For this assignment it is probably desirable that collaboration be reduced. Each of the problems has a fairly short solution that requires an insight. Sharing these insights might spoil the pleasure of coming up with them yourselves.

1. The problem of 0-1 integer programming (0-1 IP) is defined as follows. Given a set of linear inequalities with integer coefficients, is the set satisfiable by an assignment to the variables where each variable is assigned to a 0 or a 1? For example: consider the set of inequalities

\[
\begin{align*}
x + y - z &\geq 1 \\
x - y + z &\geq 2
\end{align*}
\]

This set does not have a solution because the second equation forces \(x = z = 1\) and \(y = 0\).

(a) Show that 0-1 IP is in NP.

(b) Show that 3SAT \(\leq^P_{NP}\) 0-1 IP. (Hint: you will need a variable for each Boolean variable and its complement.)

2. The independent set (IS) problem is defined as follows. Given an undirected graph \(G\) and number \(k\), is there a set \(S\) of \(k\) vertices such that there are no edge in \(G\) between vertices in \(S\)? Show that IS is NP-complete. (Hint: IS is very closely related to CLIQUE which you can use as the basis of a reduction.)

3. The two bin problem (2-BIN) is defined as follows. Given a sequence \(\{a_1, a_2, \ldots, a_n\}\) of positive integers, is there a \(X \subseteq \{1, 2, \ldots, n\}\) such that

\[
\sum_{i \in X} a_i = \sum_{i \in \overline{X}} a_i ?
\]

Show that 2-xBIN is NP-complete. (Hint: 2-BIN is closely related to subset sum which you can use as the basis of a reduction.)

4. (optional) Show that 2-SAT is in P. 2-SAT is the problem of determining if a Boolean formula in conjunctive normal form with at most 2 literals per clause is satisfiable.

5. (optional) Define the EXACT COVER problem as follows. Given a universe \(U = \{u_1, u_2, \ldots, u_n\}\) and subsets \(S_i\) of \(U\) for \(1 \leq i \leq m\), is there a set \(X \subseteq \{1, 2, \ldots, m\}\) such that \(\bigcup_{i \in X} S_i = U\) and for all \(i, j \in X\) with \(i \neq j\), \(S_i \cap S_j = \emptyset\). Show that EXACT COVER is NP-complete.