CSE 531
Assignment 5

Due November 2, 2000

For this assignment it is probably desirable that collaboration be reduced. Each of the problems has a fairly short solution that requires an insight. Sharing these insights might spoil the pleasure of coming up with them yourselves.

1. The problem of *0-1 integer programming* (0-1 IP) is defined as follows. Given a set of linear inequalities with integer coefficients, is the set satisfiable by an assignment to the variables where each variable is assigned to a 0 or a 1? For example: consider the set of inequalities

$$\begin{aligned} x + y - z &\geq 1 \\ x - y + z &\geq 2 \end{aligned}$$

   This set does not have a solution because the second equation forces $x = z = 1$ and $y = 0$.

   (a) Show that 0-1 IP is in NP.

   (b) Show that 3SAT $\leq_m^P$ 0-1 IP. (Hint: you will need a variable for each Boolean variable and its complement.)

2. The *independent set* (IS) problem is defined as follows. Given an undirected graph G and number k, is there a set S of k vertices such that there are no edge in G between vertices in S? Show that IS is NP-complete. (Hint: IS is very closely related to CLIQUE which you can use as the basis of a reduction.)

3. The *two bin problem* (2-BIN) is defined as follows. Given a sequence $\{a_1, a_2, \ldots, a_n\}$ of positive integers, is there a $X \subseteq \{1, 2, \ldots, n\}$ such that

$$\sum_{i \in X} a_i = \sum_{i \notin X} a_i \ ?$$

   Show that 2-xBIN is NP-complete. (Hint: 2-BIN is closely related to subset sum which you can use as the basis of a reduction.)

4. (optional) Show that 2-SAT is in P. 2-SAT is the problem of determining if a Boolean formula in conjuctive normal form with at most 2 literals per clause is satisfiable.

5. (optional) Define the EXACT COVER problem as follows. Given a universe $U = \{u_1, u_2, \ldots, u_n\}$ and subsets S_i of U for $1 \leq i \leq m$, is there a set $X \subseteq \{1, 2, \ldots, m\}$ such that $\bigcup_{i \in X} S_i = U$ and for all $i, j \in X$ with $i \neq j$, $S_i \cap S_j = \emptyset$. Show that EXACT COVER is NP-complete.