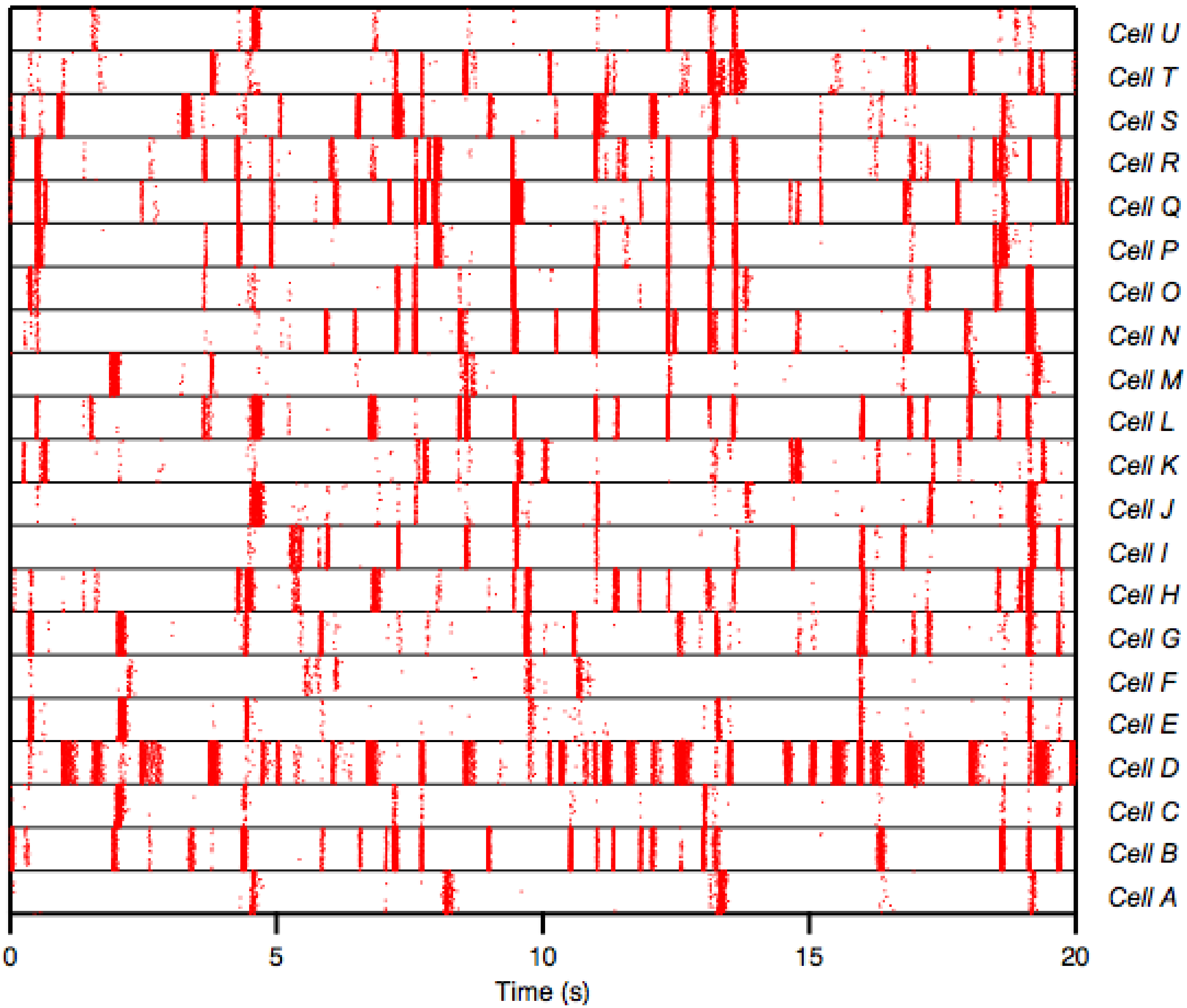
What is the neural code?



Single neurons

Populations

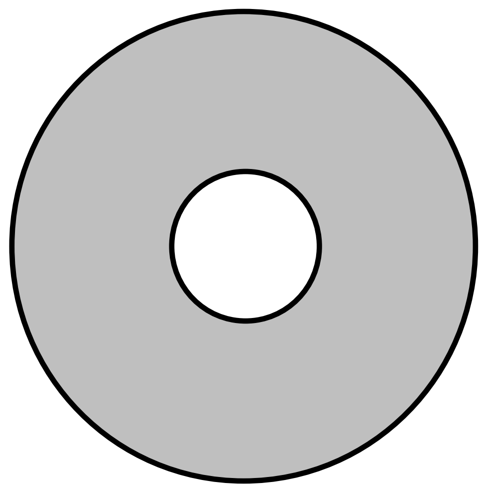
Encoding and decoding

P(response | stimulus)   *encoding*

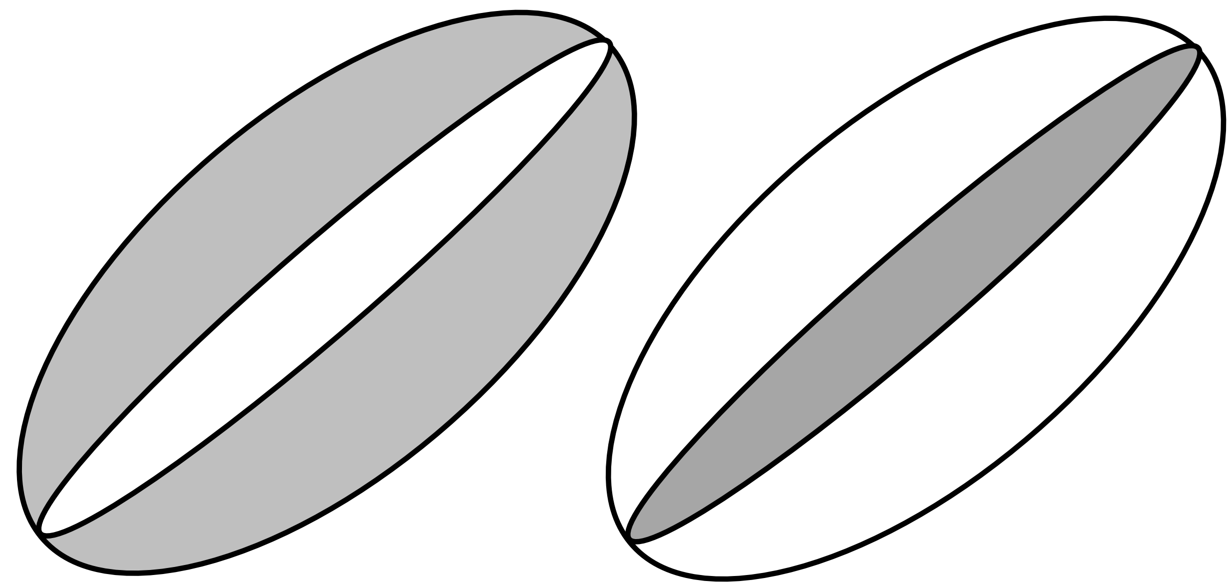
P(stimulus | response)   *decoding*

- What is response?
- What is stimulus?
- What is the function P?

Basic coding model: linear filtering



retina



Visual cortex

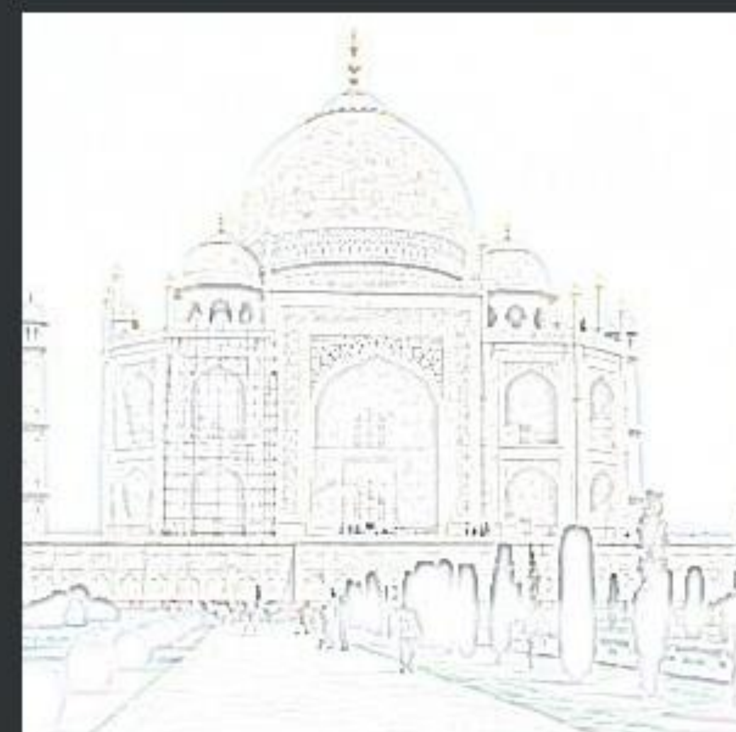
Spatial filter: $r = \iint f(x,y)\, I(x_0-x, y_0-y)\, dx\, dy$

Spatial filtering

7.2.1. Overview

Figure 16.136. Applying example for the “Difference of Gaussians” filter

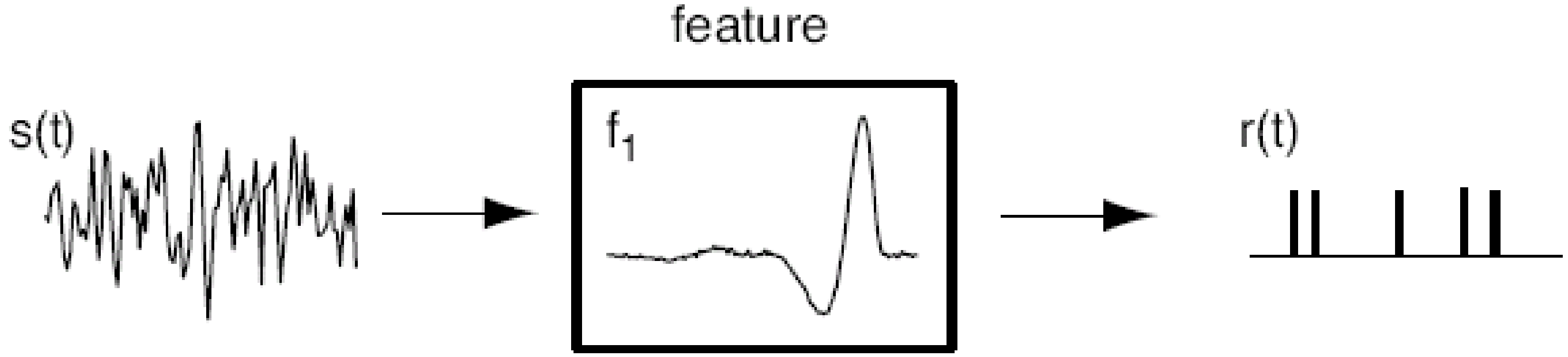
Original image



Filter “Difference of Gaussians” applied

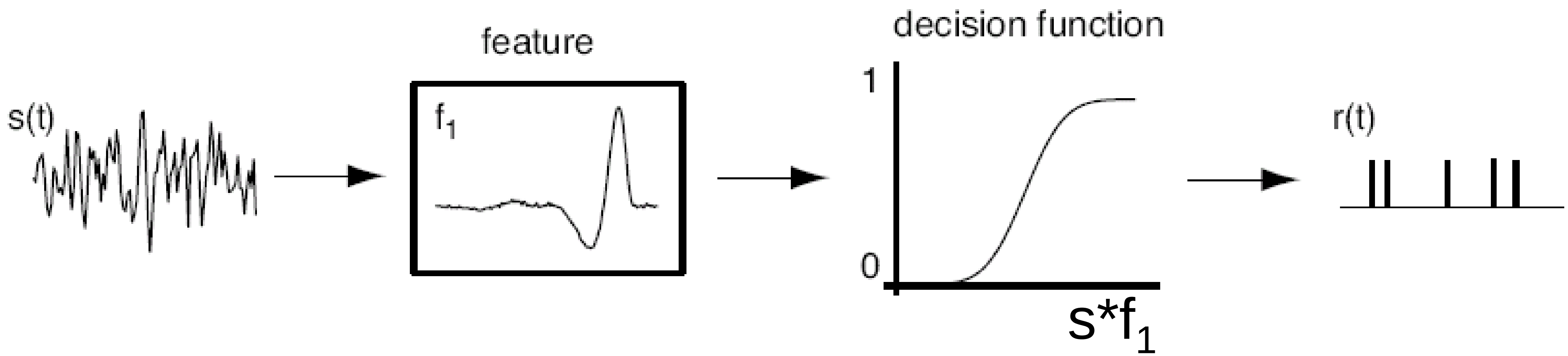
http://docs.gimp.org/2.6/en/plug-in-dog.html

Basic coding model: temporal filtering



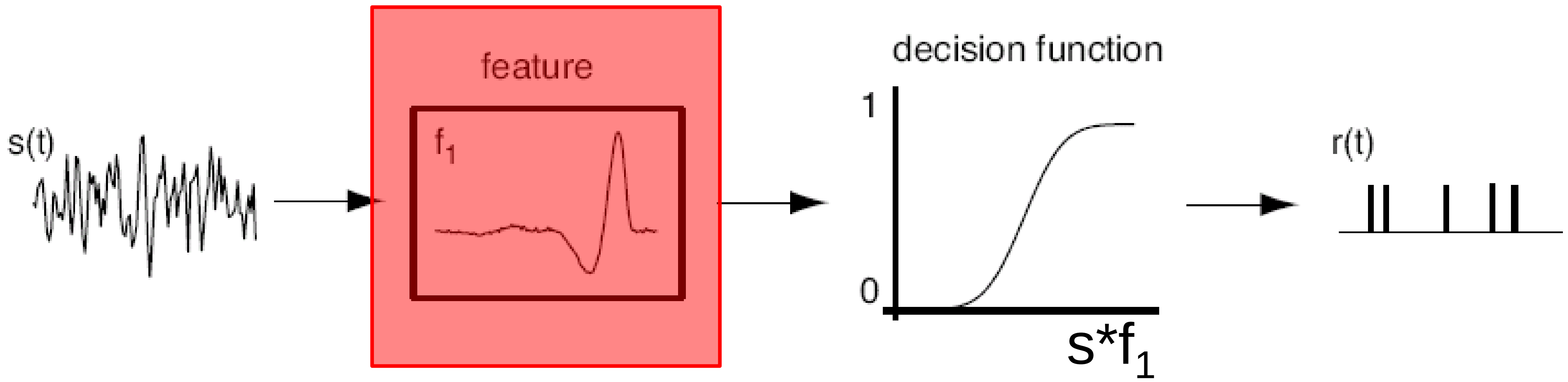
Linear filter: $r(t) = \int s(t-\tau)\, f(\tau)\, dt$

Next most basic coding model



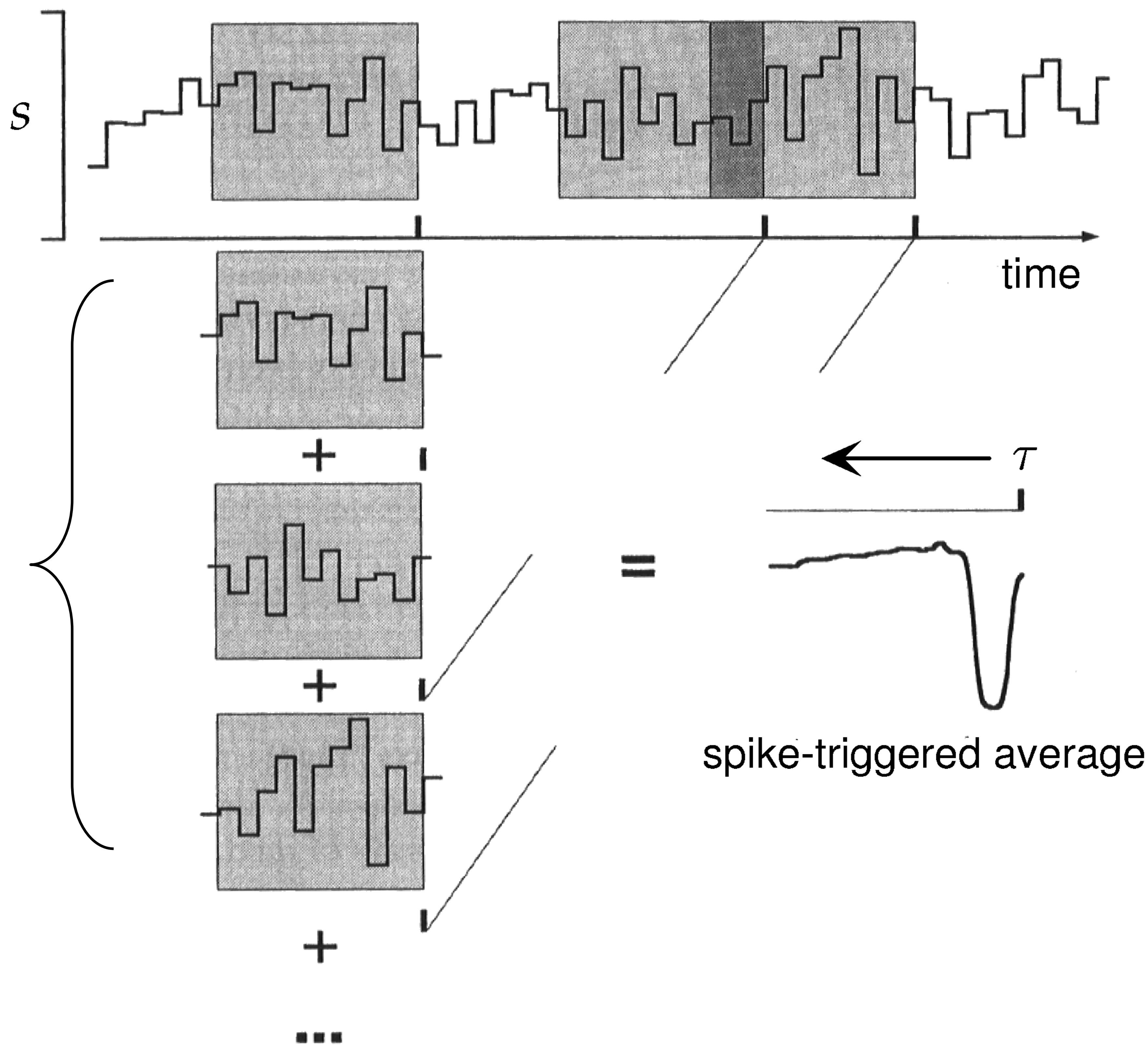
Linear filter & nonlinearity: $r(t) = g(\int s(t-\tau)\, f(\tau)\, d\tau)$

How to find the components of this model

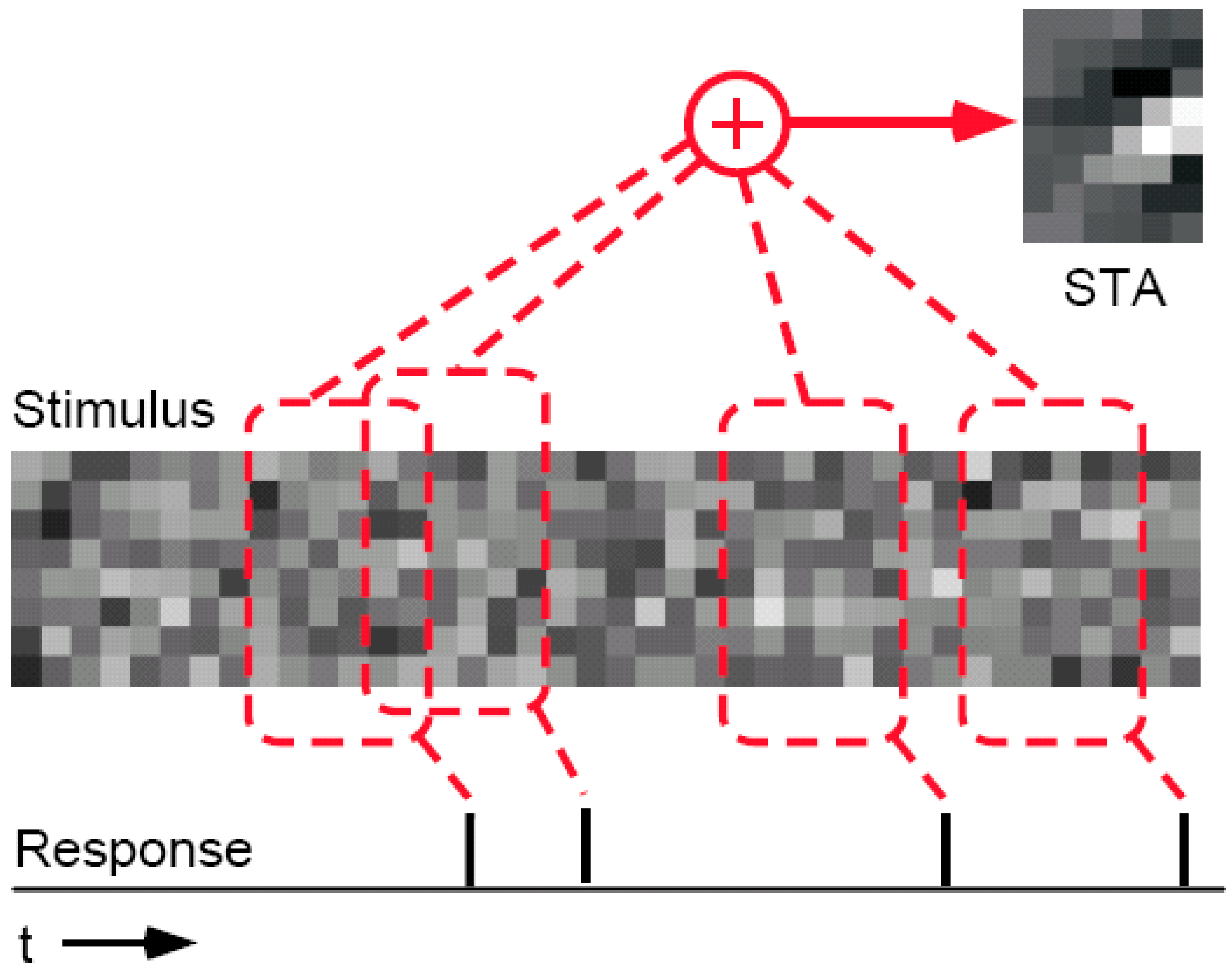


Reverse correlation: the spike-triggered average

Spike-conditional ensemble



The spike-triggered average

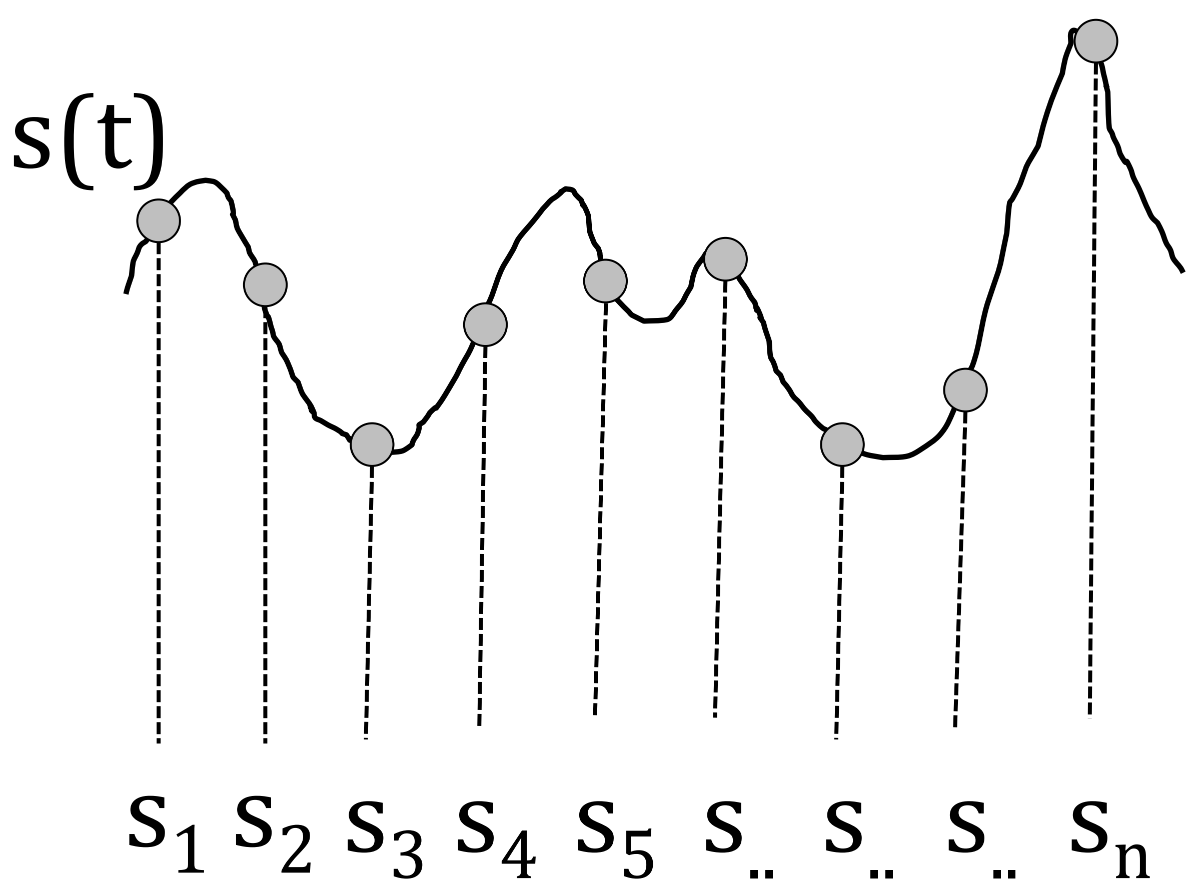


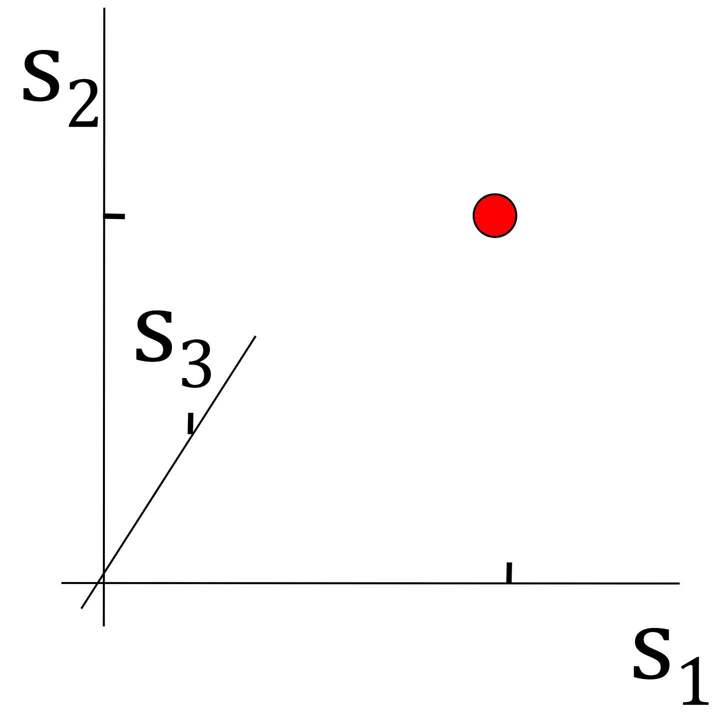
Dimensionality reduction

More generally, one can conceive of the action of the neuron or neural system as *selecting a low dimensional subset* of its inputs.

P(response | stimulus) → P(response | $s_1, s_2, .., s_n$)

Start with a very high dimensional description (eg. an image or a time-varying waveform) and pick out a small set of relevant dimensions.

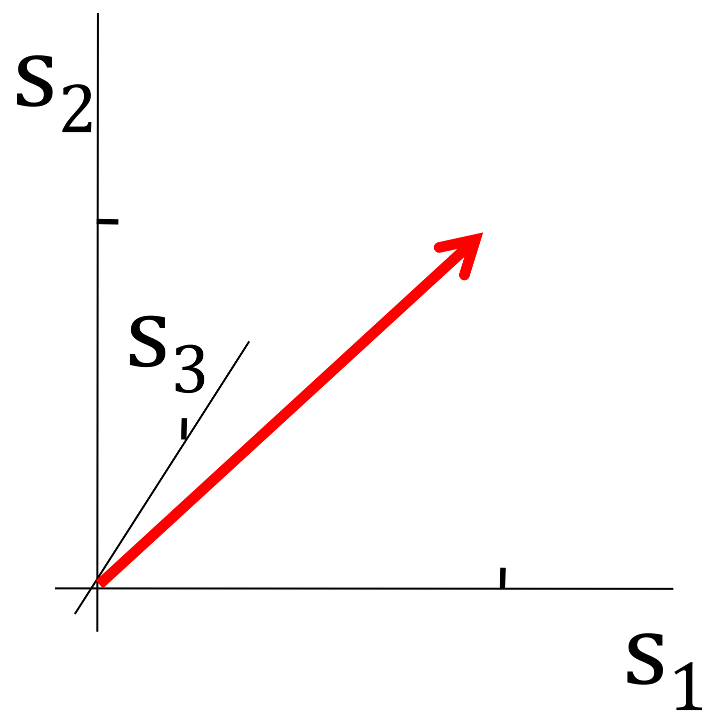




$S(t) = (S_1, S_2, S_3, \ldots, S_n)$

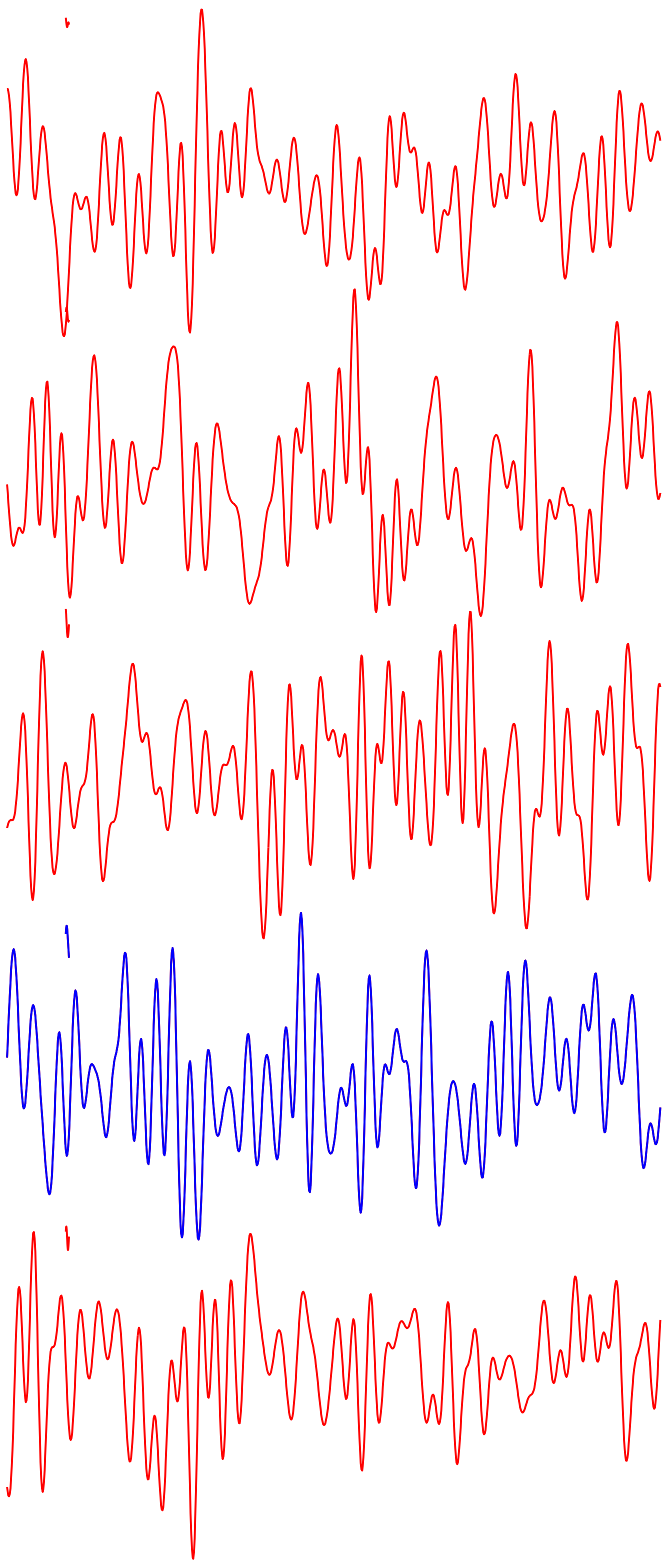
Linear filtering

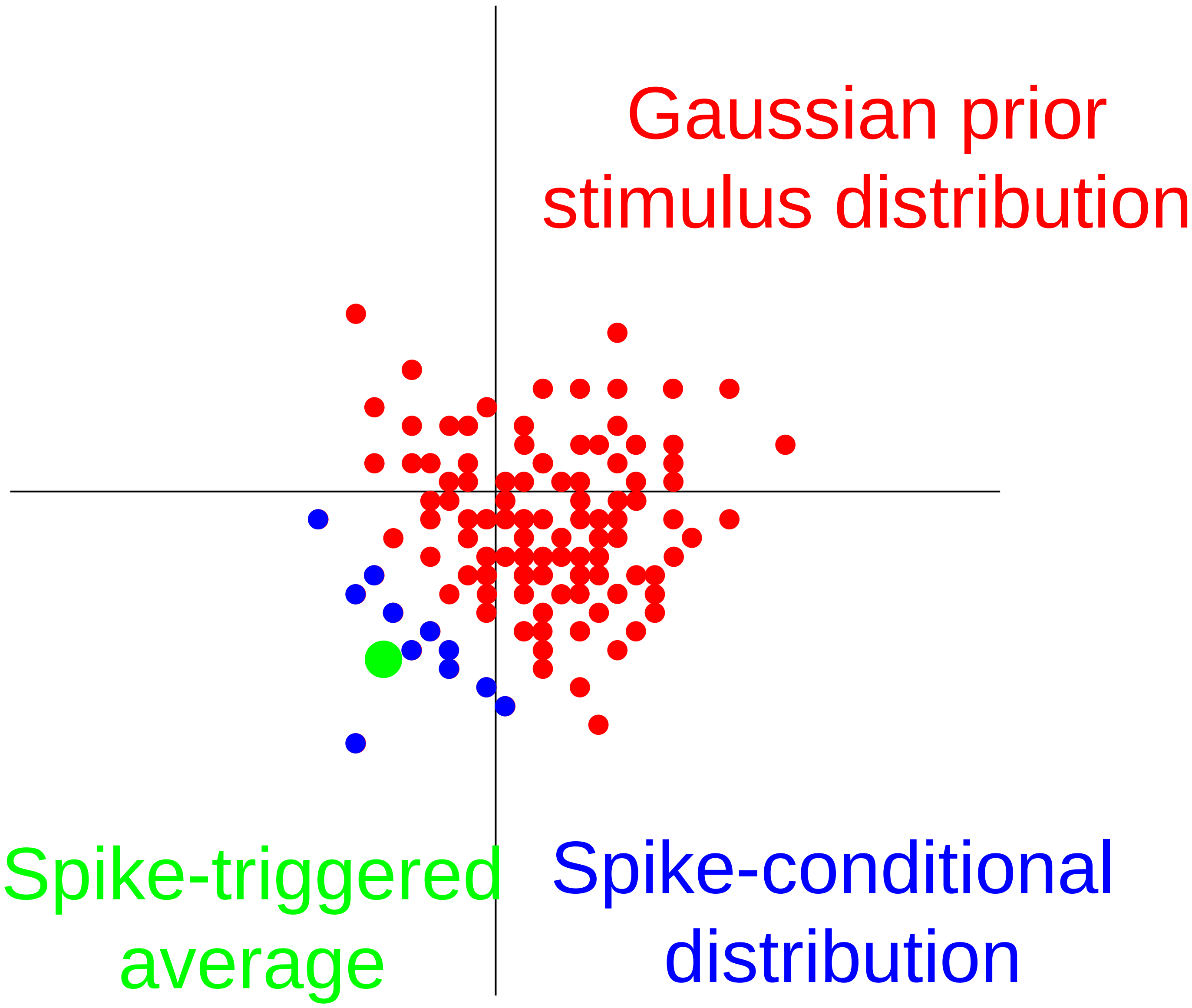
Linear filtering = convolution = projection



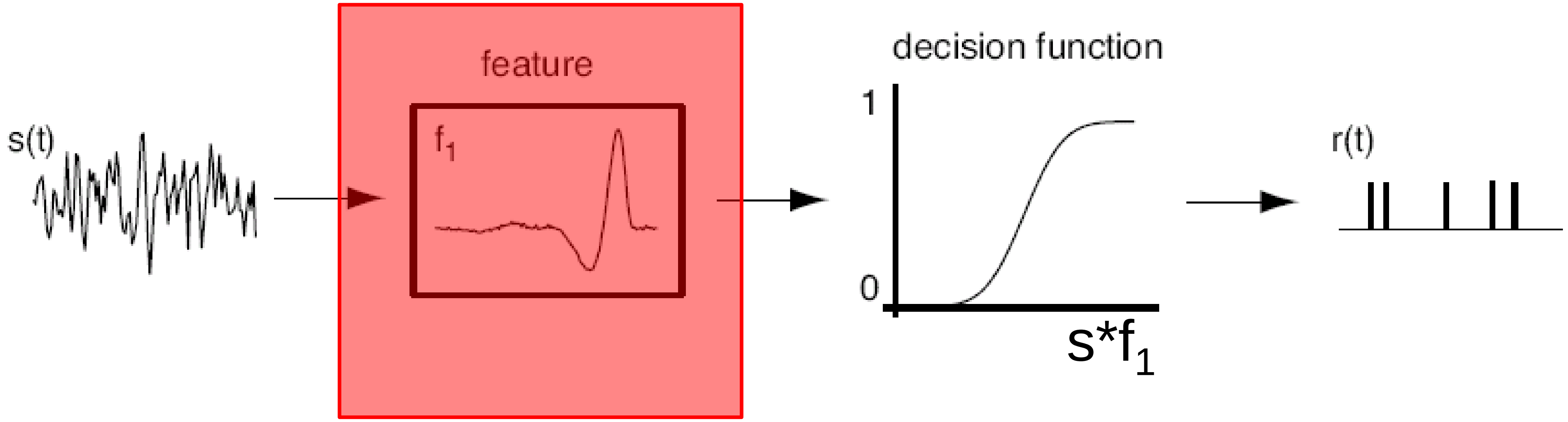
Stimulus feature is a vector in a high-dimensional stimulus space

Determining linear features from white noise





How to find the components of this model



Determining the nonlinear input/output function

The input/output function is:

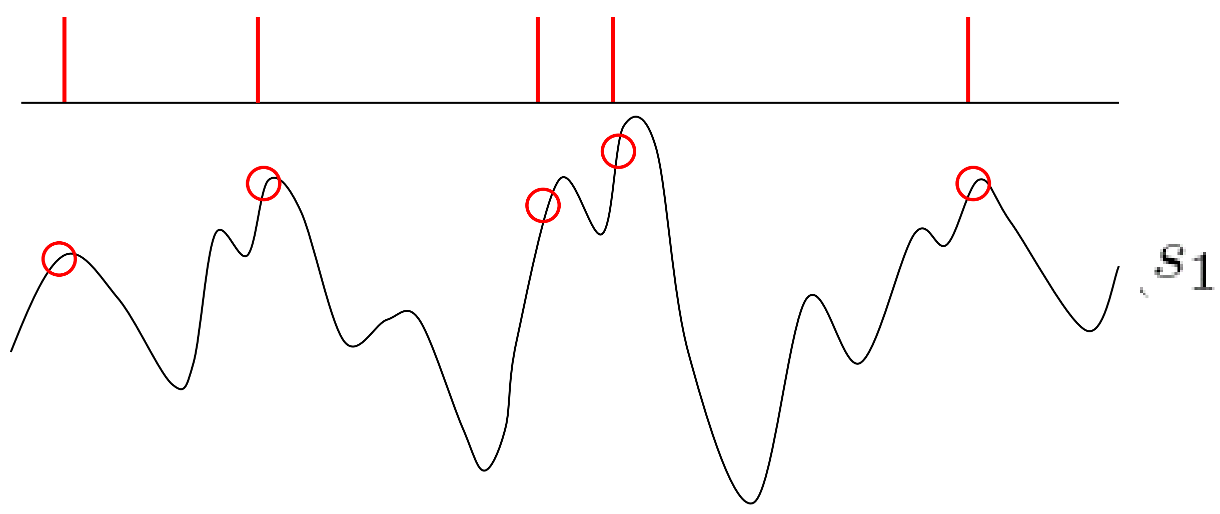
$$P(\text{spike}|\text{stimulus})$$

which can be derived from data using Bayes' rule:

$$P(\text{spike}|s_1) = \frac{P(s_1|\text{spike})P(\text{spike})}{P(s_1)}$$

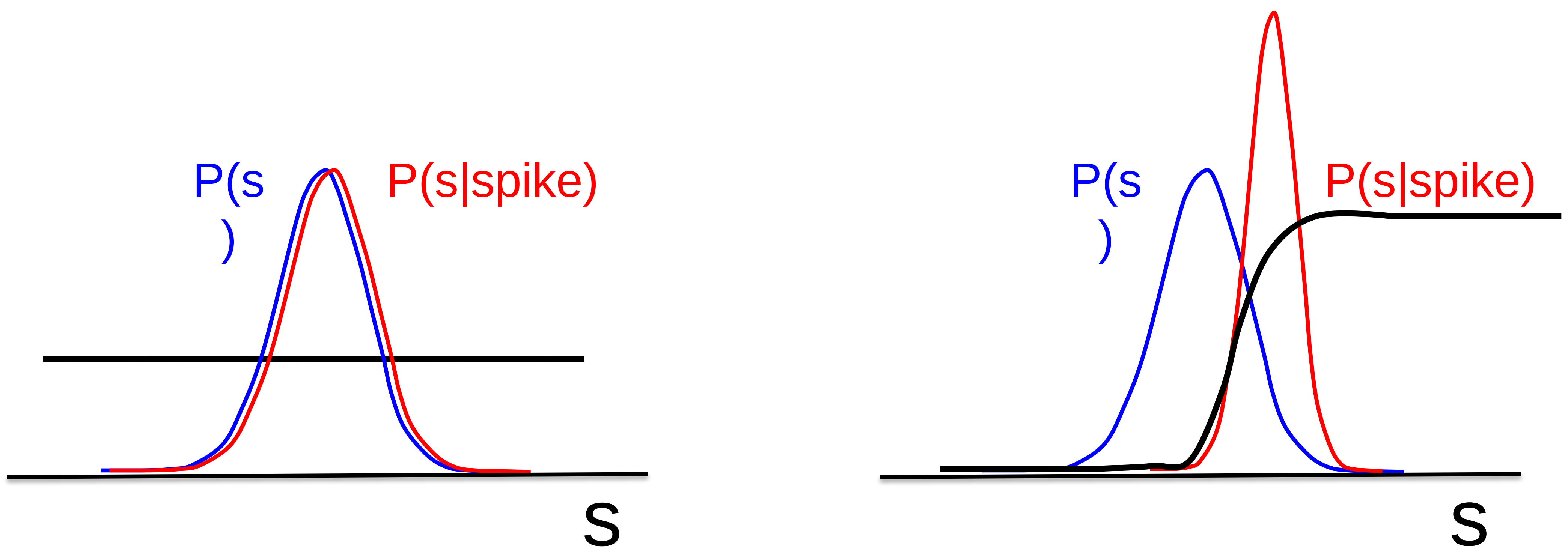
$P(s_1)$

$P(s_1|\text{spike})$

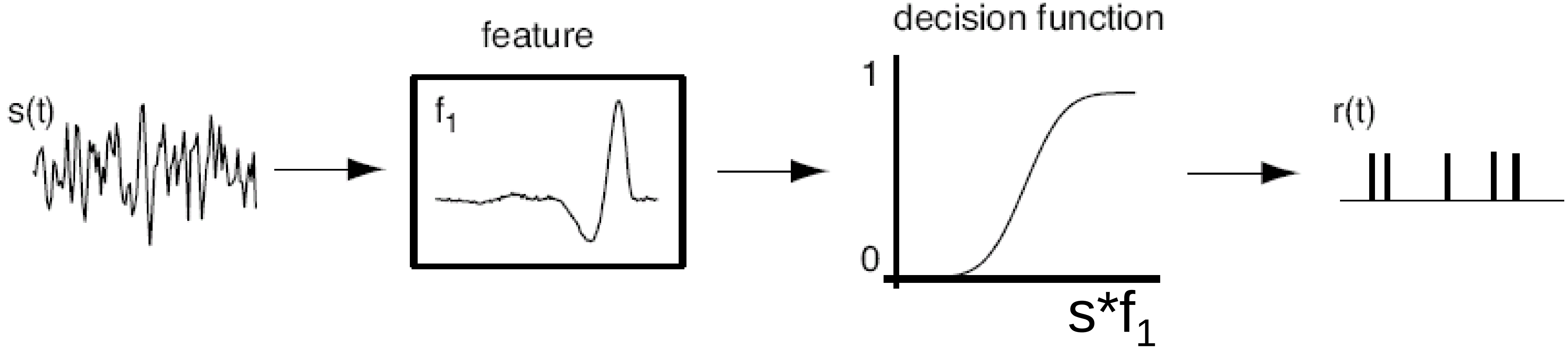


Nonlinear input/output function

Tuning curve: **P(spike|s) = P(s|spike) P(spike) / P(s)**



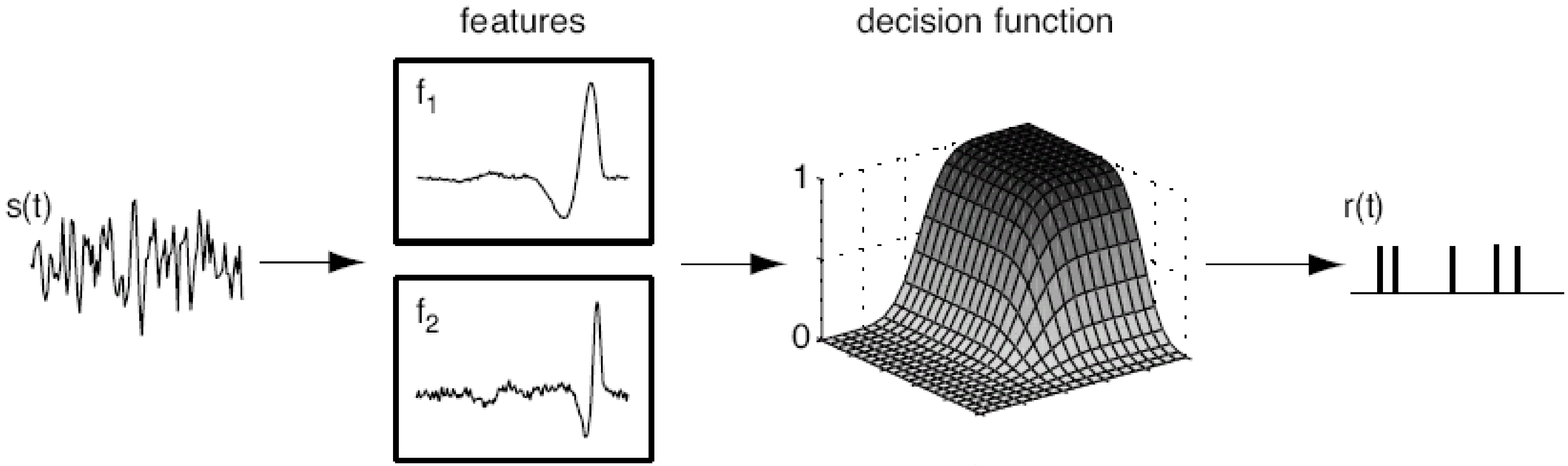
Next most basic coding model



Linear filter & nonlinearity: $r(t) = g(\int f(t-\tau)\, s(\tau)\, d\tau)$

…shortcomings?

Less basic coding models



Linear filter*s* & nonlinearity: $r(t) = g(f_1{*}s, f_2{*}s, \ldots, f_n{*}s)$

A bit of linear algebra

$\mathbf{u} \bullet \mathbf{v} = \Sigma\, u_i\, v_i$

$\mathbf{u} \bullet \mathbf{v}$ = projection of u onto v

$\mathbf{u} \bullet \mathbf{u}$ = length of u

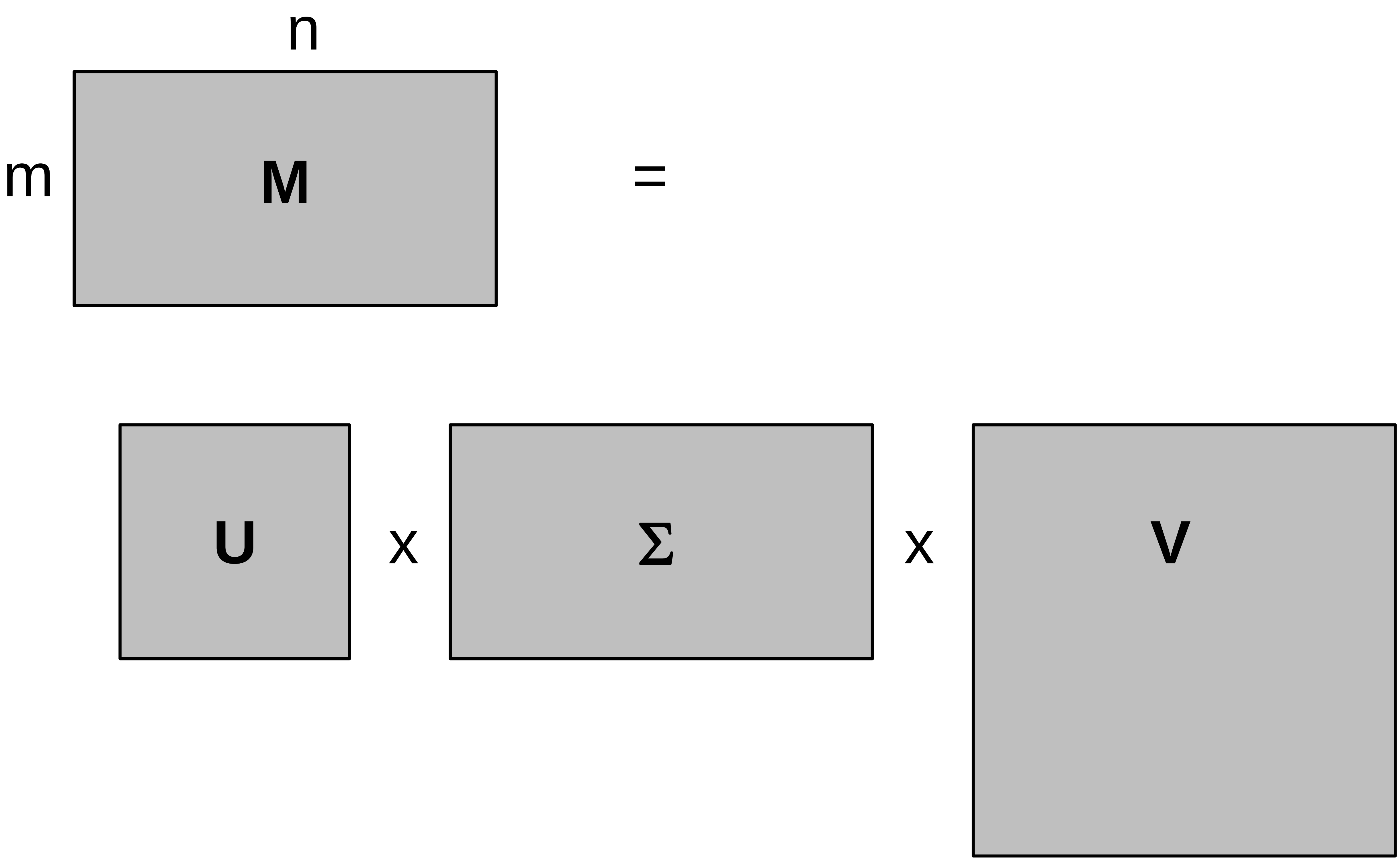
Linear filtering: $r(t) = \int f(t-\tau)\, s(\tau)\, d\tau$

$\mathbf{u} \times \mathbf{v} = \mathbf{M}$ where $M_{ij} = u_i\, v_j$

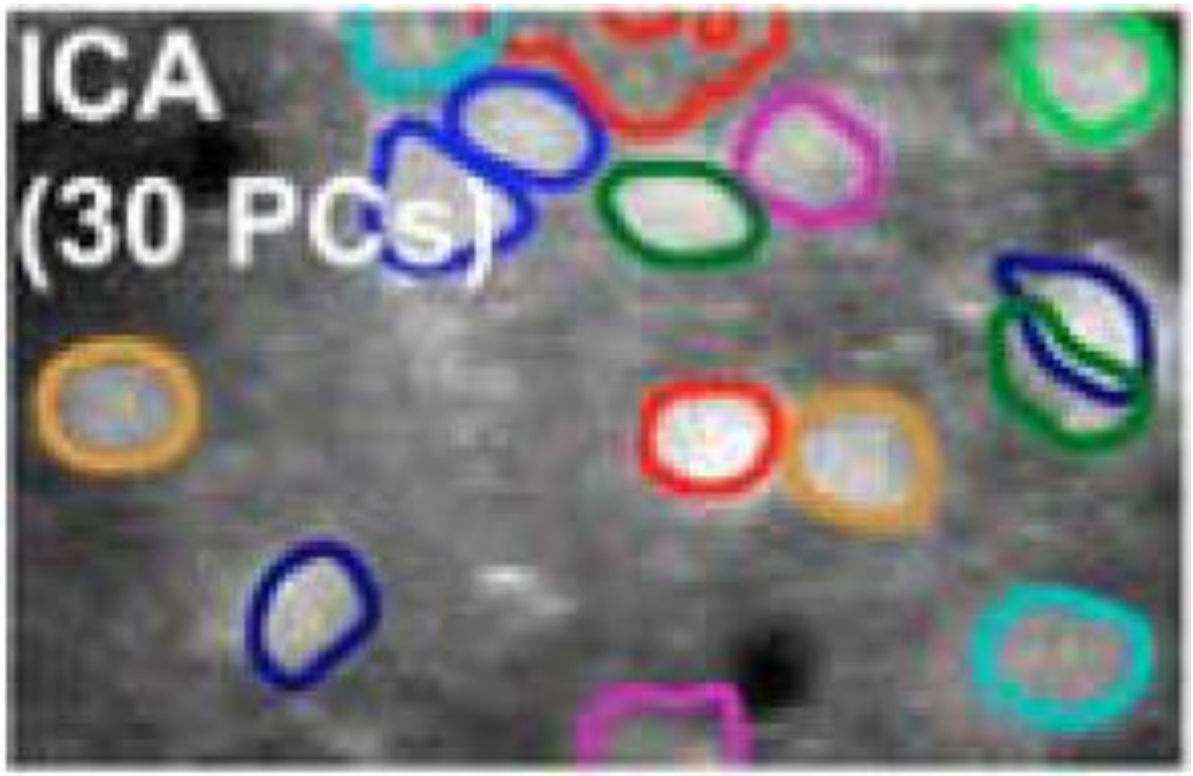
Eigenvalues and eigenvectors

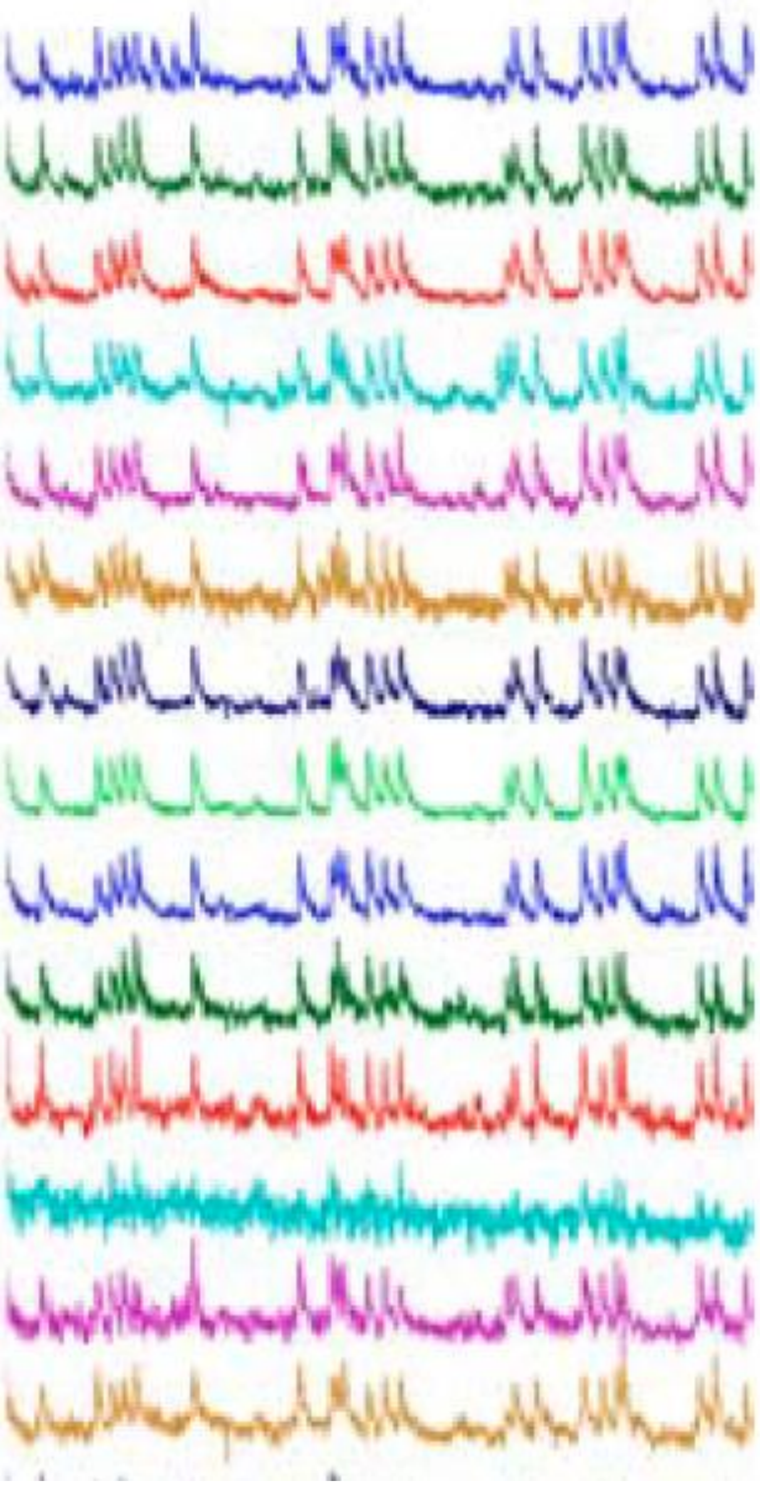
$$\mathbf{M}\,\mathbf{u} = \lambda\,\mathbf{u}$$

Singular value decomposition



Identifying dynamical processes





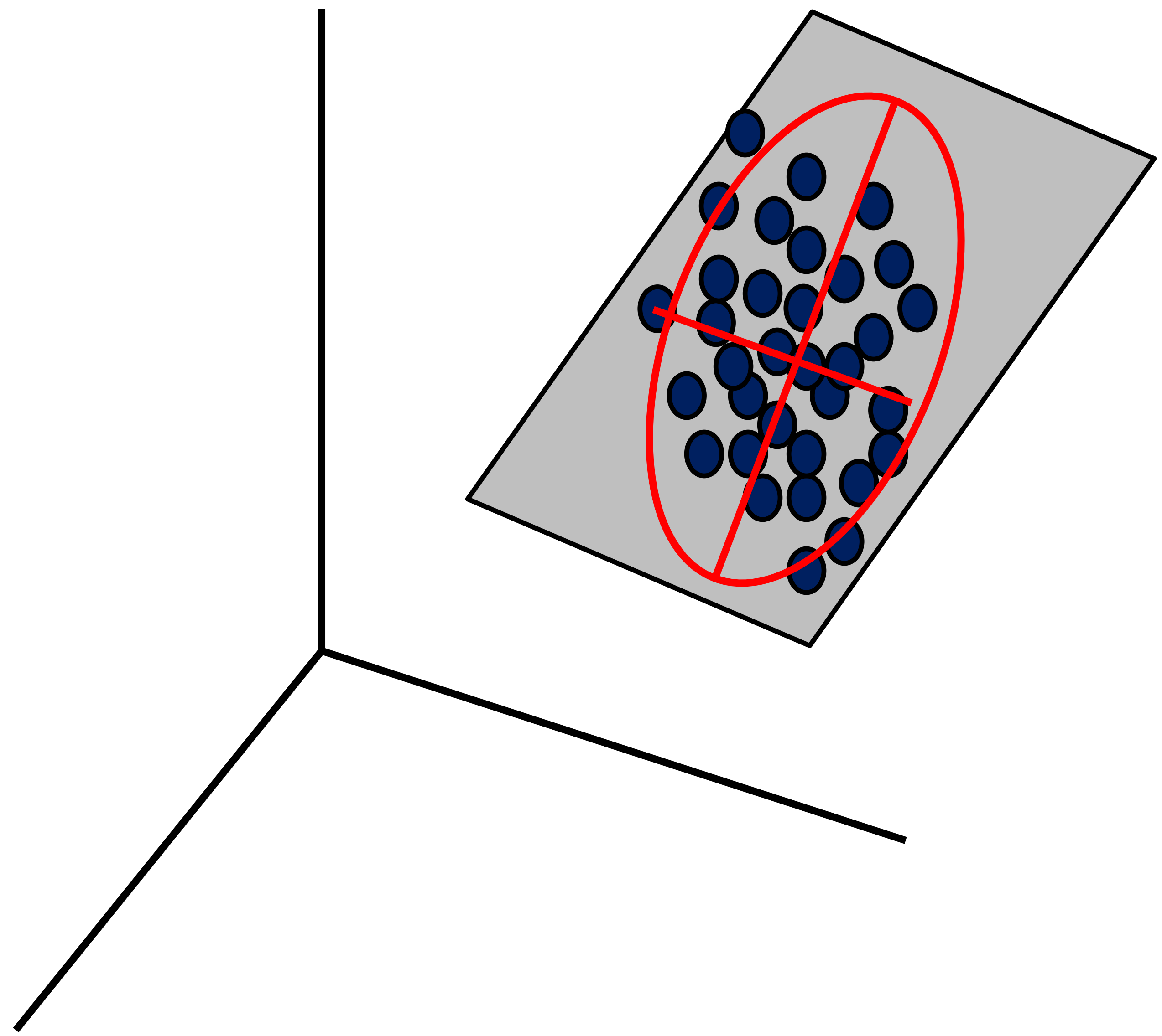
Kwan, HFSP 2010

Principal component analysis

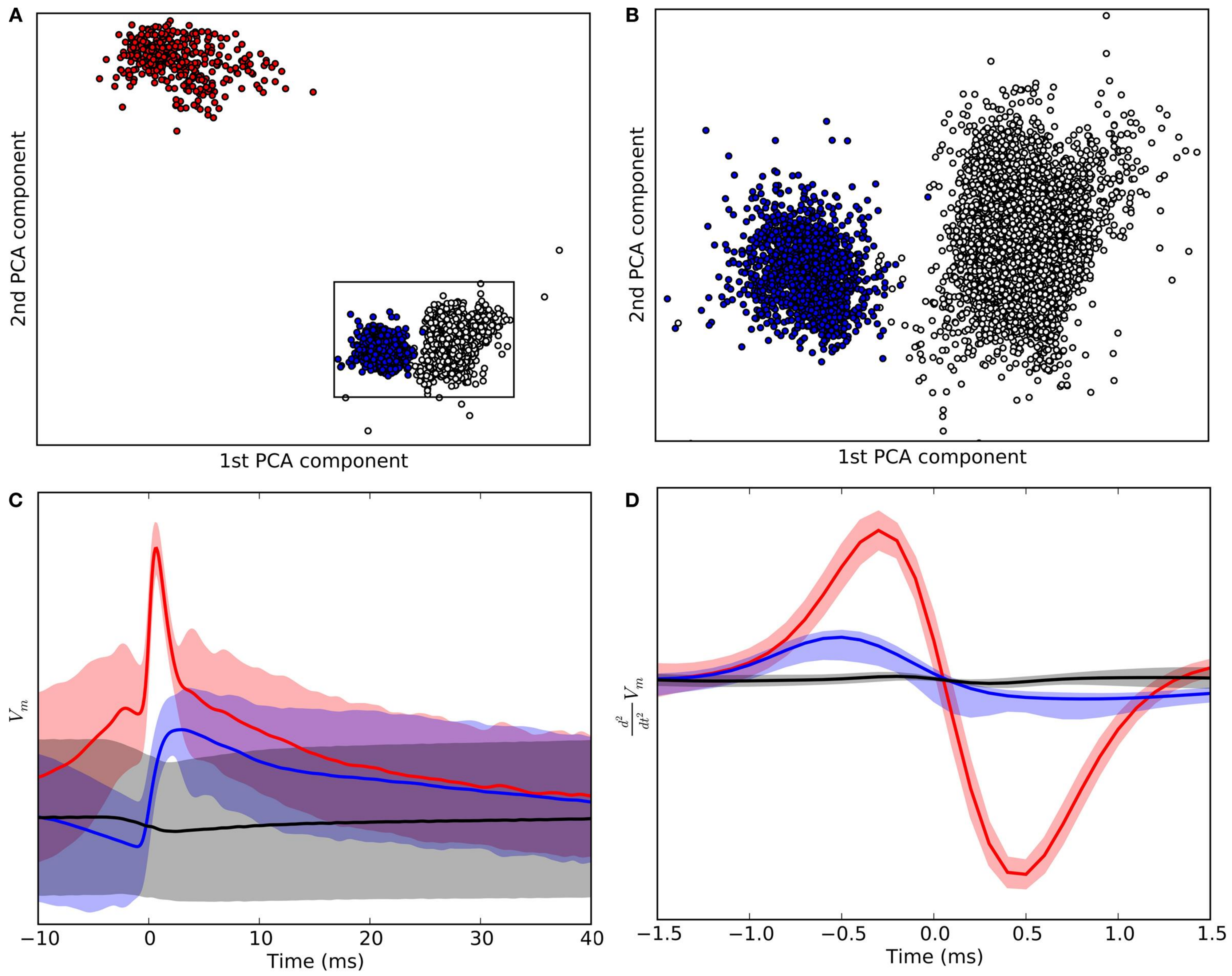
$$M M^* = (U \Sigma V)(V^* \Sigma^* U^*)$$
$$= U \Sigma (V V^*) \Sigma^* U^*$$
$$= U \Sigma \Sigma^* U^*$$

$$C = U \Lambda U^*$$

Principal component analysis

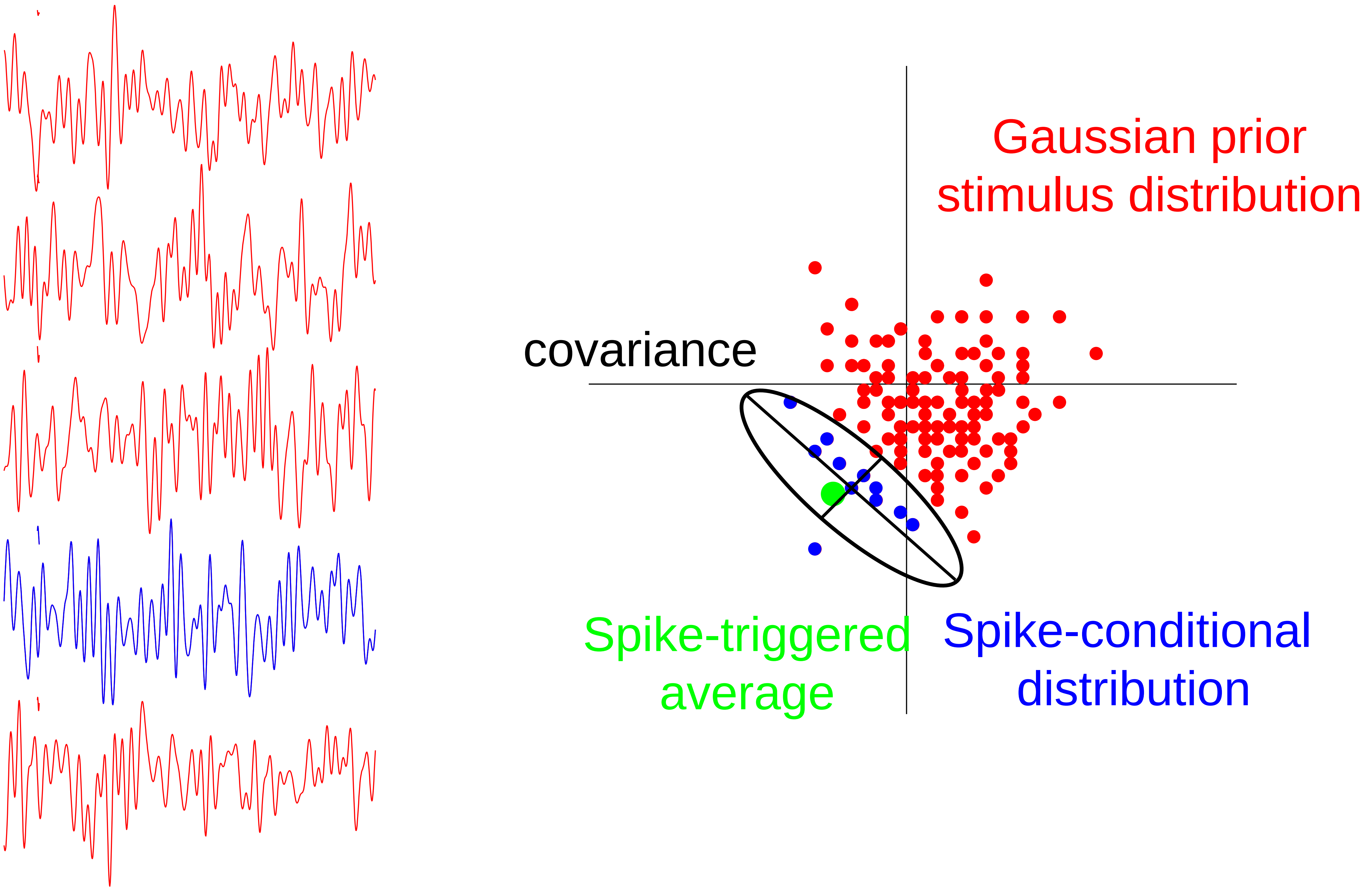


Principal component analysis



Koepsell et al., Front. Syst. Neurosci., 10 April 2009

Determining linear features from white noise



Identifying multiple features

The covariance matrix is

$$C_{ij} = \langle S(t_{\text{spike}} - t_i) S(t_{\text{spike}} - t_j) \rangle - \bar{S}(t_i)\bar{S}(t_j) - \langle I(t - t_i) I(t - t_j) \rangle$$

↑ Spike-triggered stimulus correlation

↑ Spike-triggered average

↑ Stimulus prior

Properties:

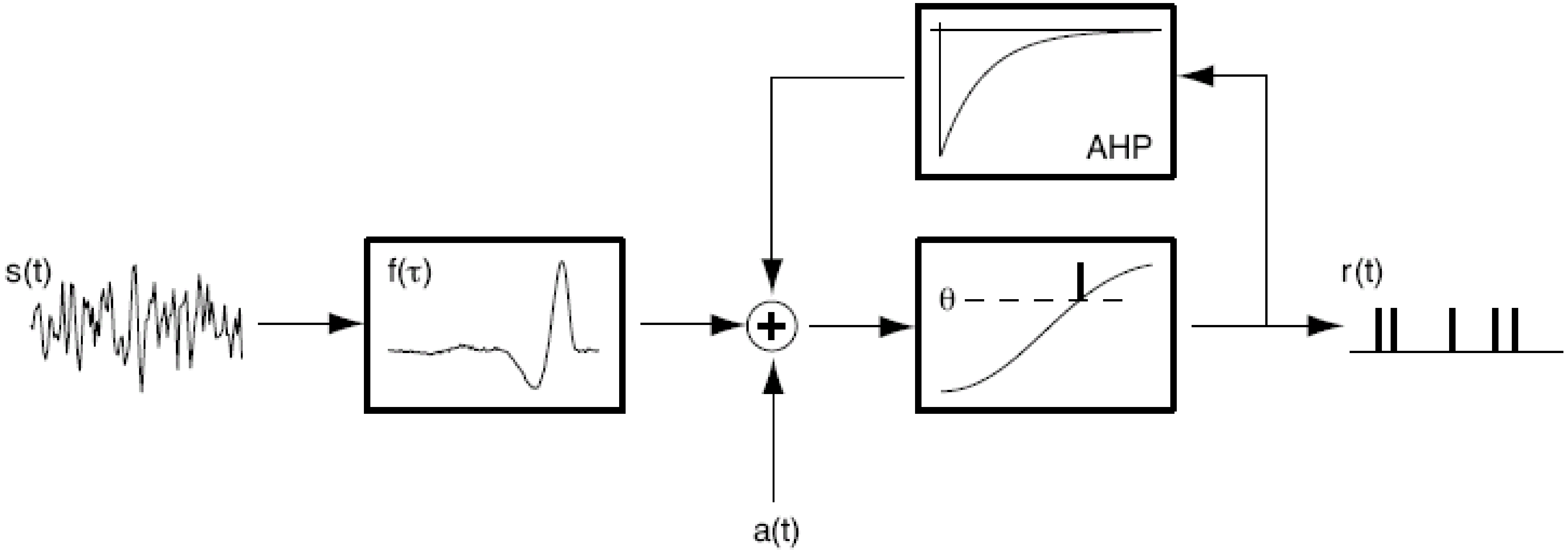
- The number of eigenvalues significantly **different from zero** is the number of relevant stimulus features

- The corresponding eigenvectors are the relevant features (or span the *relevant subspace*)

Bialek et al., 1988; Brenner et al., 2000; Bialek and de Ruyter, 2005

A toy example: a filter-and-fire model

Let's develop some intuition for how this works: a filter-and-fire threshold-crossing model with AHP

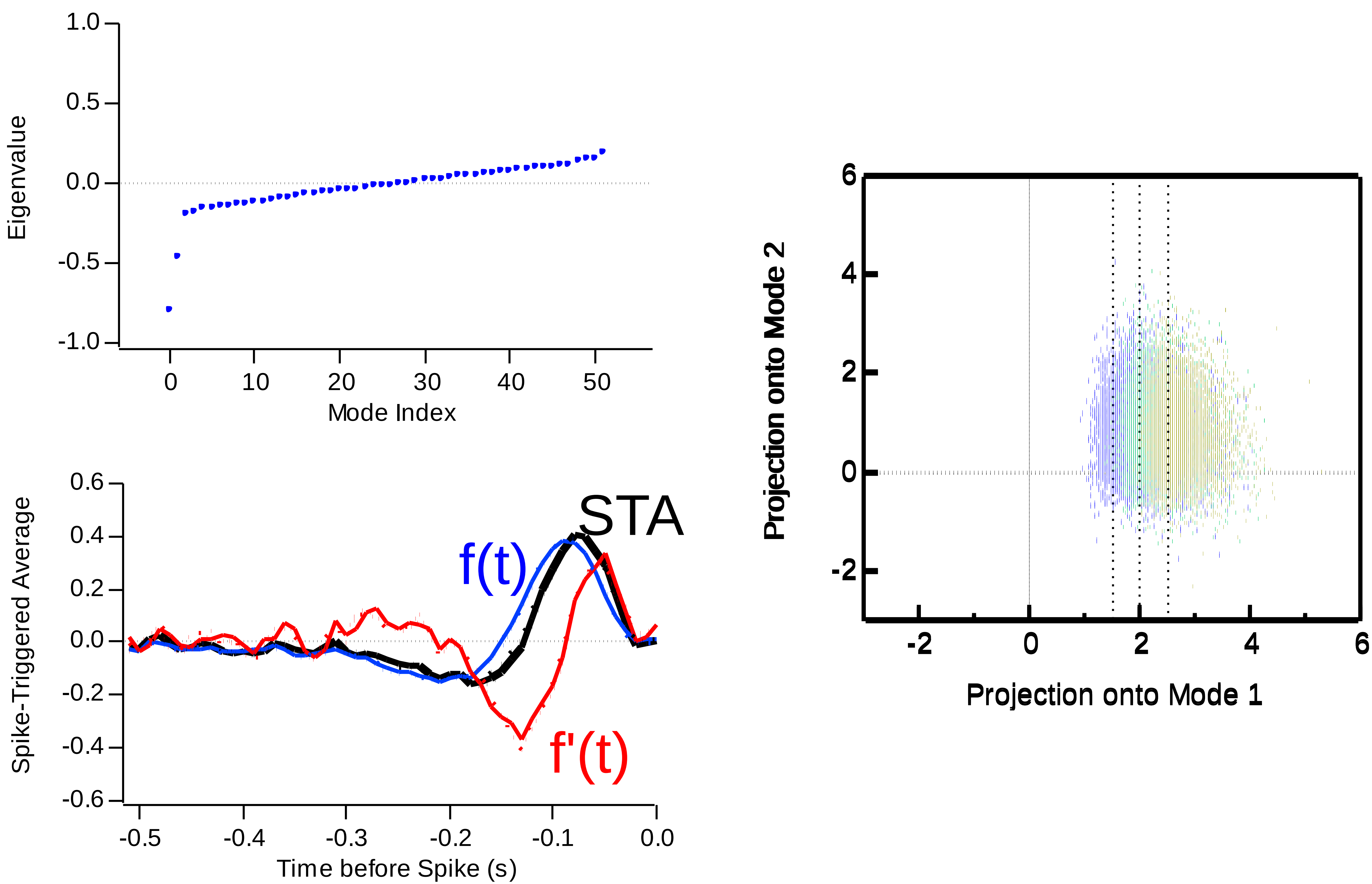


Keat, Reinagel, Reid and Meister, Predicting every spike. Neuron (2001)

- Spiking is controlled by a single filter, *f*(t)
- Spikes happen generally on an upward threshold crossing of the filtered stimulus

➔ expect 2 **relevant features**, the filter *f*(t) and its time derivative *f'*(t

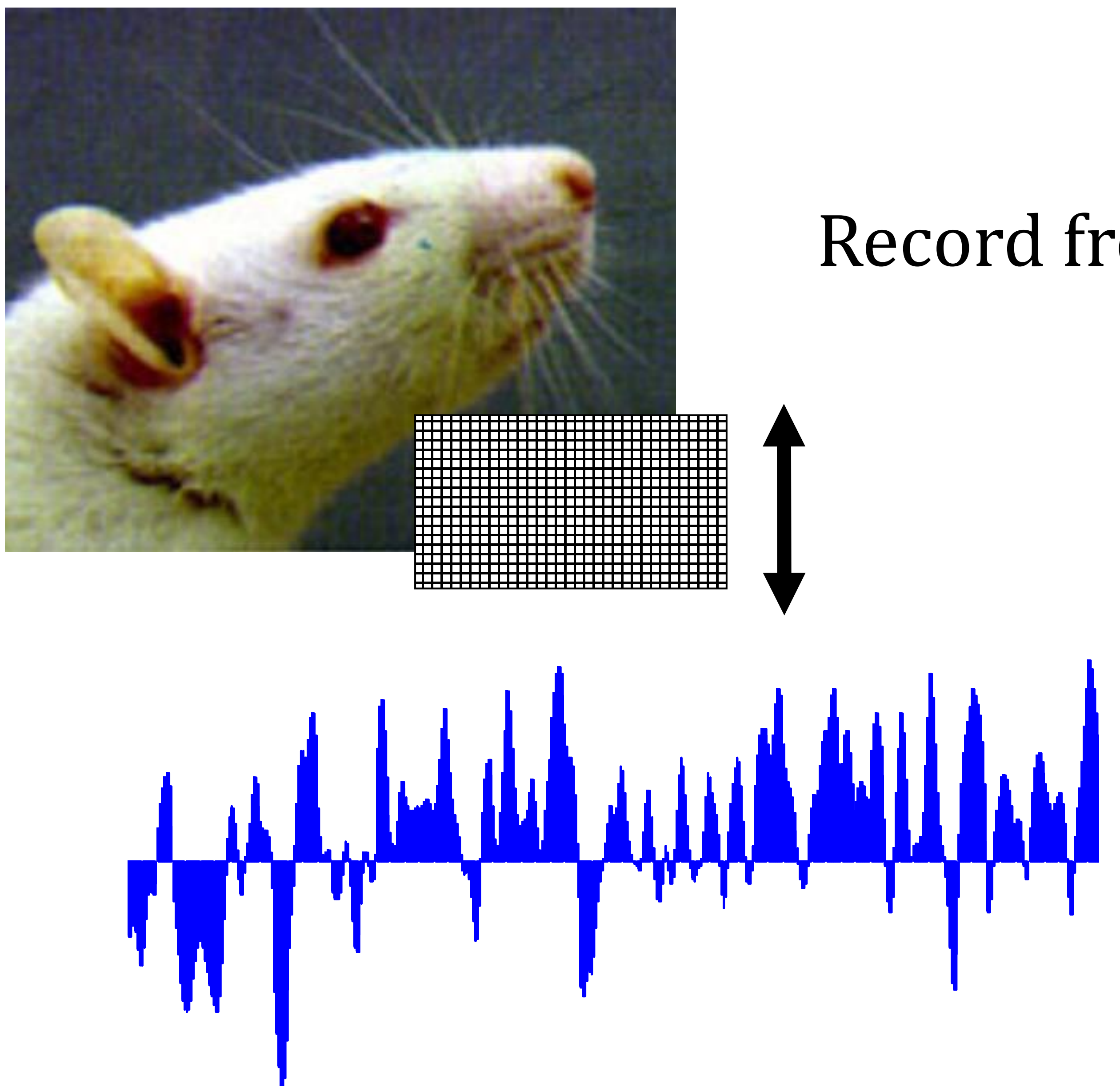
Covariance analysis of a filter-and-fire model



Let’s try it

Example: rat somatosensory (barrel) cortex

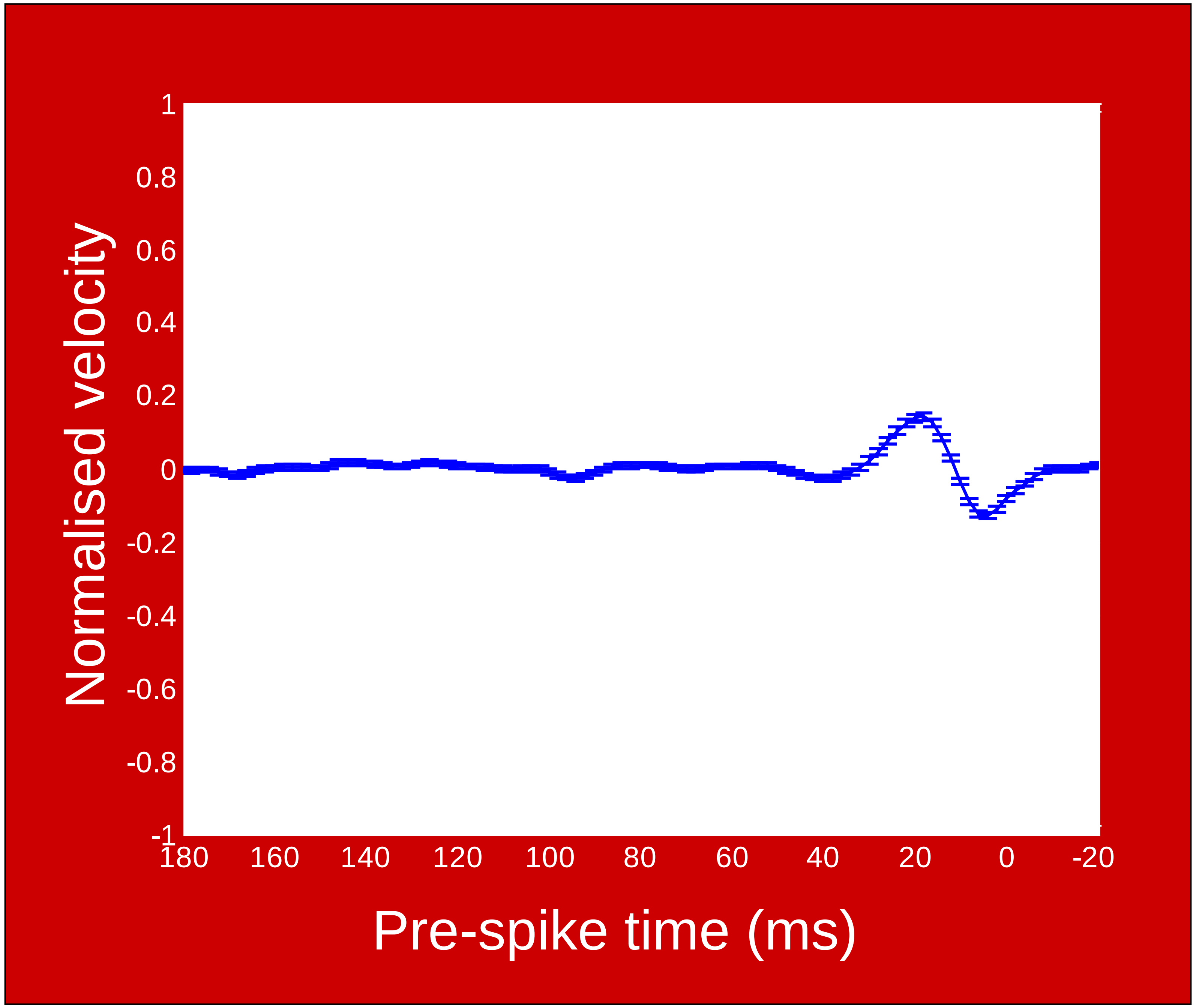
Rasmus Petersen and Mathew Diamond, SISSA



Record from single units in barrel cortex

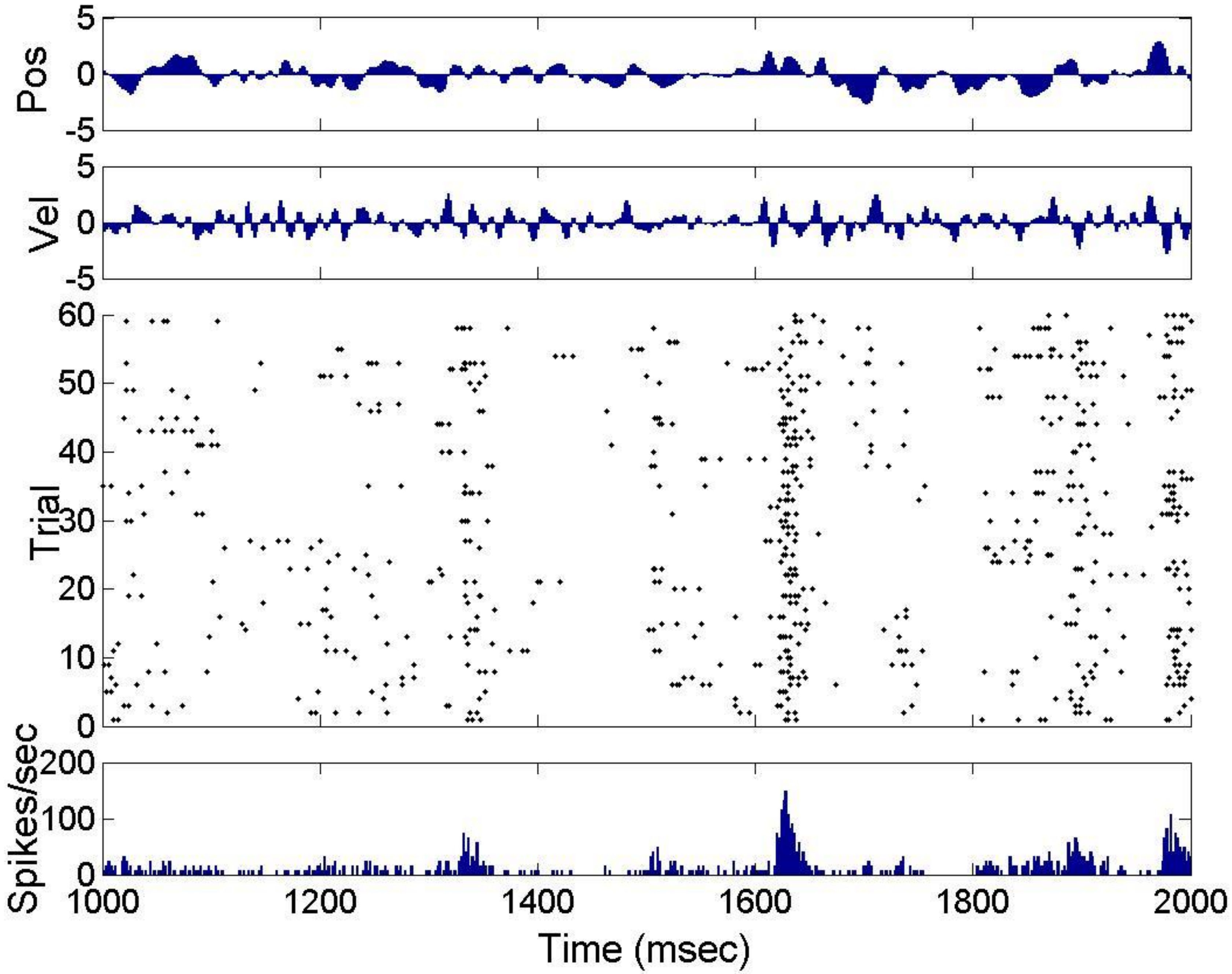
White noise analysis in barrel cortex

Spike-triggered average:



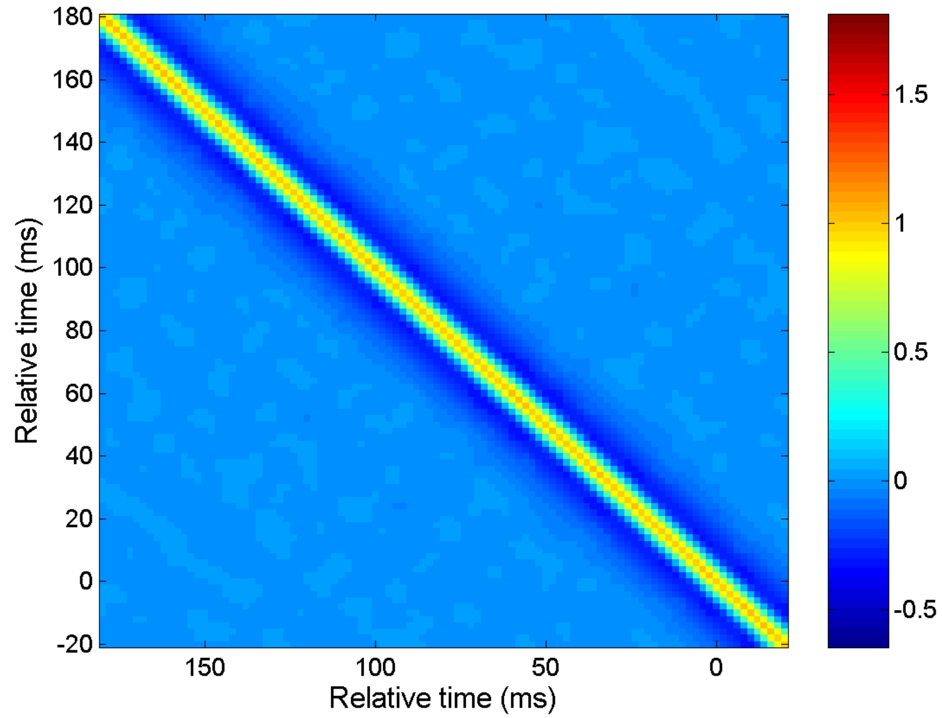
White noise analysis in barrel cortex

Is the neuron simply not very responsive to a white noise stimulus?

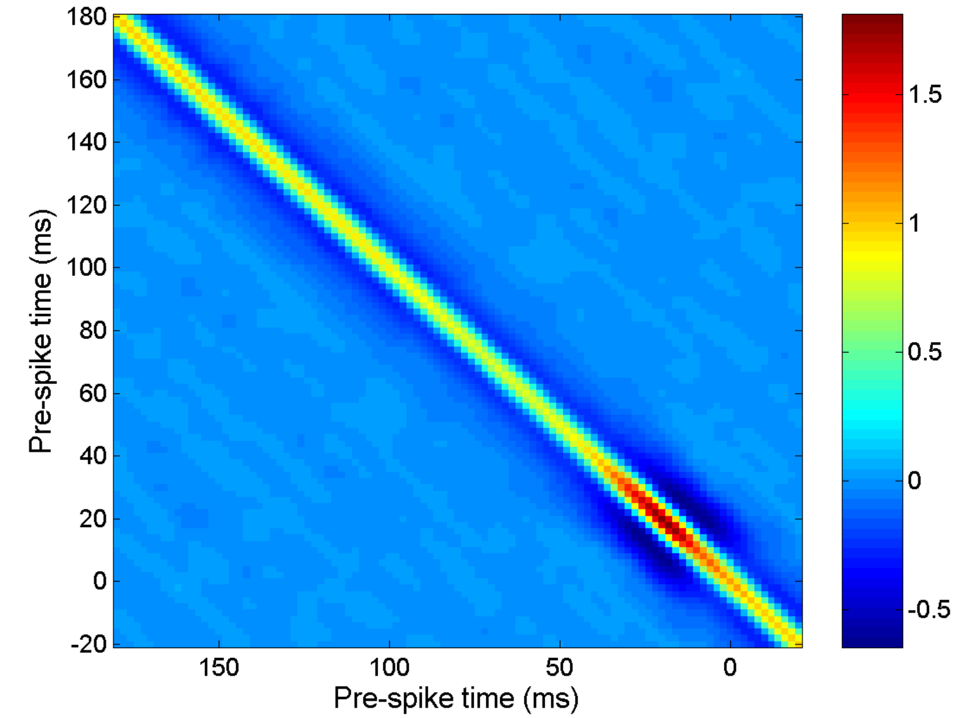


Covariance matrices from barrel cortical neurons

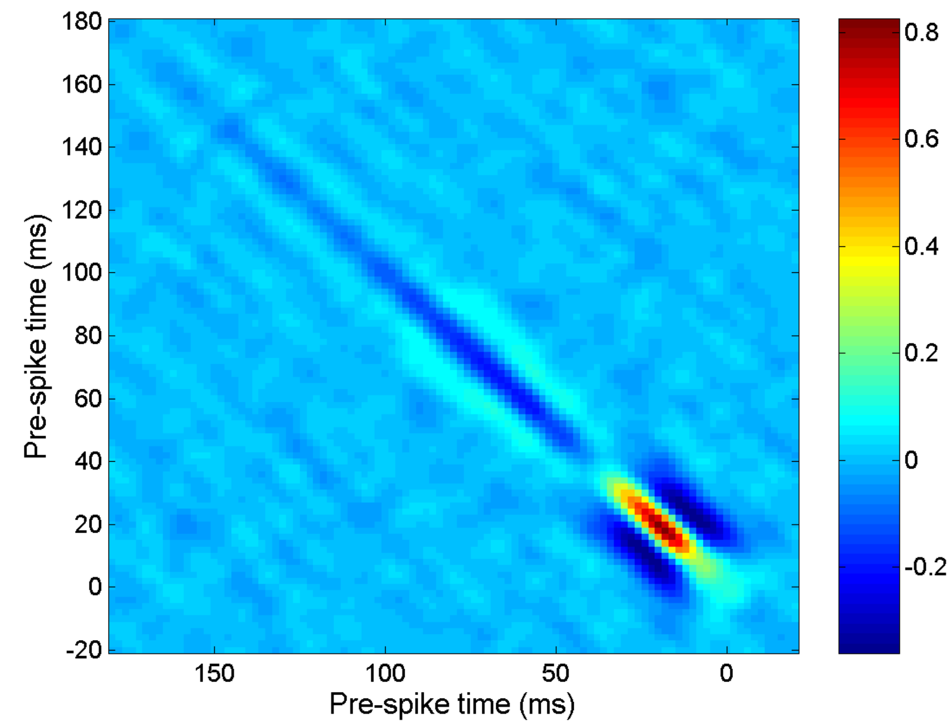
Prior



Spike-triggered



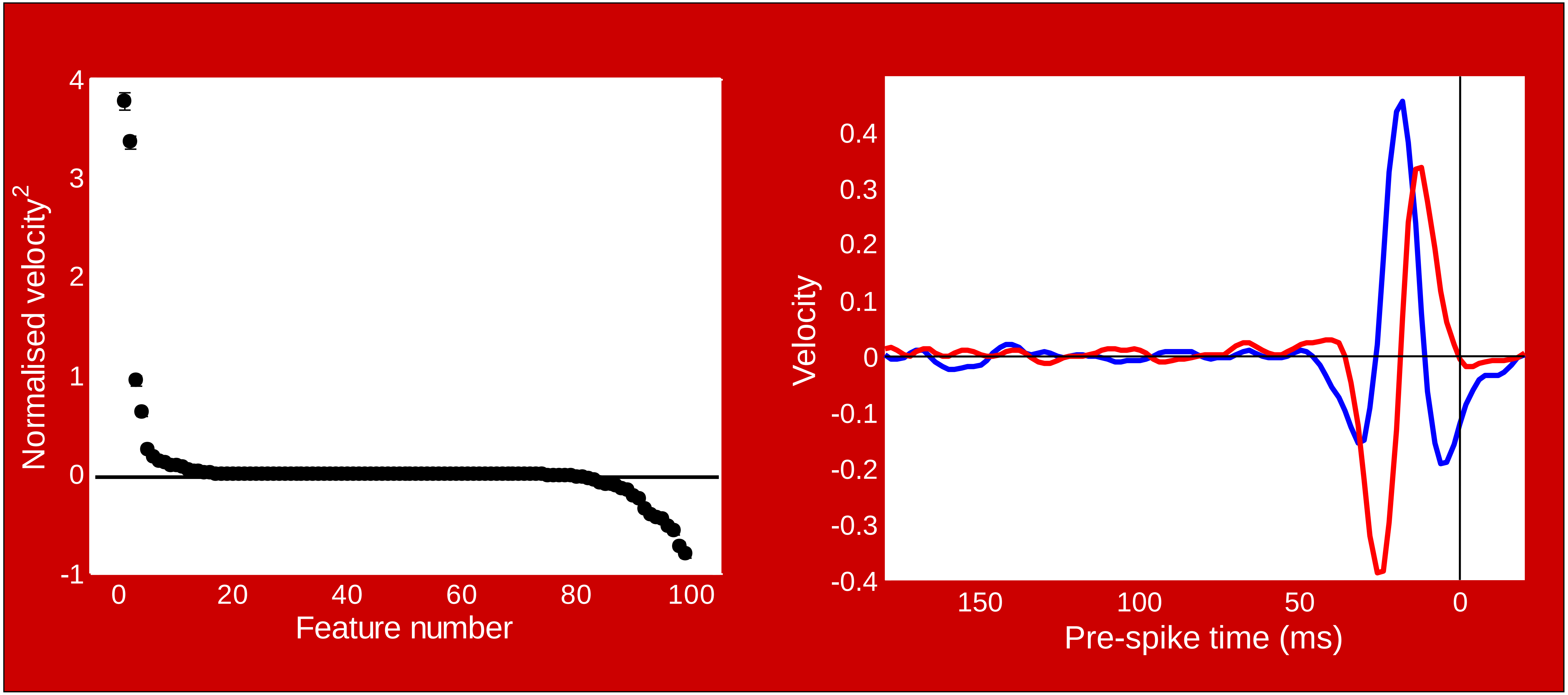
Difference



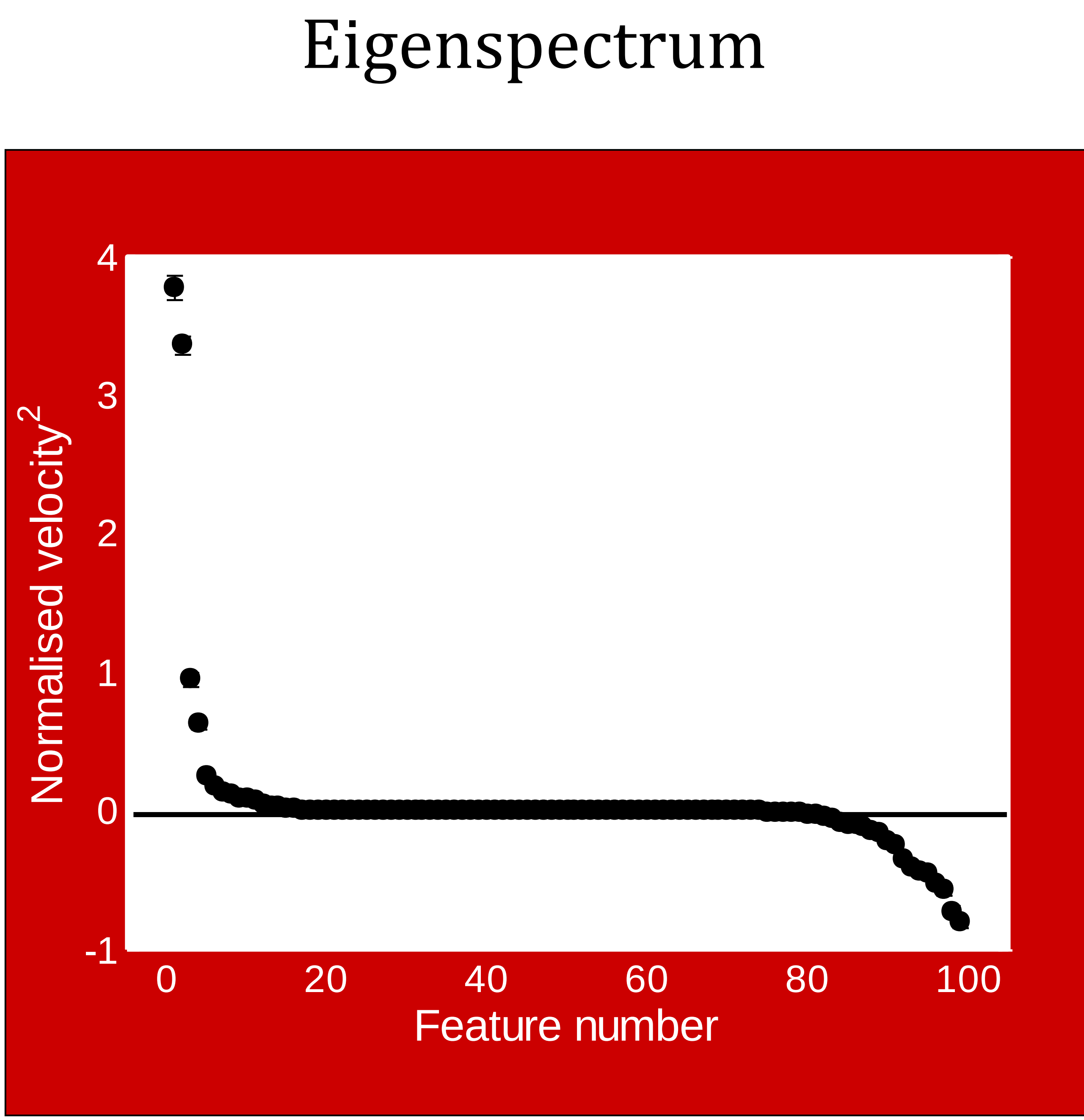
Eigenspectrum from barrel cortical neurons

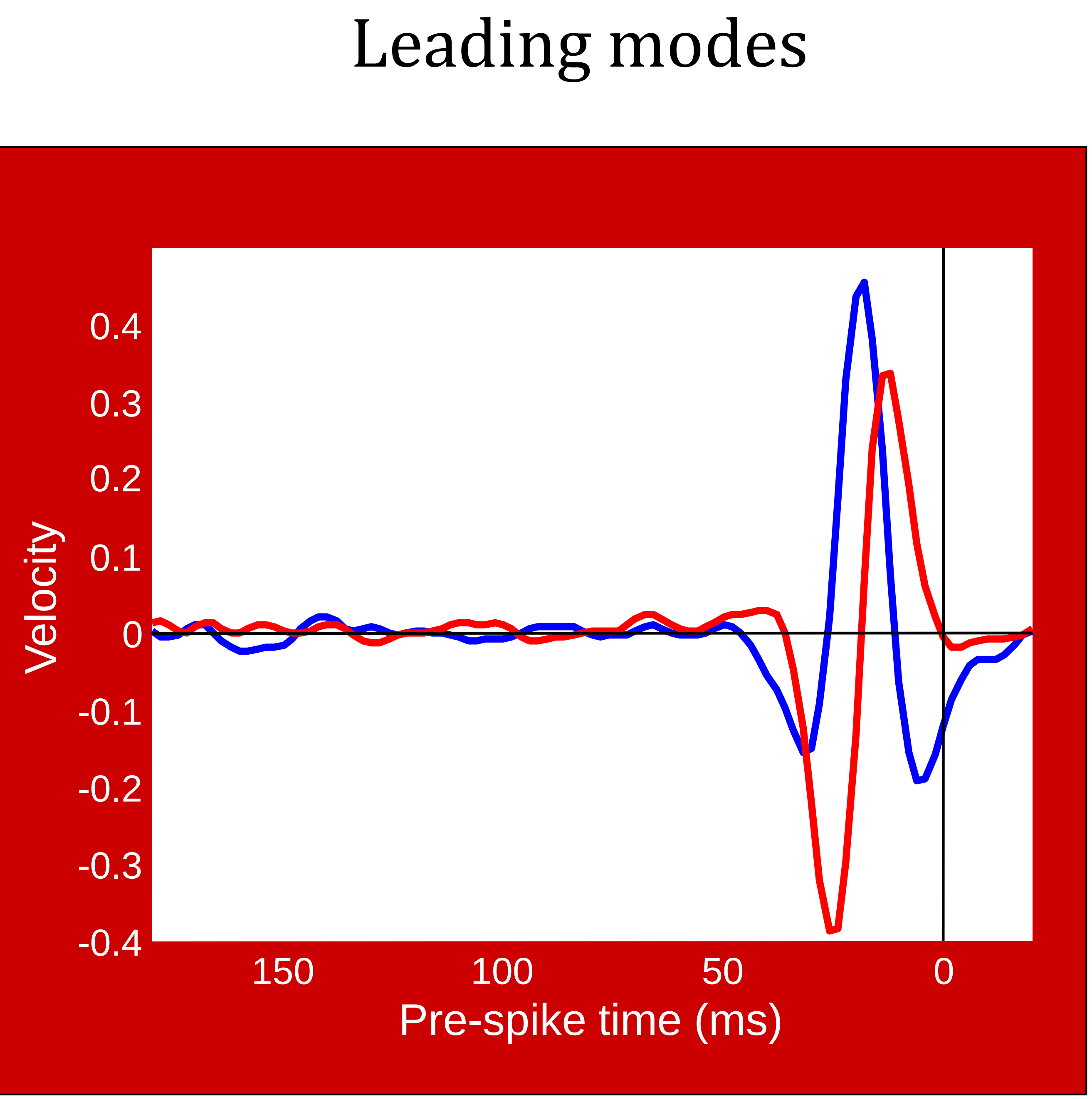
Eigenspectrum

Leading modes



Eigenspectrum from barrel cortical neurons

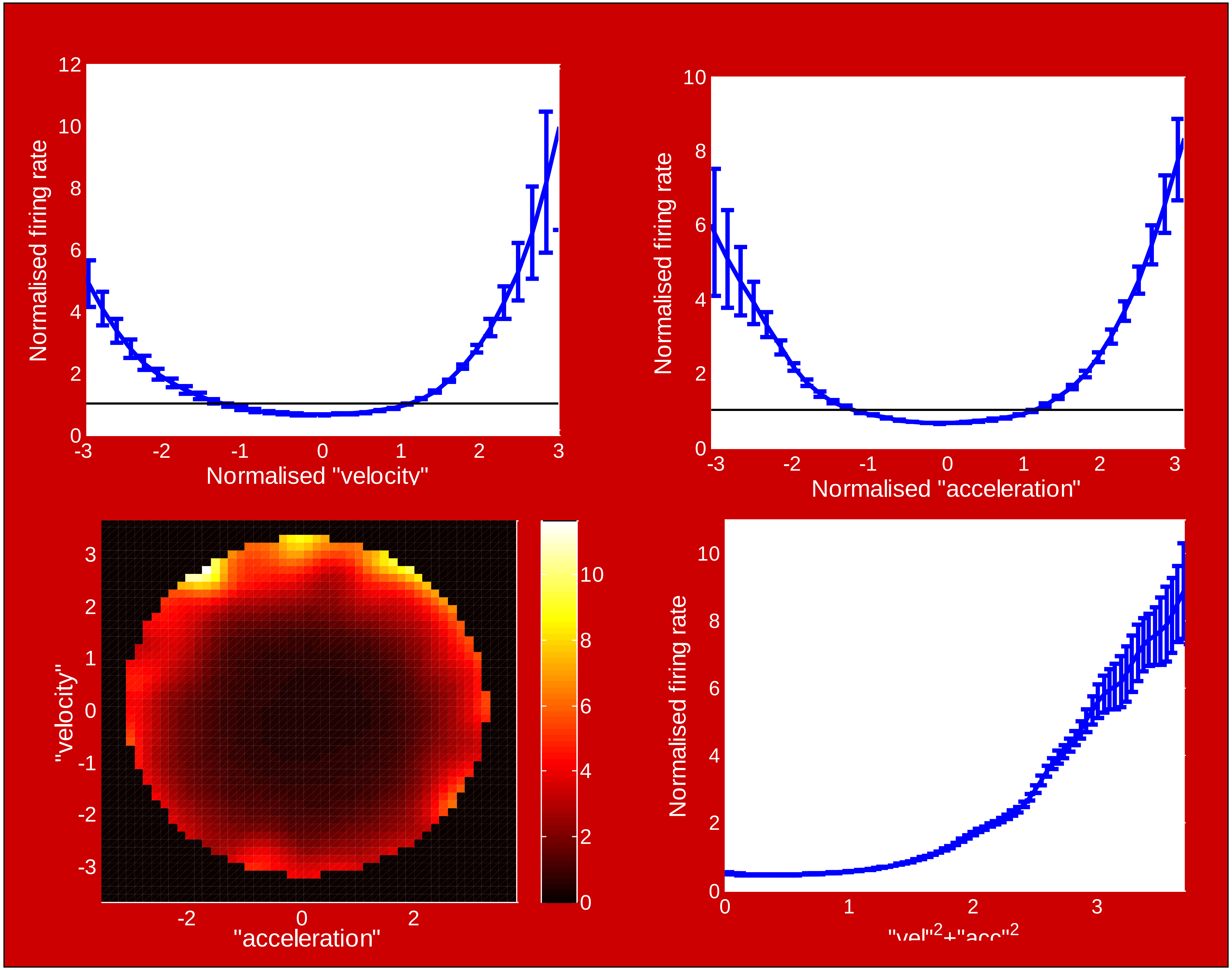




Input/output relations from barrel cortical neurons

Input/output relations wrt first two filters, alone:

and in quadrature:

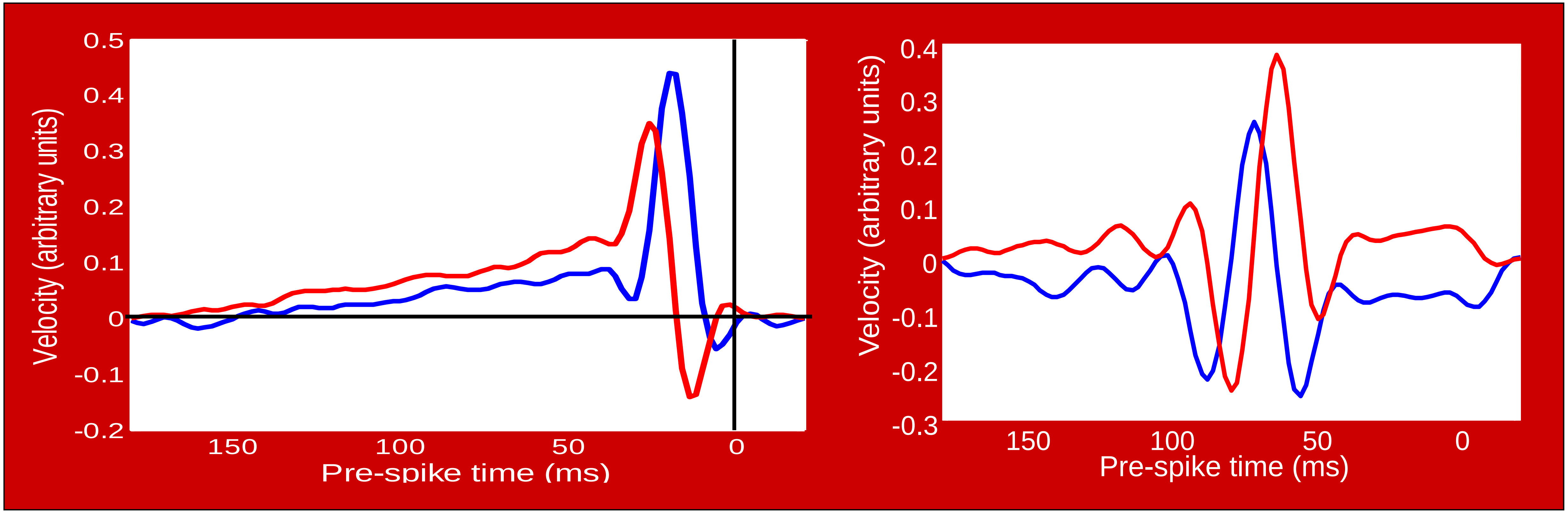


Less significant eigenmodes from barrel cortical neurons

How about the other modes?

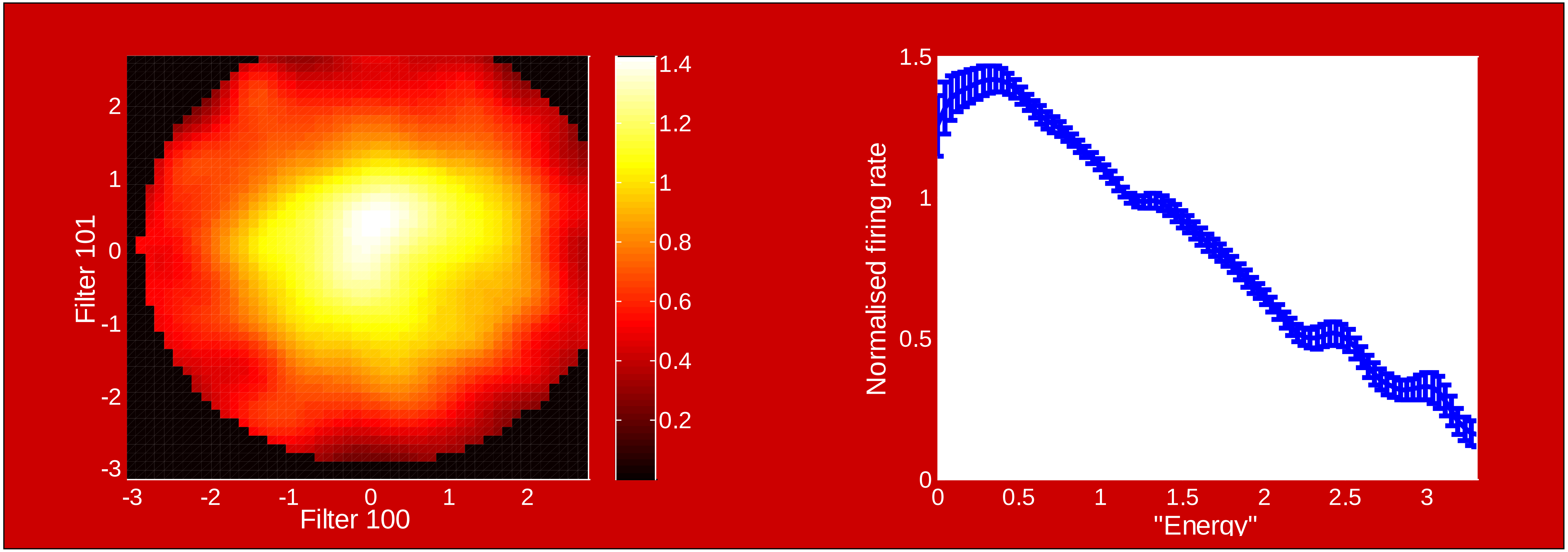
Next pair with +ve eigenvalues

Pair with -ve eigenvalues



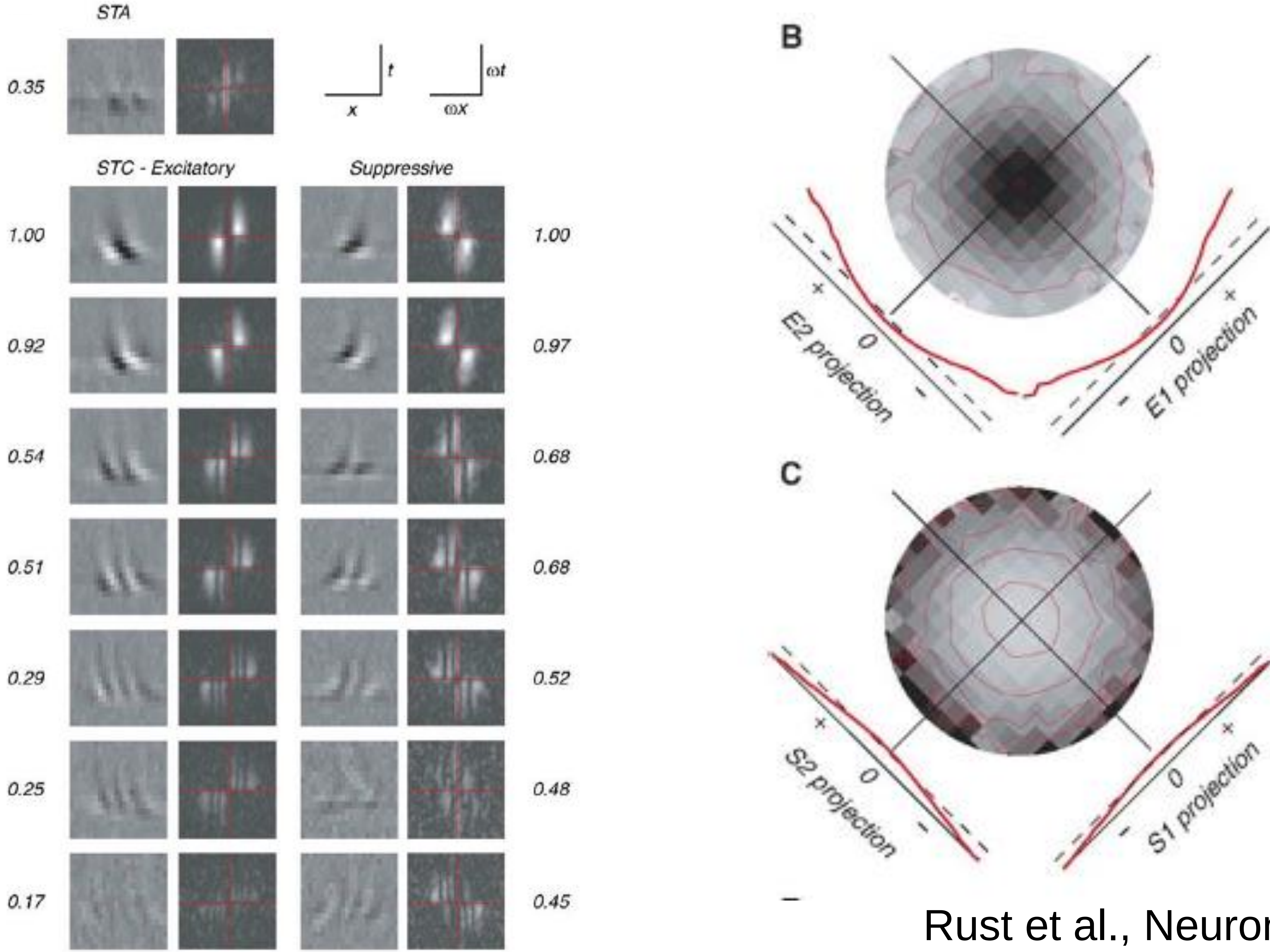
Negative eigenmode pair

Input/output relations for negative pair



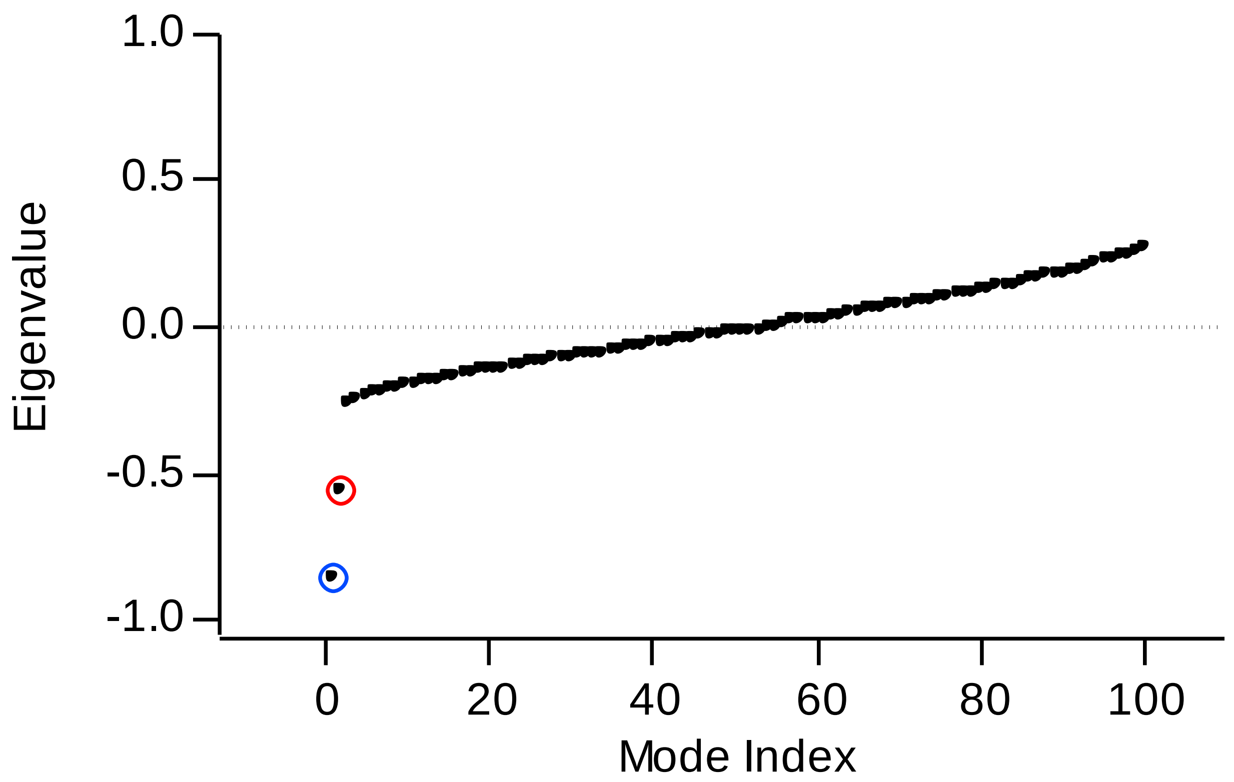
Firing rate *decreases* with increasing projection:
suppressive modes

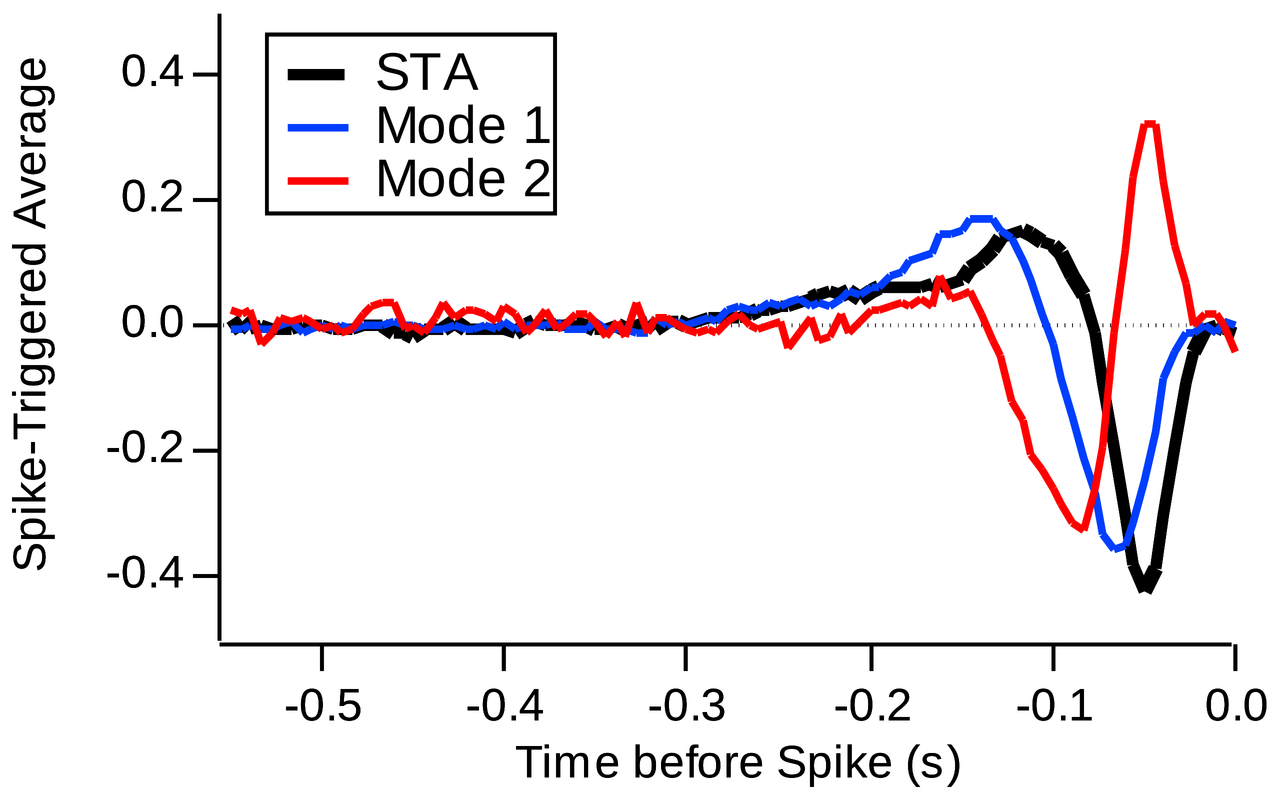
Complex cells in V1

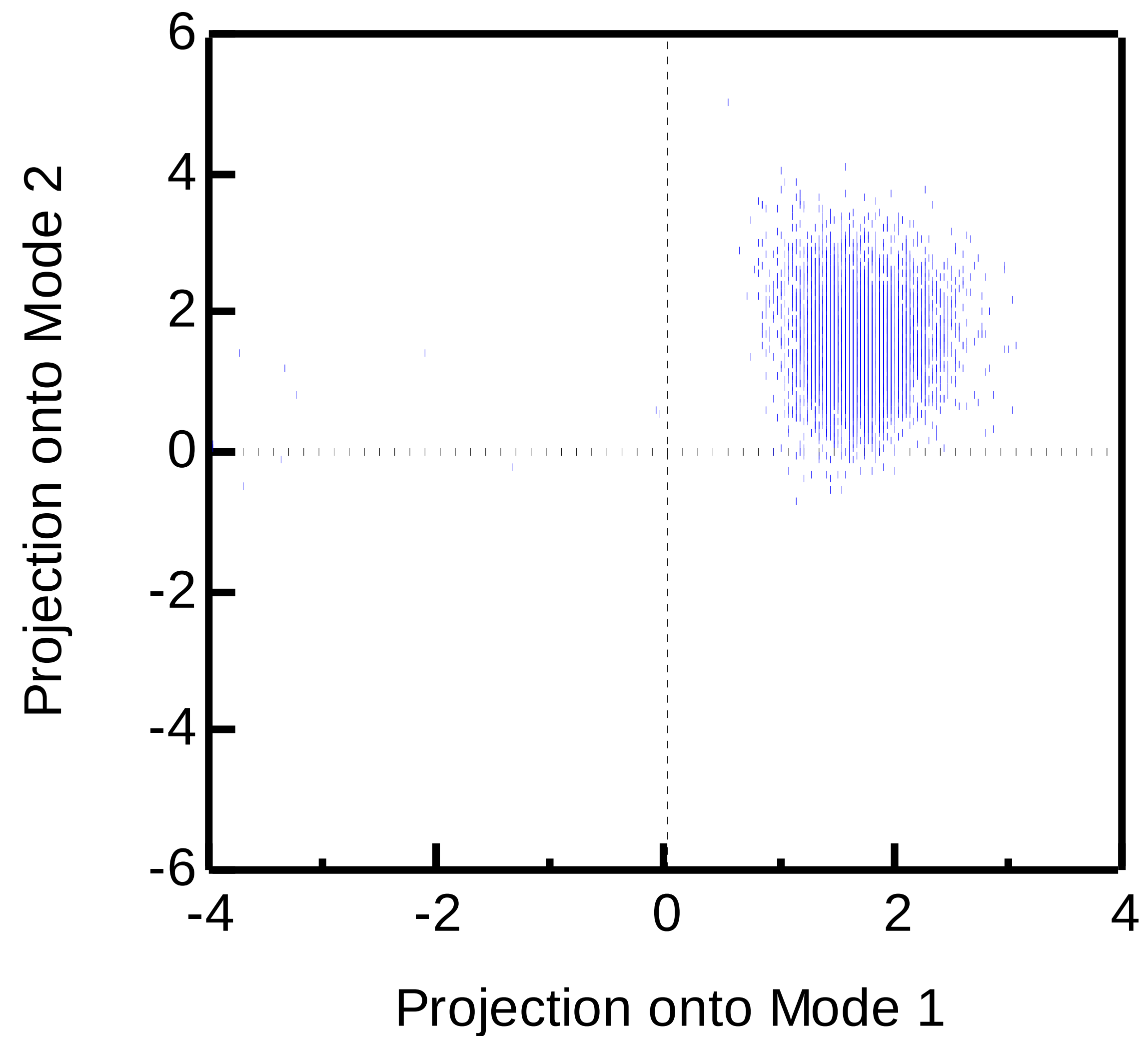


Rust et al., Neuron 2005

Salamander retinal ganglion cells perform a variety of computations

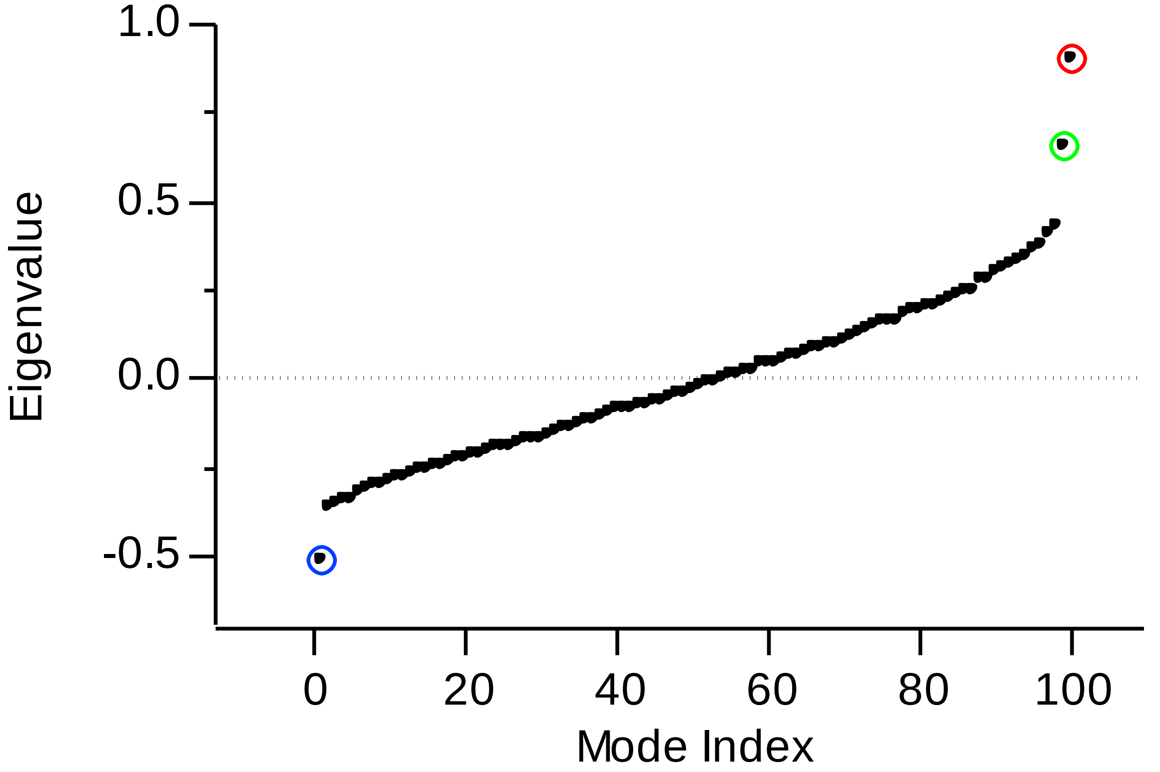


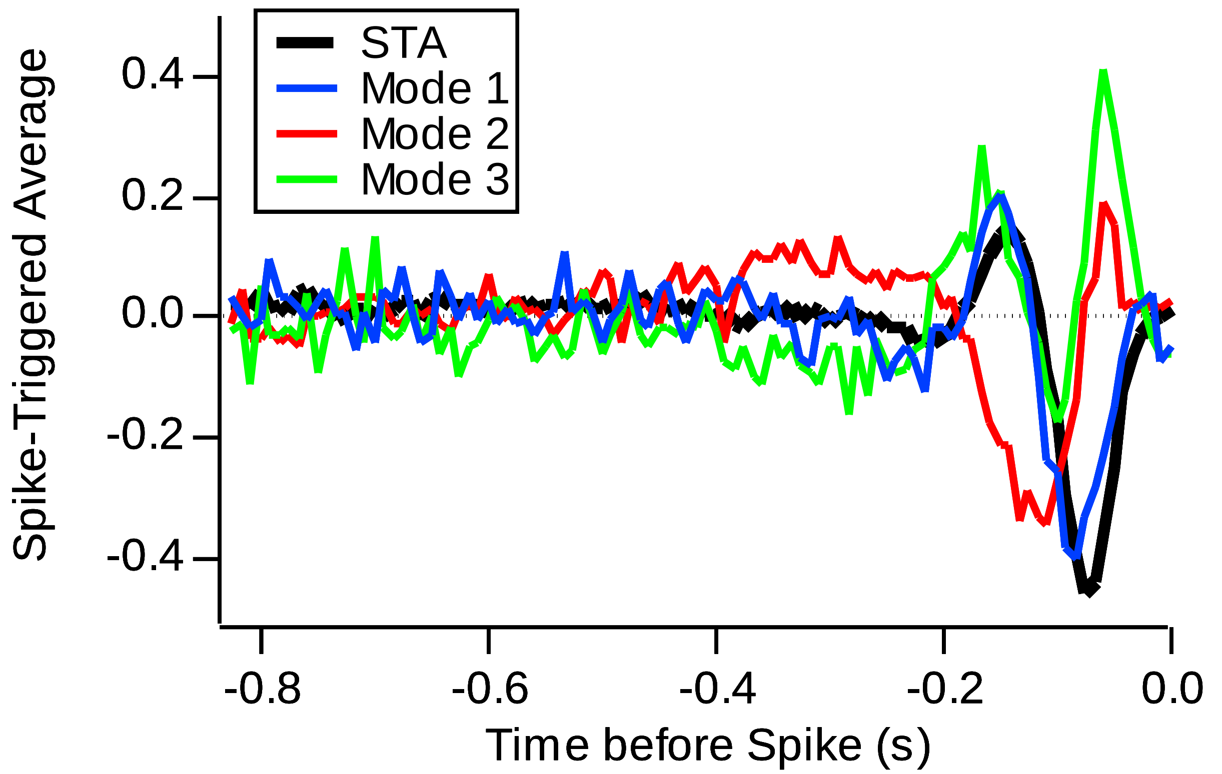


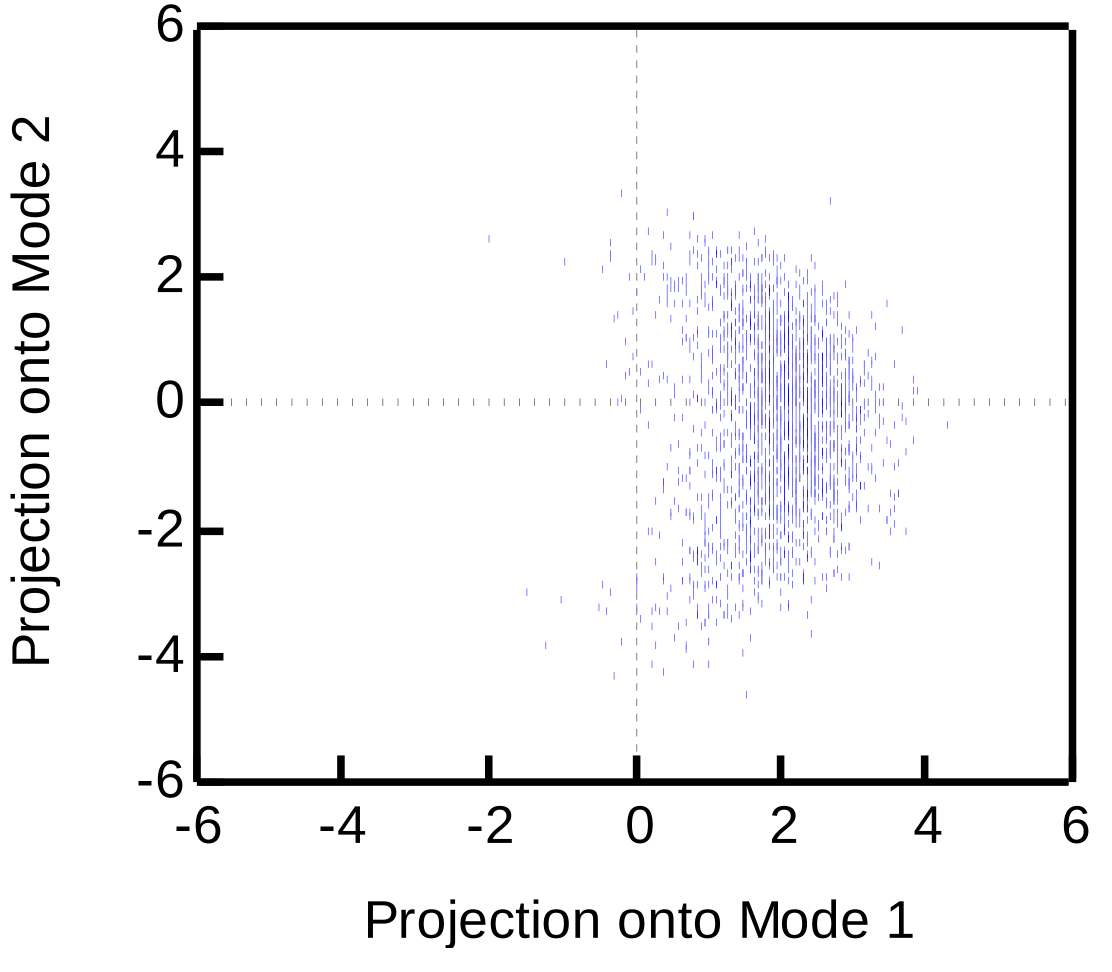


Michael Berry

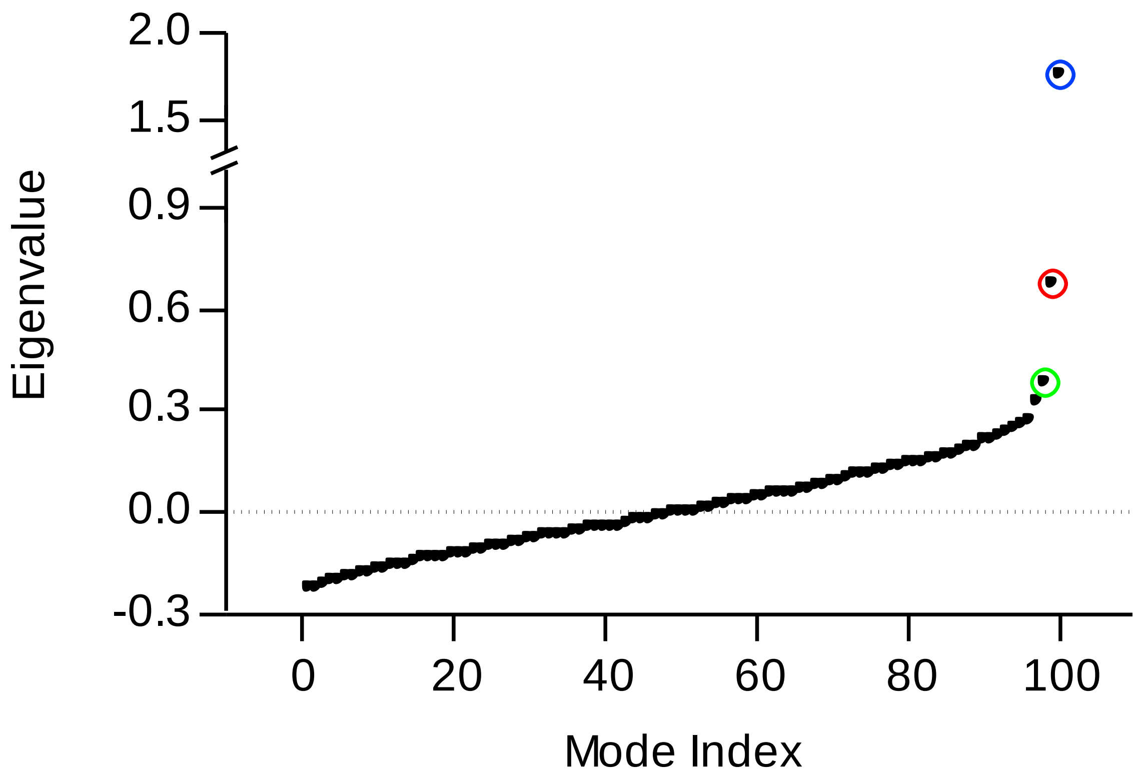
Not a threshold-crossing neuron

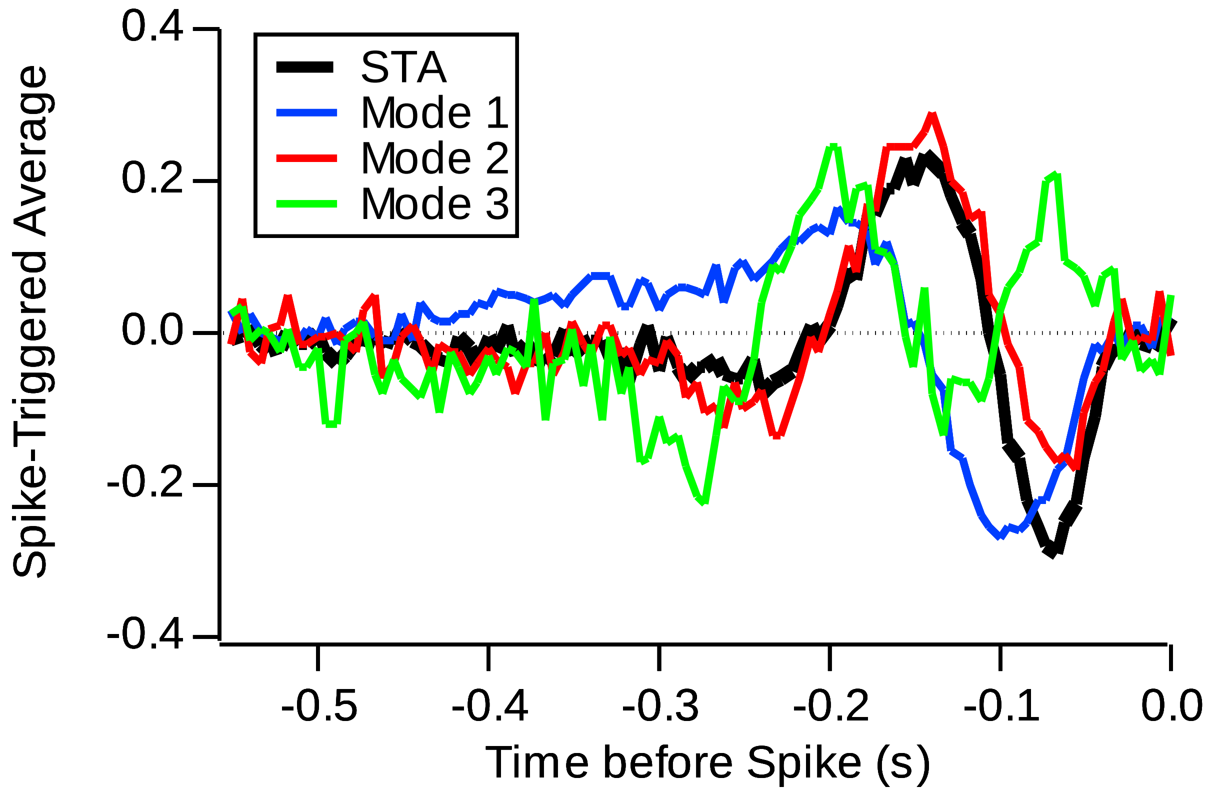


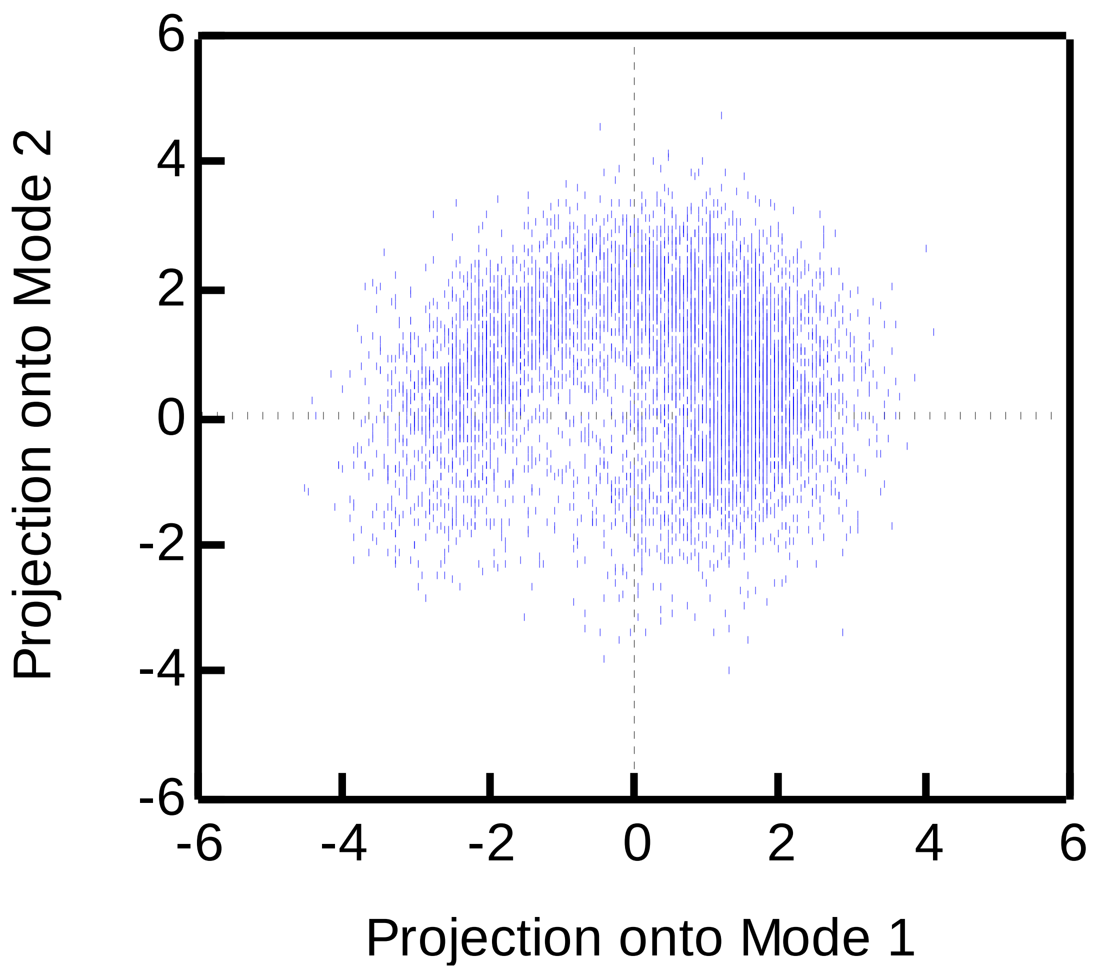




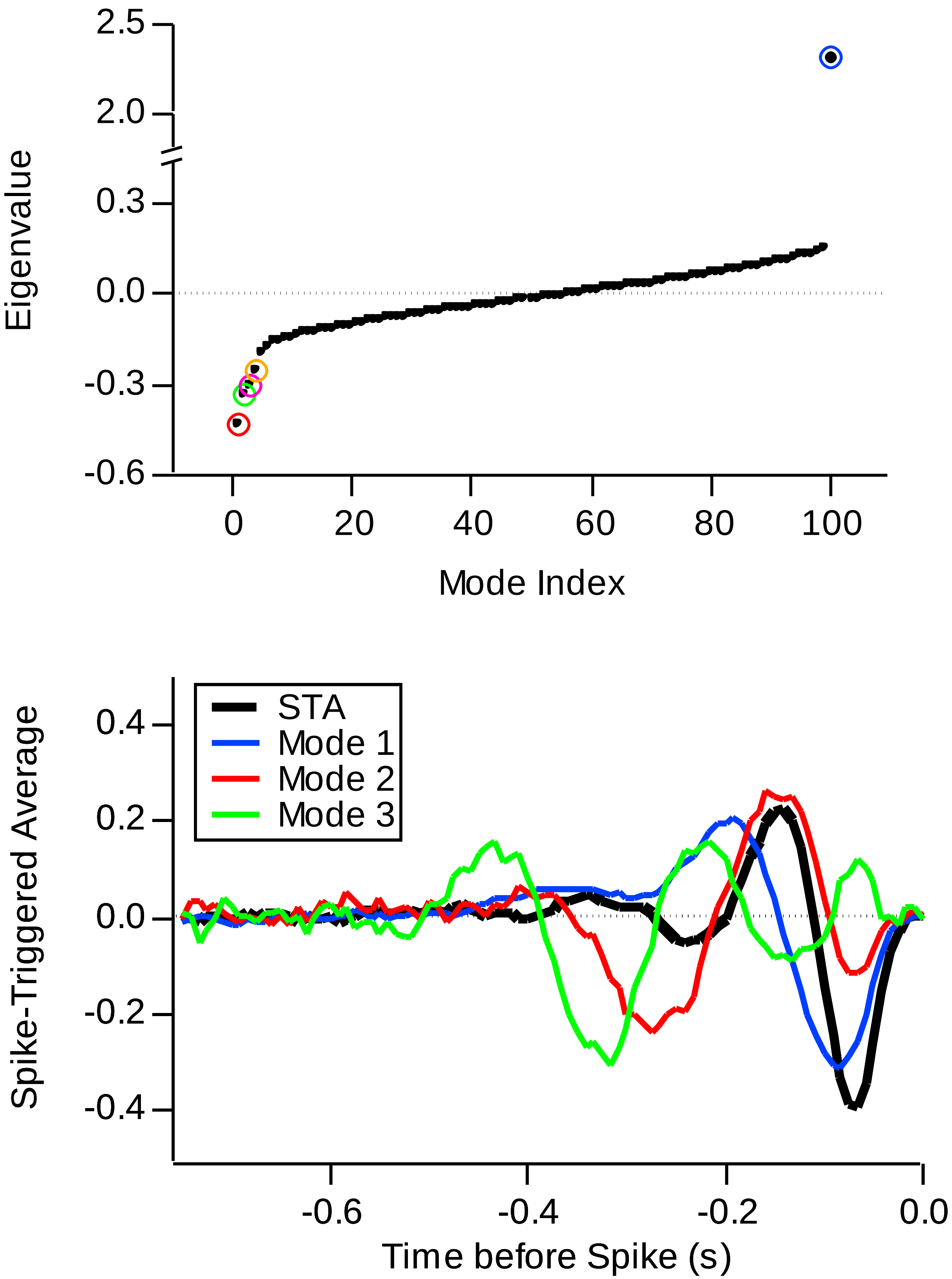
Complex cell like

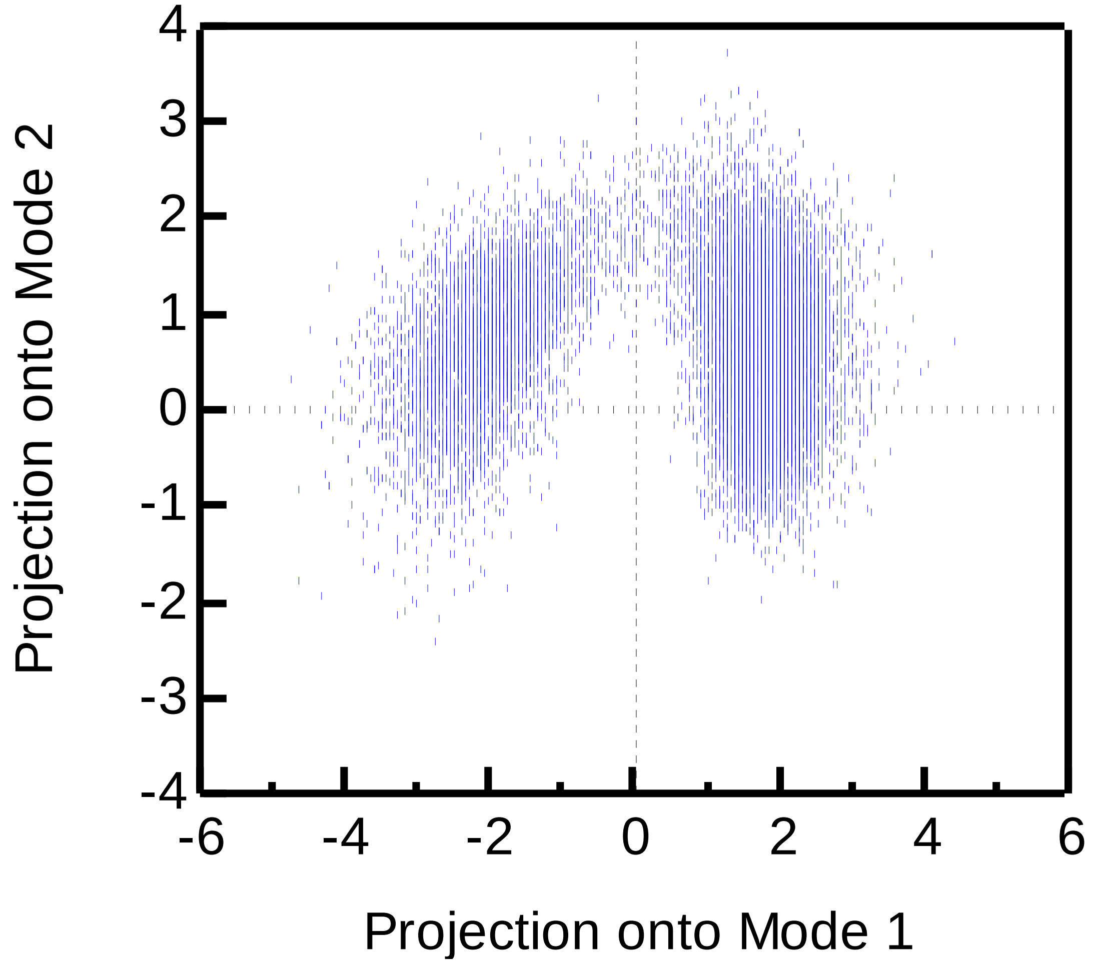






Bimodal: two separate features are encoded





Basic types of computation

- integrators

  H1, some single cortical neurons
- differentiators

  Retina, simple cells, HH neuron, auditory neurons
- frequency-power detectors

  V1 complex cells, somatosensory, auditory, retina

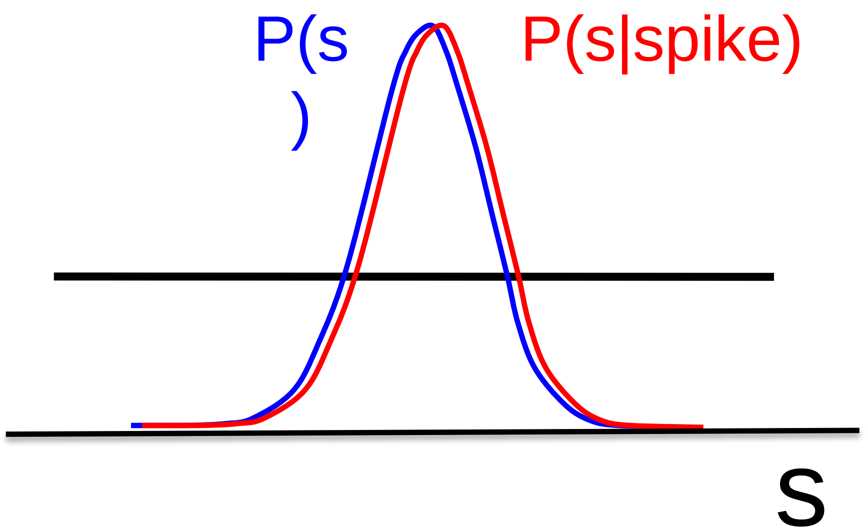
When have you done a good job?

- When the tuning curve over your variable is *interesting*.
- How to quantify interesting?

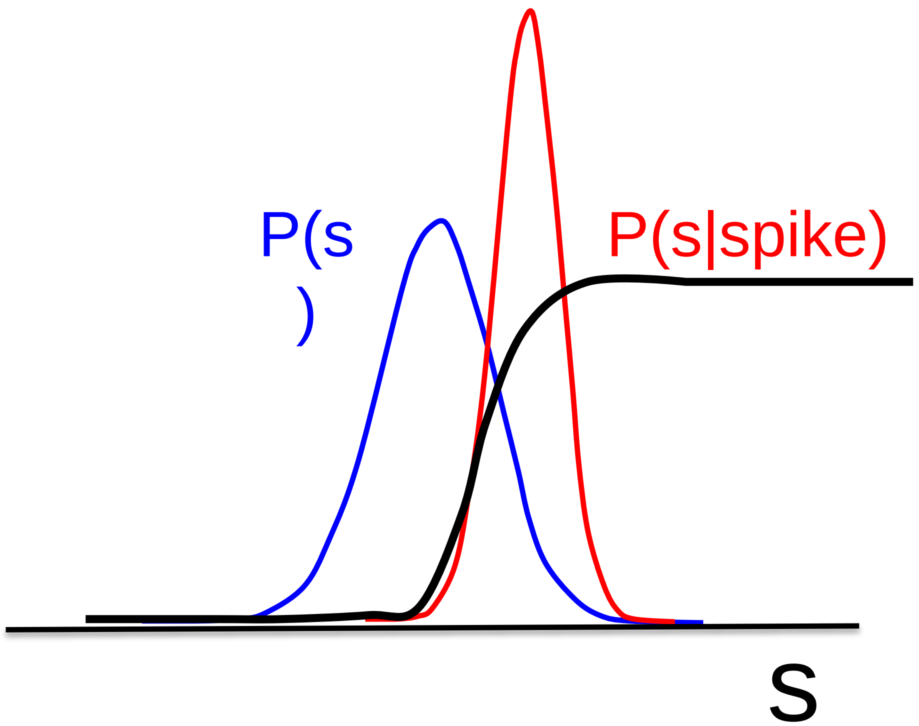
When have you done a good job?

Tuning curve: $P(\text{spike}|s) = P(s|\text{spike})\, P(\text{spike}) / P(s)$

Boring: spikes unrelated to stimulus



Interesting: spikes are selective

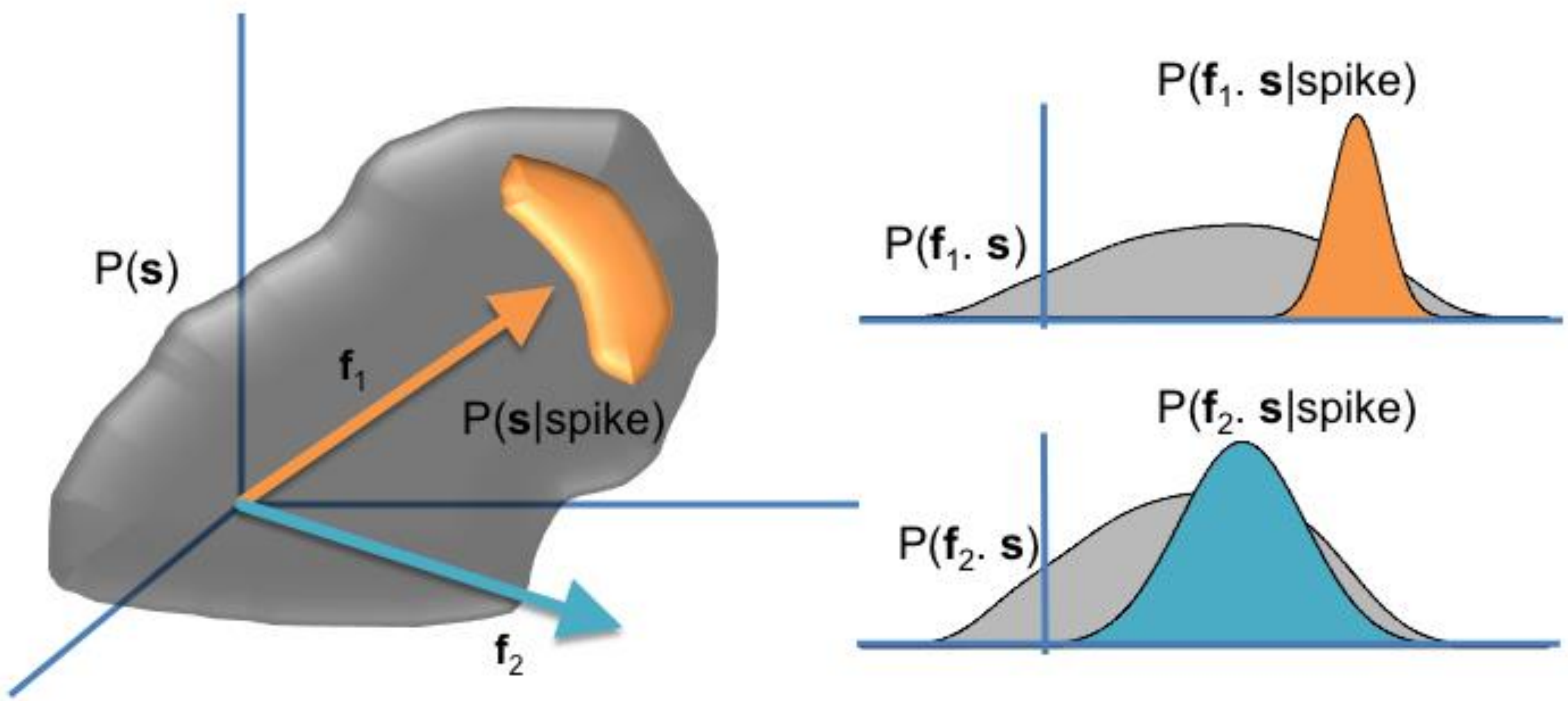


Goodness measure: $D_{KL}(P(s|\text{spike}) \mid P(s))$

Maximally informative dimensions

Sharpee, Rust and Bialek, Neural Computation, 2004

Choose filter in order to maximize D_{KL} between spike-conditional and prior distributions



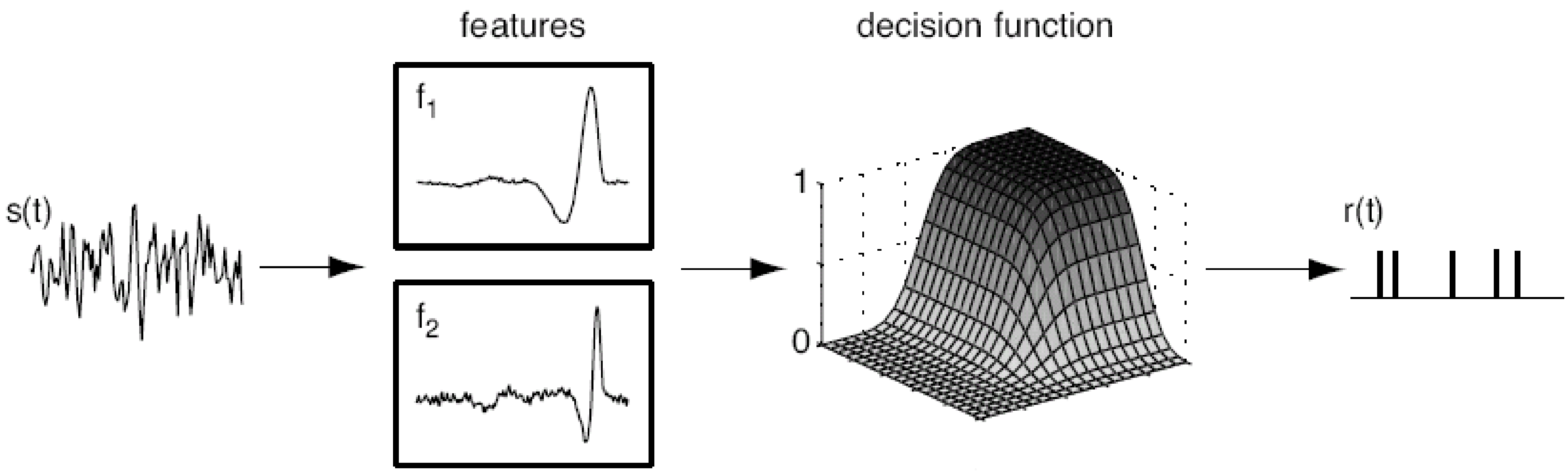
Finding relevant features

1. Single, best filter determined by the first moment

2. A family of filters derived using the second moment

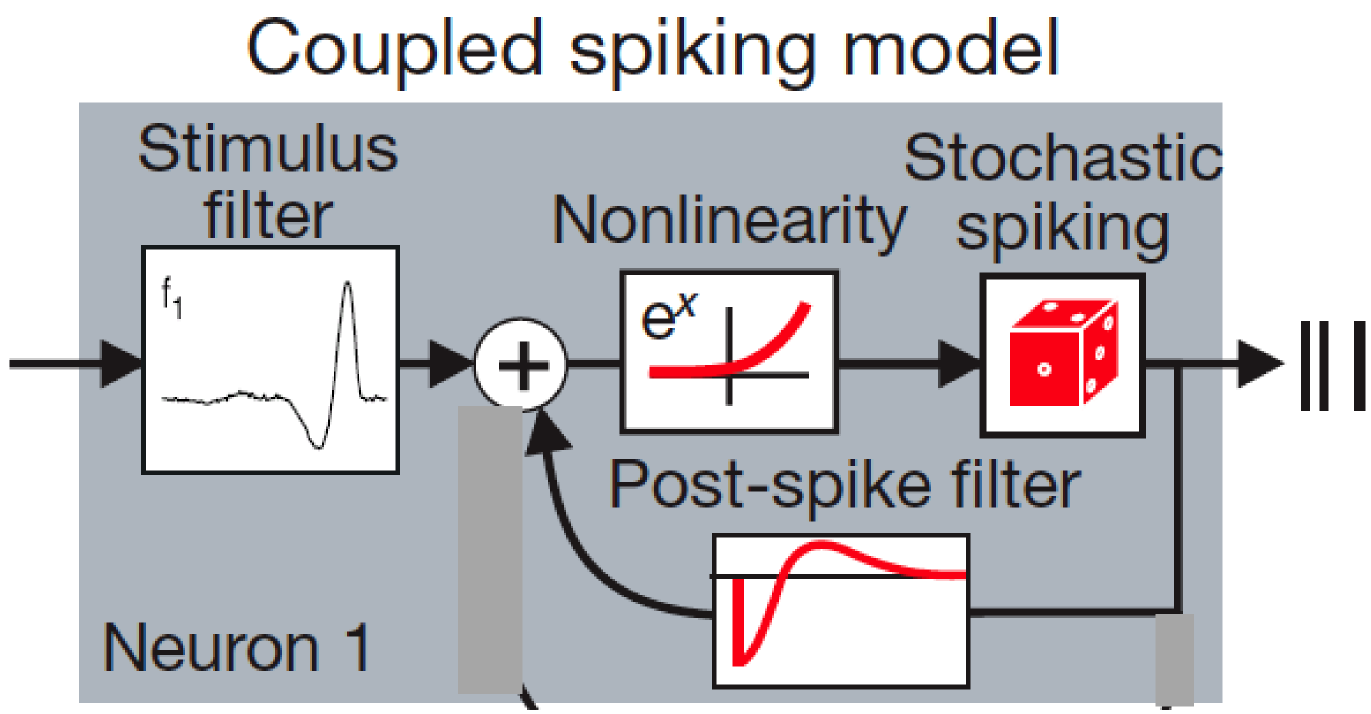
3. Use the entire distribution: information theoretic methods

Less basic coding models



Linear filter*s* & nonlinearity: $r(t) = g(f_1{*}s, f_2{*}s, \ldots, f_n{*}s)$

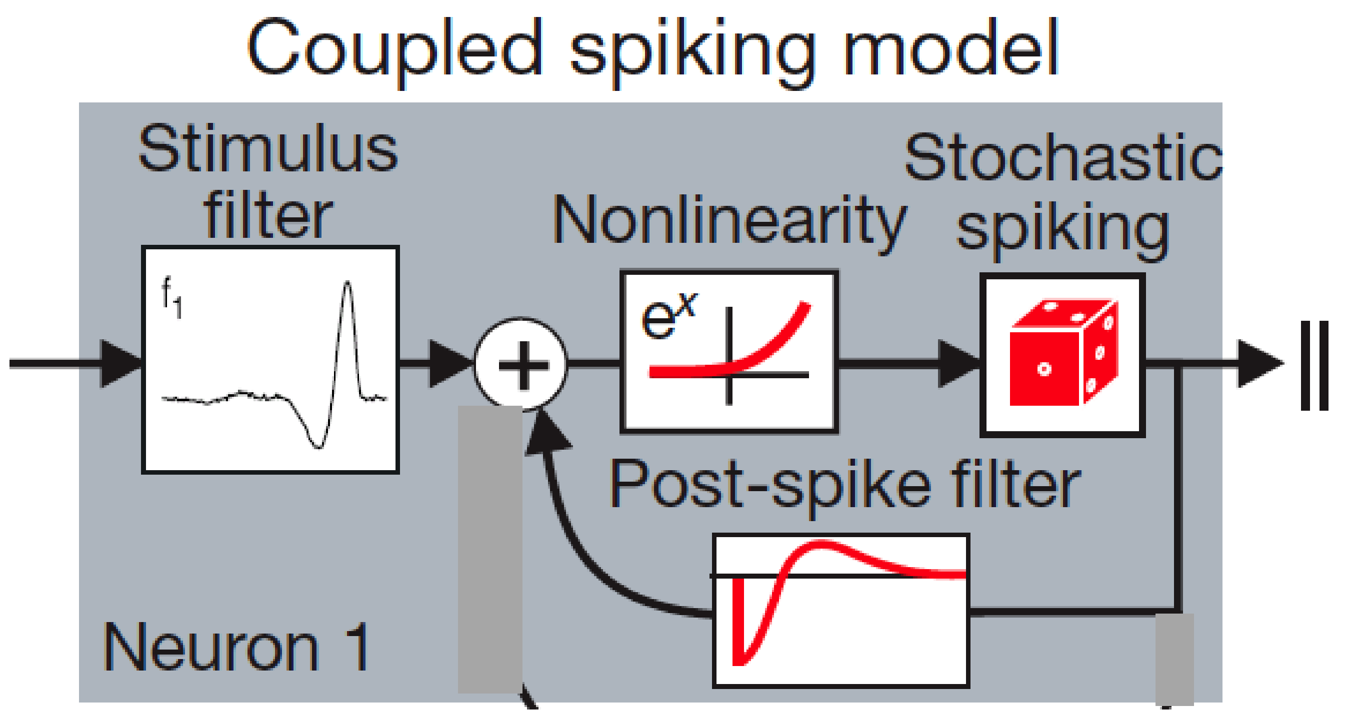
Less basic coding models



GLM: $r(t) = g(f_1 * s + f_2 * r)$

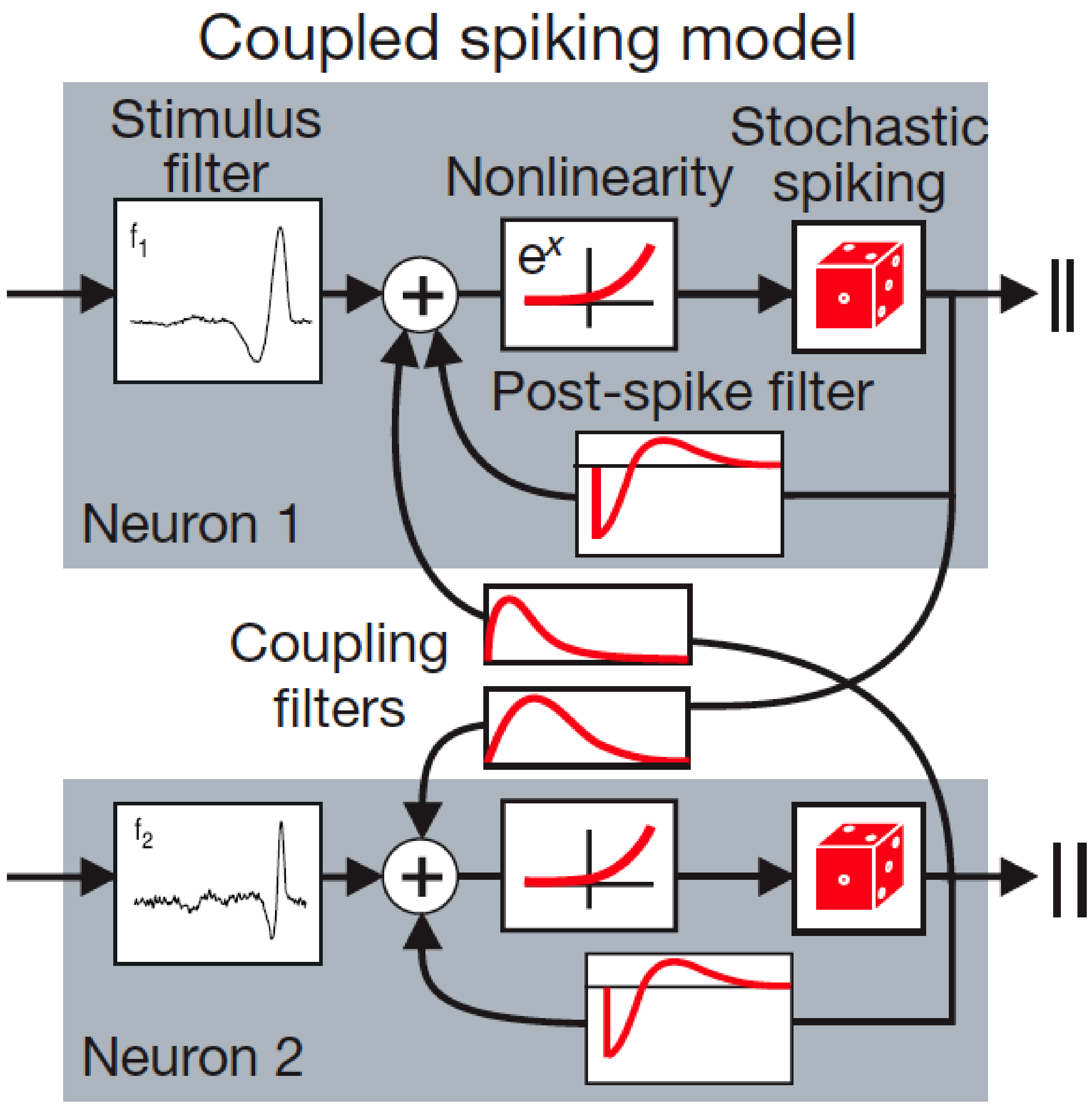
Pillow et al., *Nature* 2008; Truccolo, .., Brown, *J. Neurophysiol*. 2005

Less basic coding models



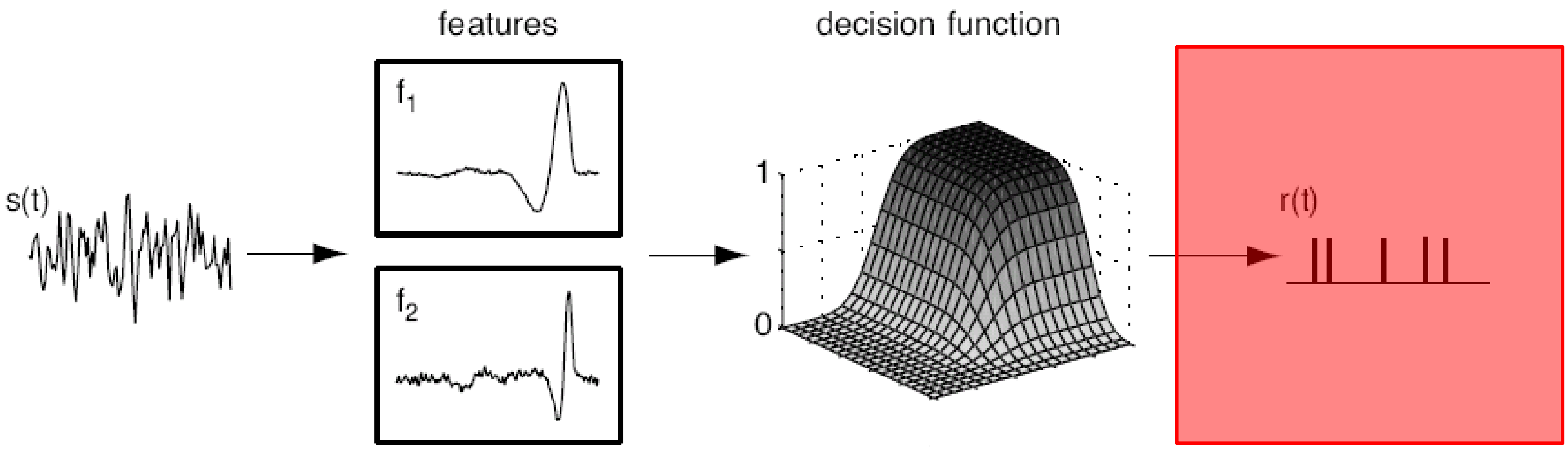
GLM: r(t) = g(f*s + h*r)

Less basic coding models



GLM: $r(t) = g(f_1 * s + h_1 * r_1 + h_2 * r_2 + \ldots)$

Point process model



Binomial spiking

Properties:

Distribution: $P_n[k] = ?$

Mean: $\langle x \rangle = ?$

Variance: $Var(x) = ?$

Binomial spiking

Properties:

Distribution: $P_T[k] = (rT)^k \exp(-rT)/k!$

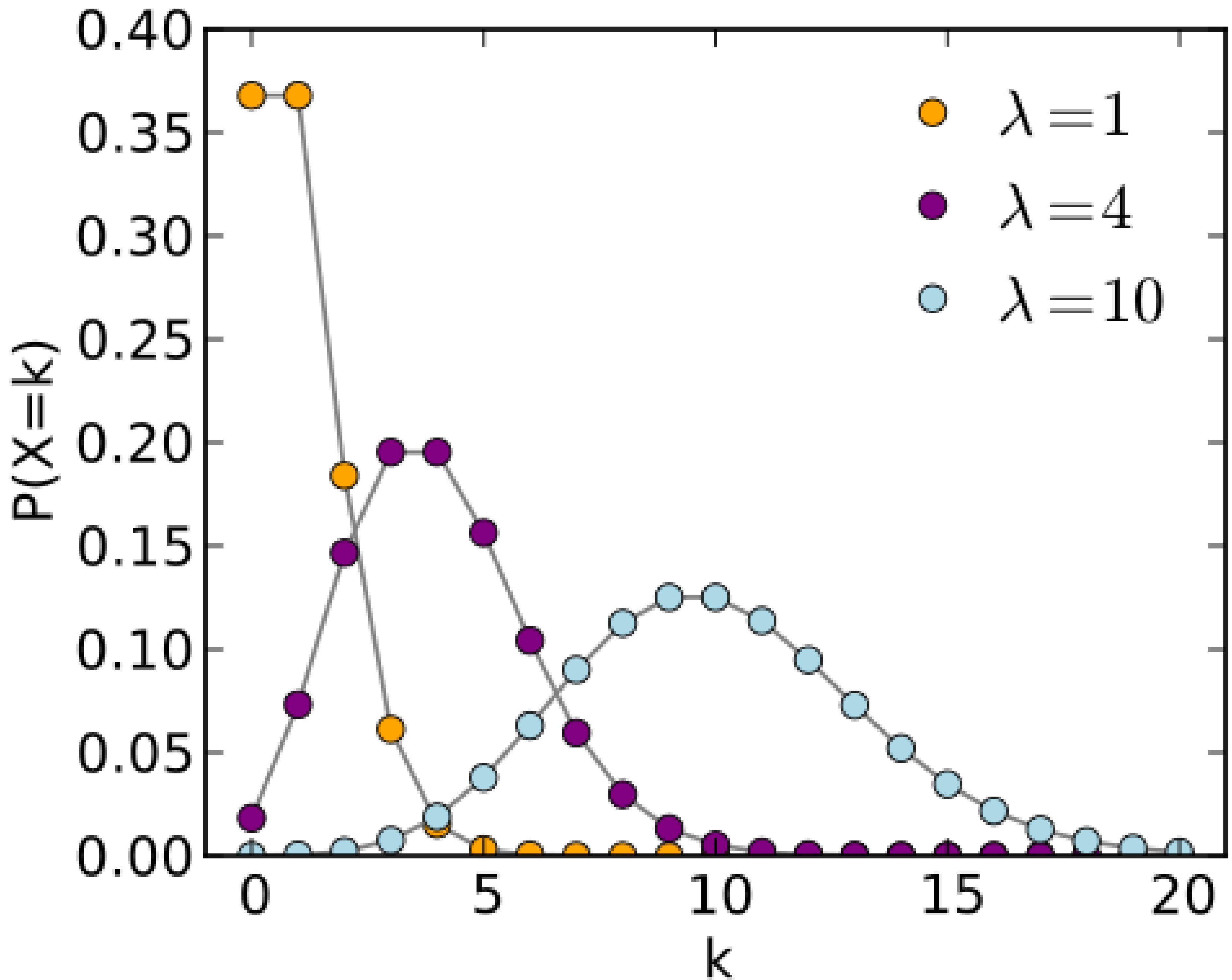
Mean: $\langle x \rangle = rT$

Variance: $\mathrm{Var}(x) = rT$

Fano factor: $F = 1$

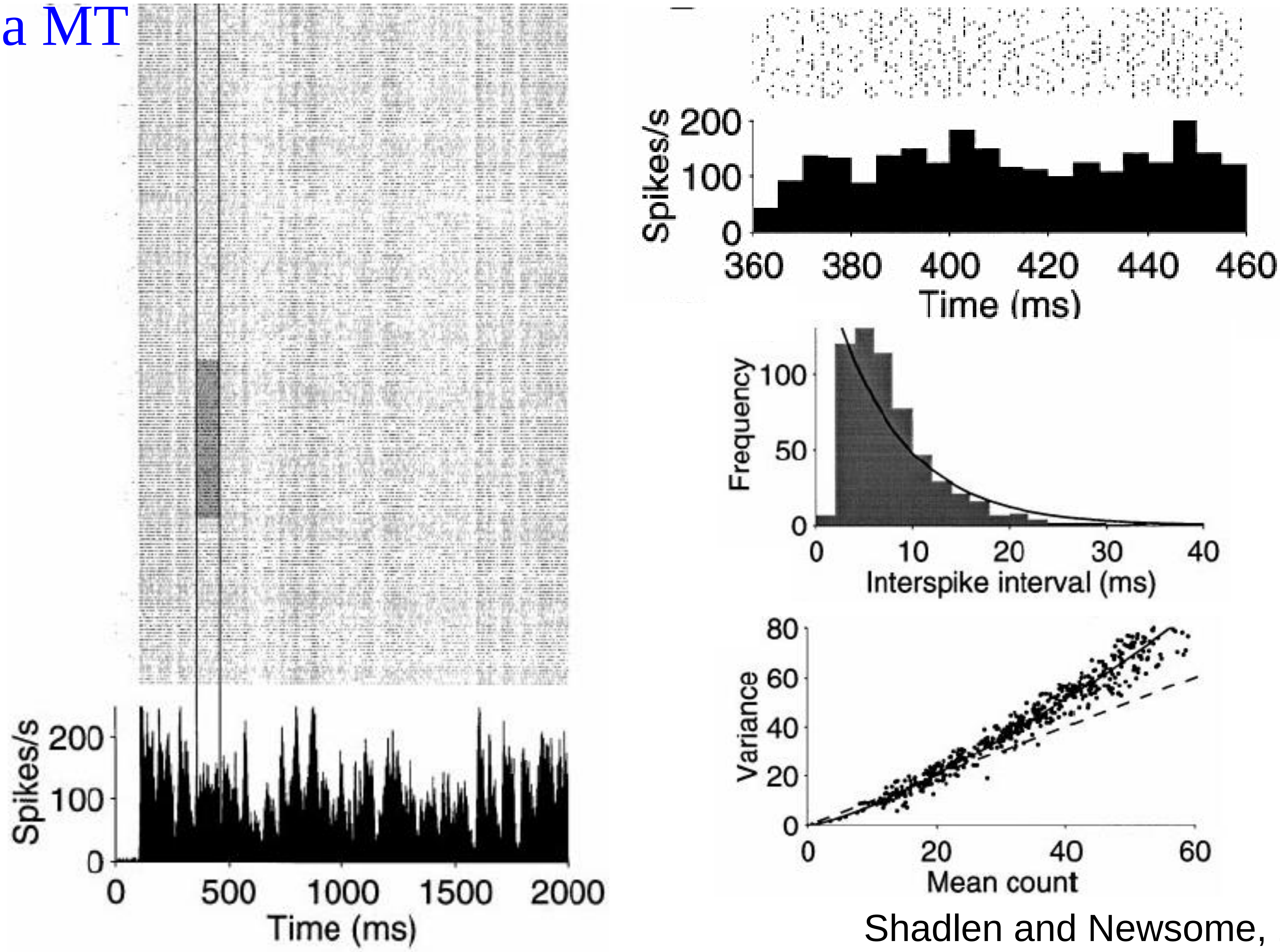
Interval distribution: $P(T) = r \exp(-rT)$

Poisson distribution



Poisson distribution

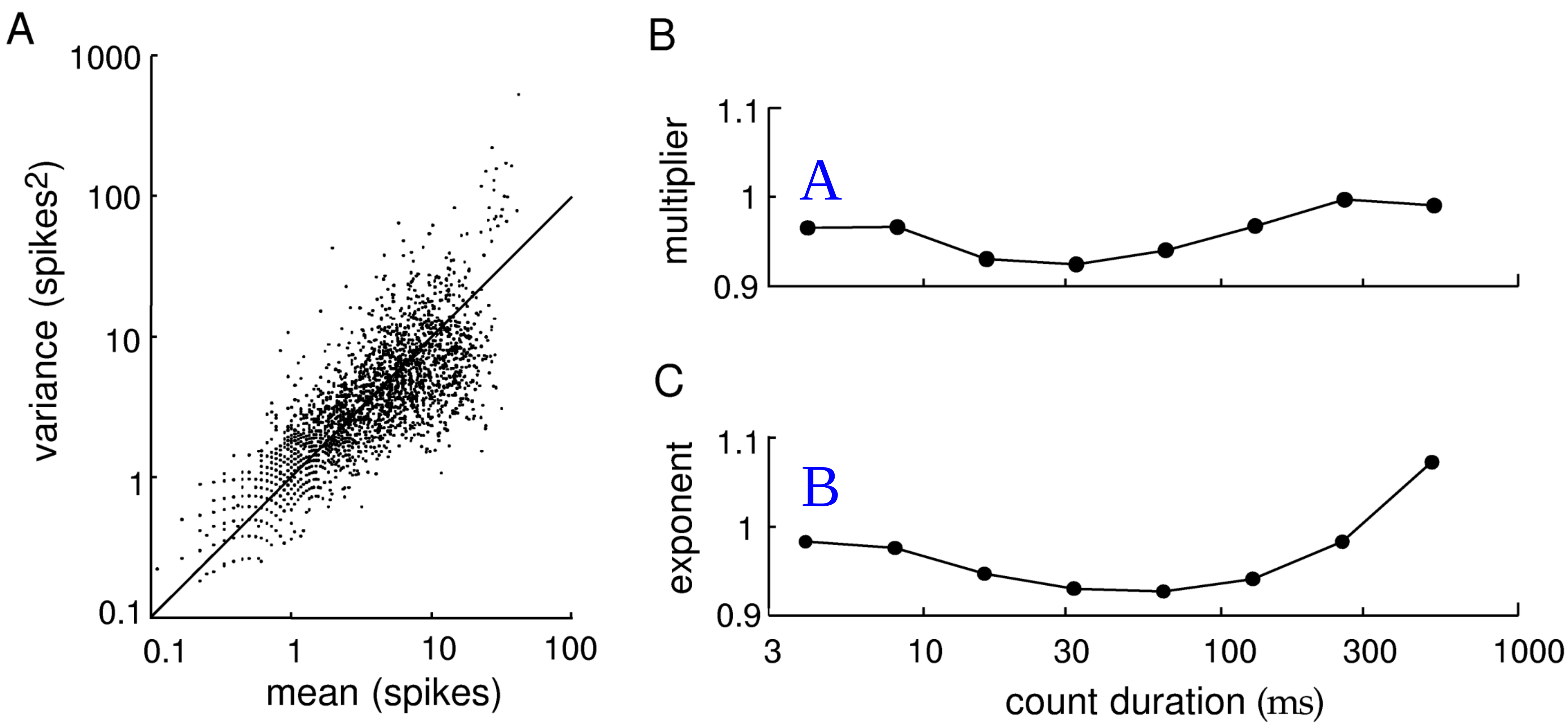
Area MT



Shadlen and Newsome, 1998

Poisson spiking in the brain

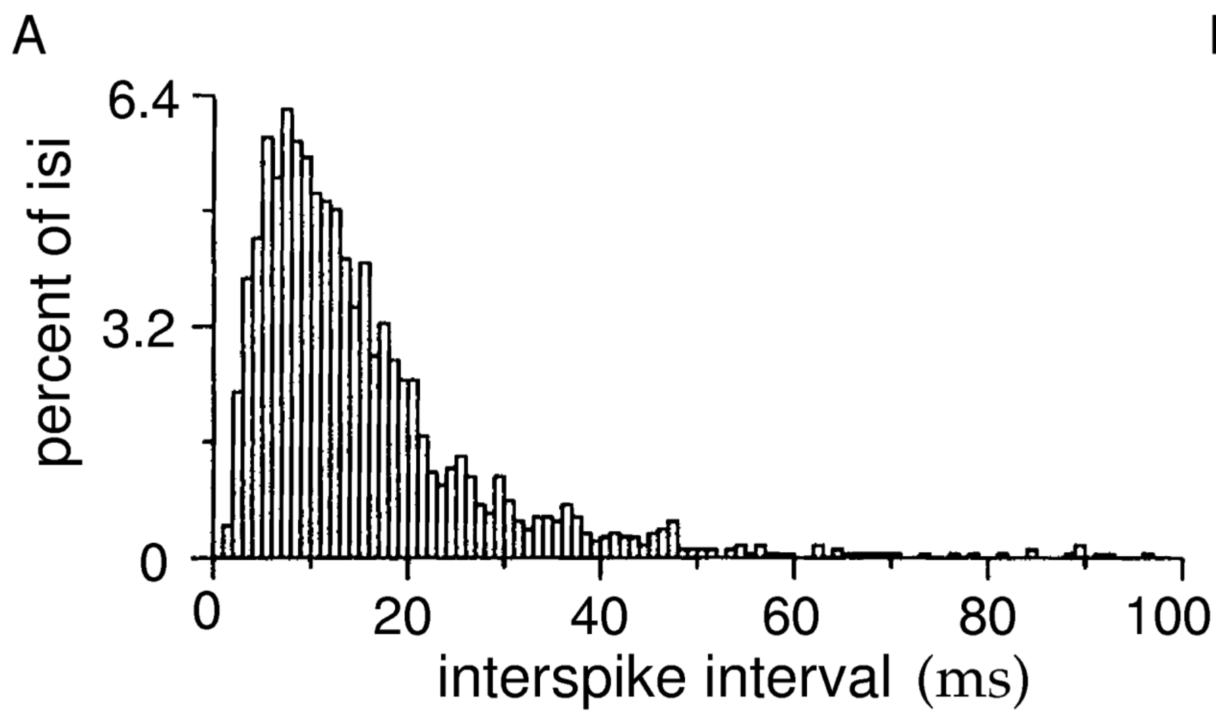
Area MT



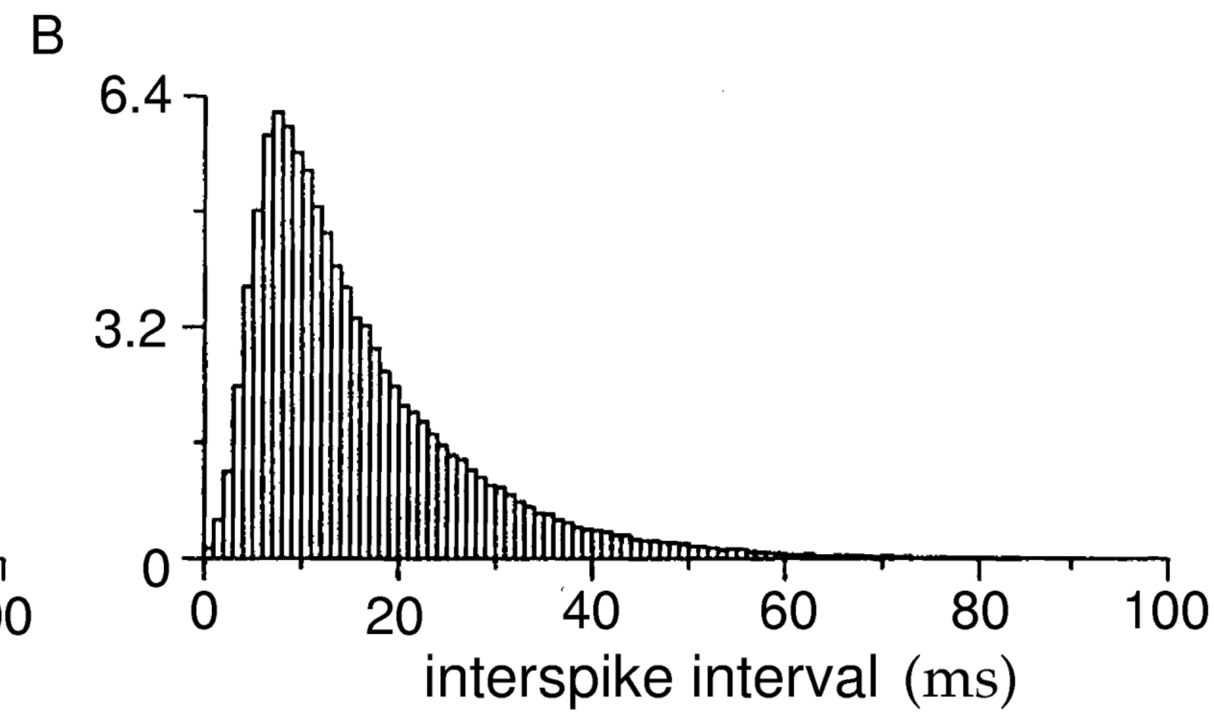
Fano factor

Data fit to:
variance = A × mean^B

Poisson spiking in the brain

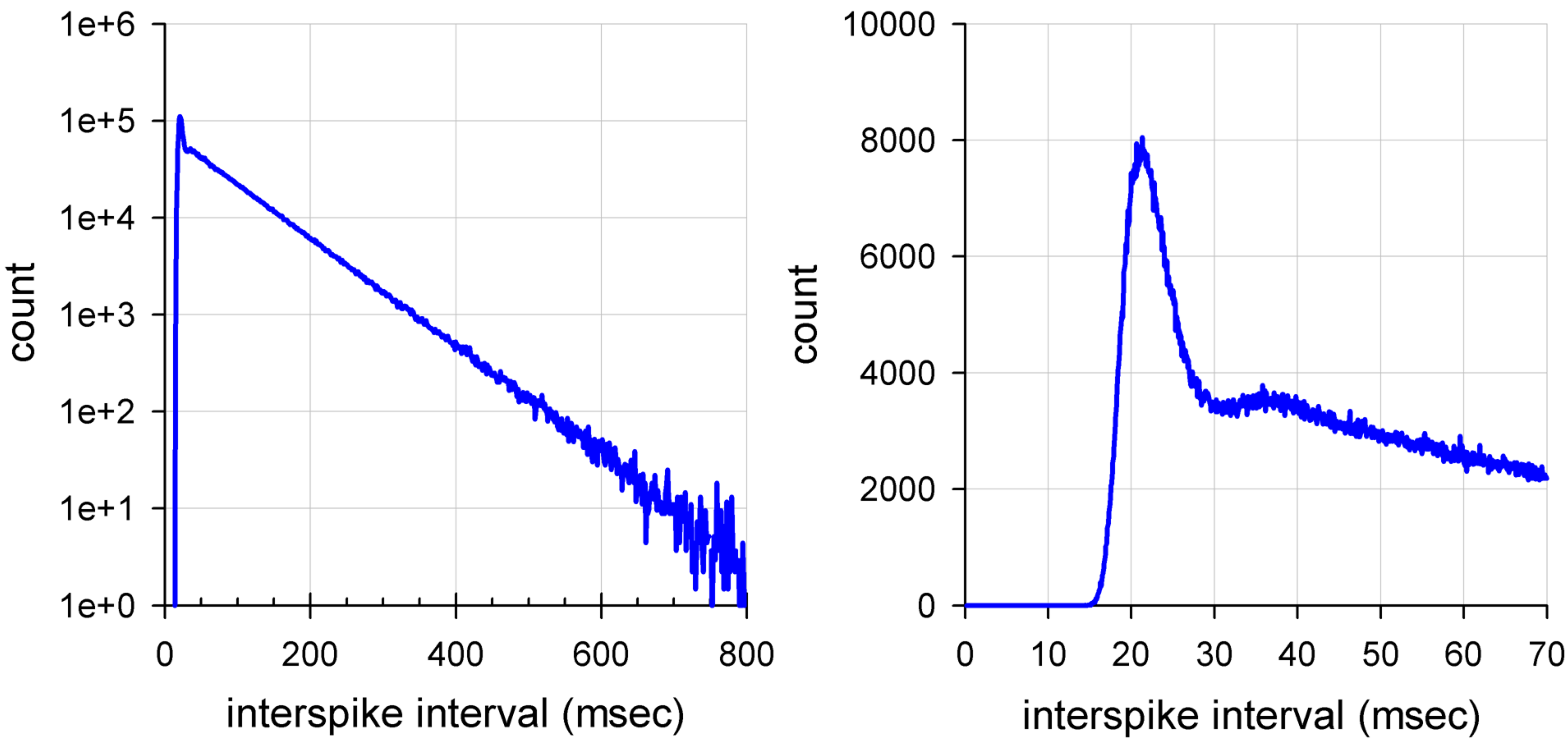


ISI Distribution from an area MT Neuron

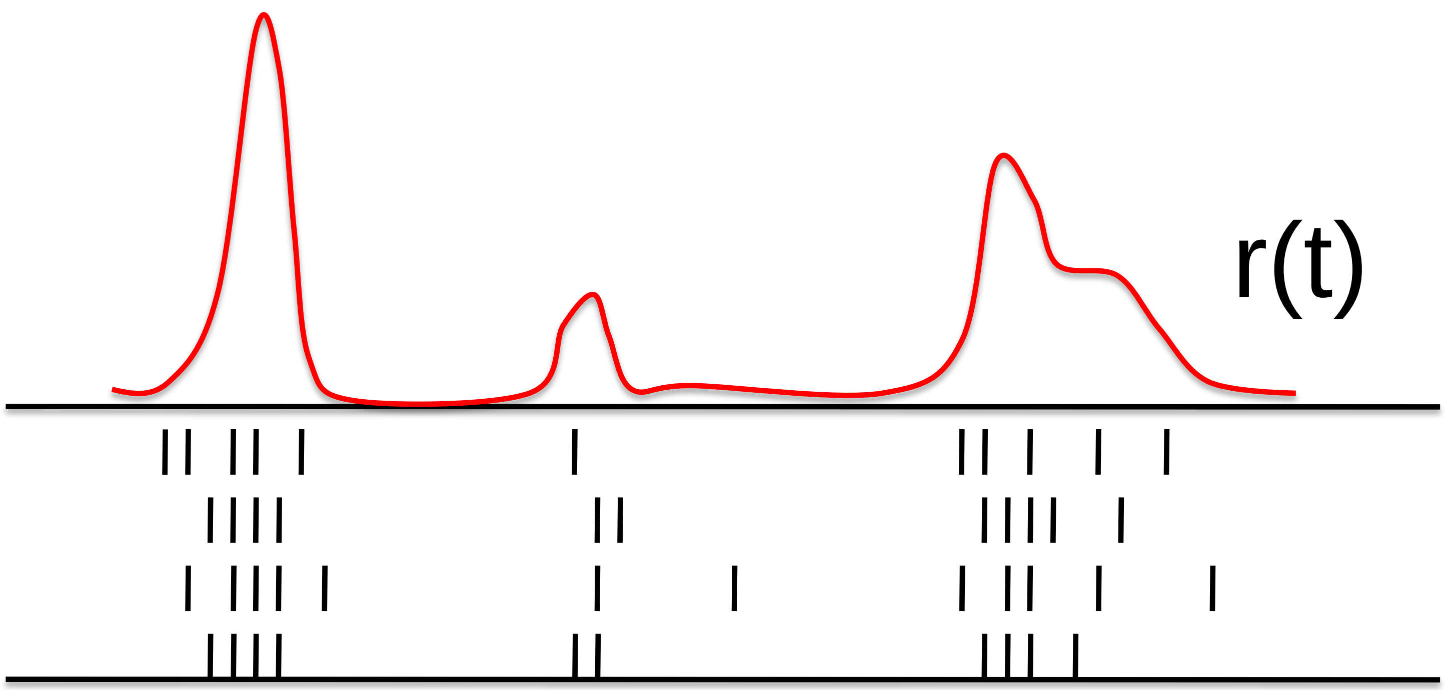


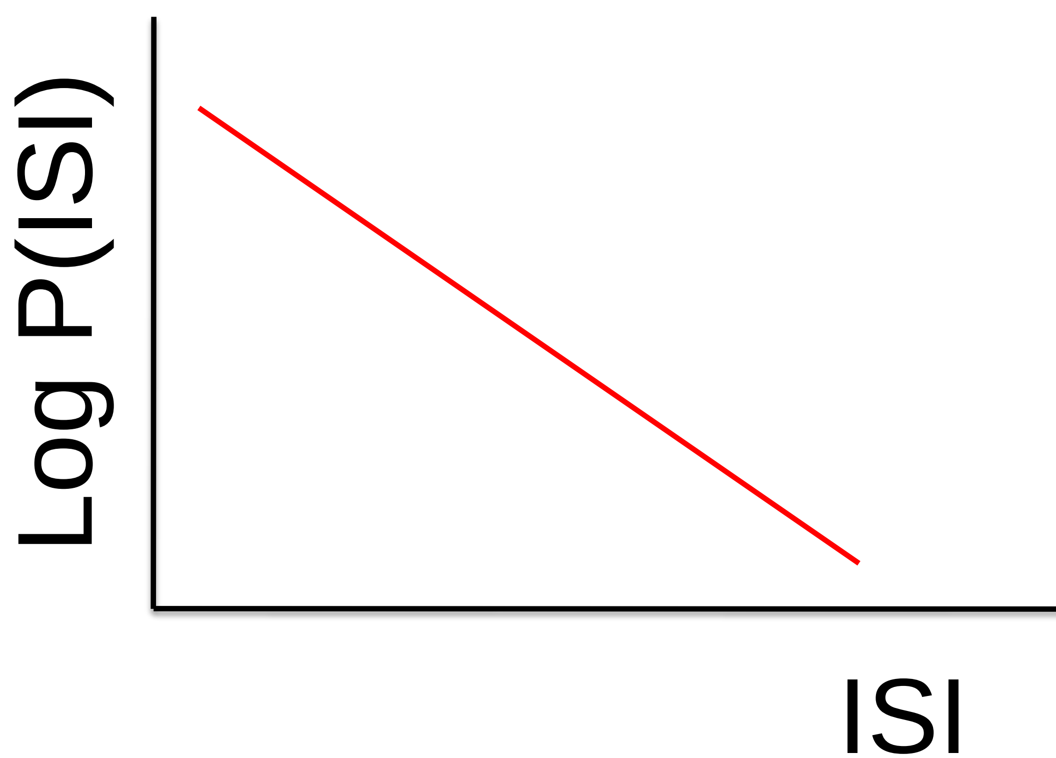
ISI distribution generated from a Poisson model with a Gaussian refractory period

Hodgkin-Huxley neuron driven by noise



Time-rescaling theorem

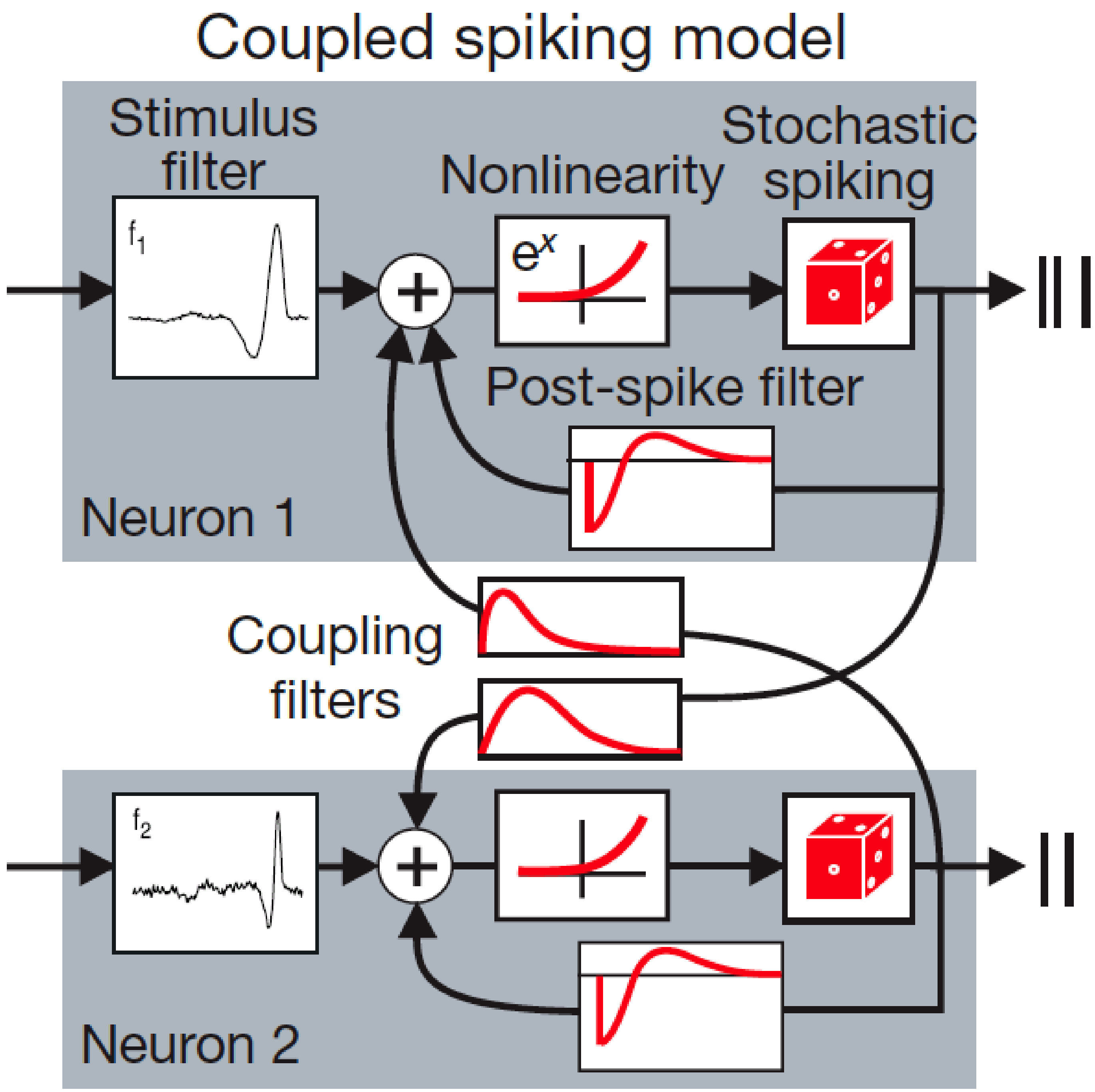




Exponential distribution of *scaled* ISIs:
$(t_{i-1-}\ t_i)\ r(t)$

Brown et al. (2001)

GLM



GLM: P(spike at *t*) = $\exp(f_1 * s + h_1 * r_1 + h_2 * r_2 + \ldots)$