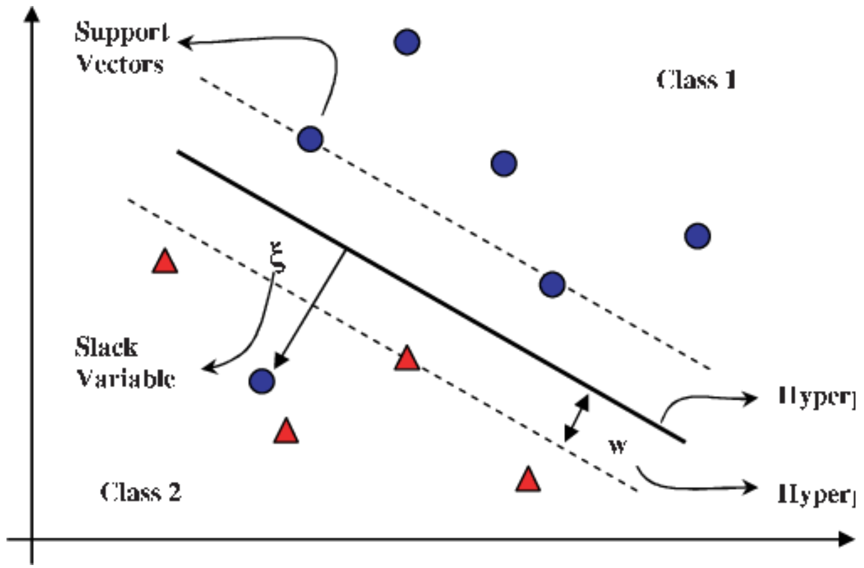
1. Uniform gap across all π, π^*
2. Noisily optimal may compromise the optimization

IRL v1 – (Fancy) Max Margin Feature Matching

Maximum margin → Structured Max-Margin + Slack

$$\min \|w\|_2$$

$$\text{s.t} \;\; w^T \mu^{\pi^*} \geq w^T \mu^{\pi} + 1, \forall \pi \in \Pi$$



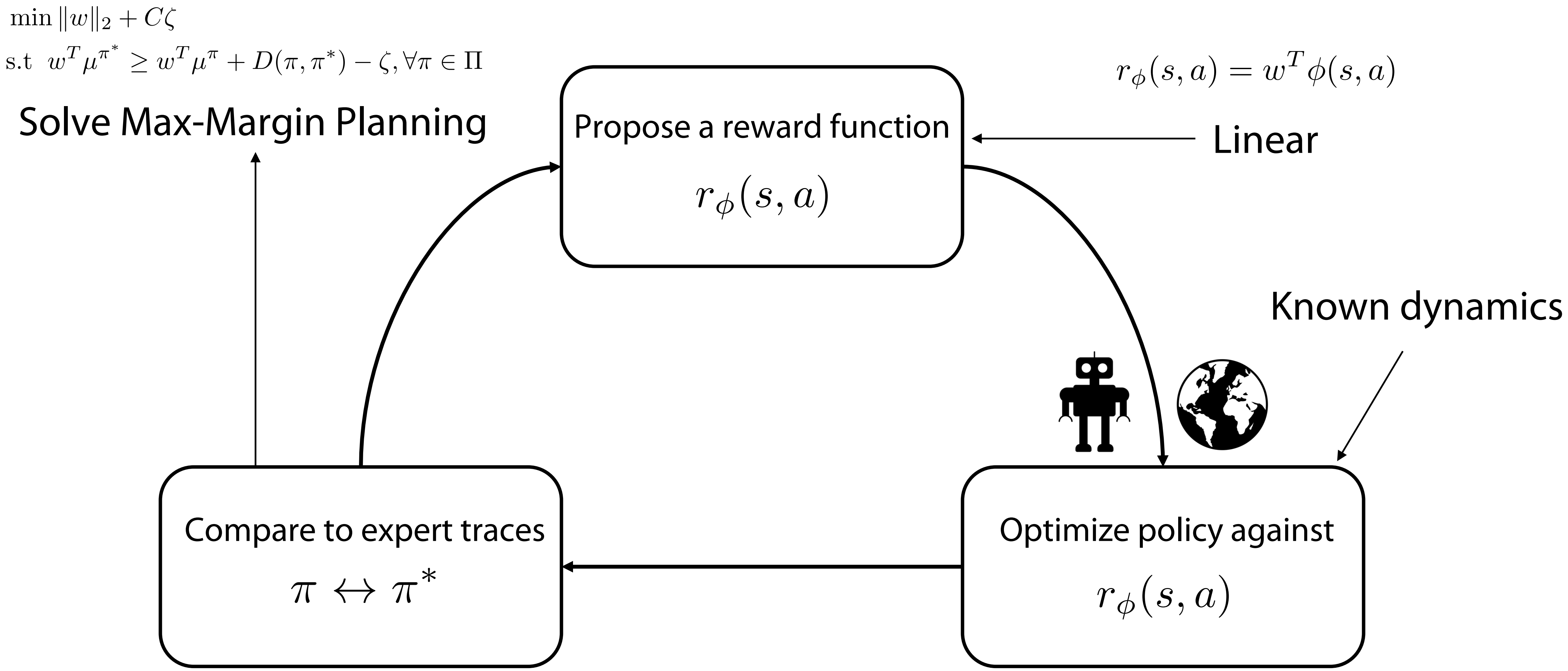
Bigger for more different policies

$$\min \|w\|_2 + C\zeta$$

$$\text{s.t} \;\; w^T \mu^{\pi^*} \geq w^T \mu^{\pi} + D(\pi, \pi^*) - \zeta, \forall \pi \in \Pi$$

Slack allows for noisy optimality

IRL v1 – Max Margin Feature Matching



$$\min \|w\|_2 + C\zeta$$

$$\text{s.t} \;\; w^T\mu^{\pi^*} \geq w^T\mu^{\pi} + D(\pi, \pi^*) - \zeta, \forall \pi \in \Pi$$

Solve Max-Margin Planning

Propose a reward function

$r_\phi(s, a)$

$r_\phi(s, a) = w^T\phi(s, a)$

Linear

Known dynamics

Optimize policy against

$r_\phi(s, a)$

Compare to expert traces

$\pi \leftrightarrow \pi^*$

IRL v1 – Max Margin Feature Matching

1. Start with a random policy π_0
2. Find the w that optimizes

$$\min_{w,\zeta} \|w\|_2 + C\zeta$$

$$\text{s.t} \;\; w^T \mu^{\pi^*} \geq w^T \mu^{\pi} + D(\pi, \pi^*) - \zeta, \forall \pi \in \{\pi_0, \pi_1, \ldots, \pi_i\}$$

3. Solve for the optimal policy against $r_\phi(s,a) = {w^{(i)}}^T \phi(s,a)$

$$\pi_{i+1} \rightarrow \text{Opt}(r_\phi(s,a), T)$$

4. Add to constraint set and repeat

Output the optimal reward function w*

Max Margin Feature Matching in Action

Lecture outline

Control as Inference to Derive Q-learning

↓

Control as Inference to Derive Model-Based RL

↓

Why inverse RL? + Problem formulation

↓

IRLv1 – max margin planning

IRL v1 – Why this may not be enough?

$$\min \|w\|_2 + C\zeta$$

$$\text{s.t} \;\; w^T \mu^{\pi^*} \geq w^T \mu^{\pi} + D(\pi, \pi^*) - \zeta, \forall \pi \in \Pi$$

May not be able to deal with scenario where true margin is quite small for some policies

Not clear if this is a good way to deal with suboptimality

Constrained optimization is tough to optimize for non-linear functions

What if we had a ”softer” notion of margin?

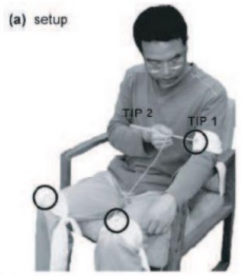
We have talked about "soft" optimality before!

We derived max-ent RL as maximum likelihood on optimality (lower bound) wrt policy

$$\max_q \mathbb{E}_{x\sim p(x)} \left[\mathbb{E}_{z\sim q(z|x)} \left[\log p(x|z) \right] - D_{KL}(q(z|x)||p(z)) \right]$$

Control as inference

$$\mathbb{E}_{\substack{s_0\sim p(s_0)\\ a_t\sim q(a_t|s_t)\\ s_{t+1}\sim p(s_{t+1}|s_t,a_t)}} \left[\sum_t \log p(\mathcal{O}_t|s_t,a_t) - \log q(a_t|s_t) \right]$$

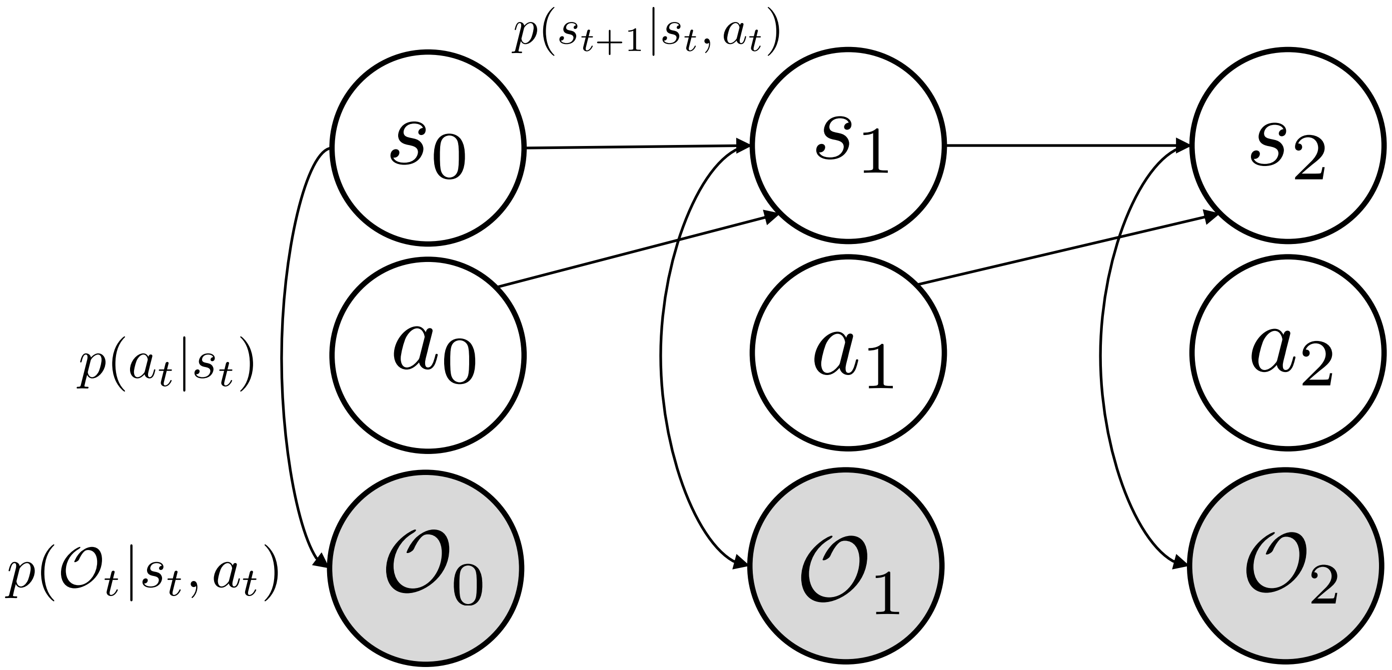


Li & Todorov '06

Ziebart '08

Can we invert this to do inverse RL with a softer notion of margin?

Let's revisit the graphical model



$$p(\mathcal{O}_t|s_t, a_t) = \exp(r(s_t, a_t))$$

$$p(\tau|\mathcal{O}_{0:T} = 1) \propto p(\tau) \exp(\sum_{t=0}^{T} r(s_t, a_t))$$

$$p(\tau)$$

Uninformed behavior according to prior/dynamics

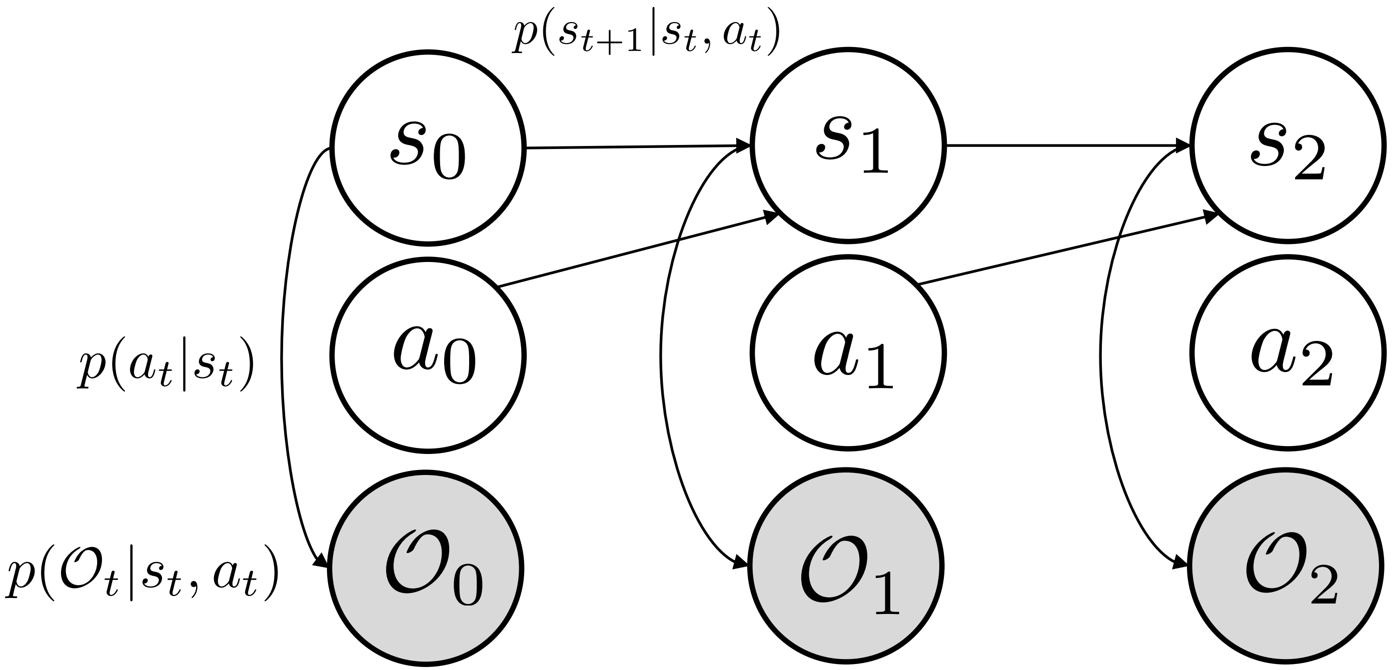
⇨

$$p(\tau|\mathcal{O}_{0:T} = 1)$$

Soft optimal behavior conditioned on optimality

We were trying to find $p(a_t|s_t, \mathcal{O}_{t:T} = 1)$ given reward

Inverse RL in CAI graphical model



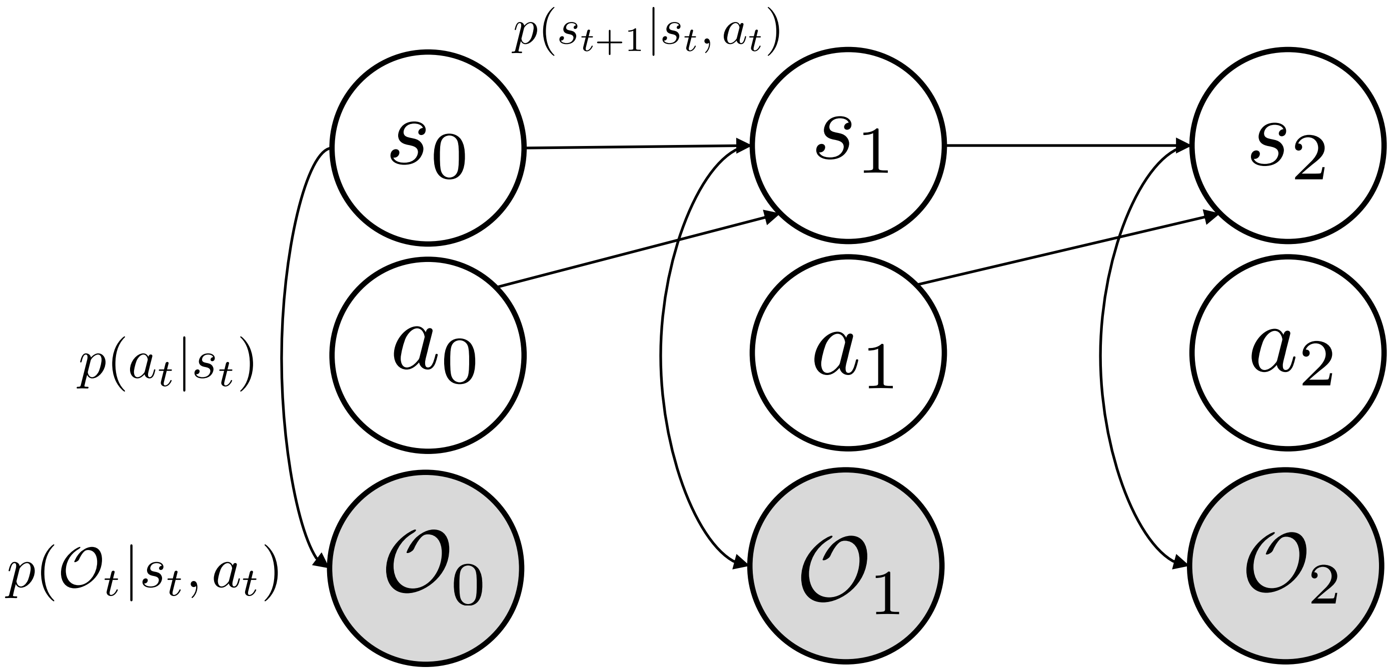
$$p(\mathcal{O}_t|s_t, a_t) = \exp(r(s_t, a_t))$$

$$p(\tau|\mathcal{O}_{0:T} = 1) \propto p(\tau)\exp(\sum_{t=0}^{T} r(s_t, a_t))$$

Now we are given (s, a) from optimal, we need to find the reward function that best explains the data

→ Maximum likelihood estimation!
(Find r, that maximizes the likelihood of (s, a) being produced on observed optimality

Inverse RL in CAI graphical model



$$p(\mathcal{O}_t|s_t, a_t) = \exp(r(s_t, a_t))$$

$$p(\tau|\mathcal{O}_{0:T} = 1) \propto p(\tau)\exp\left(\sum_{t=0}^{T} r(s_t, a_t)\right)$$

→ Maximum likelihood estimation!

(Find r, that maximizes the likelihood of (s, a) being produced on observed optimality

$$\max_{\phi} \mathbb{E}_{\tau \sim \mathcal{D}^*}\left[\log p(\tau|\mathcal{O}_{0:T} = 1)\right]$$ (Find optimality CPD that best explains observed data)

Maximum likelihood optimality estimation

$$p(\tau|\mathcal{O}_{0:T} = 1) \propto \cancel{p}(\tau) \exp(\sum_{t=0}^{T} r(s_t, a_t))$$

Independent of reward

$$= \frac{\exp(\sum_{t=0}^{T} r(s_t, a_t))}{\boxed{\int \int p(\tau) \exp(\sum_{t=0}^{T} r(s_t, a_t)) ds_{0:T} da_{0:T}}}$$

Hard to estimate – partition function (Z)

$$\Downarrow$$

$$\max_{\phi} \mathbb{E}_{\tau \sim \mathcal{D}^*} \left[\log p(\tau|\mathcal{O}_{0:T} = 1)\right]$$

Difficult to compute analytically, but it's gradient has a nice form!

Maximum likelihood optimality estimation

$$p(\tau|\mathcal{O}_{0:T} = 1) = \frac{\exp(\sum_{t=0}^{T} r(s_t, a_t))}{\int \int p(\tau) \exp(\sum_{t=0}^{T} r(s_t, a_t)) ds_{0:T} da_{0:T}}$$

$$\begin{aligned}
&\max_{\phi} \mathbb{E}_{\tau \sim \mathcal{D}^*} \left[\log p(\tau|\mathcal{O}_{0:T} = 1)\right] \\
&\quad = \mathbb{E}_{\tau \sim \mathcal{D}^*} \left[\log \left(\exp(\sum_{t=0}^{T} r_\phi(s_t, a_t)) \right) - \log Z \right] \\
&\quad = \mathbb{E}_{\tau \sim \mathcal{D}^*} \left[\sum_{t=0}^{T} r_\phi(s_t, a_t) \right] - \log Z
\end{aligned}$$

Easy to compute

Hard to compute

Maximum likelihood optimality estimation

$$p(\tau|\mathcal{O}_{0:T} = 1) = \frac{\exp(\sum_{t=0}^{T} r(s_t, a_t))}{\int \int p(\tau) \exp(\sum_{t=0}^{T} r(s_t, a_t)) ds_{0:T} da_{0:T}}$$

$$\max_{\phi} \mathbb{E}_{\tau \sim \mathcal{D}^*} \left[\log p(\tau|\mathcal{O}_{0:T} = 1)\right]$$

$$= \mathbb{E}_{\tau \sim \mathcal{D}^*} \left[\log \left(\exp(\sum_{t=0}^{T} r_\phi(s_t, a_t)) \right) - \log Z\right]$$

$$= \mathbb{E}_{\tau \sim \mathcal{D}^*} \left[\sum_{t=0}^{T} r_\phi(s_t, a_t)\right] - \log Z$$

Easy to compute

Hard to compute

Let’s take the gradient

$$\max_{\phi} \mathbb{E}_{\tau \sim \mathcal{D}^*} \left[\log p(\tau | \mathcal{O}_{0:T} = 1)\right]$$

$$\mathcal{L}(\phi) = \mathbb{E}_{\tau \sim \mathcal{D}^*} \left[\sum_{t=0}^{T} r_\phi(s_t, a_t)\right] - \log Z$$

$$\nabla_\phi \mathcal{L}(\phi) = \mathbb{E}_{\tau \sim \mathcal{D}^*} \left[\sum_{t=0}^{T} \nabla_\phi r_\phi(s_t, a_t)\right] - \nabla_\phi \log Z$$

$$\nabla_\phi \log Z = \frac{1}{Z} \nabla_\phi Z$$

$$\boxed{Z = \int p(\tau) \exp(r(\tau)) d\tau}$$

$$\nabla_\phi \mathcal{L}(\phi) = \mathbb{E}_{\tau \sim \mathcal{D}^*} \left[\sum_{t=0}^{T} \nabla_\phi r_\phi(s_t, a_t)\right] - \boxed{\frac{1}{Z} \int p(\tau) \exp(r_\phi(\tau))} \nabla_\phi r_\phi(\tau) d\tau$$

Notice this is exactly the soft optimality posterior

$$p(\tau | \mathcal{O}_{0:T} = 1) \propto p(\tau) \exp(\sum_{t=0}^{T} r(s_t, a_t))$$

Let's take the gradient

$$\mathcal{L}(\phi) = \mathbb{E}_{\tau \sim \mathcal{D}^*}\left[\sum_{t=0}^{T} r_\phi(s_t, a_t)\right] - \log Z$$

$$\nabla_\phi \mathcal{L}(\phi) = \mathbb{E}_{\tau \sim \mathcal{D}^*}\left[\sum_{t=0}^{T} \nabla_\phi r_\phi(s_t, a_t)\right] - \frac{1}{Z}\int p(\tau)\exp(r_\phi(\tau))\nabla_\phi r_\phi(\tau) d\tau$$

Notice this is exactly the soft optimality posterior

$$p(\tau|\mathcal{O}_{0:T} = 1) \propto p(\tau)\exp(\sum_{t=0}^{T} r(s_t, a_t))$$

$$\nabla_\phi \mathcal{L}(\phi) = \mathbb{E}_{\tau \sim \mathcal{D}^*}\left[\sum_{t=0}^{T} \nabla_\phi r_\phi(s_t, a_t)\right] - \mathbb{E}_{\tau \sim p(\tau|\mathcal{O}_{0:T}=1)}\left[\sum_{t=0}^{T} \nabla_\phi r_\phi(s_t, a_t)\right]$$

Push up gradients along experts

Push down gradients along soft optimal policy under current reward

Computable, with RL in the inner loop

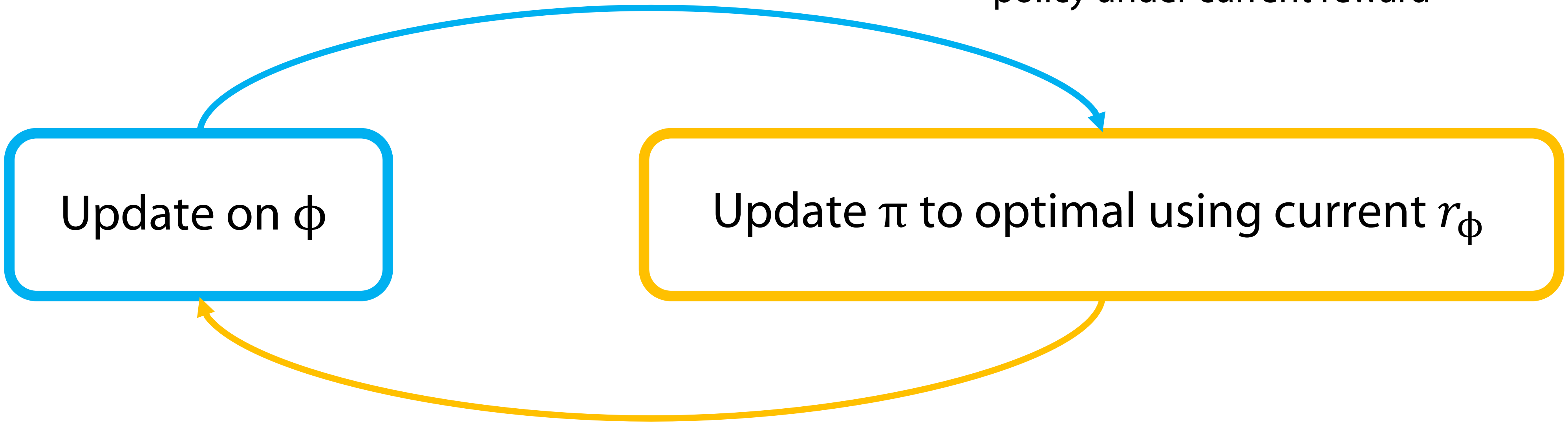
Ok so what does this mean?

$$\nabla_\phi \mathcal{L}(\phi) = \mathbb{E}_{\tau \sim \mathcal{D}^*}\left[\sum_{t=0}^{T} \nabla_\phi r_\phi(s_t, a_t)\right] - \frac{1}{Z}\int p(\tau)\exp(r_\phi(\tau))\nabla_\phi r_\phi(\tau) d\tau$$

$$\nabla_\phi \mathcal{L}(\phi) = \mathbb{E}_{\tau \sim \mathcal{D}^*}\left[\sum_{t=0}^{T} \nabla_\phi r_\phi(s_t, a_t)\right] - \mathbb{E}_{\tau \sim p(\tau | \mathcal{O}_{0:T}=1)}\left[\sum_{t=0}^{T} \nabla_\phi r_\phi(s_t, a_t)\right]$$

Push up gradients along experts

Push down gradients along soft optimal policy under current reward



Update on ϕ

Update π to optimal using current r_ϕ

Alternative intuition

Why do we even need the margin in the first place?

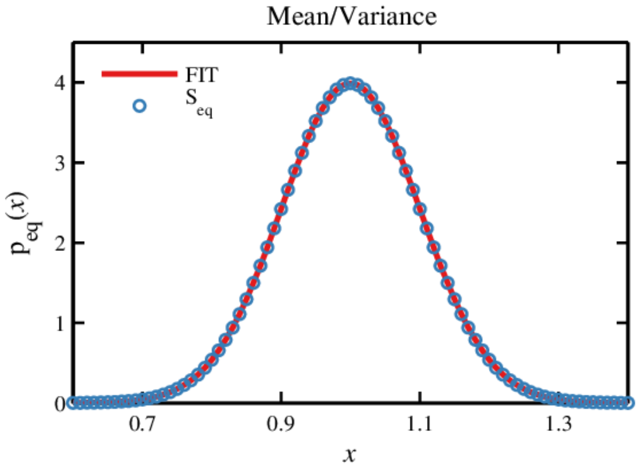
$$V_r^{\pi^*}(s) \geq V_r^{\pi}(s) \quad \forall \pi, \forall s \longleftarrow \text{underdefined}$$

Unclear how to value one suboptimal trajectory vs other → be maximally uniform!

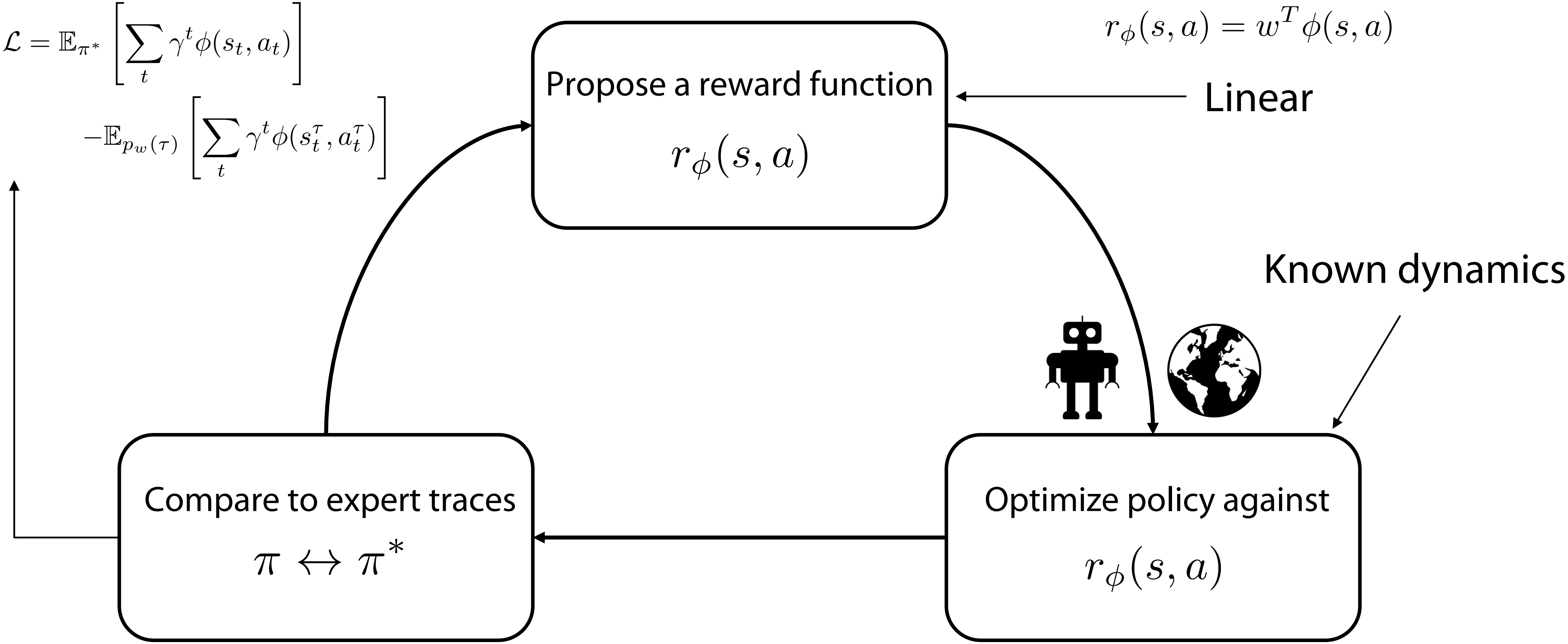
$$\max_{p(\tau)} \quad \mathcal{H}(p(\tau)) \longrightarrow \text{Maximize entropy}$$

$$\text{s.t} \quad \mathbb{E}_{p(\tau)}[\phi(s,a)] \approx \mathbb{E}_{\pi^*}[\phi(s,a)]$$

While matching features



IRL v2 – Max-Ent IRL – Put it together



Maximum Entropy

$$\nabla_w \mathcal{L} = \mathbb{E}_{\pi^*}\left[\sum_t \gamma^t \phi(s_t, a_t)\right] - \mathbb{E}_{p_w(\tau)}\left[\sum_t \gamma^t \phi(s_t^\tau, a_t^\tau)\right]$$

$$r_\phi(s,a) = w^T \phi(s,a)$$

Propose a reward function

$r_\phi(s,a)$

Linear

Known dynamics

Optimize policy against

$r_\phi(s,a)$

Compare to expert traces

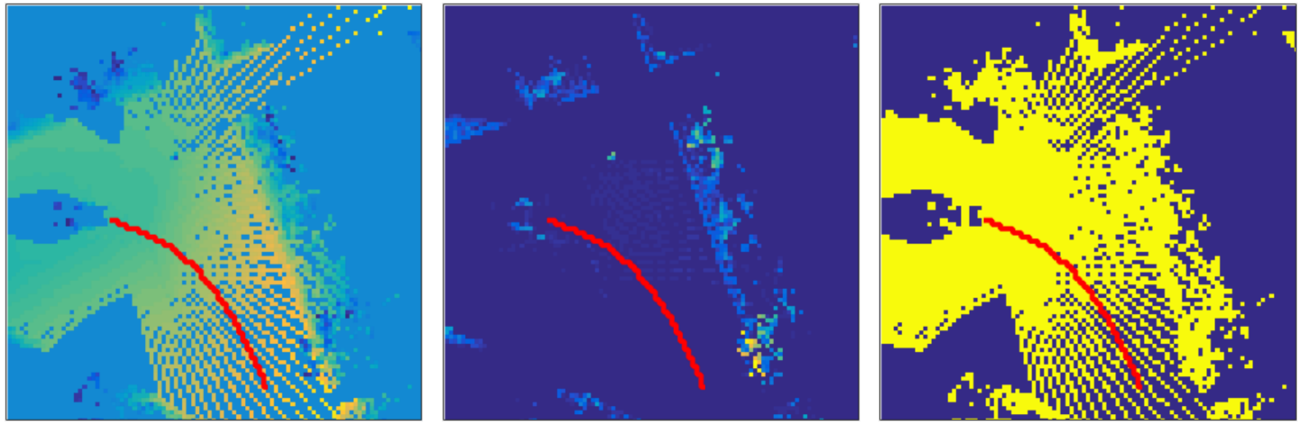
$\pi \leftrightarrow \pi^*$

IRL v2 –Max-Entropy Inverse RL (Pseudocode)

```
1. Start with a random policy $\pi_0$ and weight vector w
2. Find the “soft” optimal policy under w – $p_w(\tau)$
3. Take a gradient step on w
       $$\nabla_w \mathcal{L} = \mathbb{E}_{\pi^*}\left[\sum_t \gamma^t \phi(s_t, a_t)\right] - \mathbb{E}_{p_w(\tau)}\left[\sum_t \gamma^t \phi(s_t^\tau, a_t^\tau)\right]$$
4. Repeat
```

Output the optimal reward function w*

Max-Ent IRL in Action



Lecture Outline

Why Imitation? + Problem formulation

↓

IRLv1 – max margin planning

↓

IRLv2 – max entropy IRL

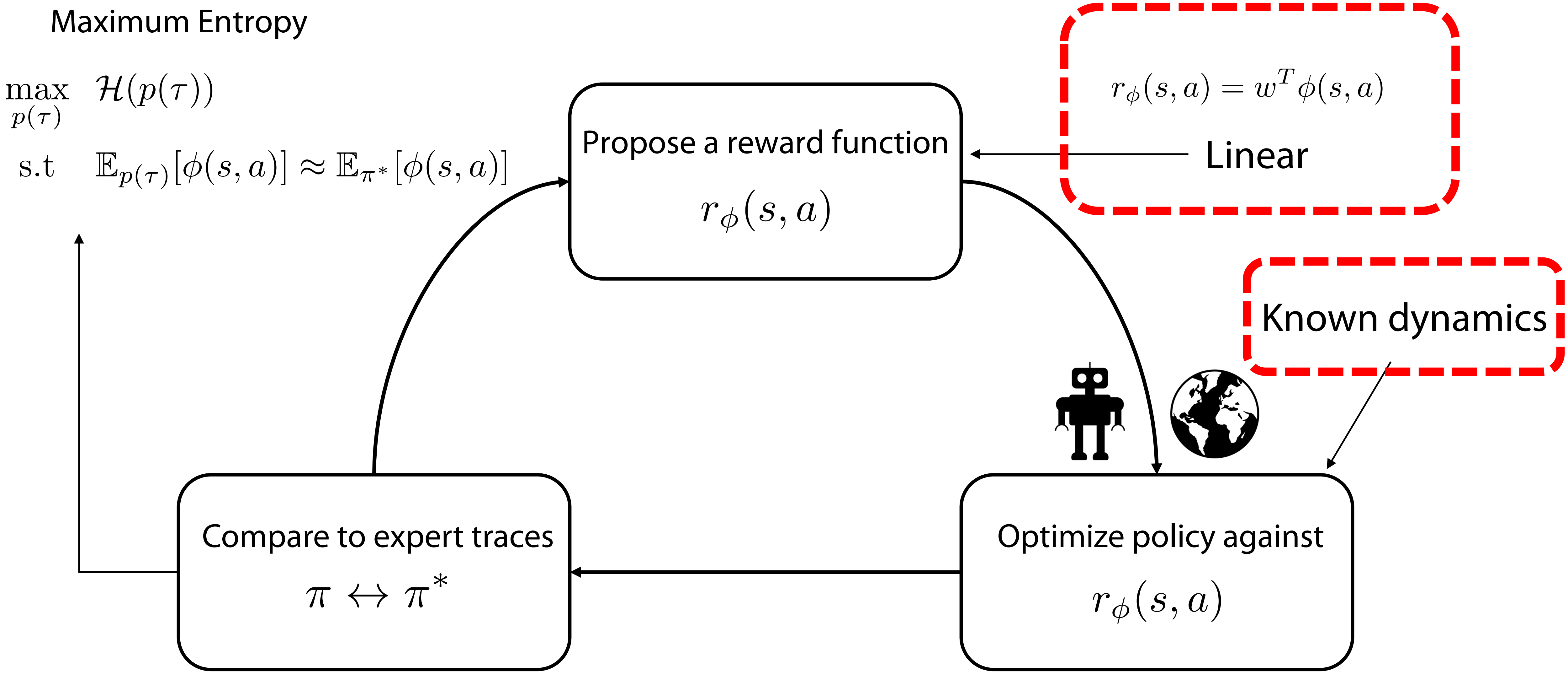
↓

IRLv3 – partial policy optimization

↓

IRLv4 – adversarial IRL

Ok but no way this could work?



Maximum Entropy

$$\max_{p(\tau)} \quad \mathcal{H}(p(\tau))$$
$$\text{s.t} \quad \mathbb{E}_{p(\tau)}[\phi(s,a)] \approx \mathbb{E}_{\pi^*}[\phi(s,a)]$$

Propose a reward function

$r_\phi(s,a)$

$r_\phi(s,a) = w^T\phi(s,a)$

Linear

Known dynamics

Optimize policy against

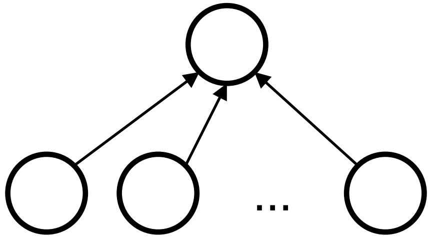
$r_\phi(s,a)$

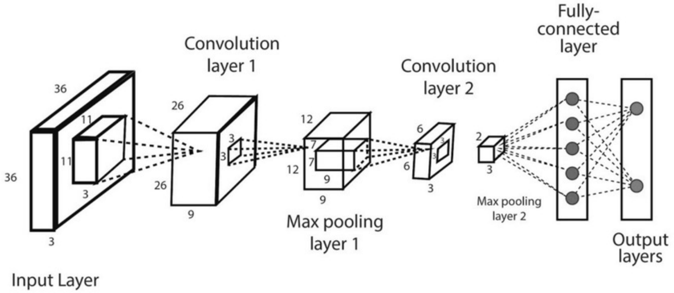
Compare to expert traces

$\pi \leftrightarrow \pi^*$

Linear Rewards → Neural Net Rewards

Max-ent IRL allows us to go from linear rewards to arbitrary neural network rewards





Linear Max-Ent IRL

$$\max_{w} \mathbb{E}_{\pi^*}\left[\sum_t w^T \gamma^t \phi(s_t, a_t)\right] - \log \int_{\tau} \left[\exp\left(\sum_t w^T \gamma^t \phi(s_t, a_t)\right)\right] d\tau$$

⇩

Non-Linear Max-Ent IRL

$$\max_{\theta} \mathbb{E}_{\pi^*}\left[\sum_t \gamma^t r_\theta(s_t, a_t)\right] - \log \int_{\tau} \left[\exp\left(\sum_t \gamma^t r_\theta(s_t, a_t)\right)\right] d\tau$$

Can simply replace, w with arbitrary θ and use autodiff!

Avoiding Complete Policy Optimization

Optimize policy against

$$r_\phi(s, a)$$

← Assumes dynamics are known so we can just do (fast) planning

What happens when dynamics are unknown!

$$\mathbb{E}_{\pi^*}\left[\sum_t \gamma^t \nabla_\theta r_\theta(s_t, a_t)\right]$$

$$-\mathbb{E}_{p_w(\tau)}\left[\sum_t \gamma^t \nabla_\theta r_\theta(s_t, a_t)\right]$$

← What if we only **improved** the policy a little bit

Biased!

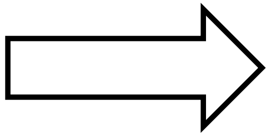
Requires complete "soft" policy optimization

Avoiding Complete Policy Optimization

Importance sampling to the rescue!

$$\mathbb{E}_{p(x)}\left[f(x)\right] = \mathbb{E}_{q(x)}\left[\frac{p(x)}{q(x)} f(x)\right]$$

$$\mathbb{E}_{\pi^*}\left[\sum_t \gamma^t \nabla_\theta r_\theta(s_t, a_t)\right]$$
$$-\mathbb{E}_{p_w(\tau)}\left[\sum_t \gamma^t \nabla_\theta r_\theta(s_t, a_t)\right]$$

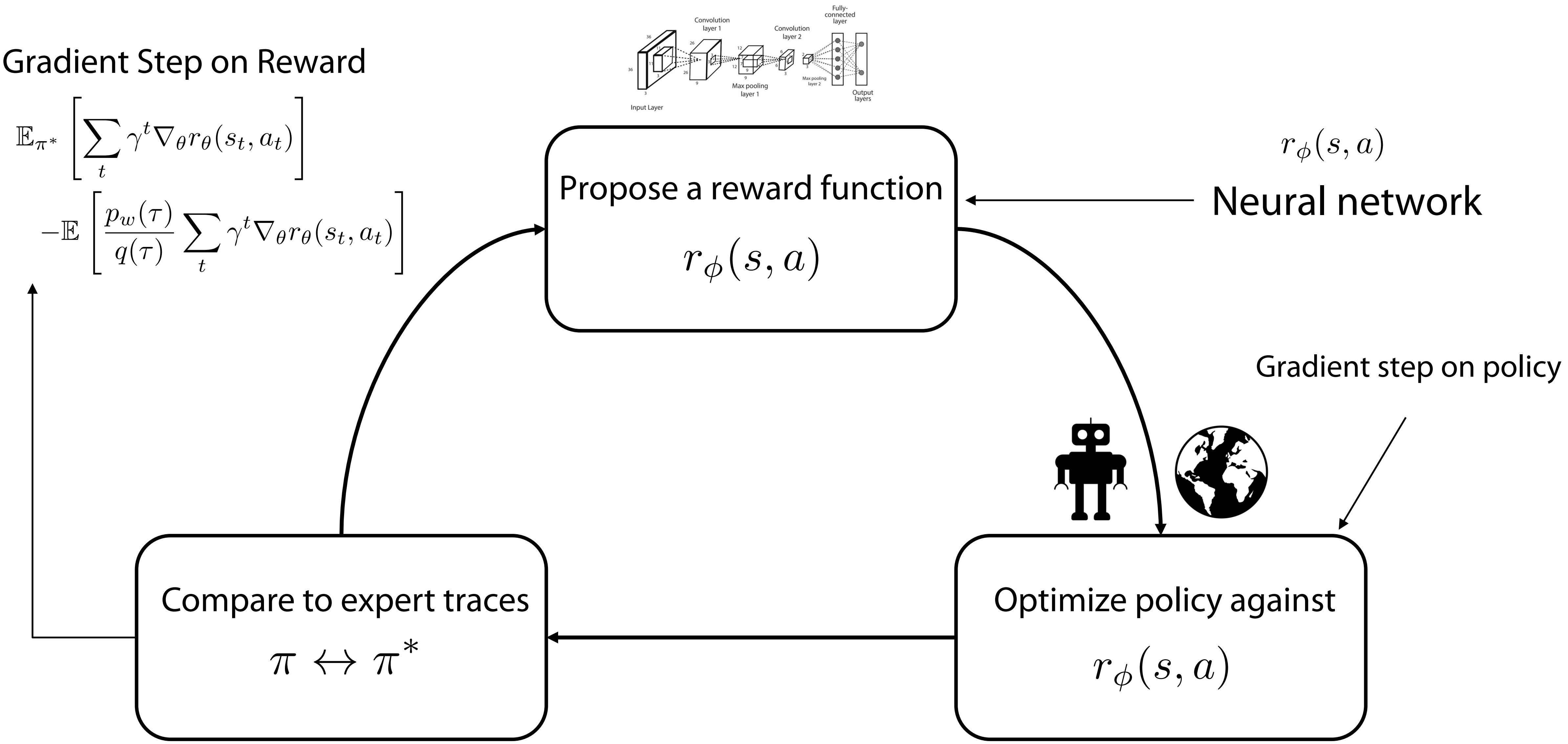


Importance Sampling

$$\mathbb{E}_{\pi^*}\left[\sum_t \gamma^t \nabla_\theta r_\theta(s_t, a_t)\right]$$
$$-\mathbb{E}_q\left[\frac{p_w(\tau)}{q(\tau)} \sum_t \gamma^t \nabla_\theta r_\theta(s_t, a_t)\right]$$
$$\hookrightarrow \frac{\exp(\sum_t r_\theta(s_t, a_t))}{\Pi_t \pi_\theta(a_t|s_t)}$$

Can transfer significantly more from iteration to iteration rather than doing full nested optimization

IRLv4 – Guided Cost Learning



Gradient Step on Reward

$$\mathbb{E}_{\pi^*}\left[\sum_t \gamma^t \nabla_\theta r_\theta(s_t, a_t)\right]$$

$$-\mathbb{E}\left[\frac{p_w(\tau)}{q(\tau)} \sum_t \gamma^t \nabla_\theta r_\theta(s_t, a_t)\right]$$

Propose a reward function

$$r_\phi(s, a)$$

$r_\phi(s, a)$

Neural network

Gradient step on policy

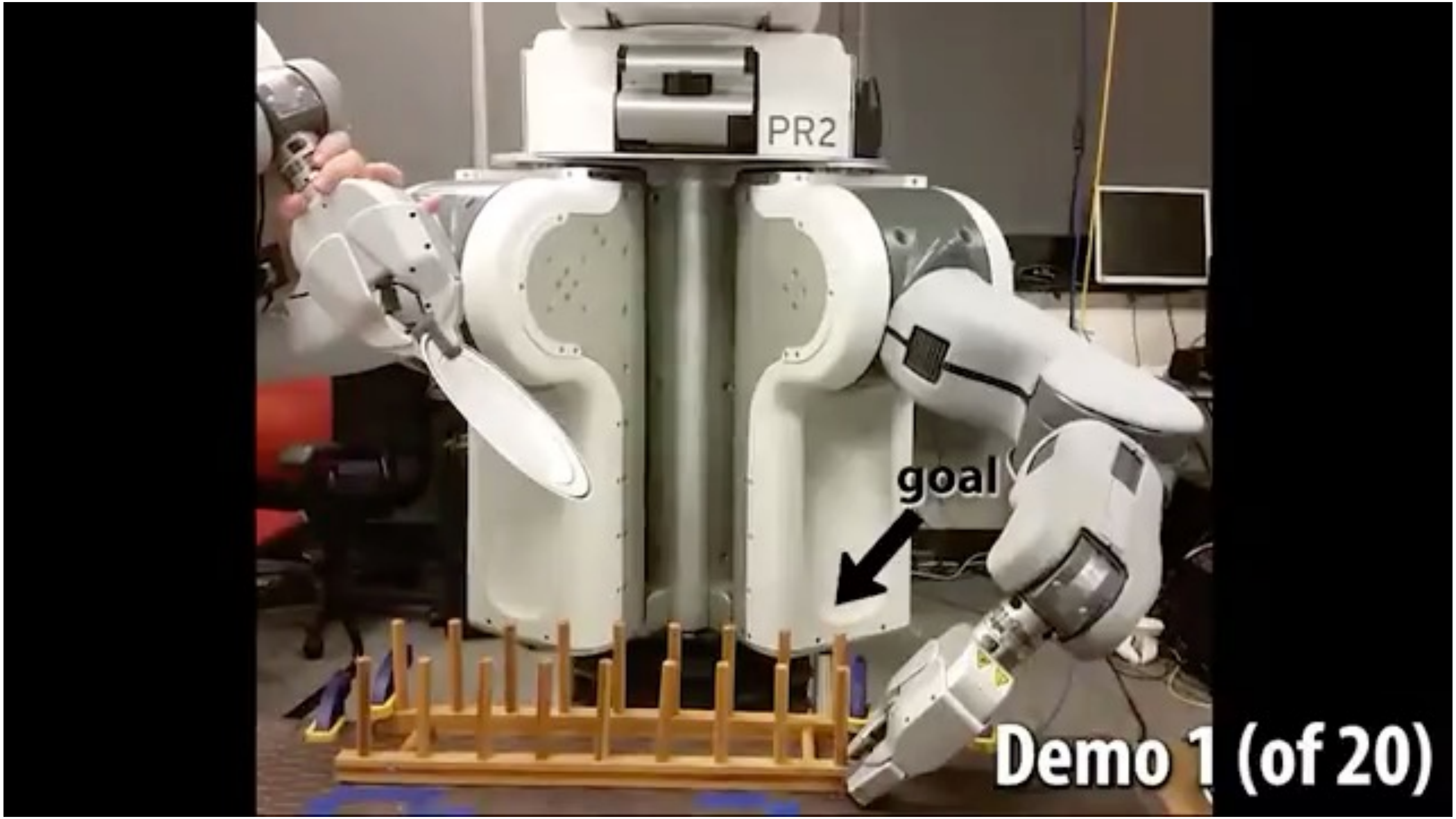
Optimize policy against

$$r_\phi(s, a)$$

Compare to expert traces

$$\pi \leftrightarrow \pi^*$$

IRLv4 – Guided Cost Learning



Lecture Outline

Why Imitation? + Problem formulation

↓

IRLv1 – max margin planning

↓

IRLv2 – max entropy IRL

↓

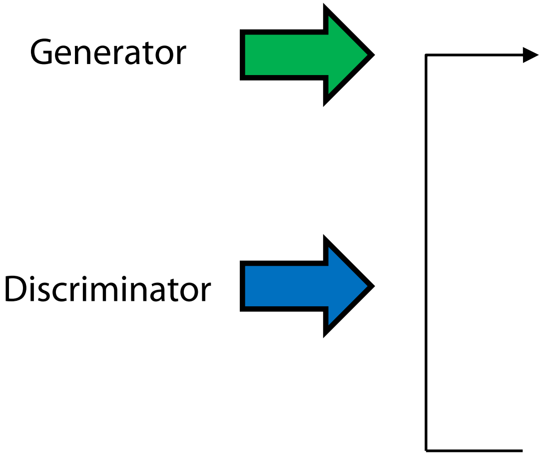
IRLv3 – partial policy optimization

↓

IRLv4 – adversarial IRL

Connecting Maximum-Entropy RL to GANs

Looks like a game



1. Start with a random policy π_0 and weight vector w
2. Take a step on “soft” optimal policy under w – $p_w(\tau)$
3. Take a gradient step on w

$$\nabla_\theta \mathcal{L} = \mathbb{E}_{\pi^*}\left[\sum_t \gamma^t \nabla_\theta r_\theta(s_t, a_t)\right] - \mathbb{E}_q\left[\frac{p_w(\tau)}{q(\tau)} \sum_t \gamma^t \nabla_\theta r_\theta(s_t, a_t)\right]$$

4. Repeat

Output the optimal reward function w*

Recasting GAIL as an IRL method

For a particular parameterization of the discriminator, GAIL recovers a reward

Max-Ent Inverse RL

$$\mathbb{E}_{\pi^*}\left[\sum_t \gamma^t \nabla_\theta r_\theta(s_t, a_t)\right]$$
$$-\mathbb{E}_q\left[\frac{p_w(\tau)}{q(\tau)} \sum_t \gamma^t \nabla_\theta r_\theta(s_t, a_t)\right]$$

Push up demos, push down policy

$\Longleftrightarrow$

With some massaging

GAIL

$$\mathbb{E}_{\pi^*}\left[D_\psi(\tau)\right] +$$
$$\mathbb{E}_{\pi_\theta}\left[(1 - D_\psi(\tau))\right]$$

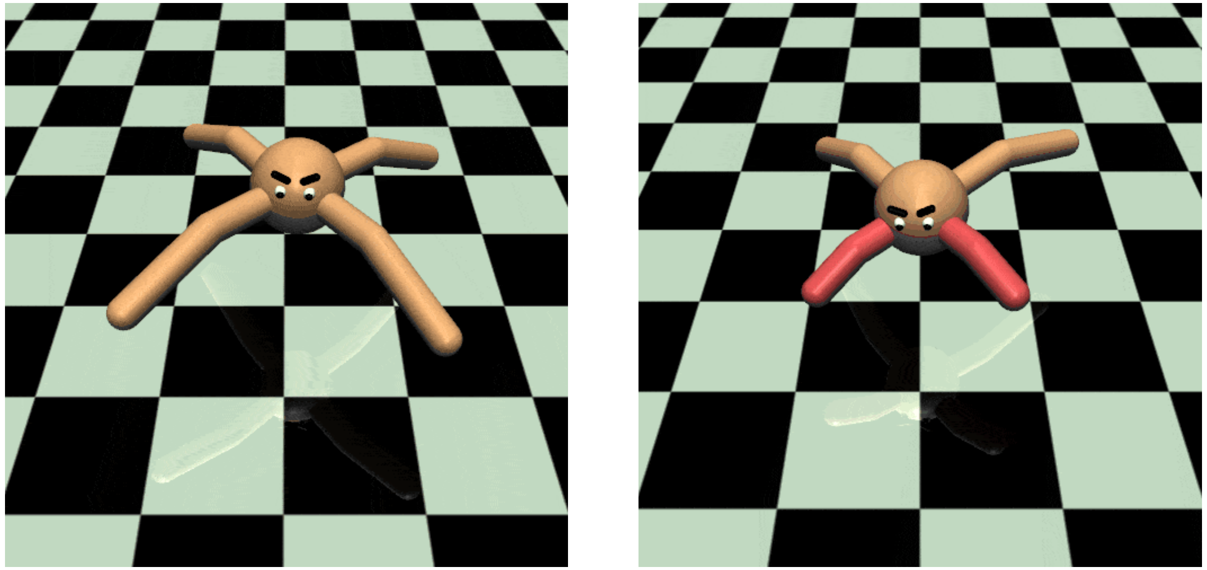
Push up real data, push down generated

$$D_\theta(\tau) = \frac{\frac{1}{Z}\exp(r_\theta(\tau))}{\frac{1}{Z}\exp(r_\theta(\tau)) + \Pi_t \pi_\theta(a_t|s_t)}$$

GAIL (which is just a GAN), recovers Max-Ent IRL

In practice, often use GAIL and just log D as reward

Adversarial IRL in Action



Lecture Outline

Why Imitation? + Problem formulation

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↓

IRLv4 – adversarial IRL

Some perspectives on IRL vs Imitation

Imitation Learning

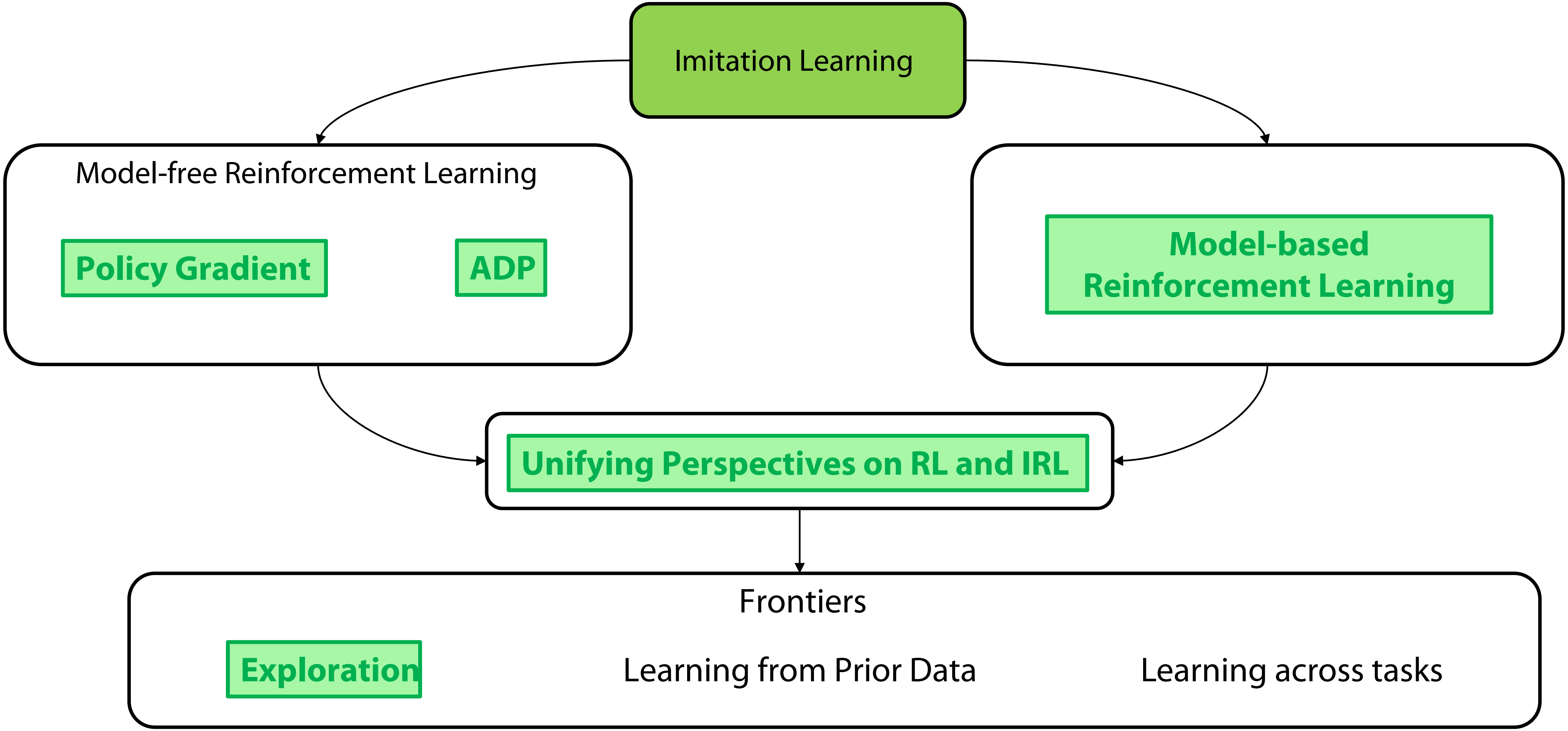
\+ simple, easy to implement
\+ no additional interaction required
- compounding error
- Multimodality
- generalization

Inverse RL

\+ potential for generalization
\+ can help with covariate shift
- Needs environment access
- Hard to implement/train
- Often works worse from images

Choose depending on the application

Class Structure



Imitation Learning

Model-free Reinforcement Learning

Policy Gradient

ADP

Model-based Reinforcement Learning

Unifying Perspectives on RL and IRL

Frontiers

Exploration

Learning from Prior Data

Learning across tasks