

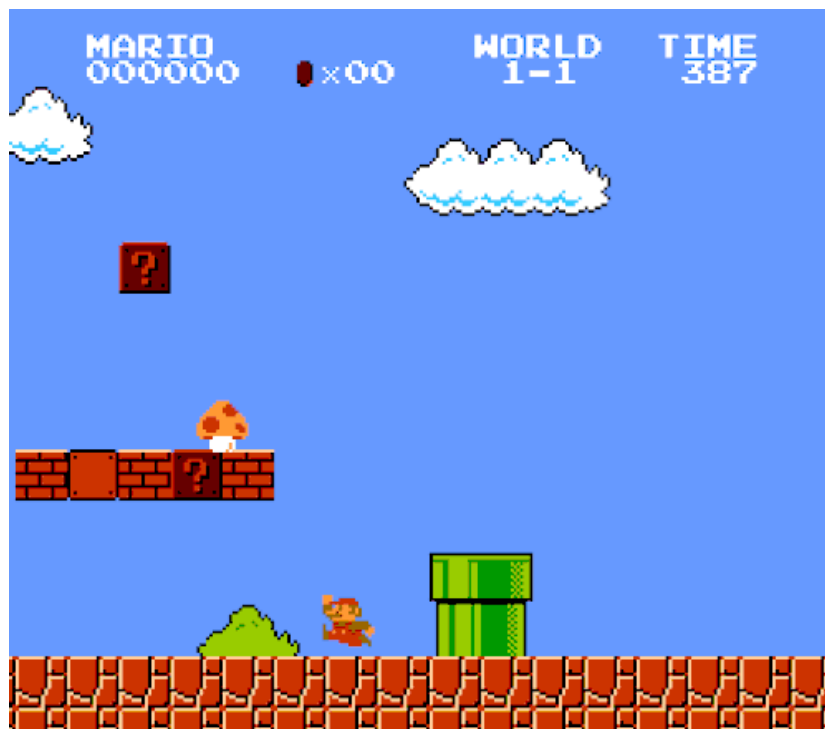


# Reinforcement Learning

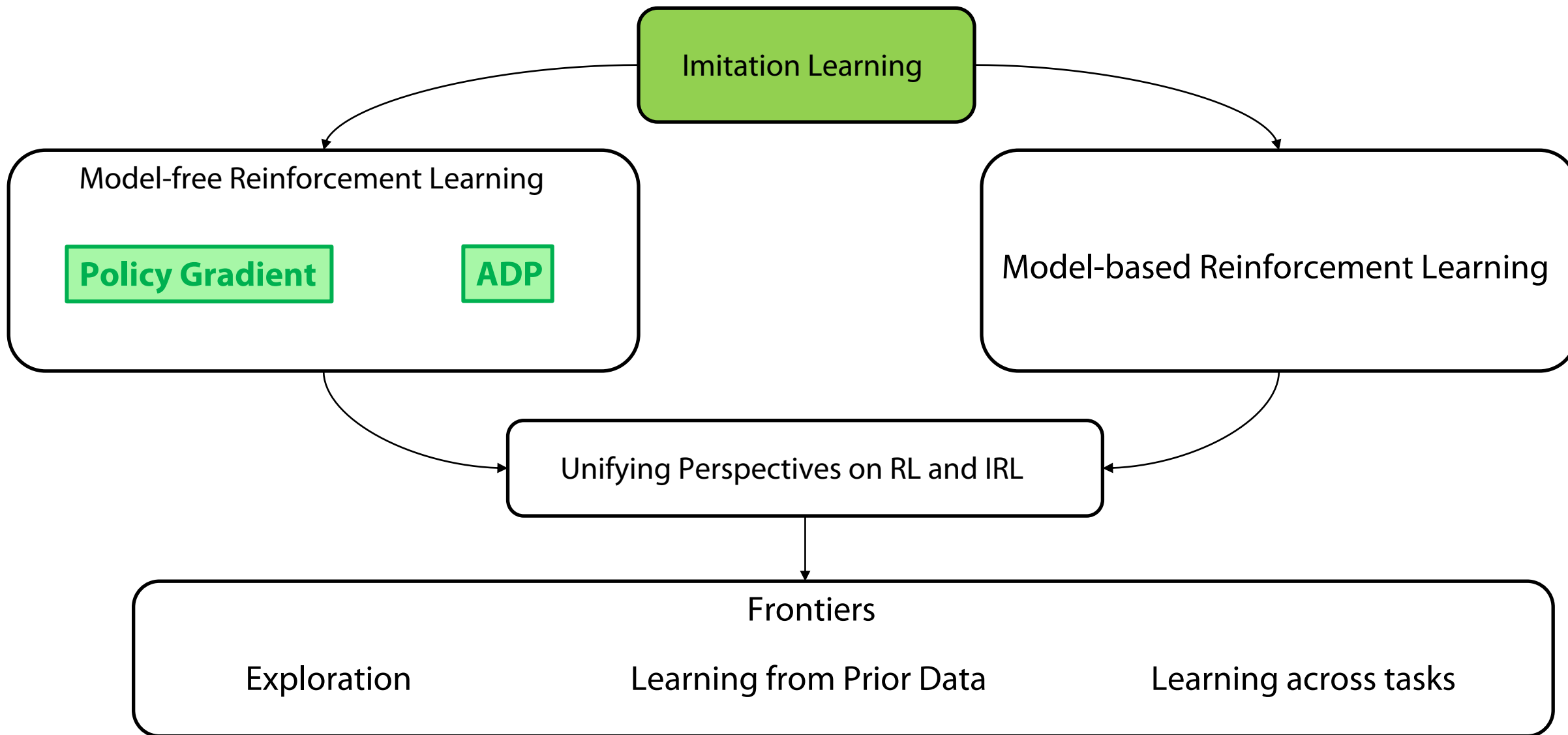
## Spring 2024

Abhishek Gupta

TAs: Patrick Yin, Qiuyu Chen



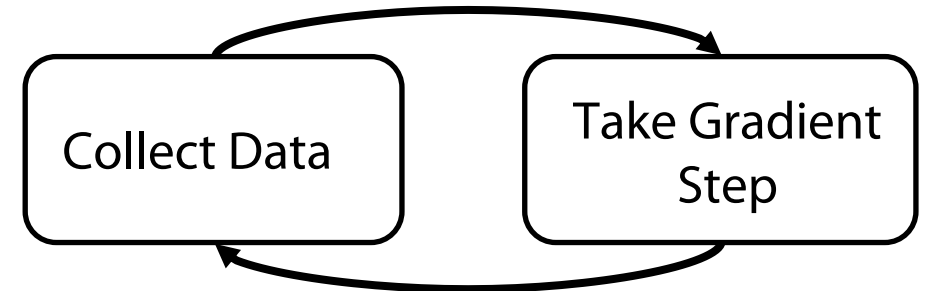
# Class Structure



# Resulting Algorithm (REINFORCE)

$$\nabla_{\theta} J(\theta) = \int p_{\theta}(\tau) \nabla_{\theta} \log p_{\theta}(\tau) d\tau$$

$$\theta_{i+1} = \theta_i + \alpha \nabla_{\theta} J(\theta) |_{\theta=\theta_i}$$



REINFORCE algorithm:

On-policy



1. sample  $\{\tau^i\}$  from  $\pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)$  (run it on the robot)
2.  $\nabla_{\theta} J(\theta) \approx \sum_i (\sum_t \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t^i | \mathbf{s}_t^i)) (\sum_t r(\mathbf{s}_t^i, \mathbf{a}_t^i))$
3.  $\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$

# Challenges in Policy Gradient

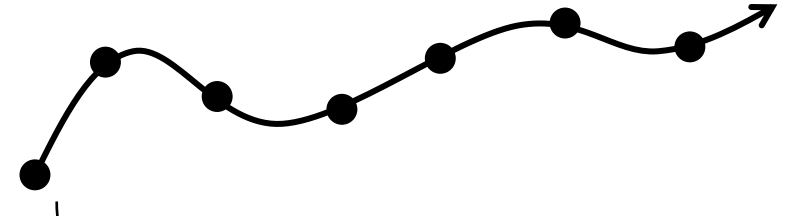
$$\nabla_{\theta} J(\theta) = \int p_{\theta}(\tau) \nabla_{\theta} \log p_{\theta}(\tau) d\tau$$

$$\approx \frac{1}{N} \sum_{i=0}^N \sum_{t=0}^T \nabla_{\theta} \log \pi_{\theta}(a_t^i | s_t^i) \sum_{t'=0}^T r(s_{t'}^i, a_{t'}^i)$$

**High variance estimator!!**

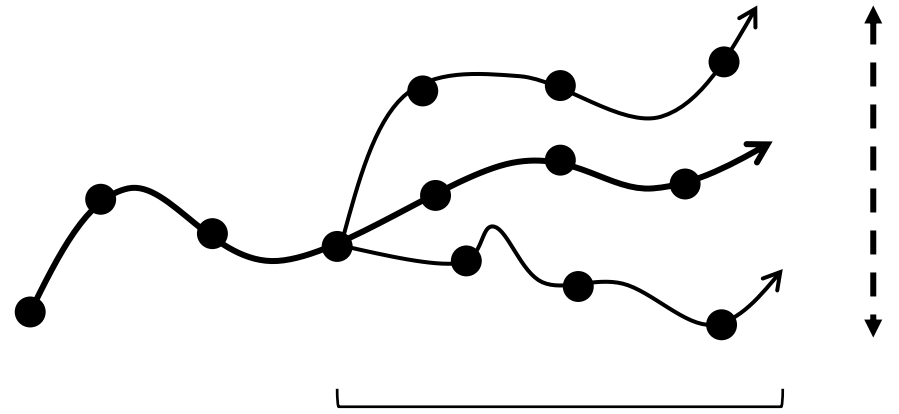
Hard to tell what matters without many samples

What we do



Single sample estimate

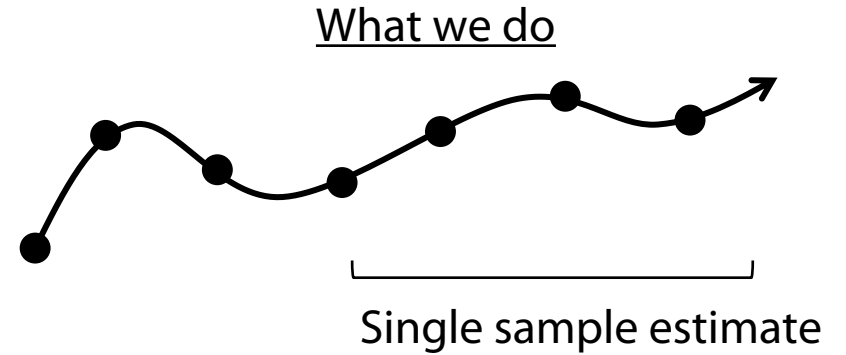
What we actually want



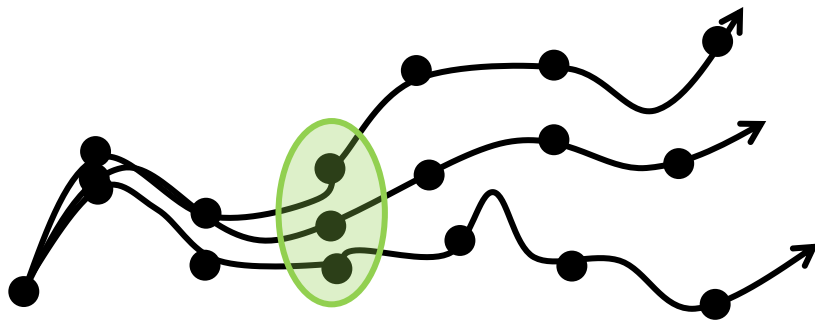
Averaged return estimate

# What can we do to lower variance?

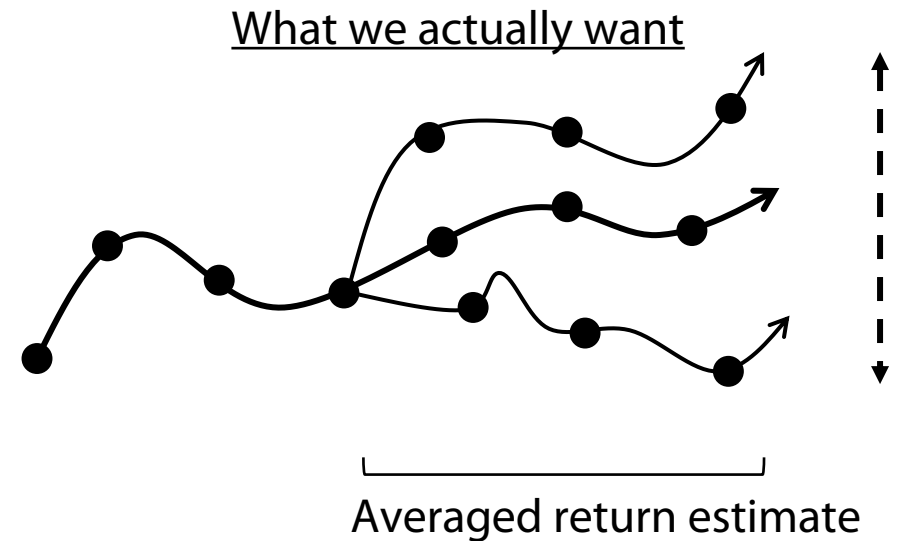
$$\begin{aligned}\nabla_{\theta} J(\theta) &= \int p_{\theta}(\tau) \nabla_{\theta} \log p_{\theta}(\tau) d\tau \\ &\approx \frac{1}{N} \sum_{i=1}^N \sum_t \nabla_{\theta} \log \pi_{\theta}(a_t^i | s_t^i) \underbrace{\sum_{t'=t}^T r(s_{t'}^i, a_{t'}^i)}\end{aligned}$$



Idea: bundle this across many (s, a) with a function approximator



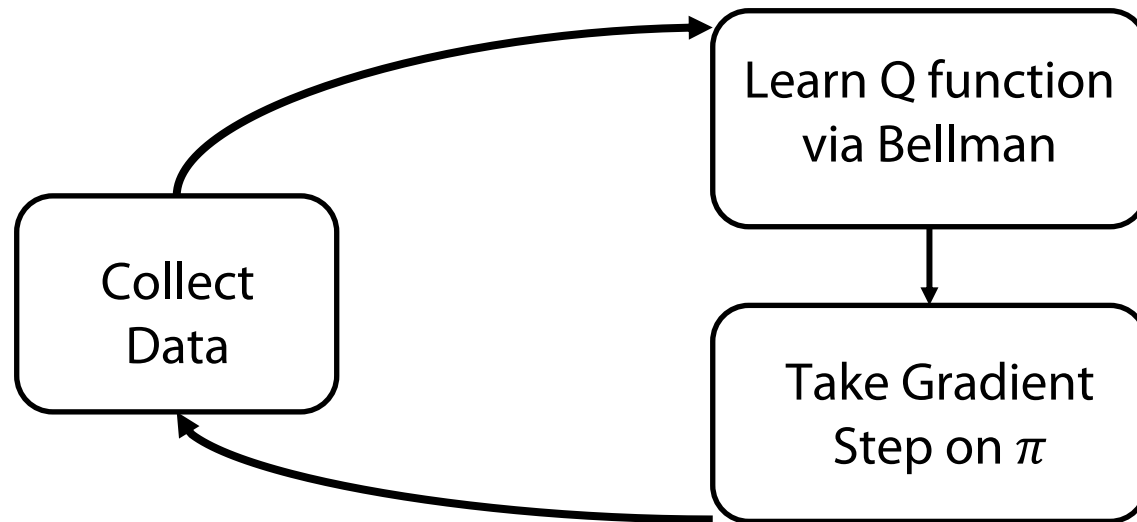
Function approximator bundles return estimates across states



# Recap of Off-Policy Reinforcement Learning

Critic: learned via the Bellman update (Policy Evaluation)

$$\min_{\phi} \mathbb{E}_{(s_t, a_t, s_{t+1}) \sim \mathcal{D}} \left( Q_{\phi}^{\pi}(s_t, a_t) - (r(s_t, a_t) + Q_{\hat{\phi}}^{\pi}(s_{t+1}, a_{t+1})) \right)^2 \quad a_{t+1} \sim \pi(\cdot | s_{t+1})$$



Lowers variance and is off-policy!

Actor: updated using learned critic (Policy Improvement)

$$\max_{\pi} \mathbb{E}_{s \sim \mathcal{D}} \mathbb{E}_{a \sim \pi(\cdot | s)} [Q^{\pi}(s, a)]$$

# Pros/Cons of Off-Policy Methods in Robotics

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## Pros:

1. Sample-efficient enough for real world
2. Can learn from images with suitable design choices
3. Off-policy, can incorporate prior data

## Cons

1. Often unstable
2. Can achieve lower asymptotic performance
3. Requires significant storage

# Lecture outline

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The Anatomy of Model-Based Reinforcement Learning



Model based RL v0 → random shooting + MPC



Model based RL v1 → MPPI + MPC



Model based RL v2 → uncertainty based models



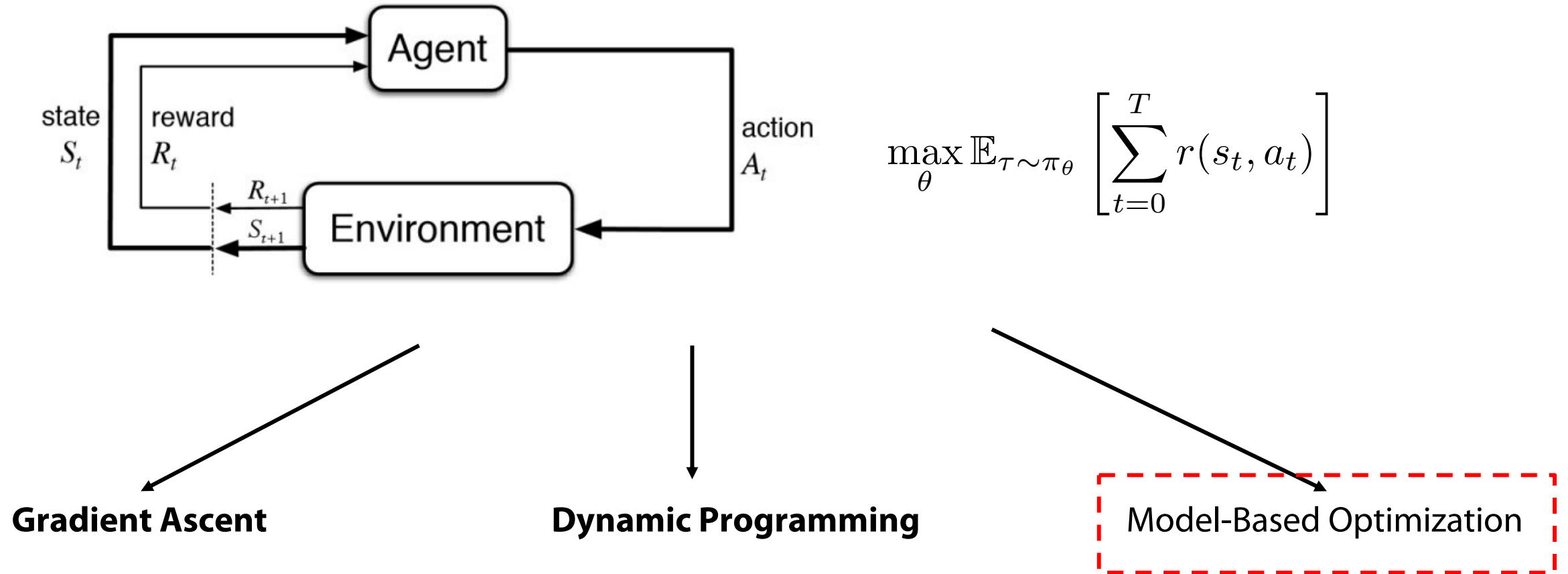
Model based RL v3 → policy optimization with models



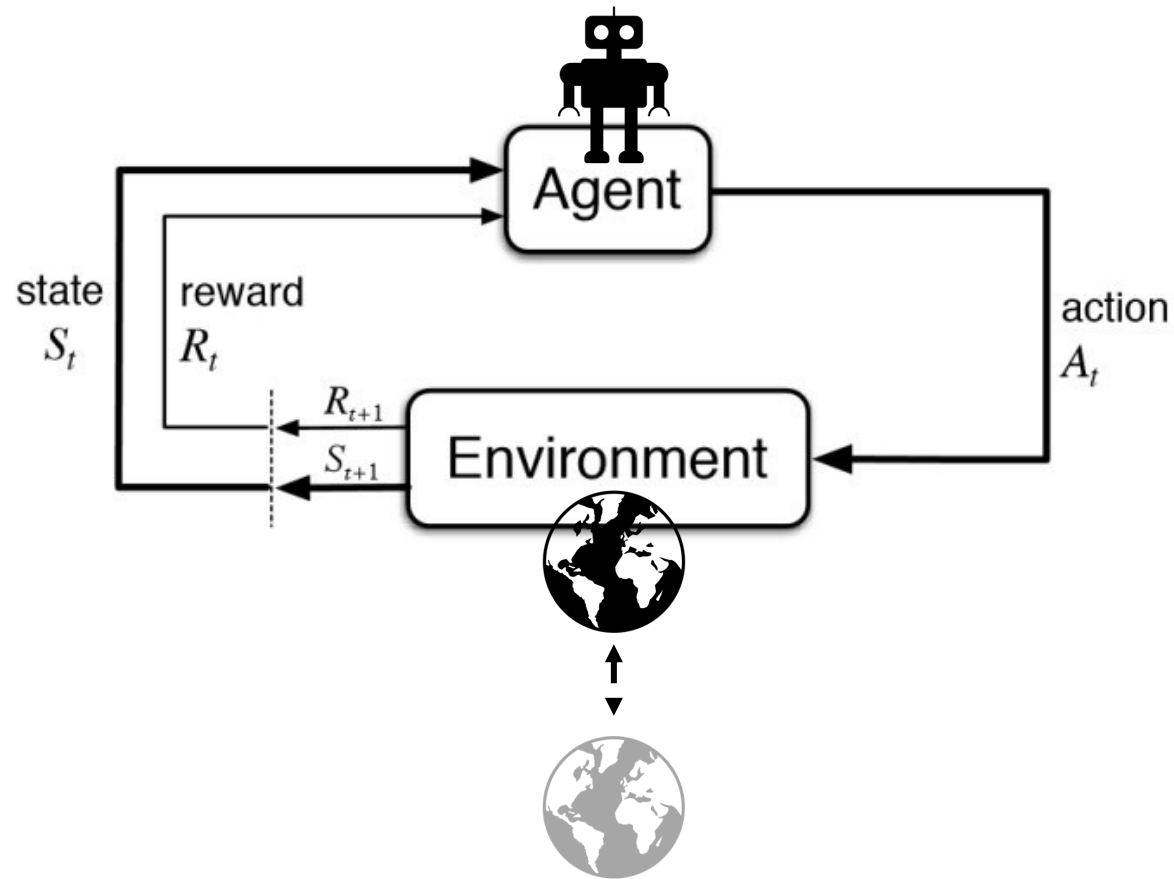
Model based RL v4 → latent space models with images



# Landscape of Reinforcement Learning Algorithms



# What if we just learned how the world worked?



$$\max_{\theta} \mathbb{E}_{\tau \sim \pi_{\theta}} \left[ \sum_{t=0}^T r(s_t, a_t) \right]$$

1. Learn a surrogate model of the transition dynamics from arbitrary off-policy data
2. Do reward maximization against this model

Intuitive: learn how the world works first and then plan in that model

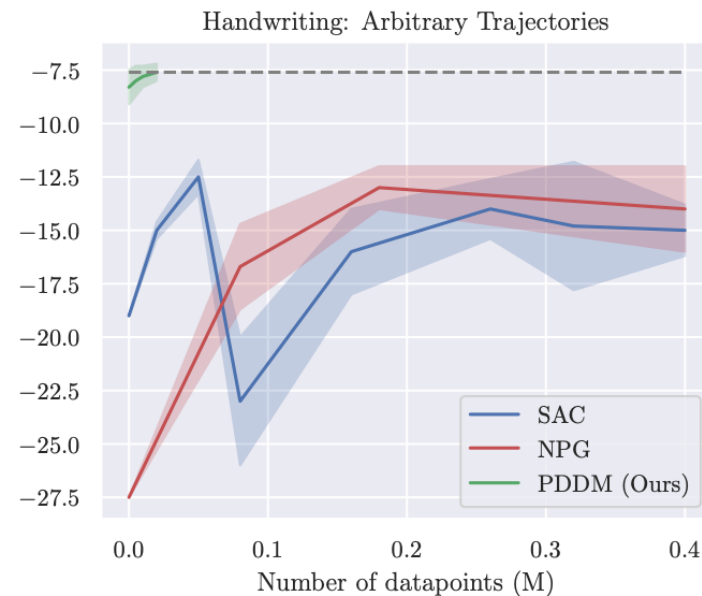
# Why do model-based RL?

Why would we do this?

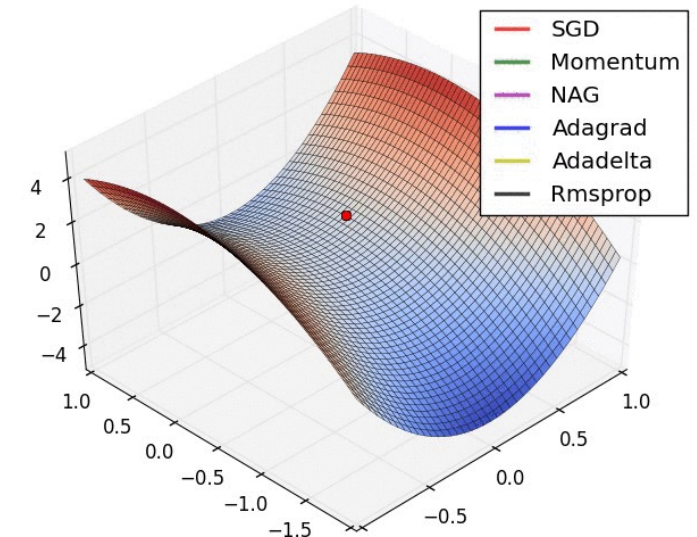
Transfer/Adaptive



Efficiency

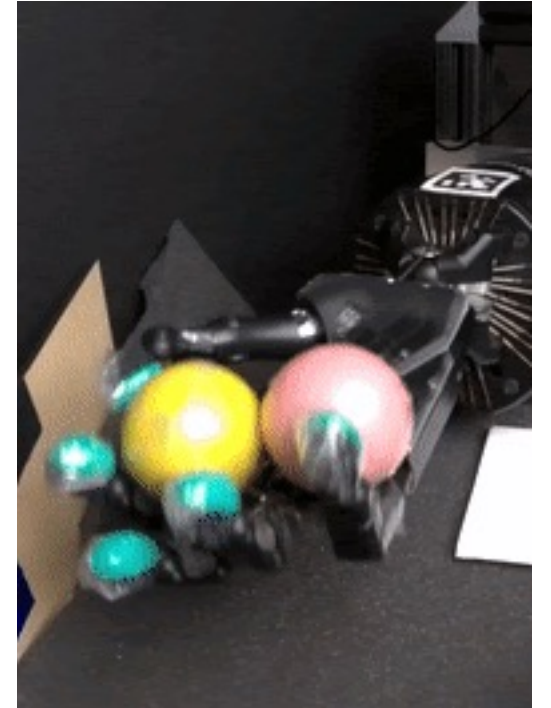


Simplicity



Naturally off-policy!

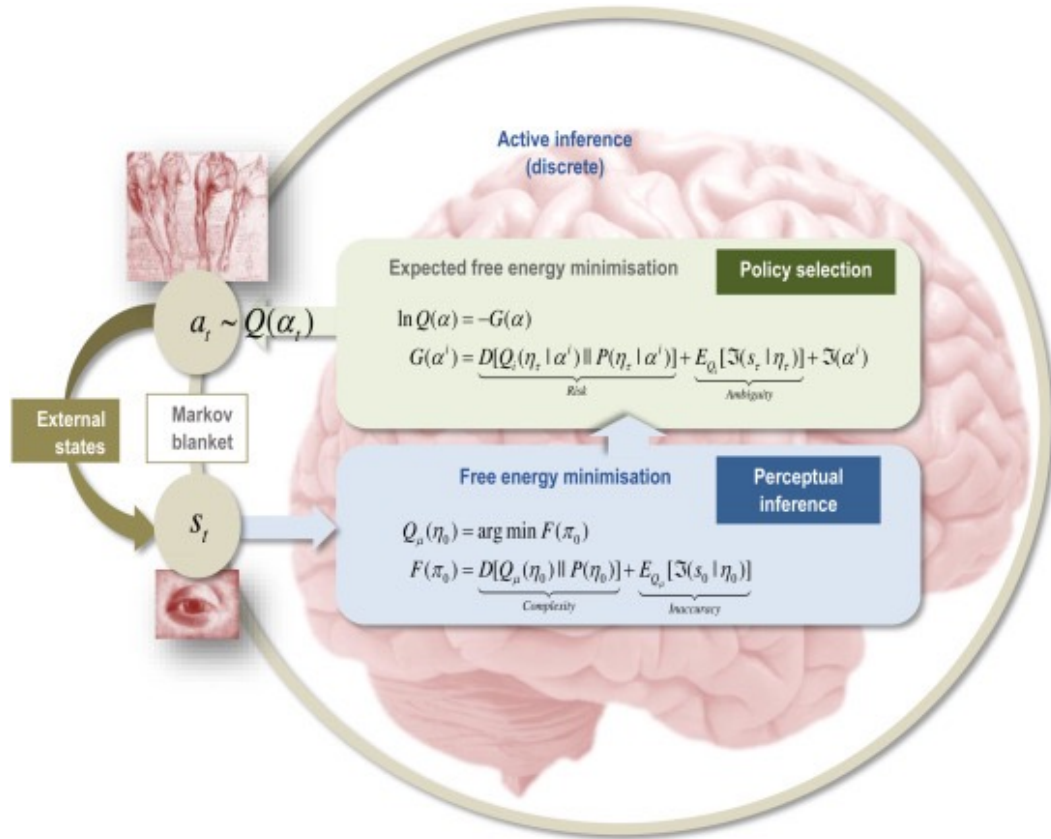
# Why do model-based RL?



Just 2 hours of real robot training

# Connections to Cognitive Science

Significant evidence for mechanisms for prediction of outcomes in neuro/cognitive science



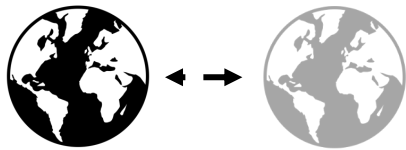
## Reinforcement learning in the brain

Yael Niv

Psychology Department & Princeton Neuroscience Institute, Princeton University

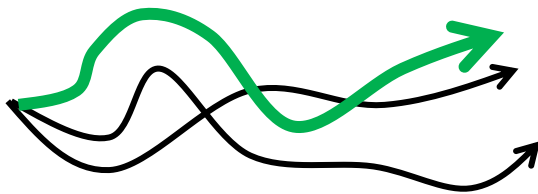
# Model Based RL – Problem Statement

Model Learning



$$\hat{p}_\theta \leftarrow \arg \min_{\hat{p}_\theta} \mathcal{L}(\mathcal{D}, \hat{p}_\theta)$$

Planning



$$\arg \max_{\pi} \mathbb{E}_{\hat{p}, \pi} \left[ \sum_t r(s_t, a_t) \right]$$

Can also just be a single trajectory

How should we instantiate these?

# What will we not cover today?

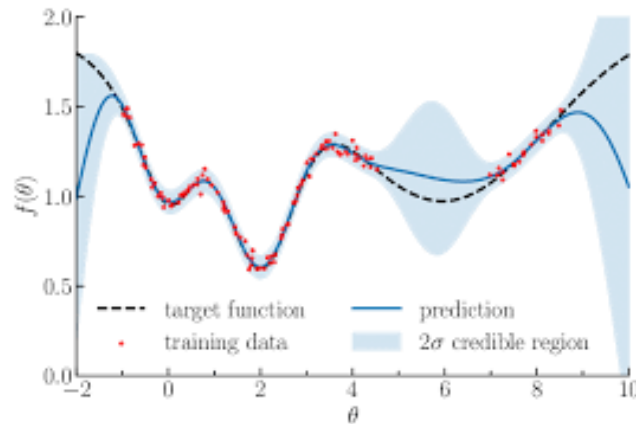
## iLQR/iLQG

$$\min_{\mathbf{u}_1, \dots, \mathbf{u}_T} \sum_{t=1}^T c(\mathbf{x}_t, \mathbf{u}_t) \text{ s.t. } \mathbf{x}_t = f(\mathbf{x}_{t-1}, \mathbf{u}_{t-1})$$

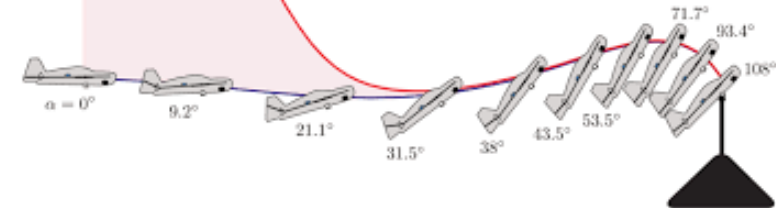
$$f(\mathbf{x}_t, \mathbf{u}_t) = \mathbf{F}_t \begin{bmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{bmatrix} + \mathbf{f}_t$$

$$c(\mathbf{x}_t, \mathbf{u}_t) = \frac{1}{2} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{bmatrix}^T \mathbf{C}_t \begin{bmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{bmatrix} + \begin{bmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{bmatrix}^T \mathbf{c}_t$$

## MBRL with GPs/Non-Parametrics



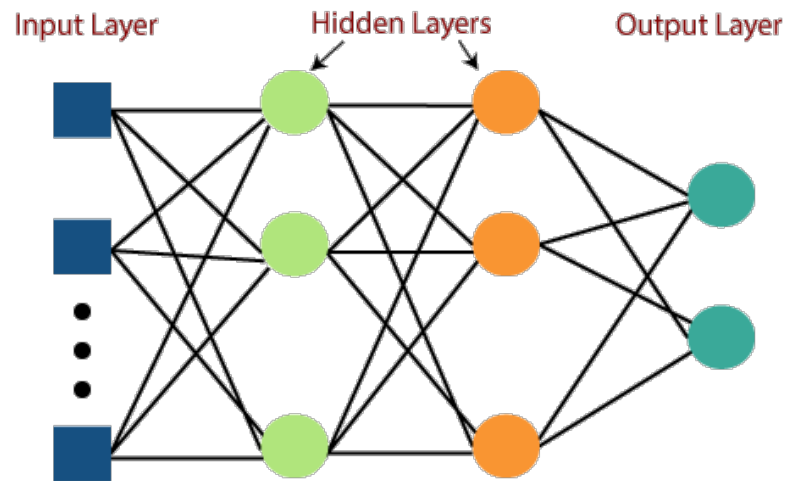
## Non-linear TrajOpt



Byron's lectures do a wonderful job, do go watch them!

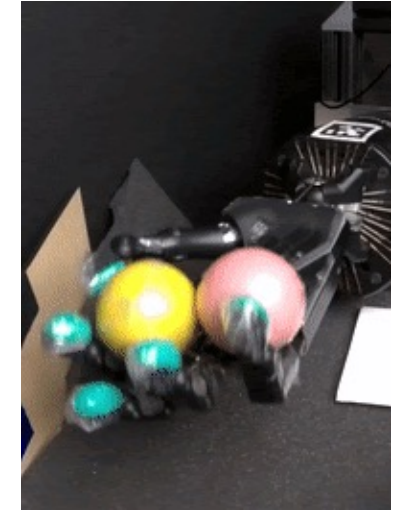
# What will we cover today?

Use neural networks as our model!



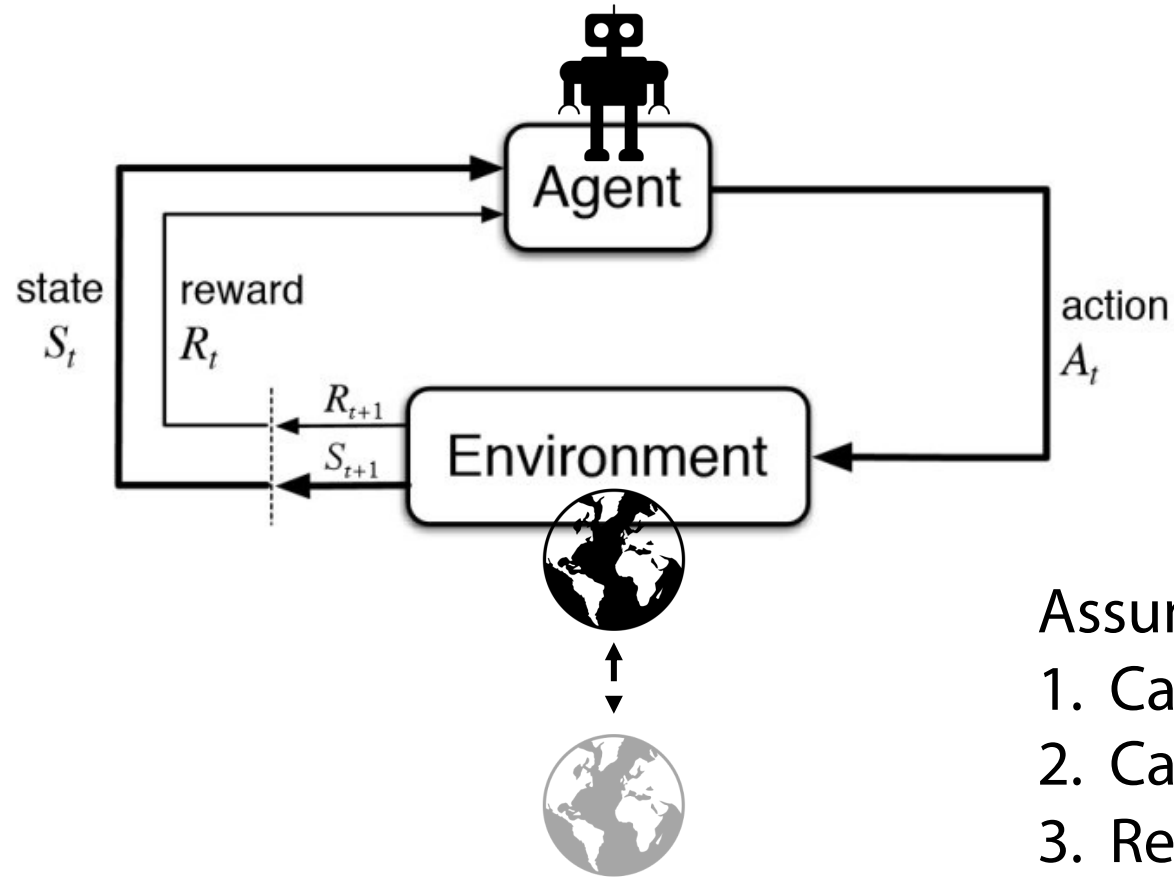
$$\hat{p}_\theta \leftarrow \arg \min_{\hat{p}_\theta} \mathcal{L}(\mathcal{D}, \hat{p}_\theta)$$

$$\arg \max_{\pi} \mathbb{E}_{\hat{p}, \pi} \left[ \sum_t r(s_t, a_t) \right]$$





# Model Based RL – Assumptions



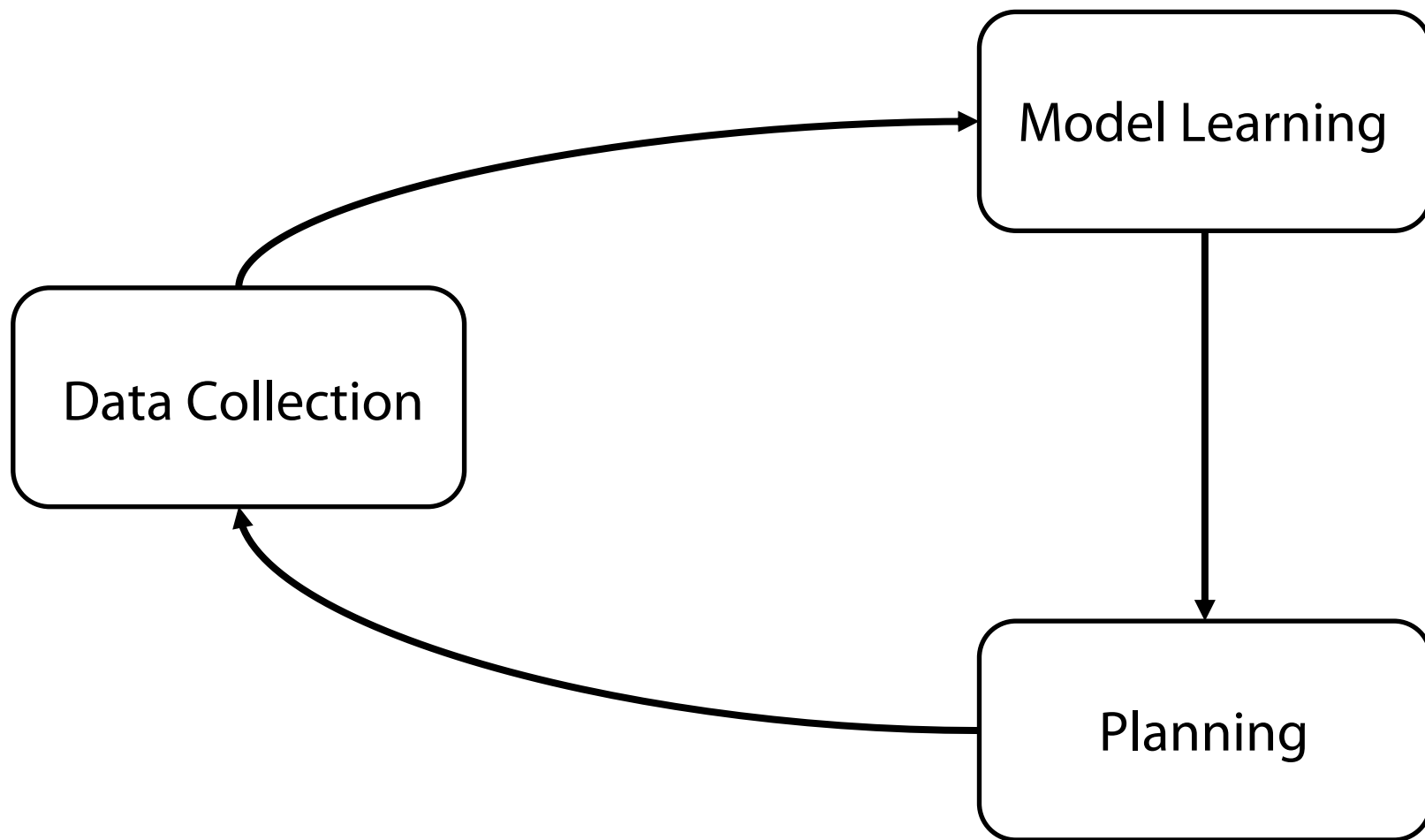
$$\max_{\theta} \mathbb{E}_{\tau \sim \pi_{\theta}} \left[ \sum_{t=0}^T r(s_t, a_t) \right]$$

Assumptions:

1. Can only **sample** from dynamics
2. Can **reset** the environment
3. Reward function is **known**

We will get into this in a later lecture!

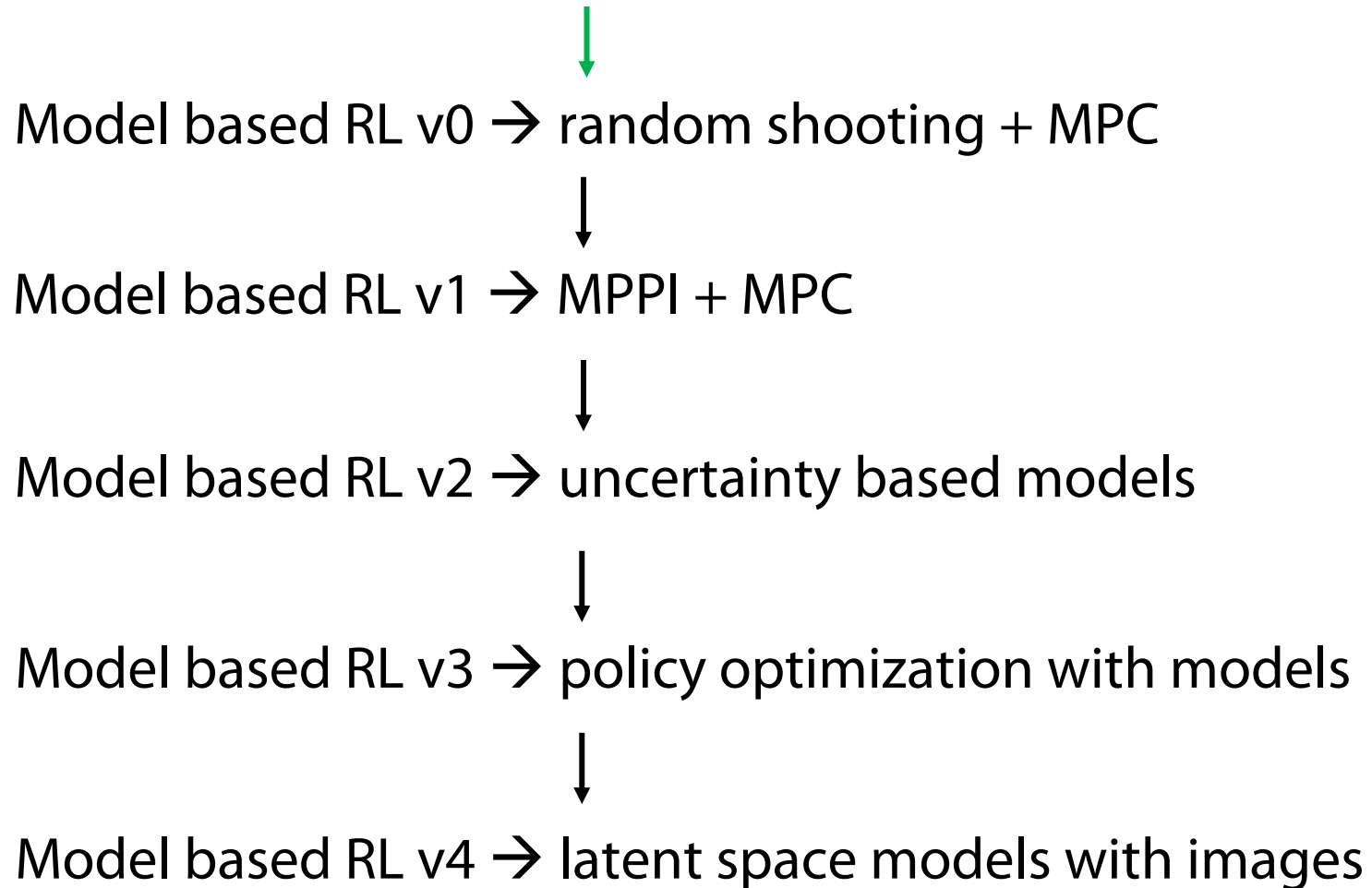
# Model Based RL – A template



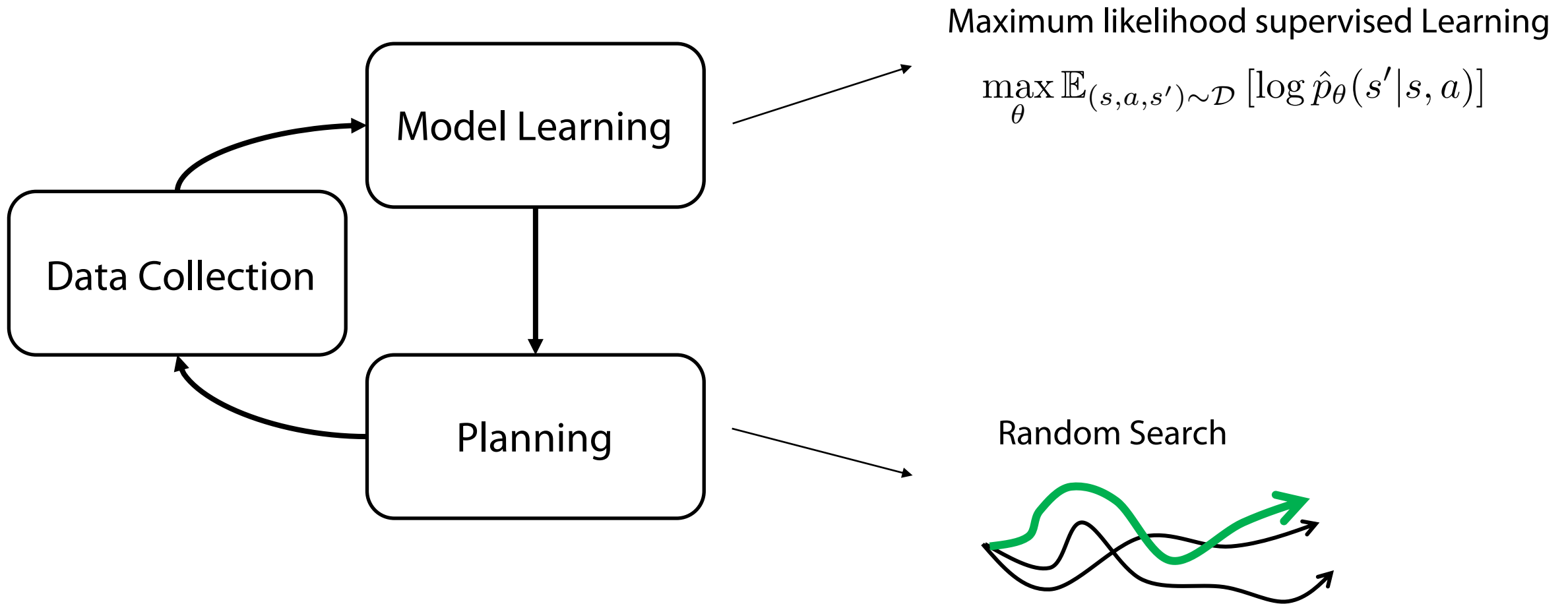
# Lecture outline

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## The Anatomy of Model-Based Reinforcement Learning



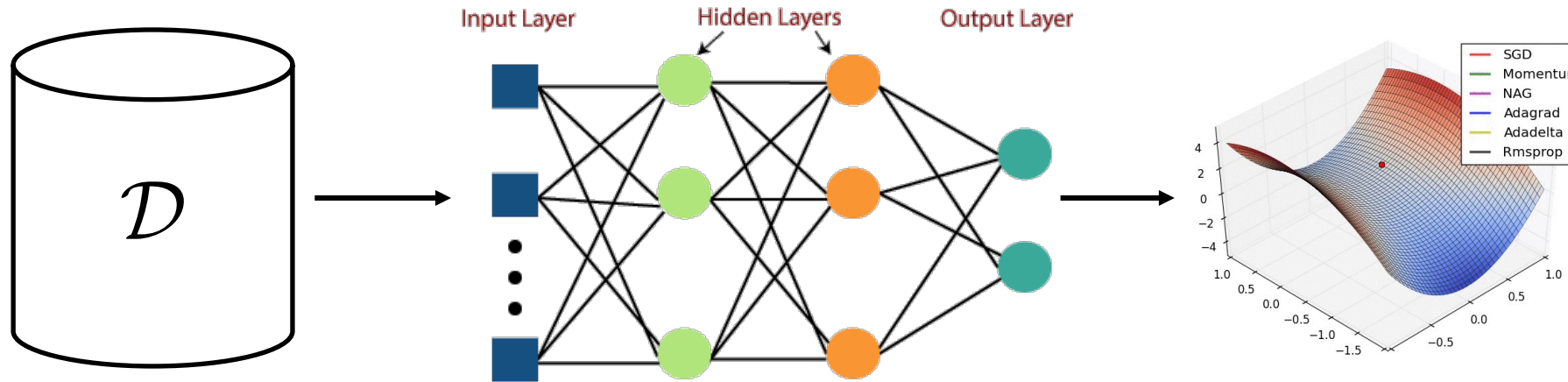
# Model Based RL – Naïve Algorithm (v0)



# Model Based RL – Naïve Algorithm (Model Learning) (v0)

$$\max_{\theta} \mathbb{E}_{(s,a,s') \sim \mathcal{D}} [\log \hat{p}_{\theta}(s' | s, a)]$$

Fit 1-step models



Trick: Model Residual's ( $s' - s$ )

Choice of  $\hat{p}_{\theta}$  distribution determines the loss function:

1. Gaussian  $\rightarrow L_2$
2. Energy Based Model  $\rightarrow$  Contrastive Divergence
3. Diffusion Model  $\rightarrow$  Score Matching

More expressive may be better, at the risk of overfitting

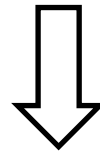
# Model Based RL – Naïve Algorithm (Planning)

Planning

$$\max_{a_0, a_1, \dots, a_T} \sum_{t=0}^T r(\hat{s}_t, a_t)$$

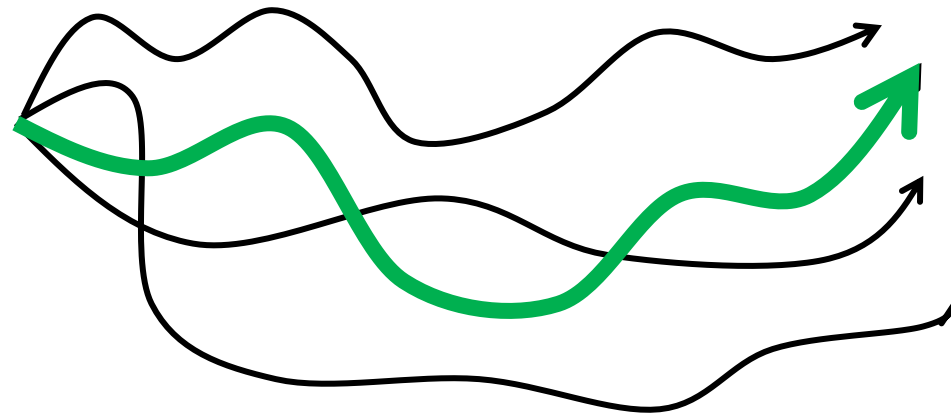
$$\hat{s}_{t+1} \sim \hat{p}_\theta(s_{t+1} | \hat{s}_t, a_t)$$

$$\hat{s}_1 \sim \hat{p}_\theta(s_{t+1} | s_0, a_0)$$



Just do random search!

$$\arg \max_{a_0^j, a_1^j, \dots, a_T^j} \sum_{t=0}^T r(\hat{s}_t^j, a_t^j)$$
$$\hat{s}_{t+1}^j \sim \hat{p}_\theta(\cdot | \hat{s}_t^j, a_t^j)$$

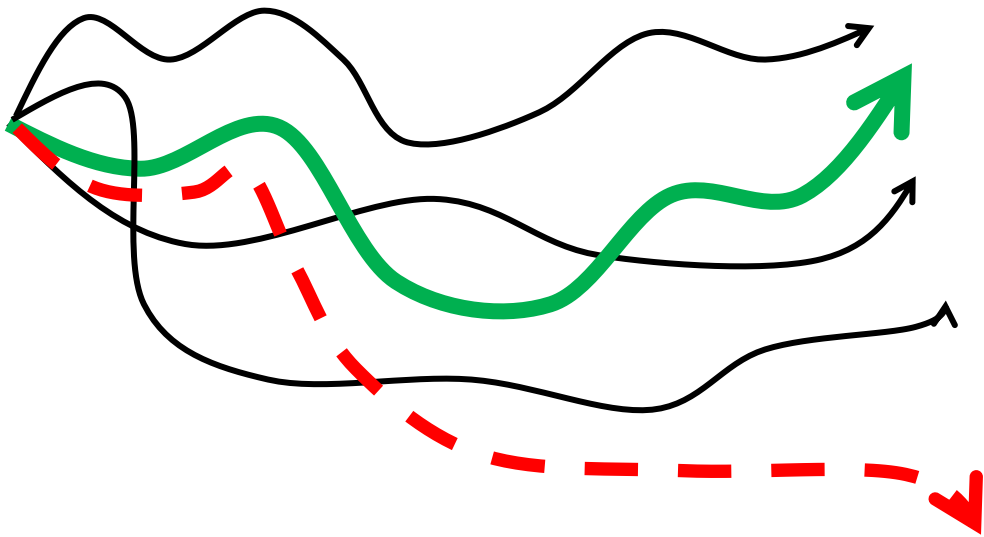


Just execute actions open loop!

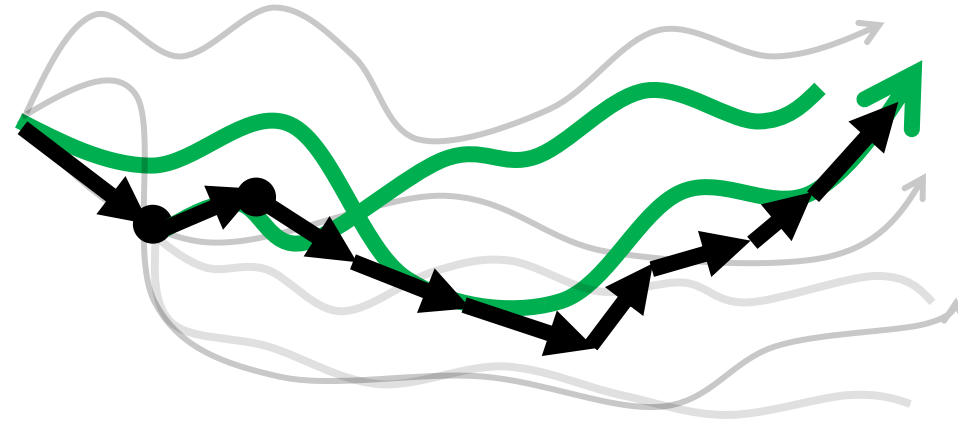
Can soften by taking softmax rather than argmax

# Model Based RL – Naïve Algorithm (MPC)

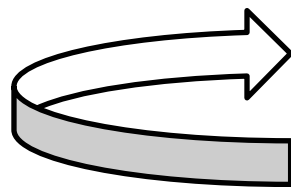
Without feedback, an open loop controller can diverge even for minimal noise



Replanning can help with divergence

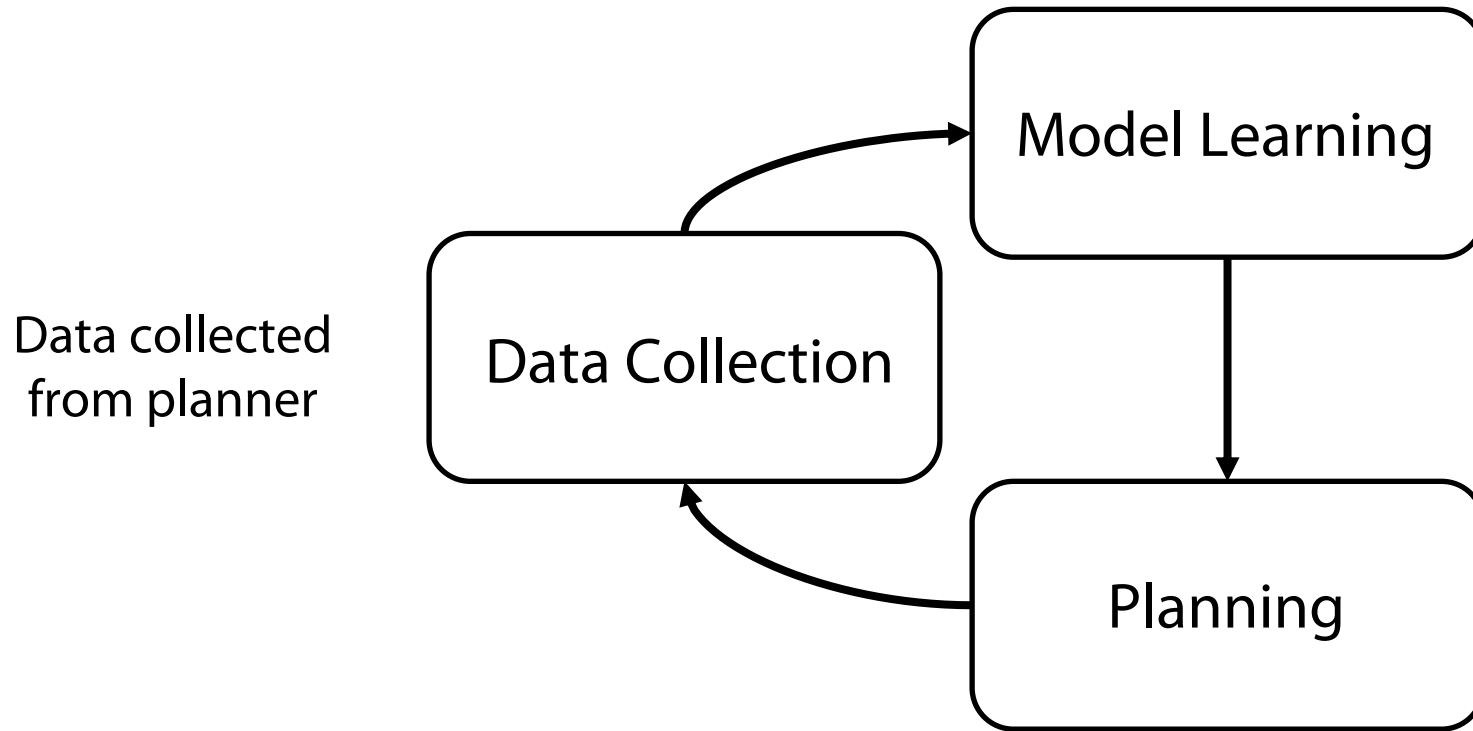


Model-Predictive/Receding Horizon Control



1. Plan with random shooting from  $s_t$
2. Execute the first action  $a_0$  and reach  $s_{t+1}$

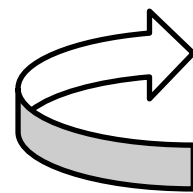
# Model Based RL – Naïve Algorithm (v0)



Maximum likelihood supervised Learning

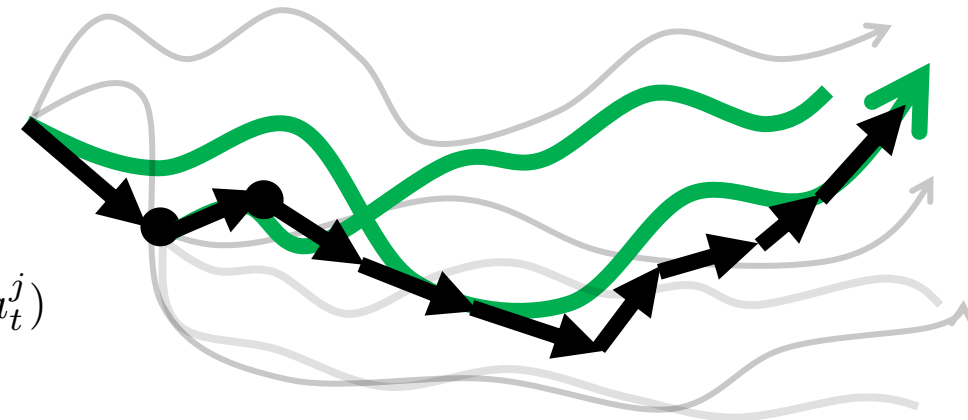
$$\max_{\theta} \mathbb{E}_{(s,a,s') \sim \mathcal{D}} [\log \hat{p}_{\theta}(s'|s,a)]$$

Better than open loop planning because of feedback



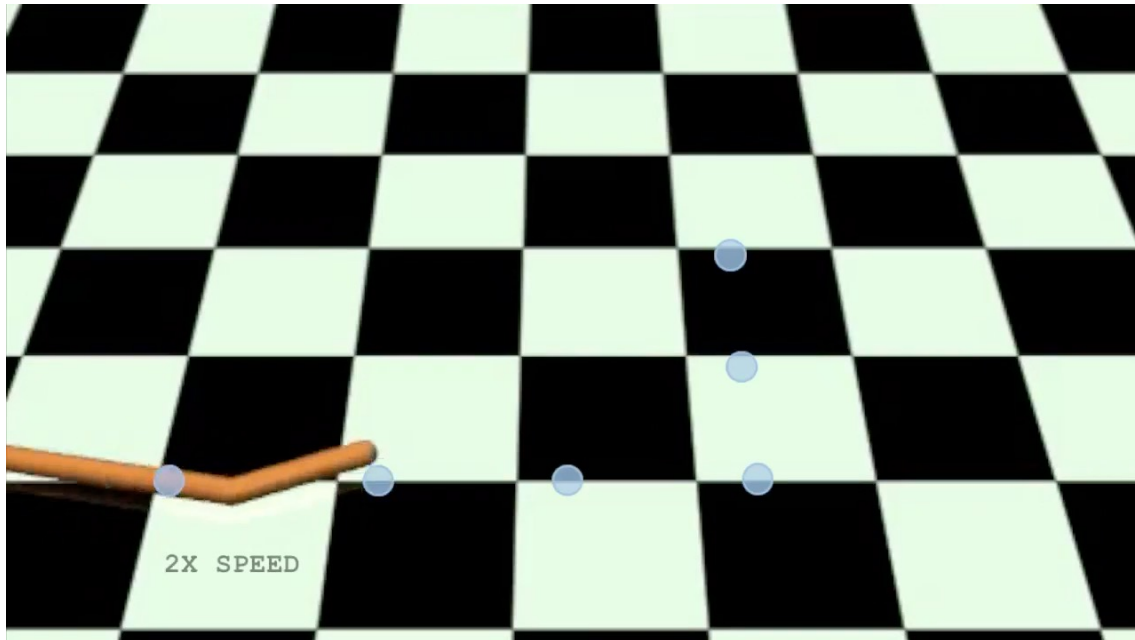
$$\arg \max_{a_0^j, a_1^j, \dots, a_T^j} \sum_{t=0}^T r(\hat{s}_t^j, a_t^j)$$
$$\hat{s}_{t+1}^j \sim \hat{p}_{\theta}(\cdot | \hat{s}_t^j, a_t^j)$$

Planning with Shooting + MPC



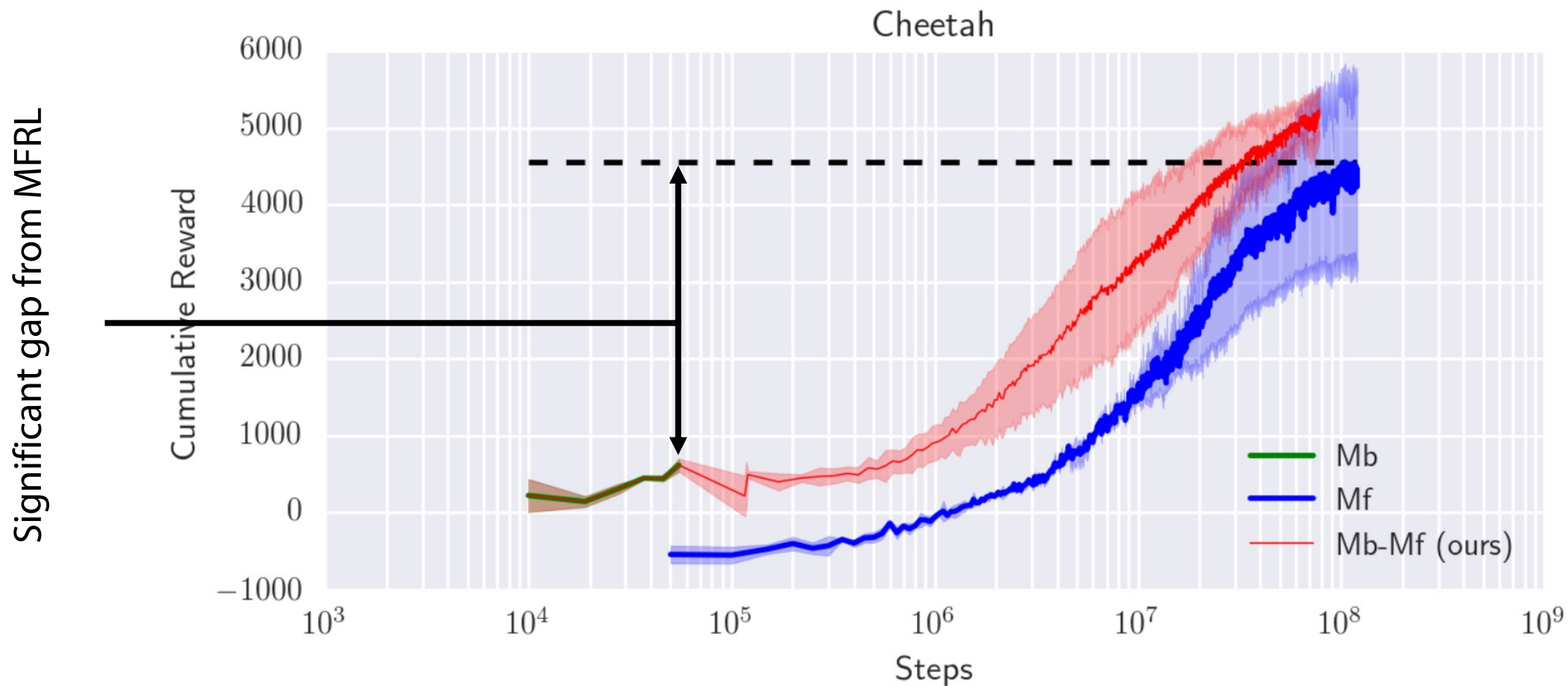


# Does it work?



Just 20 minutes of training time with random data!

# Does it work?



# Lecture outline

## The Anatomy of Model-Based Reinforcement Learning



Model based RL v0 → random shooting + MPC



Model based RL v1 → MPPI + MPC



Model based RL v2 → uncertainty based models

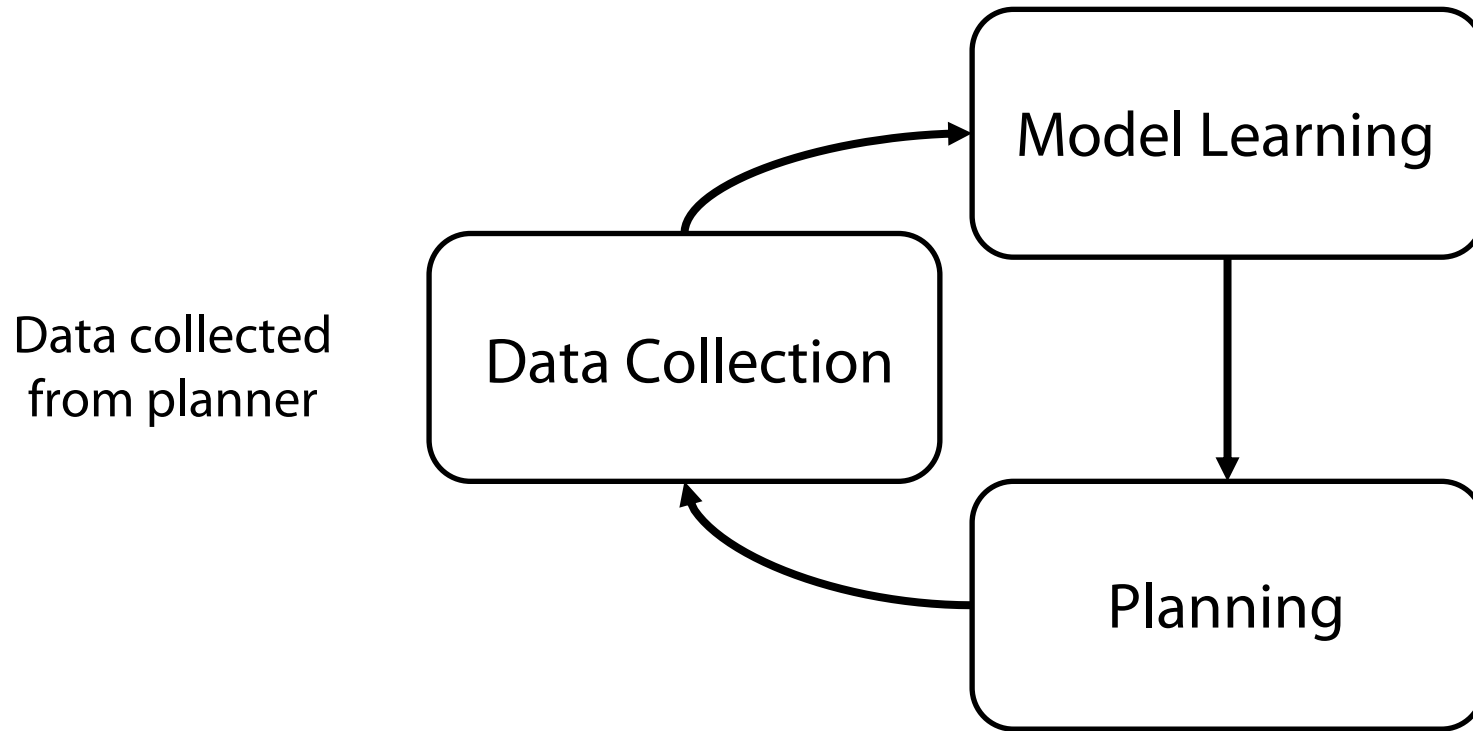


Model based RL v3 → policy optimization with models



Model based RL v4 → latent space models with images

# What might be the issue?



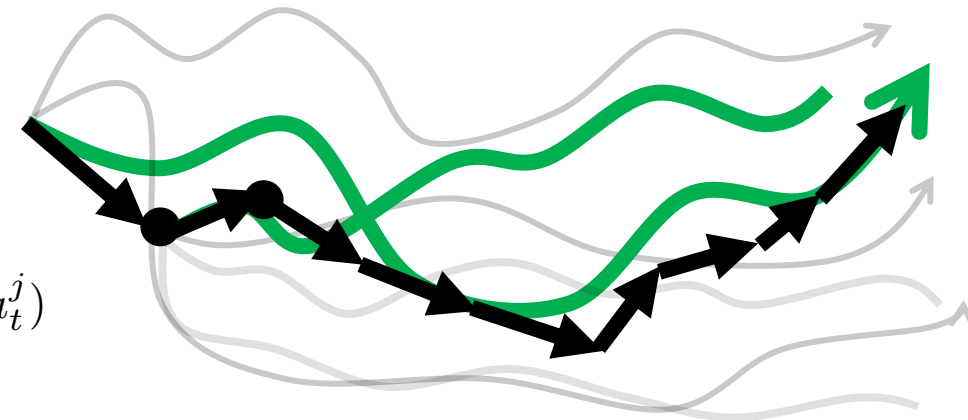
Maximum likelihood supervised Learning

$$\max_{\theta} \mathbb{E}_{(s,a,s') \sim \mathcal{D}} [\log \hat{p}_{\theta}(s'|s,a)]$$

Planning with Shooting + MPC

**Searching for a needle in a haystack by random shooting, high variance!**

$$\arg \max_{a_0^j, a_1^j, \dots, a_T^j} \sum_{t=0}^T r(\hat{s}_t^j, a_t^j)$$
$$\hat{s}_{t+1}^j \sim \hat{p}_{\theta}(\cdot | \hat{s}_t^j, a_t^j)$$

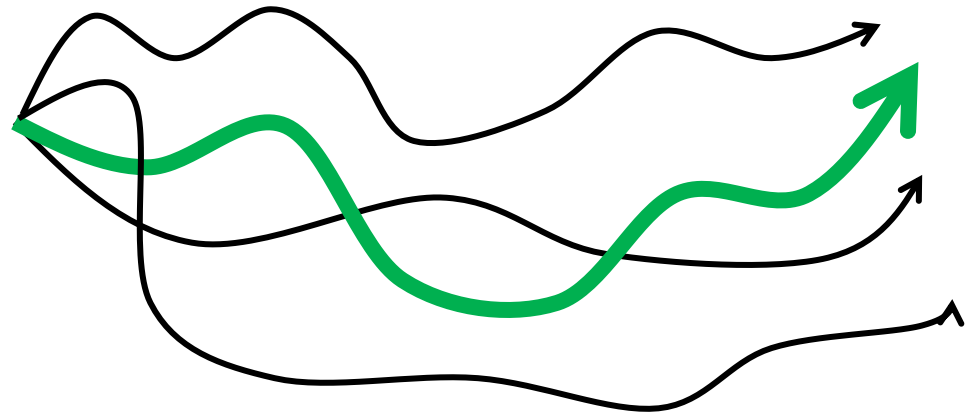
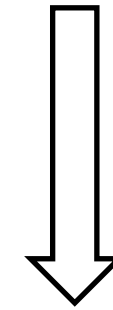


# Better Sampling Techniques for Shooting

Sampled from stationary uniform/gaussian distribution

$$\arg \max_{a_0^j, a_1^j, \dots, a_T^j} \sum_{t=0}^T r(\hat{s}_t^j, a_t^j)$$
$$\hat{s}_{t+1}^j \sim \hat{p}_\theta(\cdot | \hat{s}_t^j, a_t^j)$$

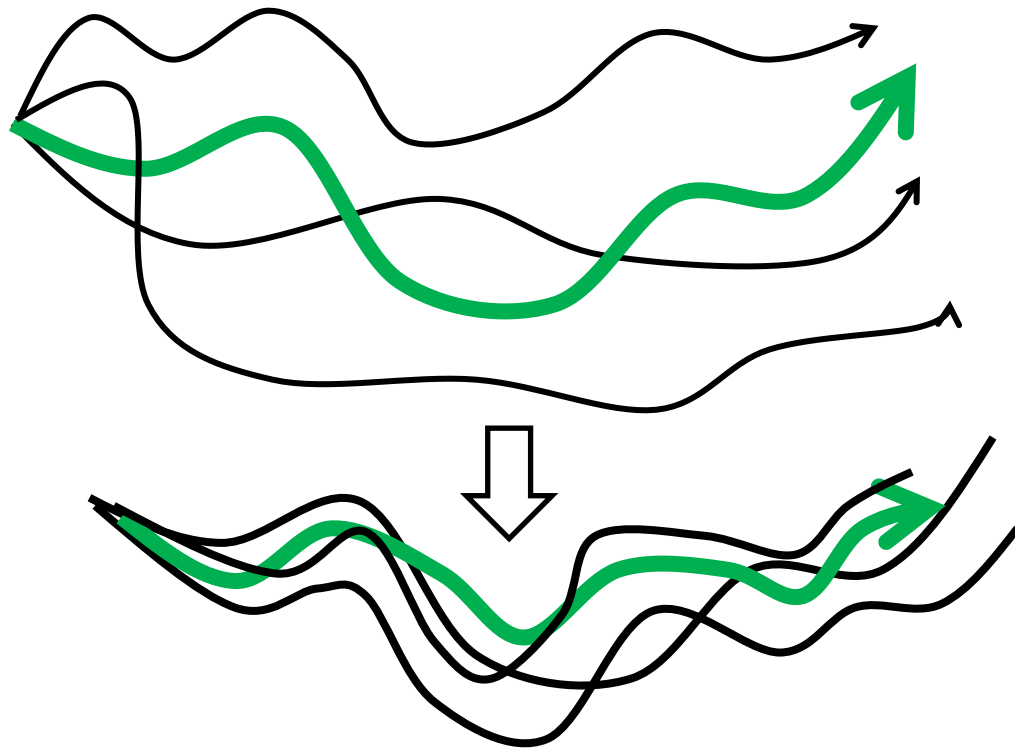
Can we inform the sampling function with the reward function?



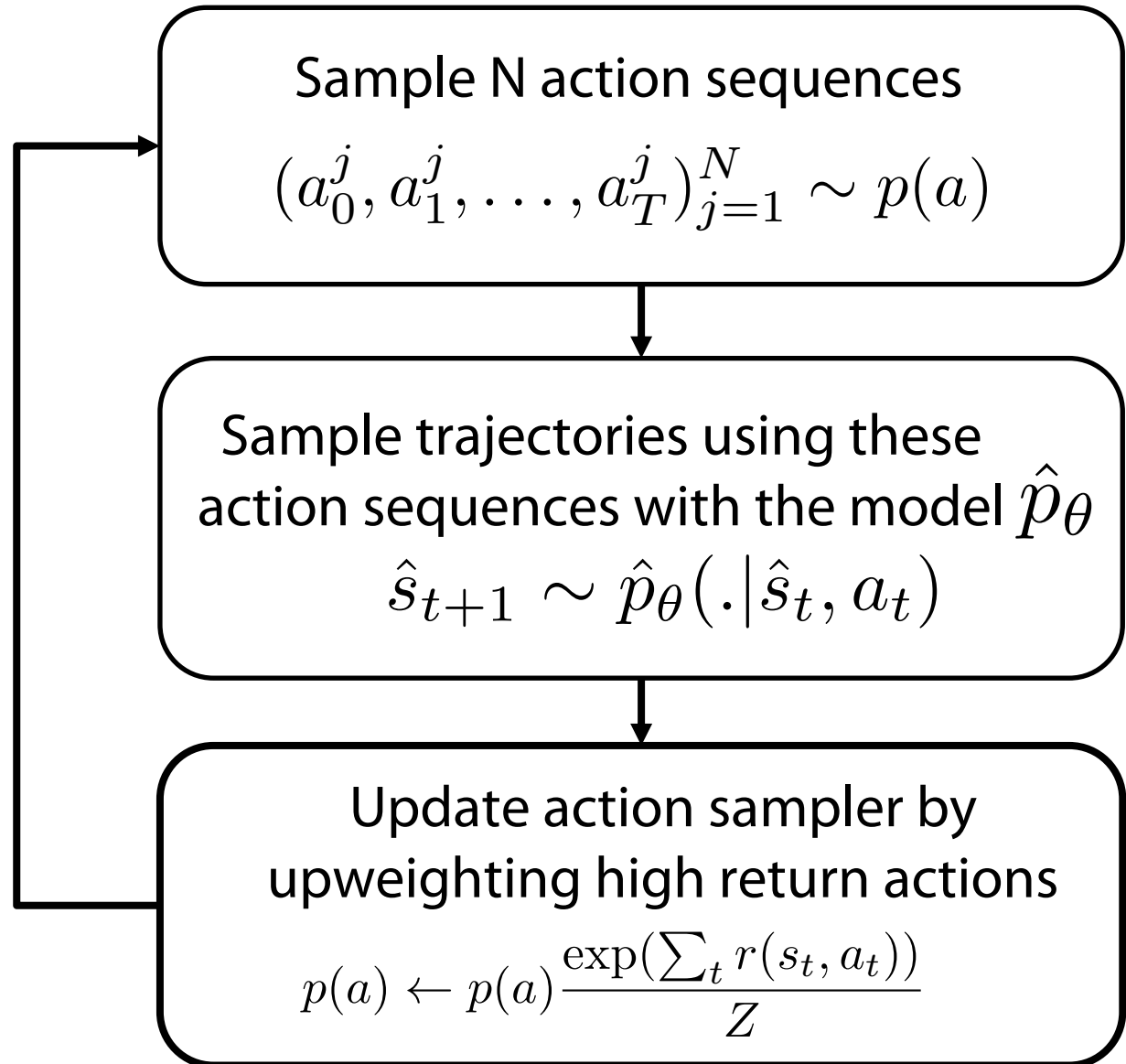
Idea: Iteratively upweight sampling distribution around the things that are higher returns

# Better Sampling Techniques for Shooting - MPPI

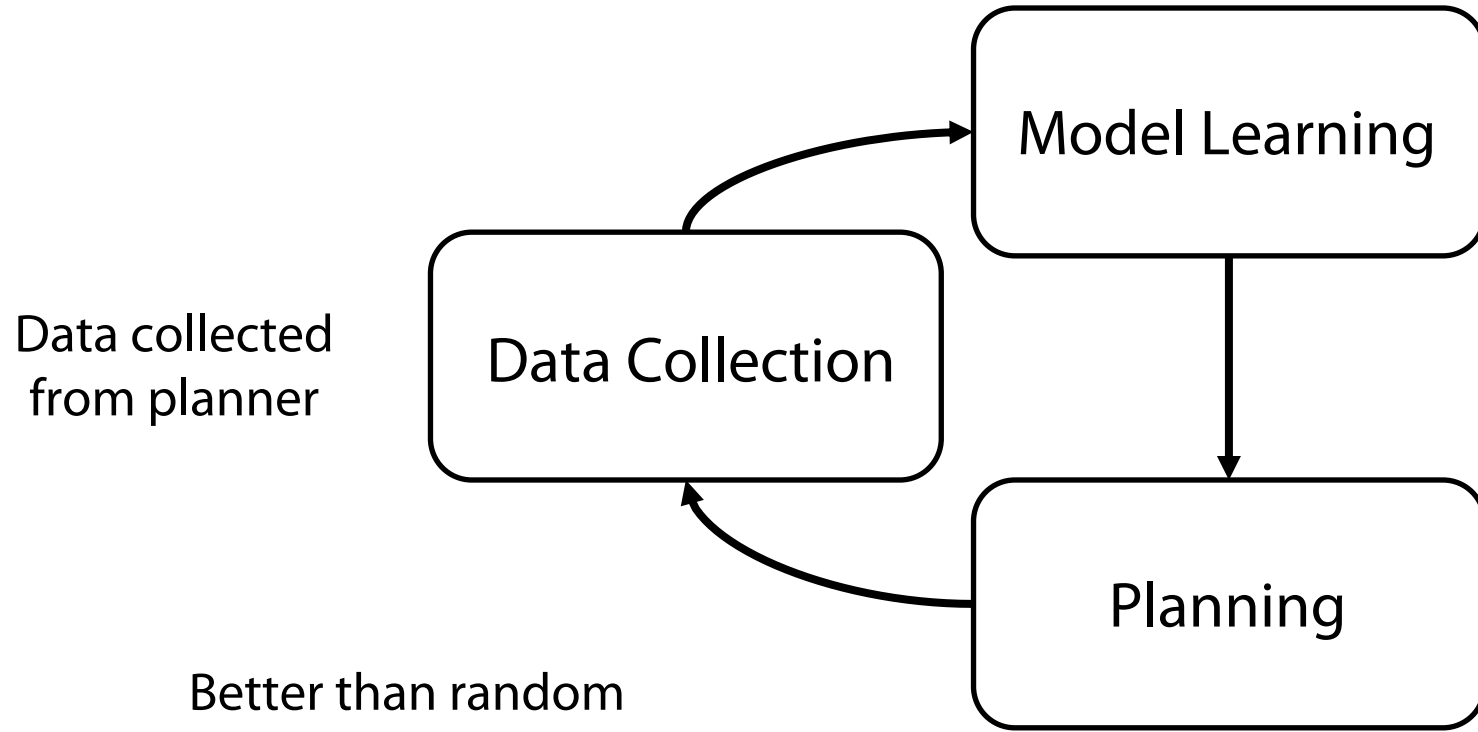
Idea: Iteratively upweight sampling distribution around the things that are higher returns



Referred to as **MPPI**, lower variance!



# Model Based RL – Better Sampling Methods (v1)



Maximum likelihood supervised Learning

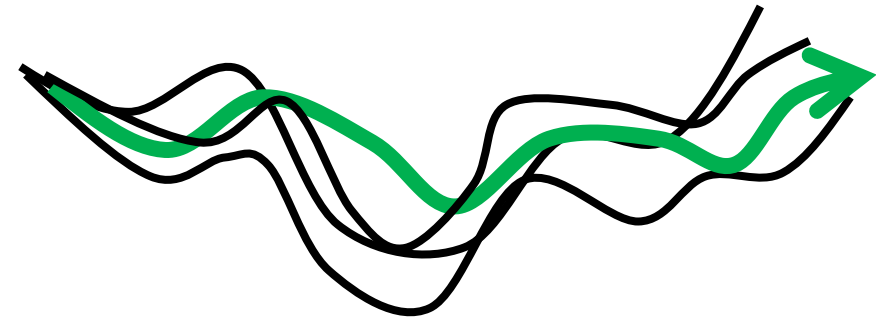
$$\max_{\theta} \mathbb{E}_{(s,a,s') \sim \mathcal{D}} [\log \hat{p}_{\theta}(s'|s,a)]$$

Better than random shooting + MPC, since lower variance!

Aside: Can derive this update trying to bring sampling distribution close to optimal distribution

$$\arg \max_{a_0^j, a_1^j, \dots, a_T^j} \sum_{t=0}^T r(\hat{s}_t^j, a_t^j)$$
$$\hat{s}_{t+1}^j \sim \hat{p}_{\theta}(\cdot | \hat{s}_t^j, a_t^j)$$
$$p(a) \leftarrow p(a) \frac{\exp(\sum_t r(s_t, a_t))}{Z}$$

Planning with MPPI + MPC



# Does it work?





# Lecture outline

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## The Anatomy of Model-Based Reinforcement Learning



Model based RL v0 → random shooting + MPC



Model based RL v1 → MPPI + MPC



Model based RL v2 → uncertainty based models



Model based RL v3 → policy optimization with models

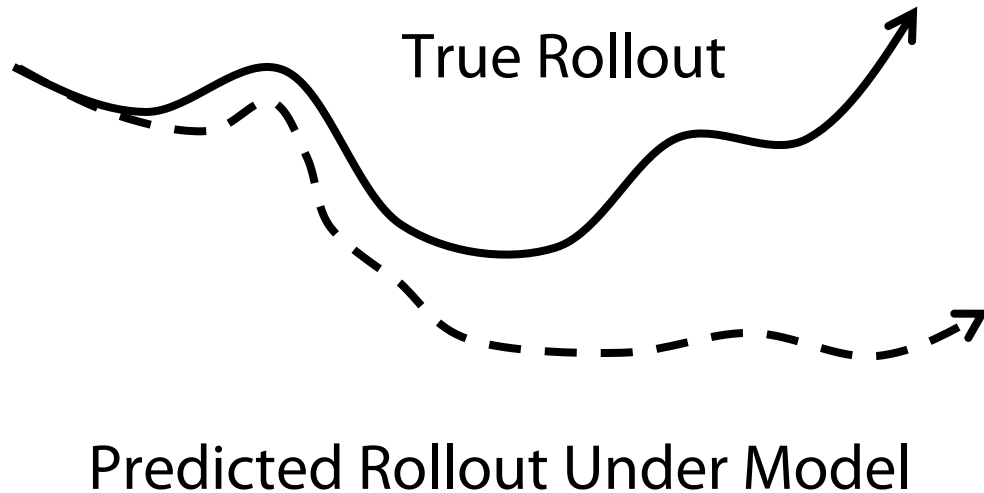


Model based RL v4 → latent space models with images

# What might be the issue?

Rollouts under learned model  $\neq$  Rollouts under true model

└─→ Model bias/compounding error



Why does this happen? → lack of data

1. Errors in state go to OOD next states
2. Deviations in actions go to OOD next states

↓  
Model is bad on OOD states!

Most trained deep models can only roll out for 5-10 steps maximum!

# How might we deal with compounding error?

Idea 1: Change the training objective of the model to directly account for this!

Equation error – 1 step prediction error

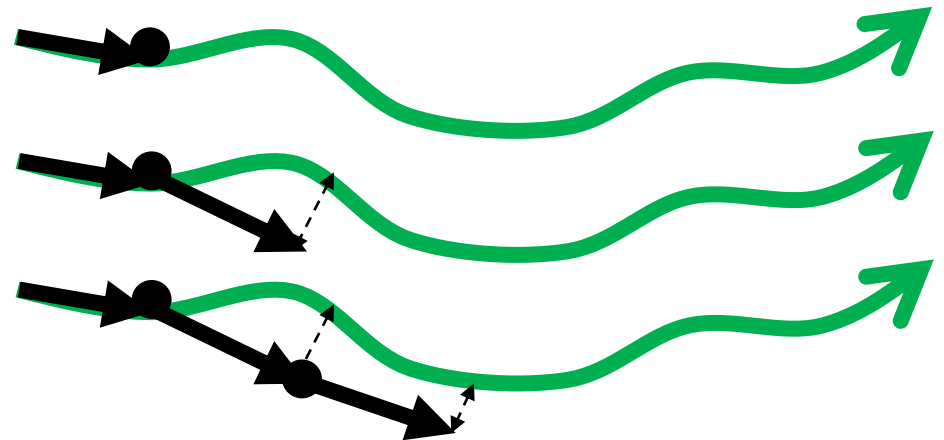
$$\max_{\theta} \mathbb{E}_{(s,a,s') \sim \mathcal{D}} [\log \hat{p}_{\theta}(s' | s, a)]$$

Simulation error – K step prediction error

$$\max_{\theta} \sum_t \log \hat{p}_{\theta}(s_{t+1} | \hat{s}_t, a_t)$$
$$\hat{s}_t \sim \hat{p}_{\theta}(\cdot | \hat{s}_{t-1}, a_{t-1})$$

Model error under learned mode  $\hat{p}_{\theta}$  rather than under true model

Can be a challenging non-convex optimization!

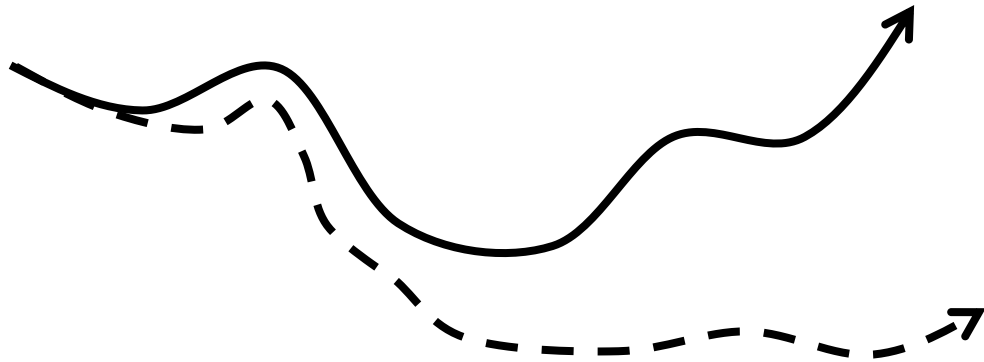


# How might we deal with compounding error?

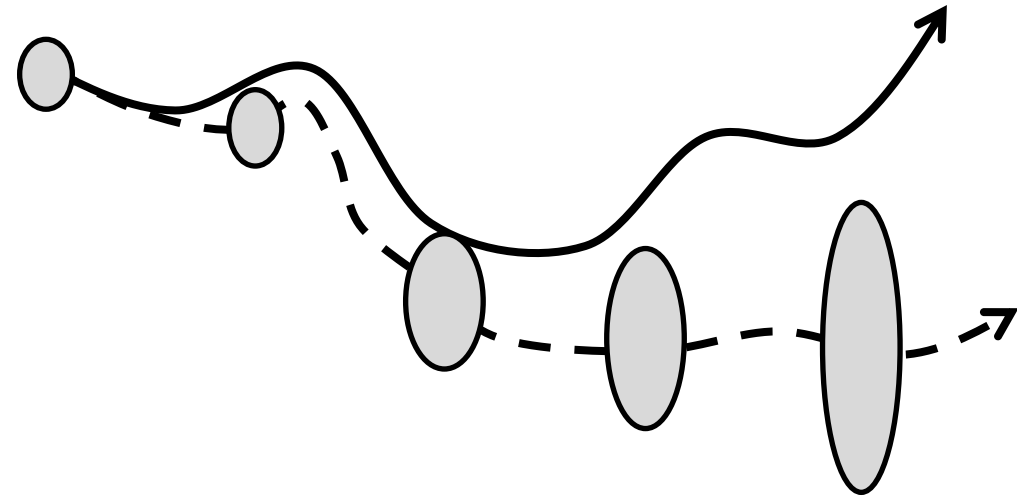
Idea 2: Estimate when OOD and account for it

└───> Measure uncertainty!

Maximum likelihood models



Uncertainty-aware models



Being aware of uncertainty allows us to account for the effects of model bias!

# What is uncertainty?

## Alleatoric Uncertainty

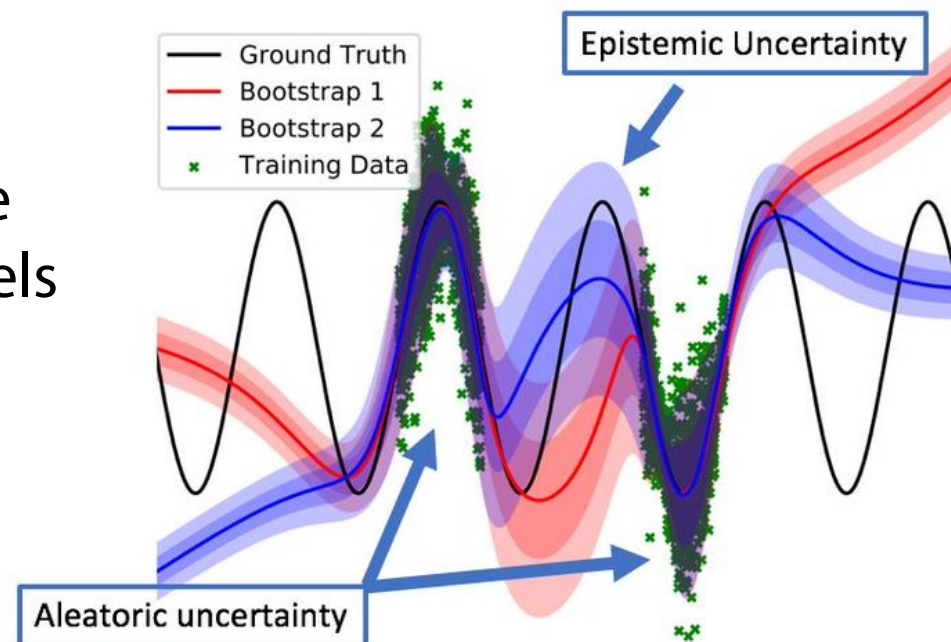
(environment stochasticity)

Easier, can use stochastic models

## Epistemic Uncertainty

(Lack of data)

More challenging, need to compute posterior



Let's largely focus on epistemic uncertainty

# How might we measure uncertainty?

$$p(\theta|\mathcal{D})$$

Difficult to estimate directly!

$$p(\theta|\mathcal{D}) = \frac{p(\mathcal{D}|\theta)p(\theta)}{\int p(\mathcal{D}|\theta')p(\theta')d\theta'}$$

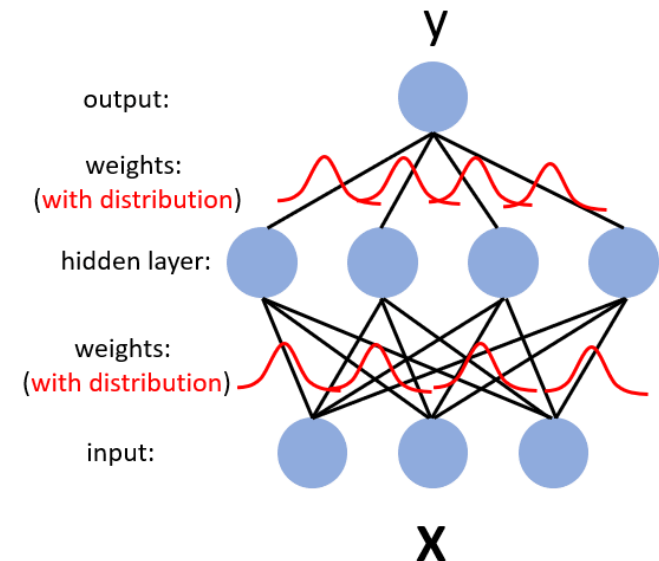
1. Bayesian neural networks
2. Ensemble methods
3. ...

Directly model posterior distribution

Use variational inference to avoid computing partition function

$$\min_{q(\theta|\mathcal{D})} D_{KL}(q(\theta|\mathcal{D}) || p(\theta|\mathcal{D}))$$

Challenge: can be difficult to express rich distributions

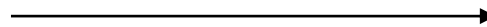


# How might we measure uncertainty?

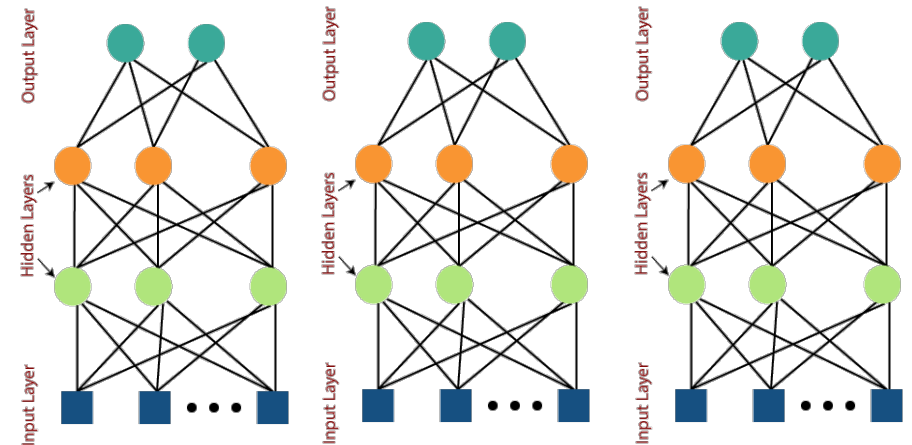
$$p(\theta|\mathcal{D})$$

Difficult to estimate directly!

1. Bayesian neural networks
2. Ensemble methods
3. ...



Learn an ensemble of models



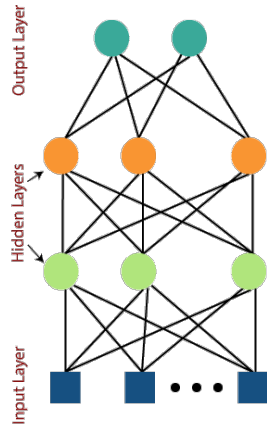
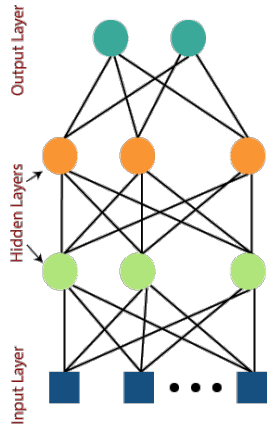
Approximate posterior

Low data regime  $\rightarrow$  high ensemble variance

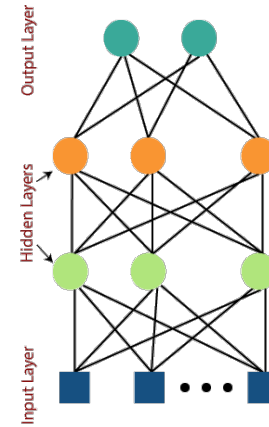
Easier and more expressive than BNNs!

# Model Based RL – Learning Ensembles of Dynamics Models

Learn ensembles of dynamics models with MLE rather than a single model



...



$$\max_{\theta} \mathbb{E}_{(s,a,s') \sim \mathcal{D}} [\log \hat{p}_{\theta}(s'|s,a)] \quad \max_{\theta} \mathbb{E}_{(s,a,s') \sim \mathcal{D}} [\log \hat{p}_{\theta}(s'|s,a)]$$

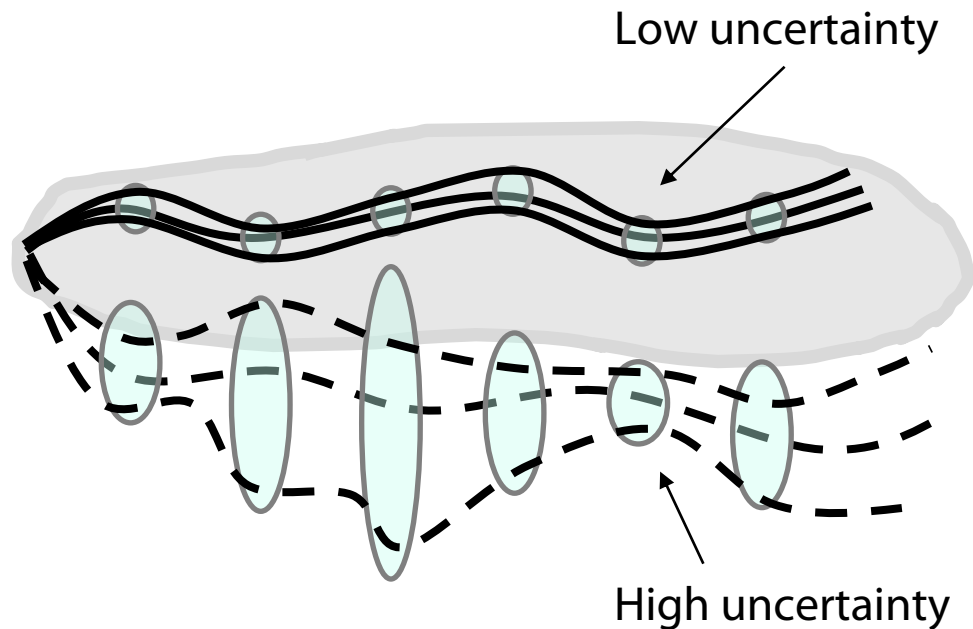
$$\max_{\theta} \mathbb{E}_{(s,a,s') \sim \mathcal{D}} [\log \hat{p}_{\theta}(s'|s,a)]$$

Learn ensembles by either subsampling the data or having different initializations



# Model Based RL – Integrating Uncertainty into MBRL (v2)

Take expected value under the uncertain dynamics



Expected value over ensemble

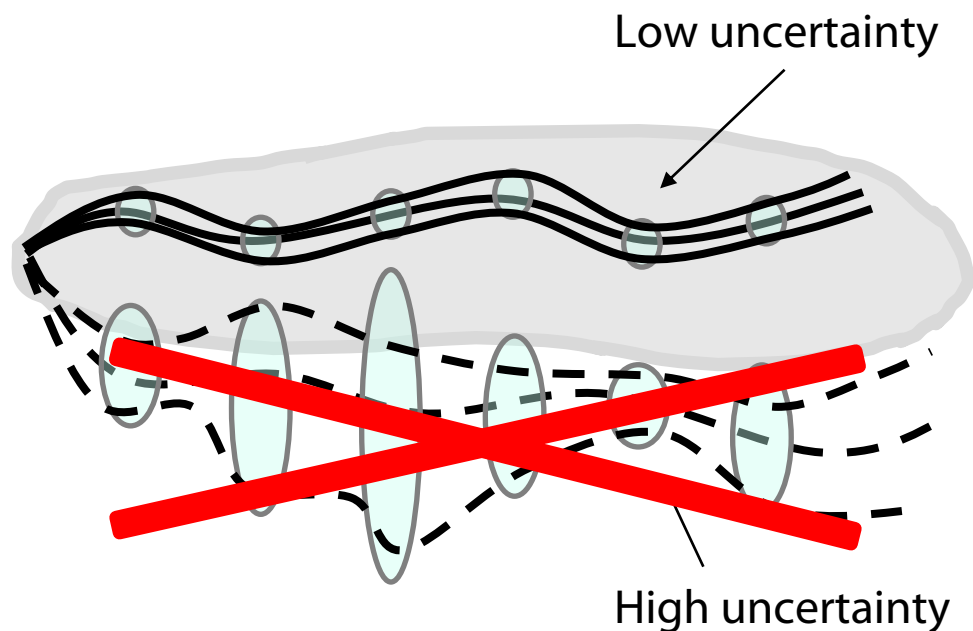
$$\arg \max_{(a_0^j, a_1^j, \dots, a_T^j)_{j=1}^N} \sum_{i=1}^K \sum_{t=0}^T r((\hat{s}_t^j)^i, a_t^j)$$
$$(\hat{s}_{t+1}^j)^i \sim \hat{p}_{\theta_i}(\cdot | (\hat{s}_t^j)^i, a_t^j)$$

Can also swap which ensemble element is propagated at every step or just pick randomly amongst them

Avoids overly OOD settings since the expected reward is affected by uncertainty

# Model Based RL – Integrating Uncertainty into MBRL (v2)

Take **pessimistic** value under the uncertain dynamics



Penalize ensemble variance

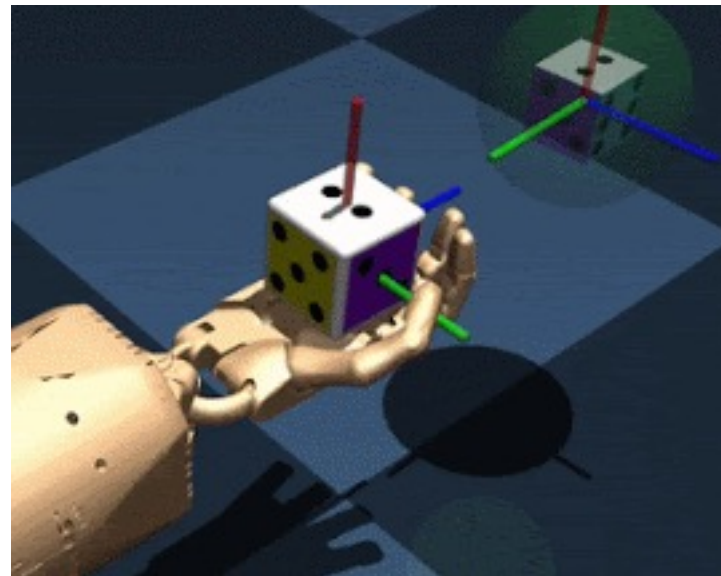
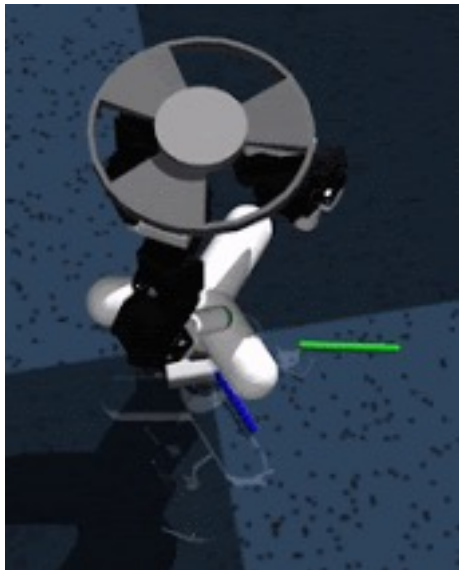
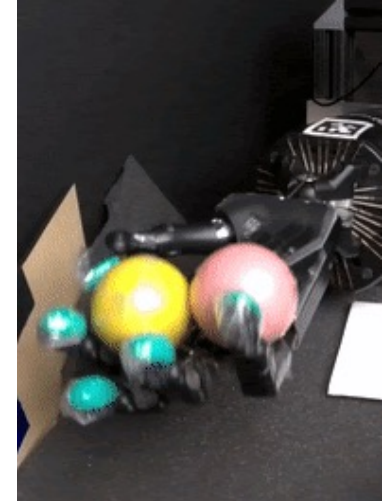
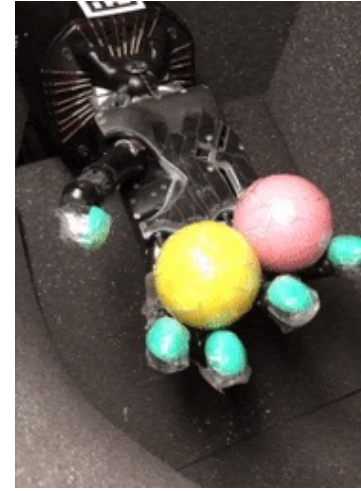
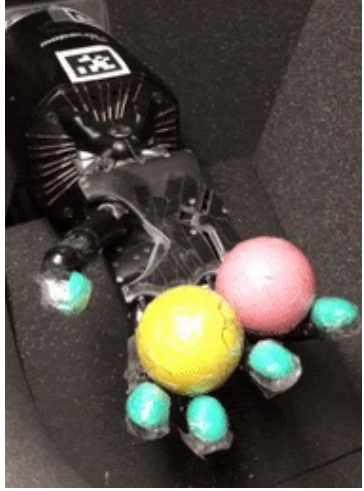
$$\arg \max_{(a_0^j, a_1^j, \dots, a_T^j)_{j=1}^N} \sum_{i=1}^K \sum_{t=0}^T r((\hat{s}_t^j)^i, a_t^j) - \lambda \text{Var}((\hat{s}_t^j)^i)$$

↓

$$(\hat{s}_{t+1}^j)^i \sim \hat{p}_{\theta_i}(\cdot | (\hat{s}_t^j)^i, a_t^j)$$

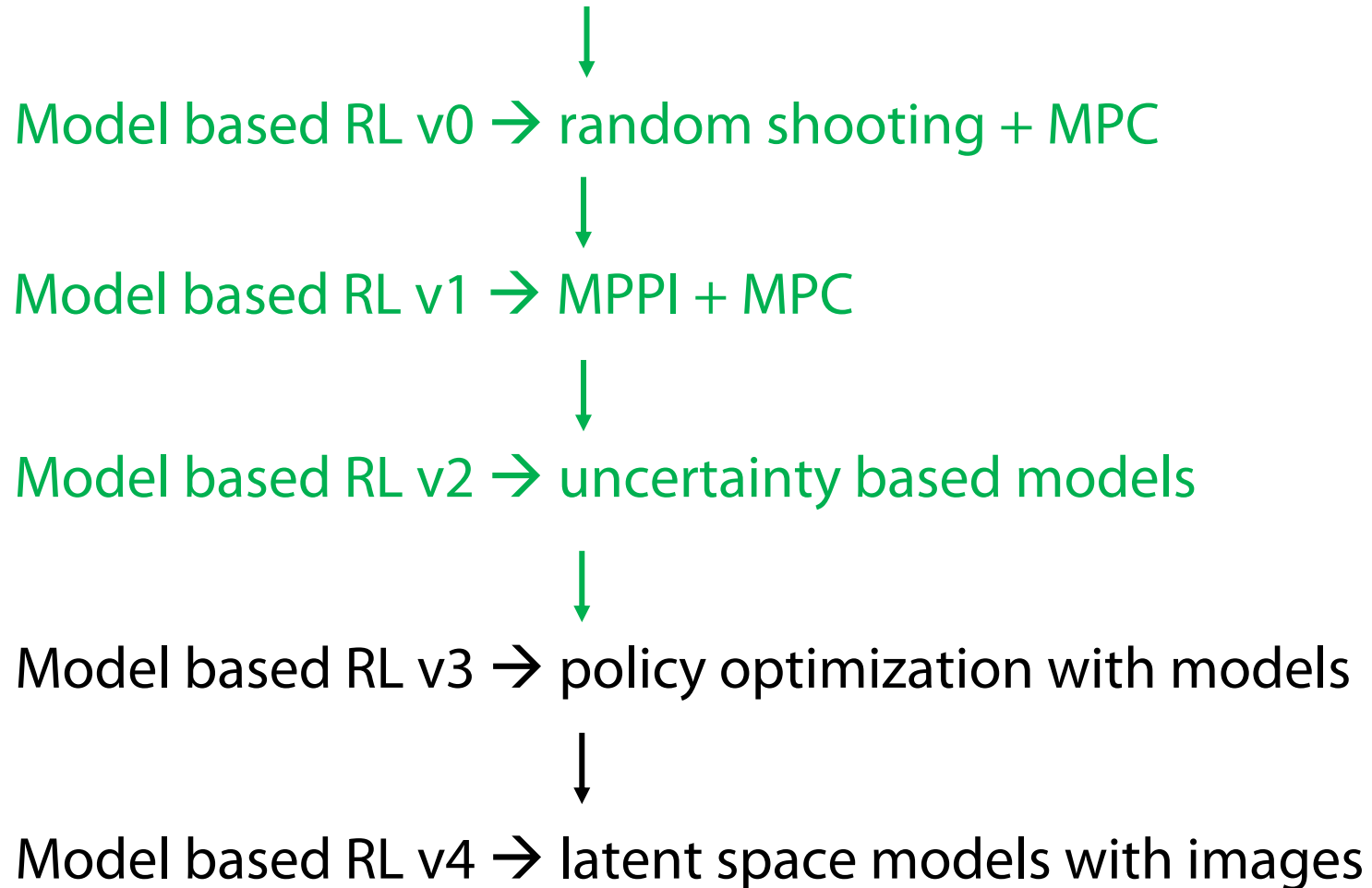
Avoids overly OOD settings since these states are explicitly penalized

# Does this work?



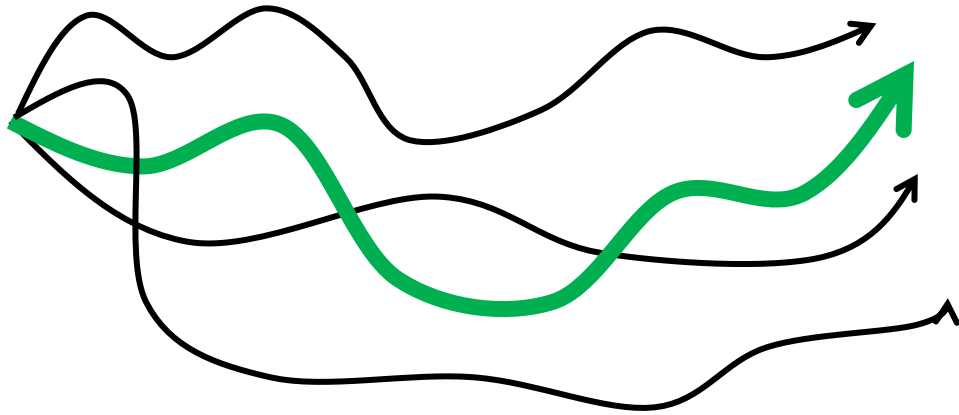
# Lecture outline

## The Anatomy of Model-Based Reinforcement Learning



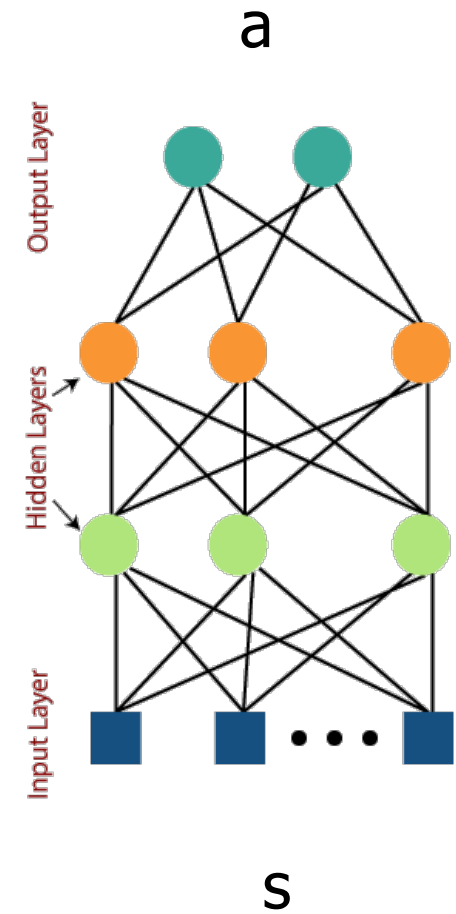
# What might be the issue?

Huge number of samples  
needed to reduce variance



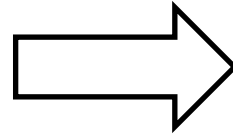
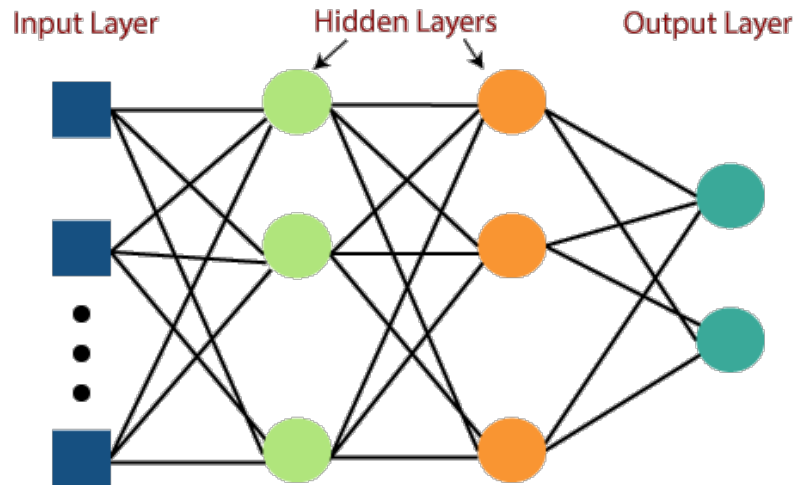
Extremely slow, hard to run in real time

Amortize planning  
into a policy

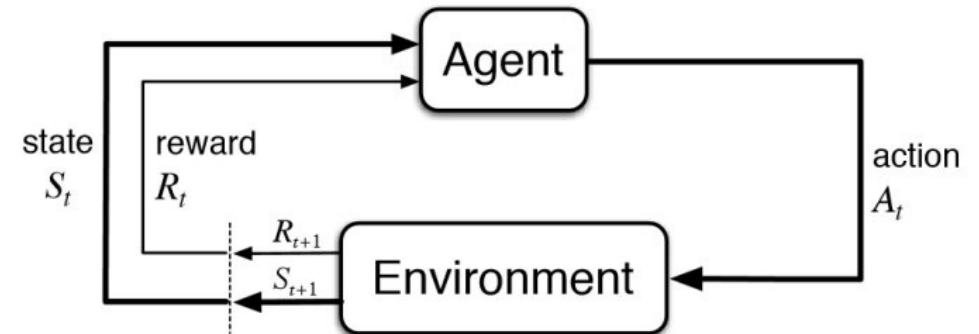


# Speeding Up Model-Based Planning

$$\max_{\theta} \mathbb{E}_{(s,a,s') \sim \mathcal{D}} [\log \hat{p}_{\theta}(s' | s, a)]$$

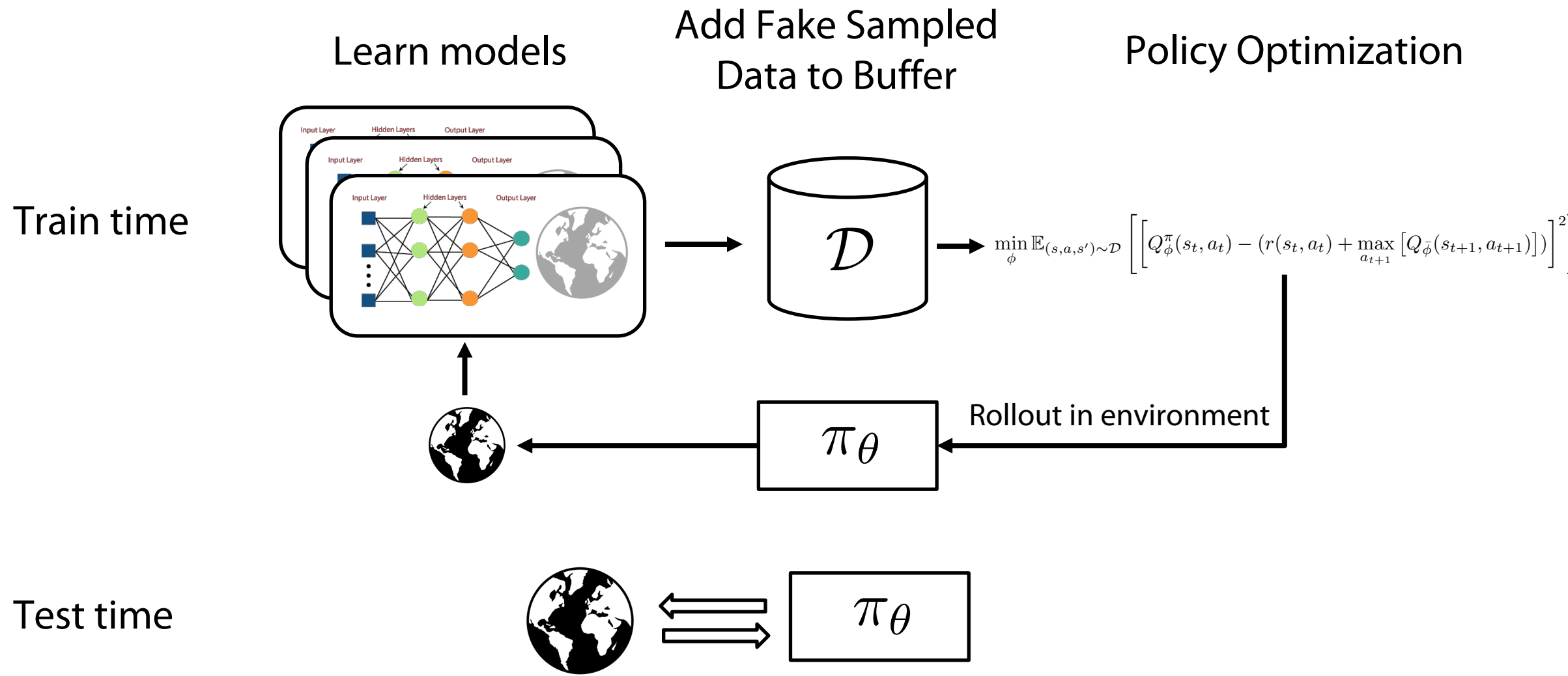


Use model(s) to generate data for policy optimization

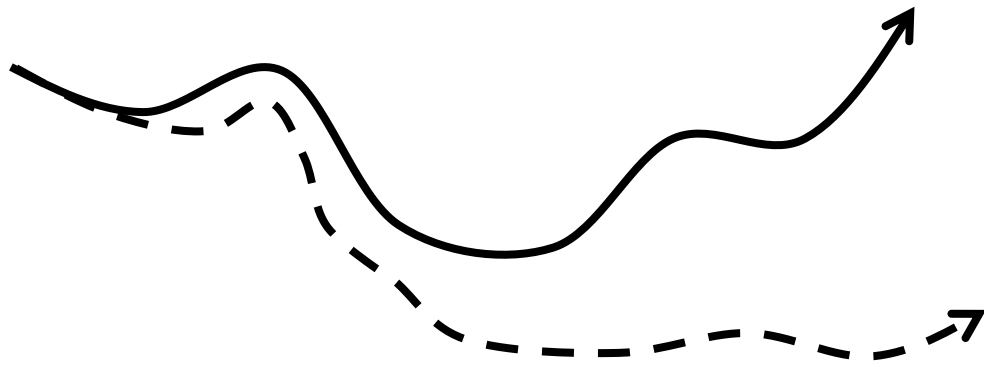


Can use PG or off-policy!

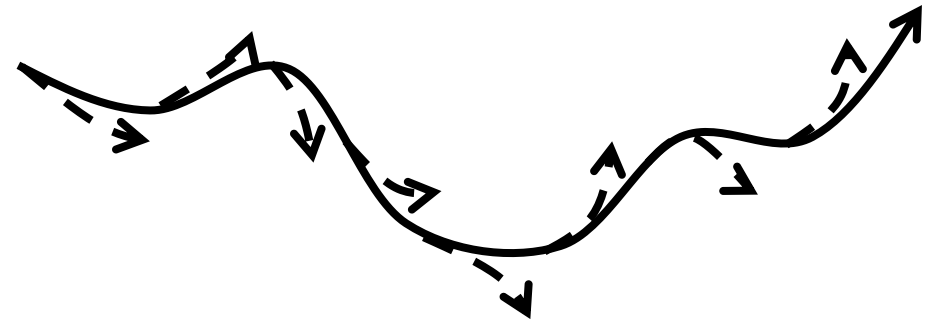
# Generating Data for Policy Optimization



# What matters in generating data from models?



Long horizon rollouts can deviate



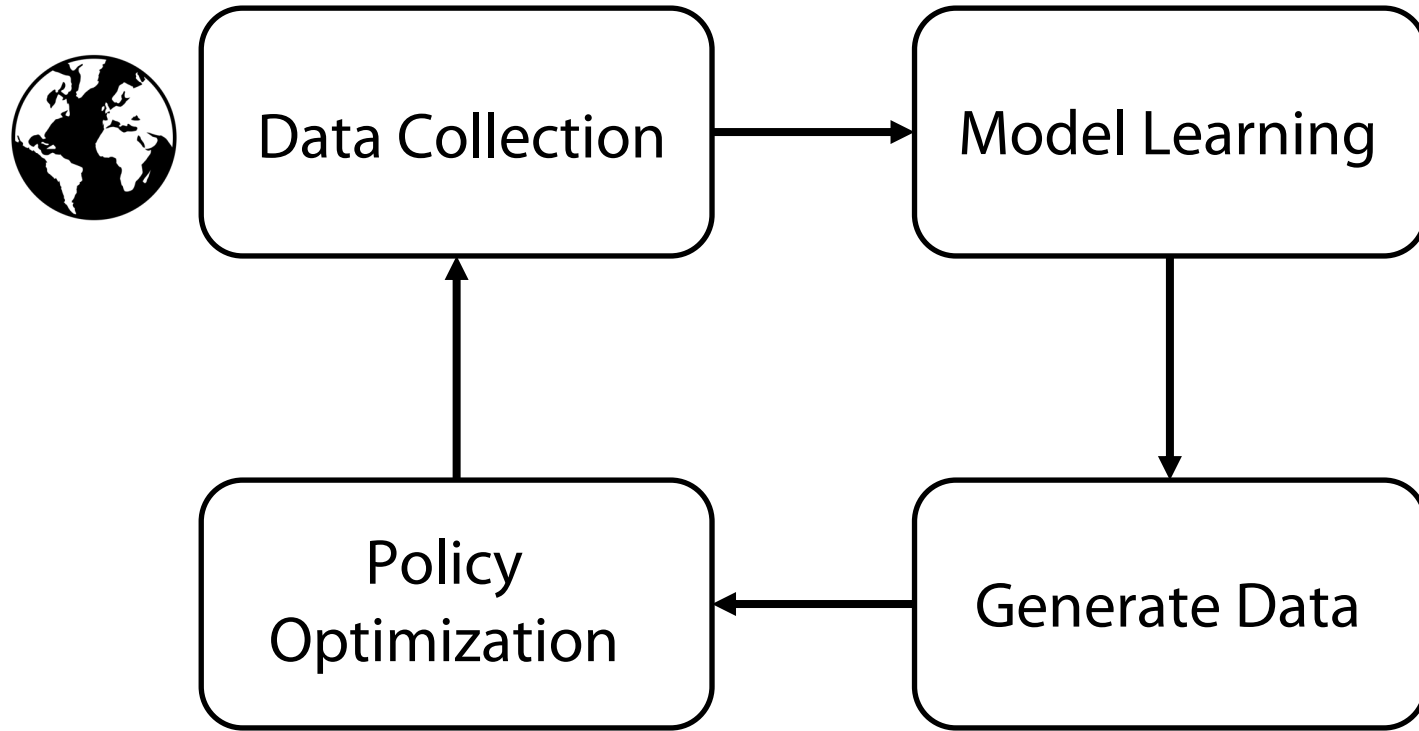
Short horizon rollouts deviate far less

Balance between off-policy coverage and compounding error

More in the readings!

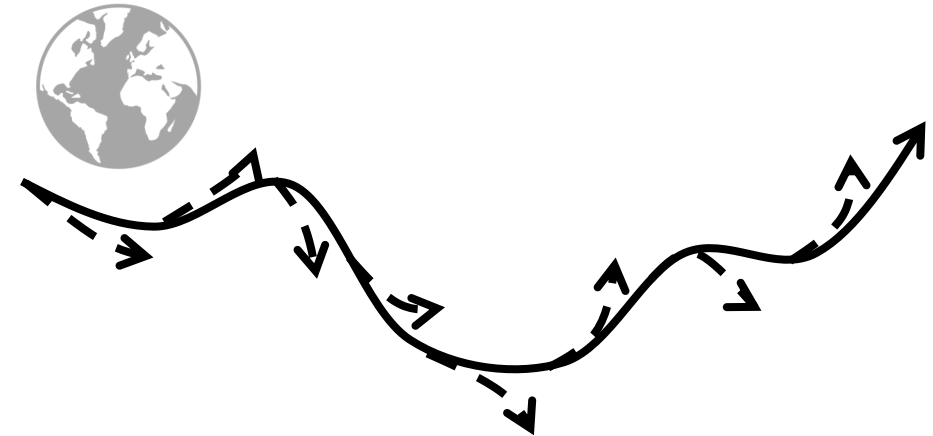


# Model Based RL – Using Models for Policy Optimization (v3)



Maximum likelihood supervised Learning

$$\max_{\theta} \mathbb{E}_{(s,a,s') \sim \mathcal{D}} [\log \hat{p}_{\theta}(s' | s, a)]$$



$$\min_{\phi} \mathbb{E}_{(s,a,s') \sim \mathcal{D}} \left[ \left[ Q_{\phi}^{\pi}(s_t, a_t) - (r(s_t, a_t) + \max_{a_{t+1}} [Q_{\bar{\phi}}(s_{t+1}, a_{t+1})]) \right]^2 \right]$$

More expensive/harder at training time, faster at test time

Does this work?



# Lecture outline

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The Anatomy of Model-Based Reinforcement Learning



Model based RL v0 → random shooting + MPC



Model based RL v1 → MPPI + MPC



Model based RL v2 → uncertainty based models

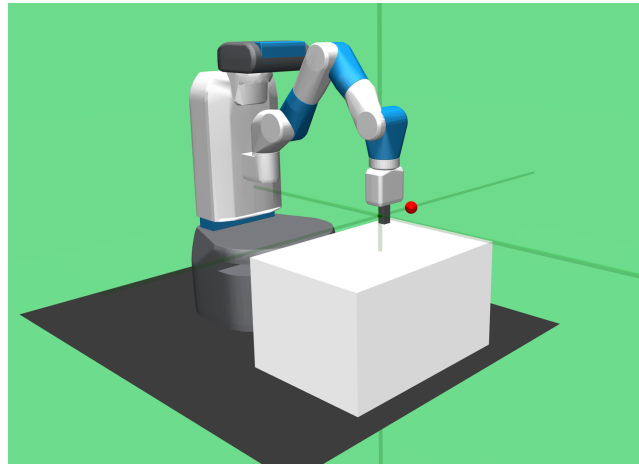
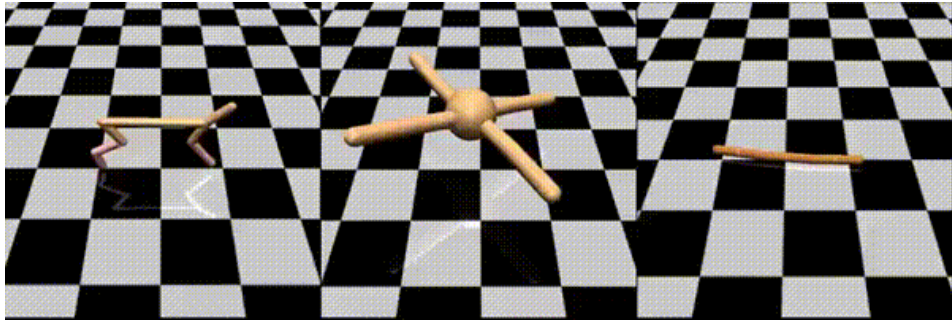


Model based RL v3 → policy optimization with models



Model based RL v4 → latent space models with images

# What about images?



State based domains

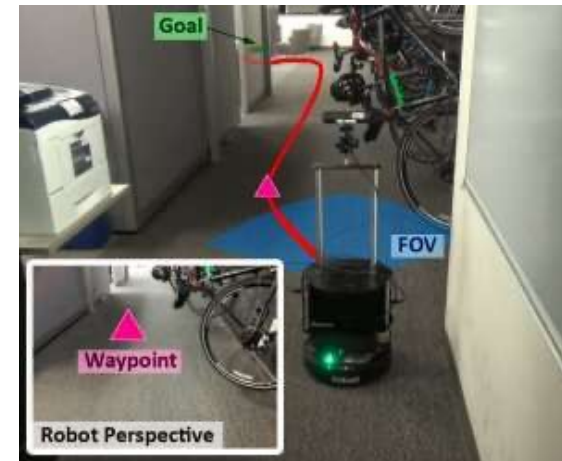
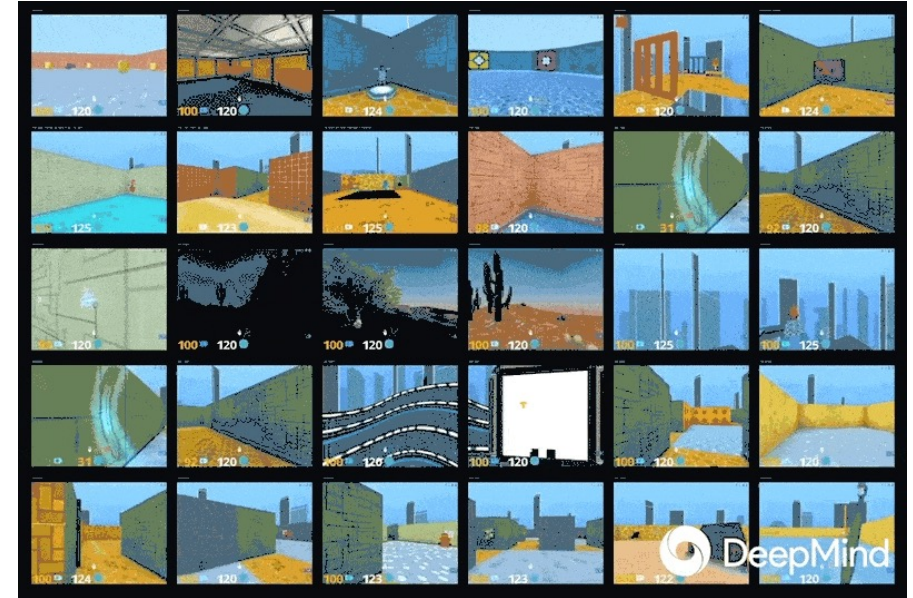
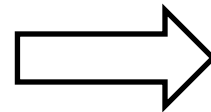
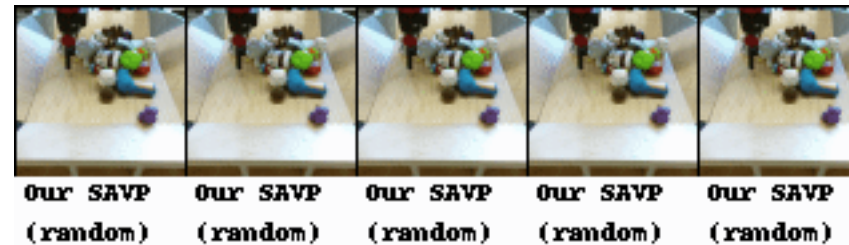


Image based domains

# Why is learning from images hard?

Generative modeling is videos, challenging to model multimodal correlated predictions



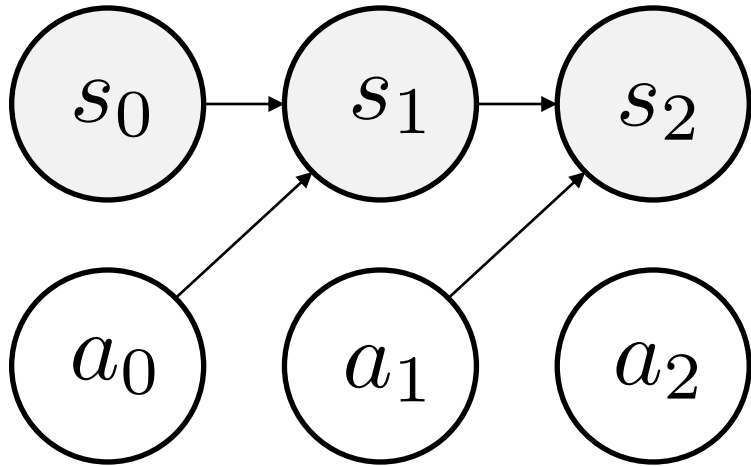
Partially observable!



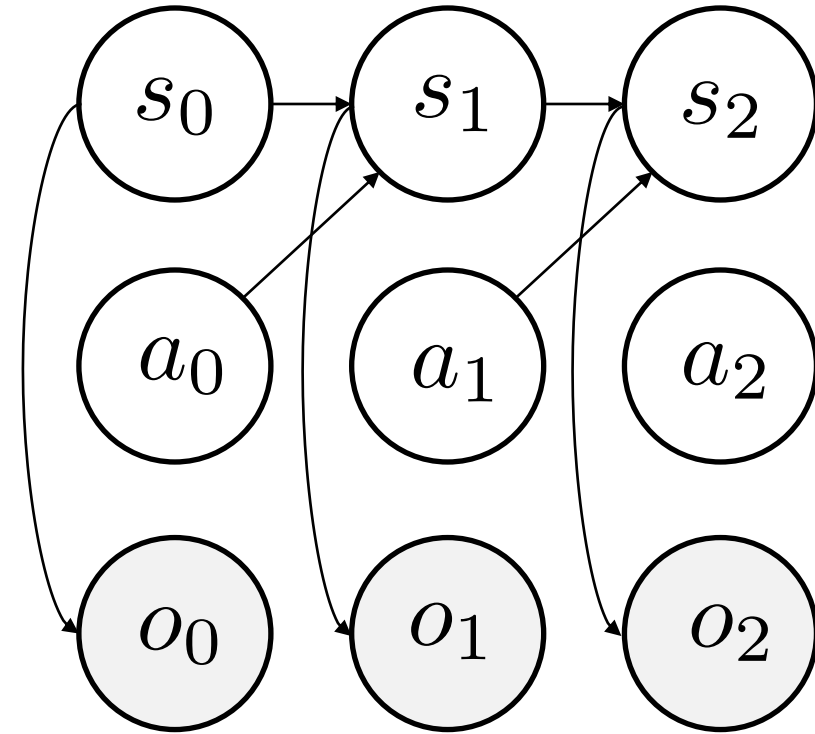
Long horizon predictions in video space can be challenging!

# Model Based RL – Latent Space Models for Image Based RL (v4)

Fully observed – Markovian case



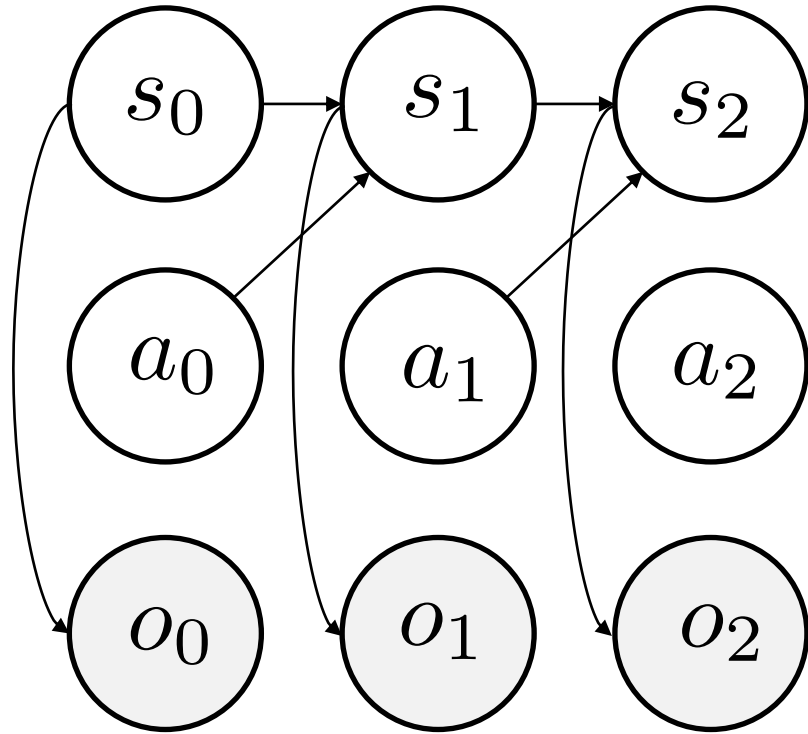
Partially observed – Non-Markovian case



If we can infer latent state and learn dynamics,  
then we can plan in a much smaller space

How do we infer latent state and learn dynamics in this space?

# How do we train latent space models?



Learn latent encoder to infer latent state from observations  $q_\phi(s_t | o_{1:t})$

Learn action conditioned latent transition model  $p_\eta(s_{t+1} | s_t, a_t)$

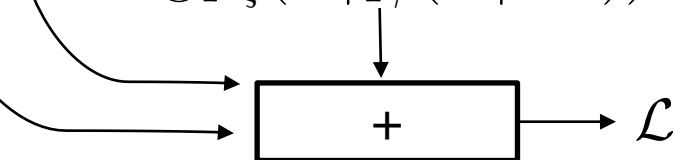
$$\log p_\eta(q_\phi(s_{t+1} | o_{1:t+1}) | q_\phi(s_t | o_{1:t}), a_t)$$

Learn latent decoder to reconstruct observations  $p_\psi(o_t | s_t)$

$$\log p_\psi(o_t | s_t)$$

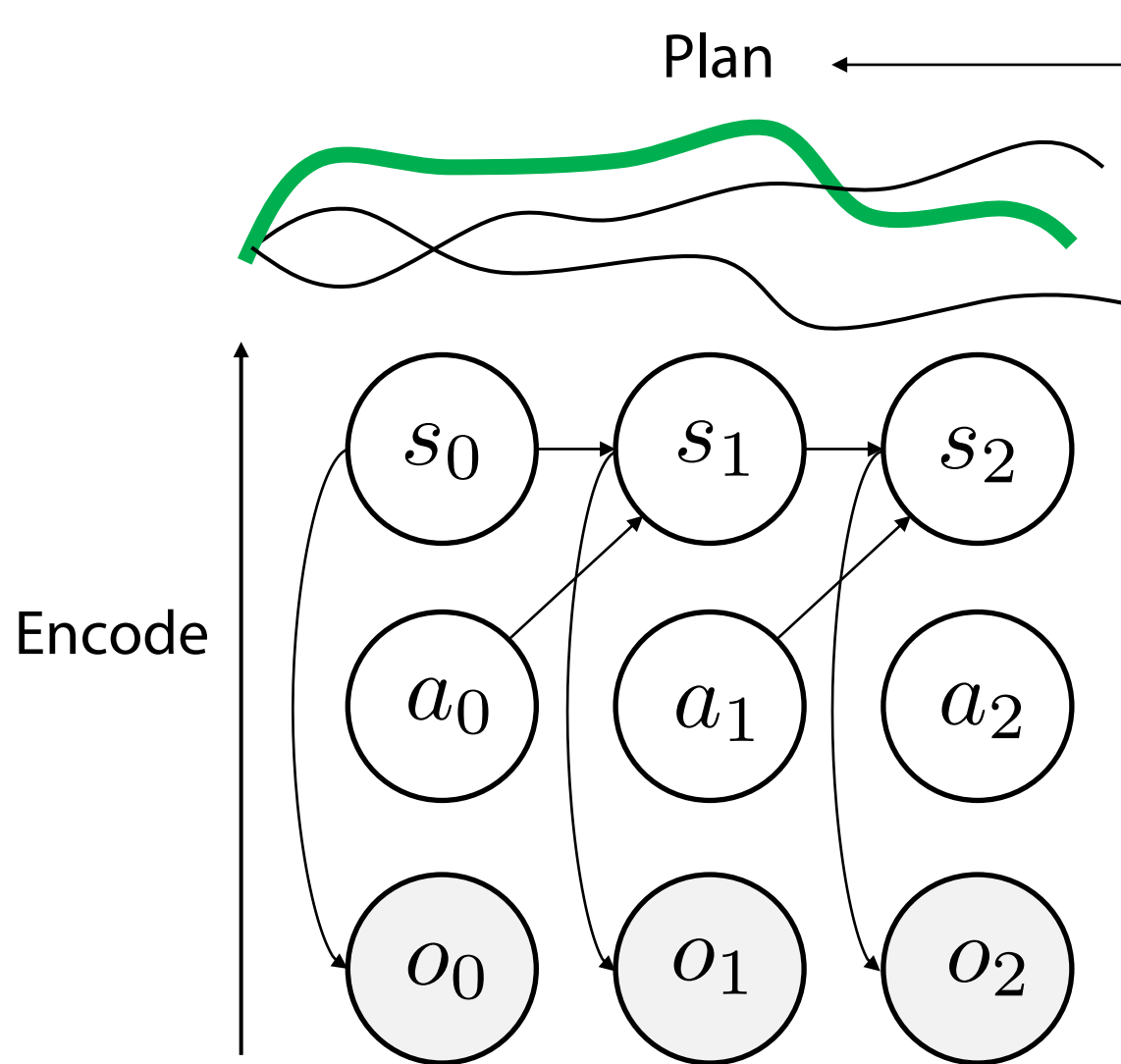
Learn reward predictor from latent state  $p_\zeta(r_t | s_t)$

$$\log p_\zeta(r_t | q_\phi(s_t | o_{1:t}))$$



Can derive the whole thing from first principles using variational inference!

# How do we use latent space models?

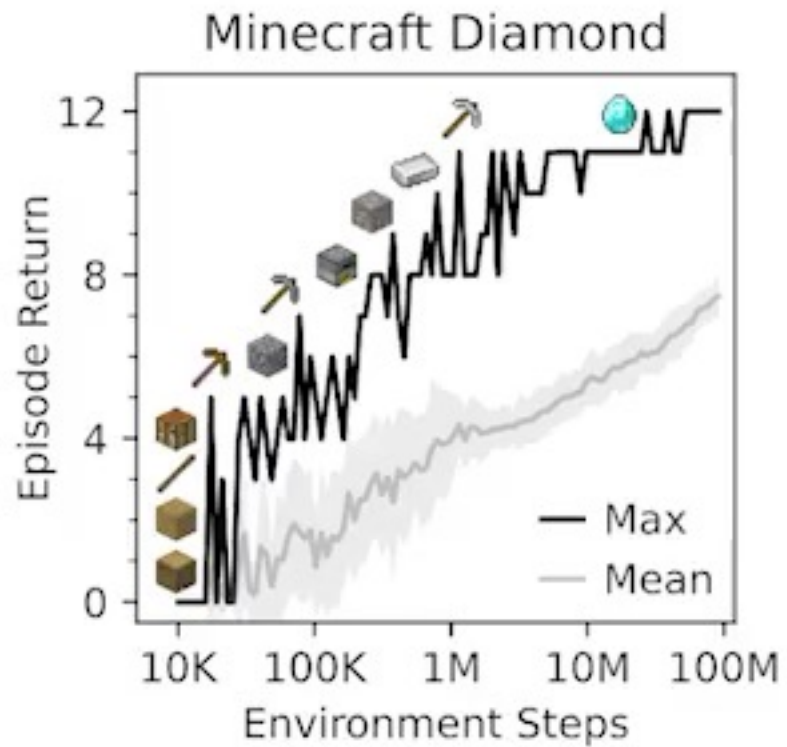


Apply any of the methods from this lecture, just in latent space!

1. Avoids predicting image frames at planning time
2. Scales much better than image prediction
3. Allows for longer horizon predictions



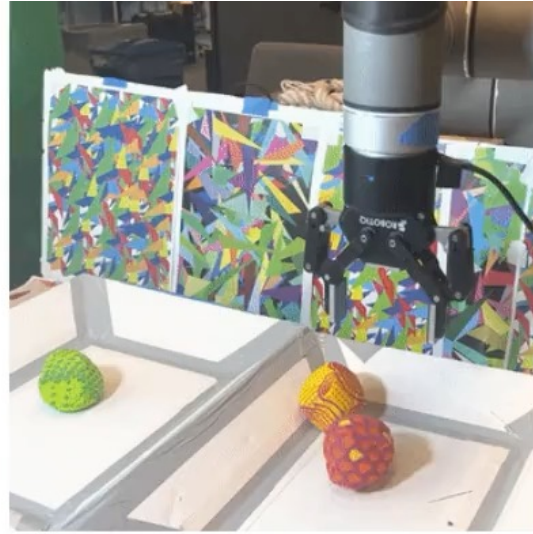
# Does this work?



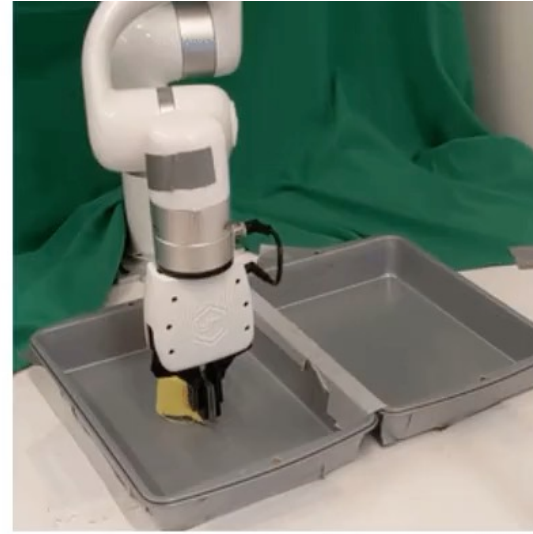
# Does this work?



A1 Quadruped  
Walking



UR5 Multi-Object  
Visual Pick Place



XArm Visual Pick  
and Place



Sphero Ollie Visual  
Navigation

Training from images in < 1 hour!

# Lecture outline

The Anatomy of Model-Based Reinforcement Learning



Model based RL v0 → random shooting + MPC



Model based RL v1 → MPPI + MPC



Model based RL v2 → uncertainty based models



Model based RL v3 → policy optimization with models



Model based RL v4 → latent space models with images

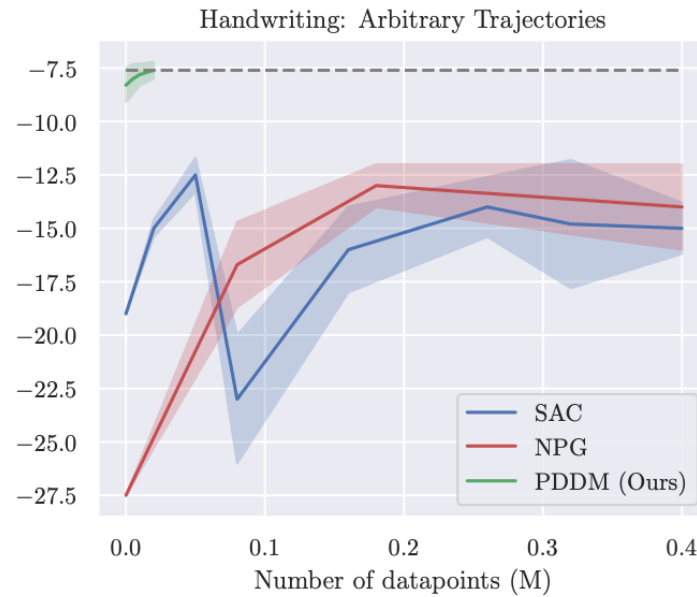
# Why should you care?

Model based RL **may be** a much more practical path to real world robotics

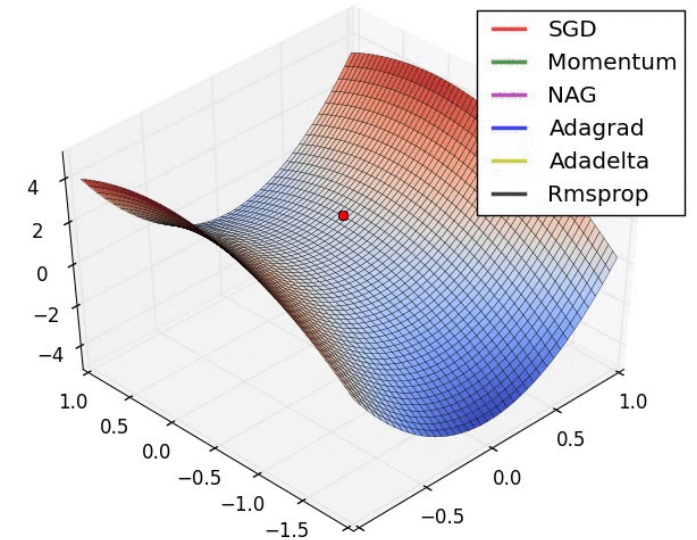
Transfer/Adaptive



Efficiency



Simplicity



← Likely to be the most future proof one!

# Are models really that different than Q-functions?

## Models

## Q-functions

Similar

1. Off-policy
2. Models the future

Very different than PG methods → on-policy, models current given future

Different

1. 1-step modeling
2. Models states
3. Can evaluate arbitrary policies
4. Parametric storage of training data

1. Cumulative modeling
2. Models returns
3. Can evaluate only policy  $\pi$
4. Non-parametric storage of data

# Class Structure

