Problem set 5

CSE 534 Autumn 2026

Due date: November 9th 11:59 pm

Instructions: Solutions should be legibly handwritten or typeset. Mathematically rigorous solutions are expected for all problems unless explicitly stated. You are encouraged to collaborate on problems in small teams; however, each team member must write and submit their own individual solution. Read the AI tools policy on the course webpage and adhere to it. For grading purposes, start each problem on a new page.

Problem 1 (Alice, Bob, and Bob play CHSH)**. (6 points)** The CHSH game is described as a game between non-communicating Alice and Bob where Alice receives an input bit x, Bob receives an input bit y, and they answer with bits a and b, respectively, such that $a \oplus b = xy$.

Consider the following game between non-communicating Alice, Bob_1, and Bob_2: The referee first flips a coin $i \in \{1, 2\}$ and then sends Alice an input bit x, and sends Bob_i the input bit y, and they answer with bits a, b, respectively. The other Bob neither receives a question nor answers. The winning conditioning is still $a \oplus b = xy$.

Prove that there exists a $\delta > 0$, such that any quantum strategy succeeds with probability $\leq \cos^2(\pi/8) - \delta$. For half-credit, prove that no quantum strategy wins with probability $= \cos^2(\pi/8)$.

Problem 2 (Odd cycle game)**.** Alice and Bob approach you claiming that they can 2-color any n-cycle for odd n. Amazed at this remark, you ask them to show you their solution. Having a proprietary solution$^{\text{TM}}$ to the problem, they don't want to reveal the full coloring, but they agree to the following game: Alice and Bob will be put in separate rooms (with no communication possible between them), and you can prompt each of them with a vertex $v \in [n]$ and they will respond with $c(v)$, the purported color of v.

You agree and set up the following game: with half probability, you ask Alice and Bob about the same uniformly random vertex $v \in [n]$, and check that Alice's answer $a \in \{0, 1\}$ matches that of Bob's answer $b \in \{0, 1\}$. And with half probability, you ask Alice about a uniformly random v and Bob about $(v + 1) \pmod n$, and check that $a \neq b$.

1. **(1 point)** Calculate the optimal classical value of the game when n is odd.
2. **(4 points)** What about the optimal quantum value to the game? Recall our in-class exploration of the CHSH game. We can characterize Alice's strategy by an observable $\{A_v\}$ for every $v \in [n]$ and Bob's strategy by an observable $\{B_v\}$ for every $v \in [n]$ with a shared state $|\psi\rangle_{AB}$. Construct a quantum strategy that achieves a success probability of at least $1 - \frac{\pi^2}{8n^2}$.
   Hint: They only need to share an EPR pair.
3. **(Optional)** Can you improve your previous strategy to get a success probability of at least $1 - \frac{\pi^2}{16n^2}$? Show how.

4. **(2 points)** Do you think that the quantum value of the odd-cycle game is 1? Give an intuitive argument why or why not (no need for a rigorous proof). You can assume, without proof, that the optimal strategy for Alice and Bob is to hold a maximally entangled state of dimension d: $\frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |i\rangle_A \otimes |i\rangle_B$ — this state is equal to the tensor product of m EPR pairs when $d = 2^m$.