NAME: ______________________________

CSE 531
Automata, Computability, Complexity
Old Final Exam

1. For each of the following questions answer true or false and JUSTIFY your answer.

   (a) If L is r.e. then there is a Turing machine that generates L in canonical order.

   (b) If A is $\mathcal{NP}$-complete and $A \leq_m^p B$ then B is $\mathcal{NP}$-complete.

   (c) $DSPACE(\log n) \subseteq \mathcal{P}$.

   (d) Every regular language is in $DSPACE(c)$ for any $c \geq 1$.

   (e) The language $\{\langle M \rangle \mid M \text{ accepts } \langle M \rangle\}$ is r.e.

2. Let $L = \{\langle M \rangle \mid M \text{ accepts 1 but does not accept 0}\}$.
   In this question you will show that L is neither r.e nor co-r.e. by showing that $A \leq_m L$ and $\overline{A} \leq_m L$ for some non-recursive language A.

   (a) Show that $L_u \leq_m L$.

   (b) Show that $\overline{L_u} \leq_m L$.

3. Prove that $L = \{0^m 1^n \mid m \text{ is an integer multiple of } n\}$ is not regular.

4. Prove that $\mathcal{NP}$ is closed under

   (a) Union

   (b) Concatenation

   (c) Kleene star

5. Define $PSPACE = \cup_{c>0} DSPACE(n^c)$.

   (a) Show that if $A \leq_m^p B$ and $B \in PSPACE$ then $A \in PSPACE$.

   We say that A is *log-space many-one reducible to* B, $A \leq_m^{log} B$, if and only if there is a function f computable in deterministic space $O(\log n)$ such that $x \in A \Leftrightarrow f(x) \in B$.

   (b) Show that if $B \in \mathcal{P}$ and $A \leq_m^{log} B$ then $A \in \mathcal{P}$.

6. Do ONE of the following two questions. You can use the following page for your answer.

   (a) Prove that the SET COVER problem defined below is $\mathcal{NP}$-complete:
   Given a collection of sets $S_1, \ldots, S_m \subseteq \{1, \ldots, n\}$ and an integer k, is there a collection of k of these sets whose union is all of $\{1, \ldots, n\}$? That is, do there exist $i_1, \ldots, i_k \leq m$ such that $\bigcup_{j \leq k} S_{i_j} = \{1, \ldots, n\}$?
   (Hint: Use the fact that DOMINATING SET is $\mathcal{NP}$-complete.)
   NOTE: Before worrying about exactly HOW to show this, make sure that you describe exactly WHAT you are going to try to do.

(b) Prove that the ZERO COST SIMPLE CYCLE problem defined below is $\mathcal{NP}$-complete: Given a *directed* graph $G = (V, E)$ of n vertices with integer weights on its edges (possibly negative), is there a simple directed cycle of length > 0 in G with total weight 0?
(A cycle is *simple* if every vertex on the cycle appears exactly once. The total weight of a cycle is the sum of the weights on its edges.)
Hint: Use the fact that HAMILTONIAN CIRCUIT is $\mathcal{NP}$-complete.
NOTE: Before worrying about exactly HOW to show this, make sure that you describe exactly WHAT you are going to try to do.