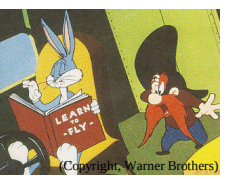
CSE/NB 528

Lecture 11: Plasticity and Learning

(Chapter 8)

Gameplan for Today



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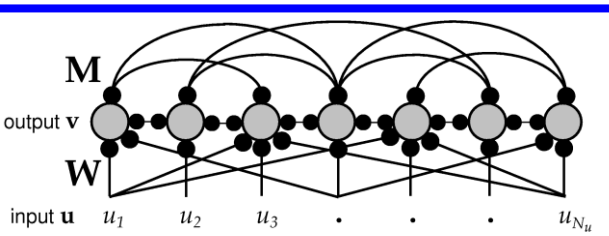
- Plasticity and Learning
  - Types: Unsupervised, Supervised, and Reinforcement learning
- Unsupervised Learning
  - Hebb rule and its variants (Covariance, Oja rule)
  - Mathematical formulation
  - Stability analysis of learning rules

So far, we have been analyzing networks with *fixed* sets of synaptic weights W and M
(based on eigenvalues of M etc.)

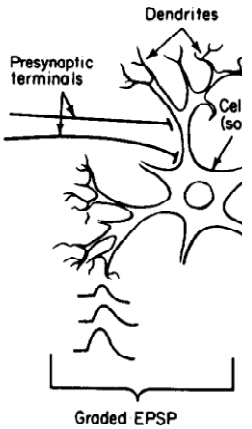
Can synaptic weights be adapted in response to inputs?

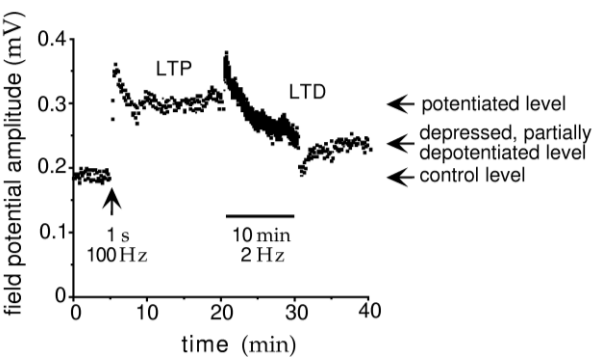
Plasticity and Learning: Adapting the Connections



- Question 1: How do we adapt the synaptic weights W and M to solve useful tasks?
- Question 2: How does the brain do it?

Synaptic Plasticity in the Brain

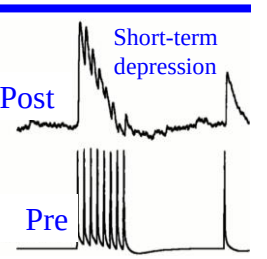


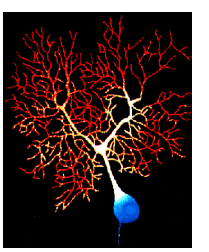


LTP = Long Term Potentiation
LTD = Long Term Depression

Other Forms of Plasticity in the Brain

- Short-Term Synaptic Plasticity
  - Short-term depression/facilitation
  - Dynamics may change on a long-term basis via LTP/LTD
- Changes to intrinsic excitability of cell
  - Density and distribution of various channels (ionic conductances)
  - Currently active research area
- Growth and morphological changes in dendrites
  - Currently active research area
- Addition of new neurons?
  - Hot topic of research in recent years…





The Theory: Classification of Learning Algorithms

- **Unsupervised Learning**
  - Synapses adapted based solely on inputs
  - Network self-organizes in response to *statistical patterns* in input
  - Similar to Probability Density Estimation in statistics
- **Supervised Learning**
  - Synapses adapted based on inputs and desired outputs
  - External "teacher" provides desired output for each input
  - Goal: Function approximation
- **Reinforcement Learning**
  - Synapses adapted based on inputs and (delayed) reward/punishment
  - Goal: Pick outputs that *maximize total expected future reward*
  - Similar to optimization based on Markov decision processes

Let's start with Unsupervised Learning

Consider a single neuron receiving feedforward inputs from other neurons (e.g. from the retina)

The Grand-Daddy of Unsupervised Learning

- Rule hypothesized by Donald Hebb in 1949
- Hebb's learning rule:

"If neuron A frequently contributes to the firing of neuron B, then the synapse from A to B should be strengthened"

- Related Mantra: *Neurons that fire together wire together*
- Hebb's goal: Produce clusters of neurons ("*cell assemblies*") that fire together in response to a stimulus

Mathematical Formulation of Hebb's Rule

On-Board Derivation

Formalizing Hebb's Rule

- Consider a linear neuron (steady state): $v = \mathbf{w}^T\mathbf{u} = \mathbf{u}^T\mathbf{w}$
- Basic Hebb Rule: $\tau_w \frac{d\mathbf{w}}{dt} = \mathbf{u}v$ (or $\mathbf{w} \leftarrow \mathbf{w} + \varepsilon \cdot \mathbf{u}v$)
- What is the average effect of this rule?

$$\tau_w \frac{d\mathbf{w}}{dt} = \langle \mathbf{u}v \rangle_{\mathbf{u}} = \langle \mathbf{u}\mathbf{u}^T\mathbf{w} \rangle_{\mathbf{u}} = \langle \mathbf{u}\mathbf{u}^T \rangle_{\mathbf{u}} \mathbf{w} = Q\mathbf{w}$$

- Q is the input correlation matrix: $Q = \langle \mathbf{u}\mathbf{u}^T \rangle$

Variants of Hebb's Rule

- Pure Hebb only increases synaptic weights (LTP)
  - What about LTD?
- Covariance rule:

$$\tau_w \frac{d\mathbf{w}}{dt} = \mathbf{u}(v - \theta_v)$$

(Note: LTD for low or no output and some input)

where θ_v can be set to <v>, the average value of v.

- Why is this called the covariance rule?

Are these learning rules stable?

On Board Analysis, leading up to Oja's rule

What does the Hebb rule do anyway?

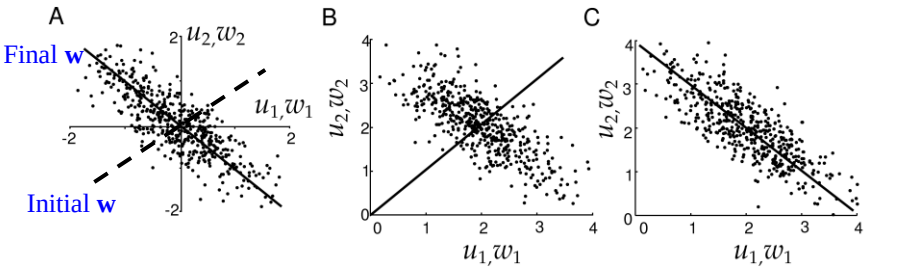
Eigenvector analysis of Hebb rule…

Hebb Rule implements Principal Component Analysis (PCA)!

Pure Hebb
Input mean = (0,0)

Pure Hebb
Input mean = (2,2)

Covariance Rule
Input mean = (2,2)



Hebb rule *rotates* weight vector to align with principal eigenvector of input correlation/covariance matrix (i.e. direction of maximum variance)

Next Class: Unsupervised and Supervised Learning

- Things to do:
  - Finish Chapter 8 and Start Chapter 10
  - Homework 3 due on Sunday May 26
  - Start group project