

Two halves of q. computation

① Theoretical model

- idealized, consistent, plausible model
- "implementation agnostic"
- algorithmic testbed

② Implementations

- ion traps, photonic system, neutral atoms, etc.
- models trying to simulate theory

Lecture 1: Axioms of q. computation

Notation:

Kets $|v\rangle$ and bras $\langle v|$ for $|v\rangle \in \mathbb{C}^d$.

Example (inner product):

$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \quad |\phi\rangle = \begin{pmatrix} \gamma \\ \delta \end{pmatrix}$$

$$\langle \phi | \psi \rangle = \gamma^* \alpha + \delta^* \beta \quad \text{and}$$

$$|\phi\rangle\langle\psi| = \begin{pmatrix} \gamma_{\alpha}^* & \gamma_{\beta}^* \\ \delta_{\alpha}^* & \delta_{\beta}^* \end{pmatrix}$$

Single qubit quantum + bit = qubit

Axiomatic approach to q. computation:

Axiom 1 One qubit system is one whose state can be expressed as a vector $|\psi\rangle \in \mathbb{C}^2$ of unit norm.

If so, $|\psi\rangle$ is a one-qubit q. state.

Common one qubit states: $|0\rangle$ and $|1\rangle$

- orthonormal $\Rightarrow$ basis, standard or $\mathbb{Z}$ -basis.
- if state is $|0\rangle$ or $|1\rangle \Rightarrow$ classical.

Same for $|+\rangle, |-\rangle$.

Ex. $|0\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}}$, $|1\rangle = \frac{|+\rangle - |-\rangle}{\sqrt{2}}$

expand out and show in lecture.

Axiom 2 U unitary $\in \mathbb{C}^{2 \times 2}$.

Transform a system by U : $|\psi\rangle \rightarrow U|\psi\rangle$.

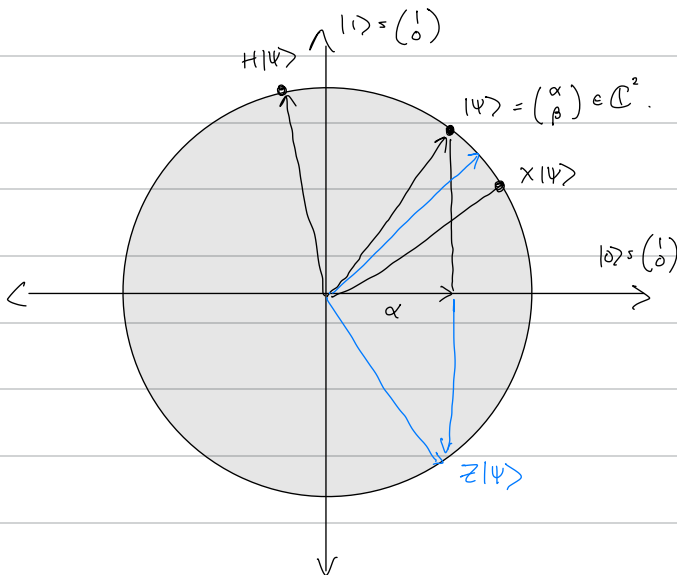
Def (Unitary)

$U^\dagger U = \mathbb{1}$, $U^{-1} = U^\dagger$, cols of U are orthonormal,
rows of U are orthonormal, U preserves norms

Common unitaries. $\mathbb{1}$, X , Z , H

Z -basis and X -basis.

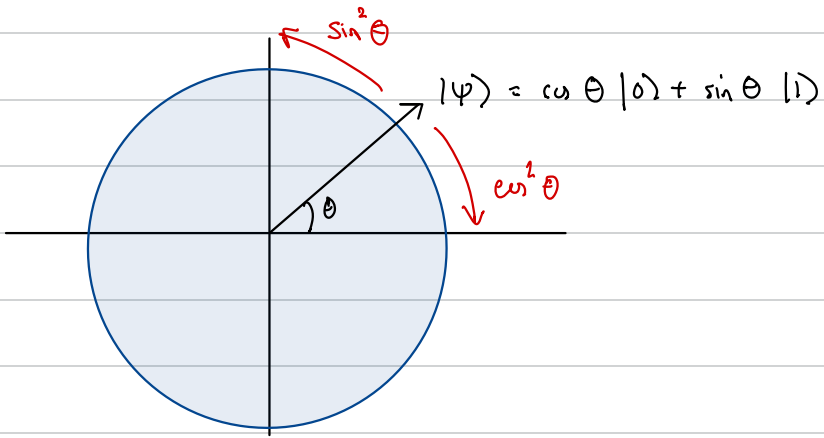
H maps between both bases.



Axiom 3 (measurement) $|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha|0\rangle + \beta|1\rangle$

transform $|\psi\rangle$ to $\begin{cases} w & \text{pr } |\alpha|^2, & |0\rangle \\ w & \text{pr } |\beta|^2, & |1\rangle \end{cases}$

+ classical value "0" or "1" read out.



Question for class: Measuring twice. Notion of classical states.
Classical objects: measurements/observations do not change state.

Thm/Lem $e^{i\theta}|\psi\rangle$ and $|\psi\rangle$ cannot be distinguished by measurement.

show $\text{Pr}[e^{i\theta}|\psi\rangle \text{ collapses to } j] = \text{Pr}[|\psi\rangle \text{ collapses to } j]$.

$e^{i\theta}$ = "global phase".

In our axioms, measurement is the only way to extract info

Axiom 4 Initialize a system as $|0\rangle \in \mathbb{C}^L$.

Then show alg for random number generator.