CSE 531: Computability and Complexity — Autumn 2003
Problem Set #5 — Instructor: Venkatesan Guruswami
Due on Wednesday, **December 10, 2003**.

Instructions: Same as Problem Set 1.

---

1. Prove that $\mathsf{LOGSPACE} \neq \mathsf{TIME}(n^2)$.

2. Prove that every language in BPP has a circuit family of polynomial size that decides it. (Hint: Use the amplification lemma to reduce the error on input x to less than $2^{-|x|}$. Then try to "hardwire" the randomness into the circuit.)

3. Prove that if $\mathsf{PH} = \mathsf{PSPACE}$, then the polynomial time hierarchy has only finitely many distinct levels, i.e., $\mathsf{PH} = \Sigma_k^P$ for some $k \geq 1$.

4. Define $\mathit{UNIQUESAT} = \{\langle \phi \rangle \mid \phi \text{ is a CNF formula that has a unique satisfying assignment}\}$. Prove that $\mathit{UNIQUESAT} \in \mathsf{P}^{SAT}$.

5. Prove that if $\mathsf{NP} \subseteq \mathsf{BPP}$, then $\mathsf{NP} = \mathsf{RP}$.

6. Prove that there exists an oracle C for which $\mathsf{NP}^C \neq \mathsf{coNP}^C$.