CSE/NEUBEH 528

Lecture 9: Computation by Networks

(Chapter 7)

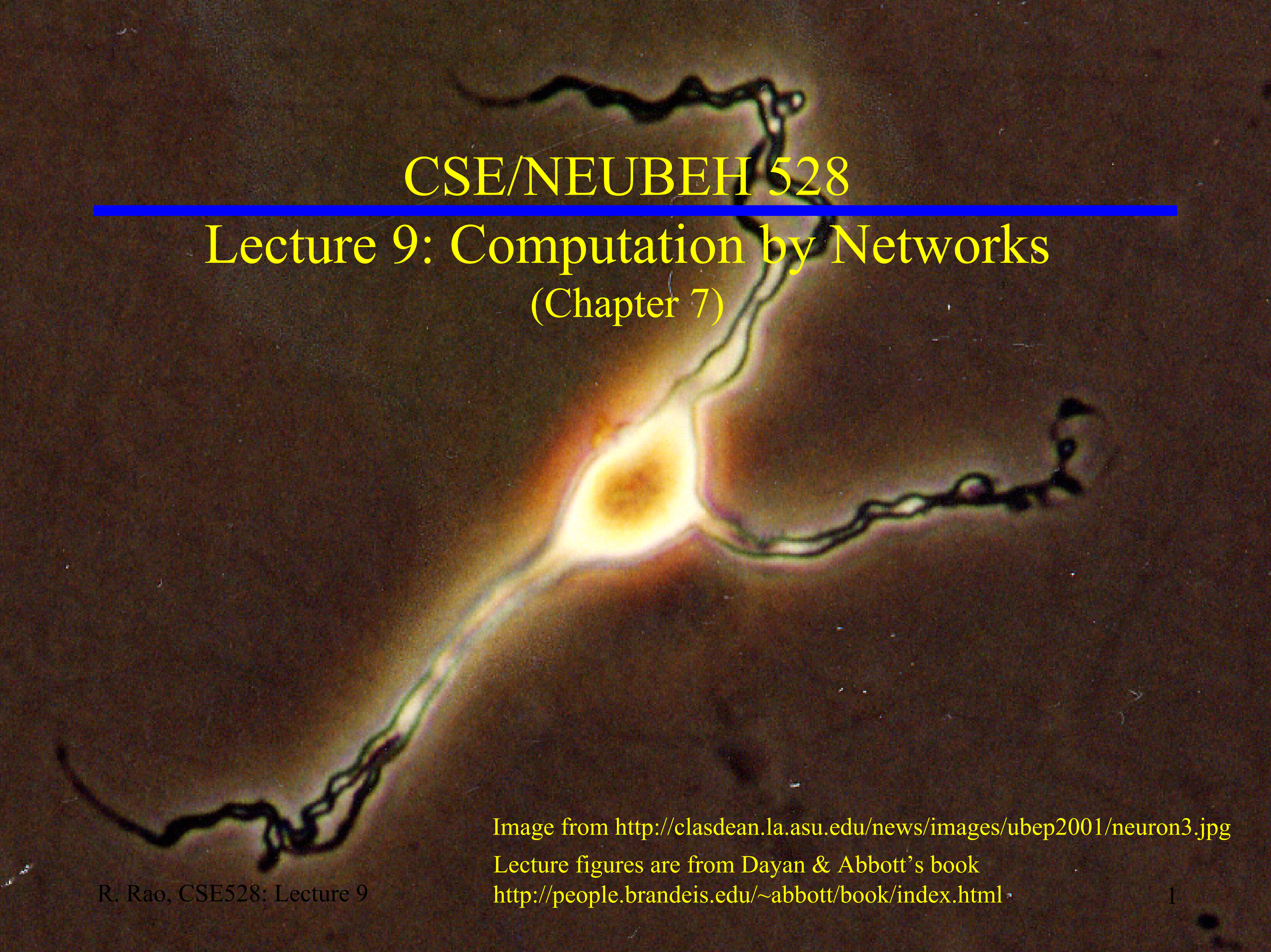


Image from http://clasdean.la.asu.edu/news/images/ubep2001/neuron3.jpg

Lecture figures are from Dayan & Abbott's book http://people.brandeis.edu/~abbott/book/index.html

Course Summary (thus far)

- Neural Encoding
  - What makes a neuron fire? (STA, covariance analysis)
  - Poisson model
- Neural Decoding
  - Stimulus Discrimination based on firing rate
  - Spike-train based decoding of stimulus
  - Population decoding (Bayesian estimation)
- Single Neuron Models
  - RC circuit model of membrane
  - Integrate-and-fire model
  - Conductance-based and Compartmental Models

Today's Agenda

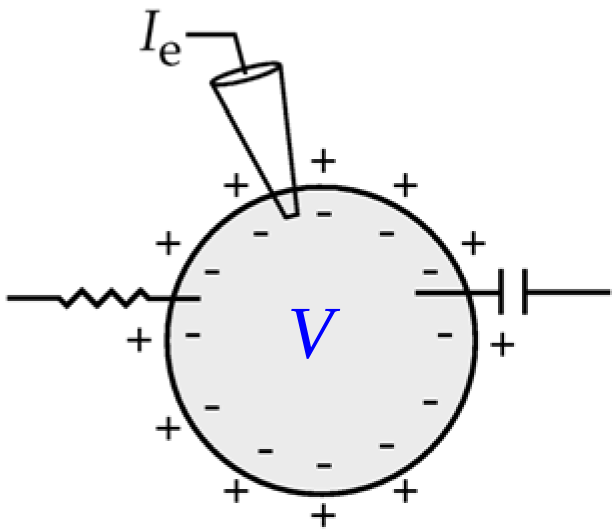
- Computation in Networks of Neurons
  - From spiking to firing-rate based networks
  - Feedforward Networks
    - E.g. Coordinate transformations in the brain
  - Linear Recurrent Networks
    - Can amplify inputs
    - Can integrate inputs
    - Can function as short-term memory

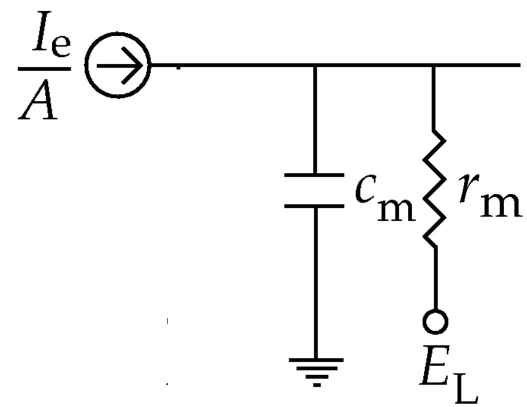
Modeling Networks of Neurons

- Option 1: Use *spiking* neurons (e.g. I & F neurons)
  - *Advantages*: Allows computation and learning based on:
    - Spike Timing
    - Spike Correlations/Synchrony between neurons
  - *Disadvantages*: Computationally expensive
- Option 2: Use neurons with *firing-rate outputs*
  - *Advantages*: Greater efficiency, scales well to large networks
  - *Disadvantages*: Ignores spike timing issues
- Question: How are these two approaches related?

Flashback

1-Compartment Membrane Model



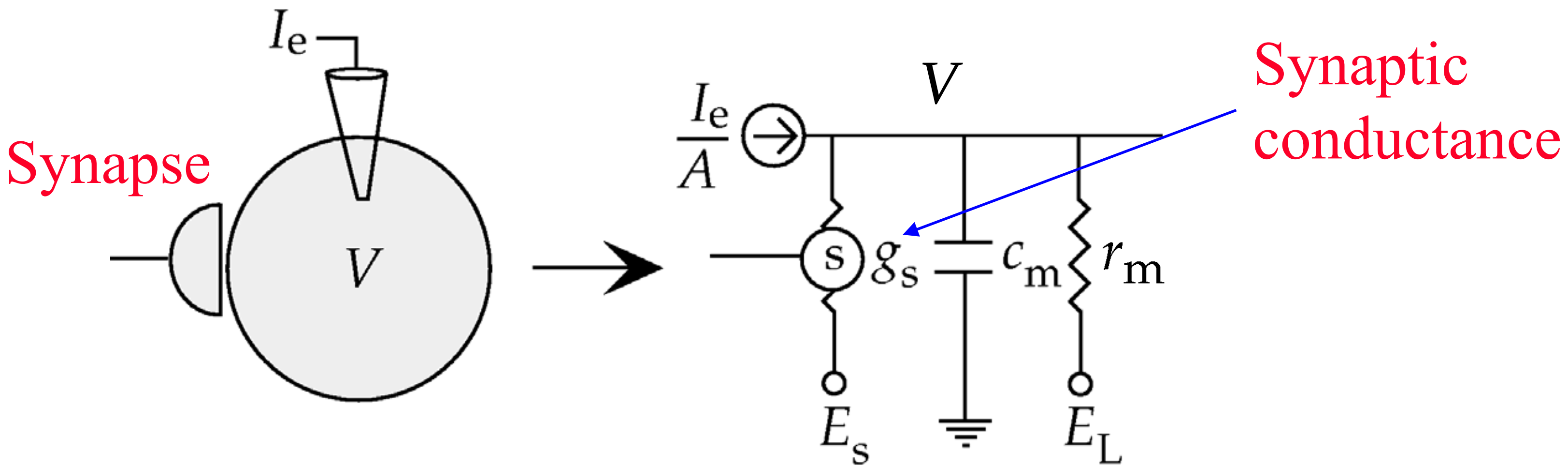


$$c_m \frac{dV}{dt} = -\frac{(V - E_L)}{r_m} + \frac{I_e}{A}, \text{ or equivalently}$$

$\tau_m = r_m c_m =$ membrane time constant

$$\boxed{\tau_m \frac{dV}{dt} = -(V - E_L) + I_e R_m}$$

Flashback Modeling Synaptic Inputs from other Neurons



$$\tau_m \frac{dV}{dt} = -(V - E_L) - r_m g_s (V - E_s) + I_e R_m$$

$$g_s = g_{s,\max} P_{rel} P_s$$

P_s: Probability of postsynaptic channel opening (= fraction of channels opened)

P_{rel}: Probability of transmitter release given an input spike

Basic Synapse Model

- Assume $P_{rel} = 1$
- Model the effect of a single spike input on P_s
- Kinetic Model: $\text{closed} \xrightarrow{\alpha_s} \text{open}$

$$\text{open} \xrightarrow{\beta_s} \text{closed}$$

$$\frac{dP_s}{dt} = \alpha_s(1 - P_s) - \beta_s P_s$$

Fraction of channels open

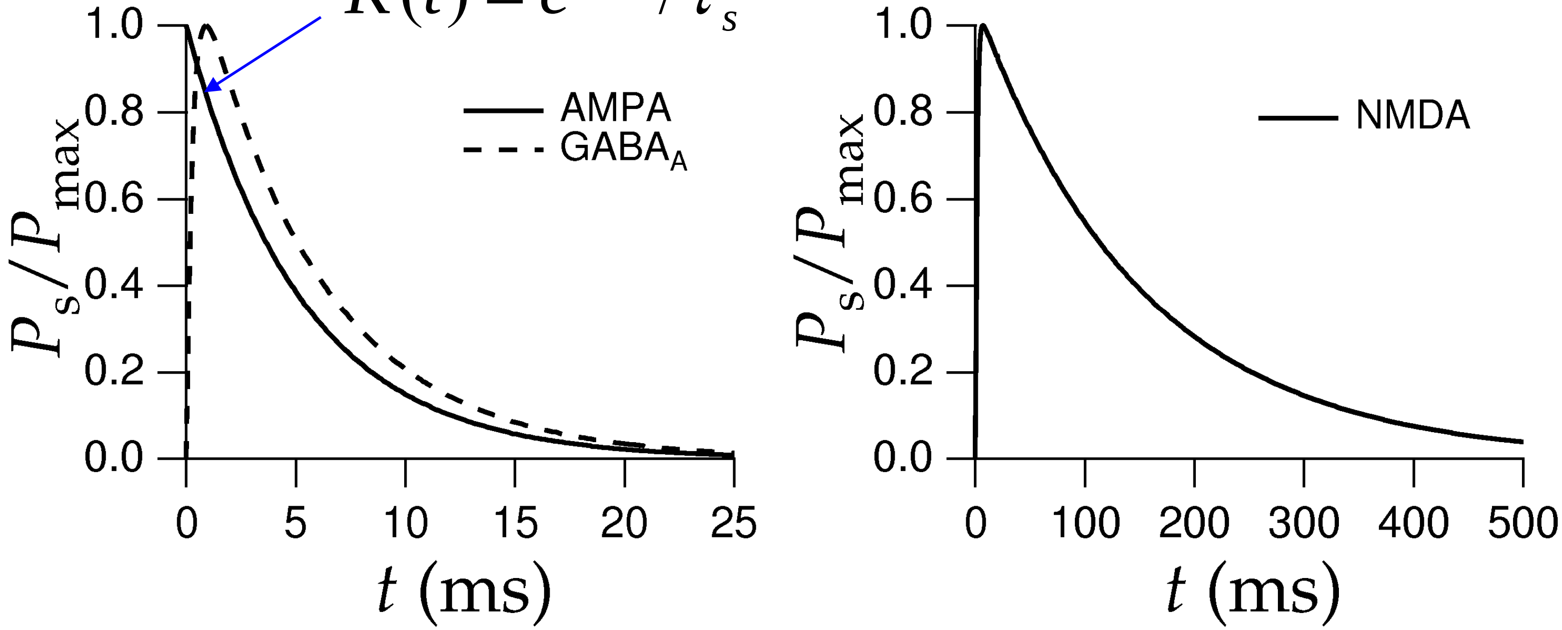
Opening rate

Closing rate

Fraction of channels closed

Postsynaptic Data

$$K(t) = e^{-\frac{t}{\tau_s}} / \tau_s$$



Exponential function $K(t)$ gives reasonable fit to biological data (other options: difference of exponentials, "alpha" function)

Modeling Synaptic Input Current

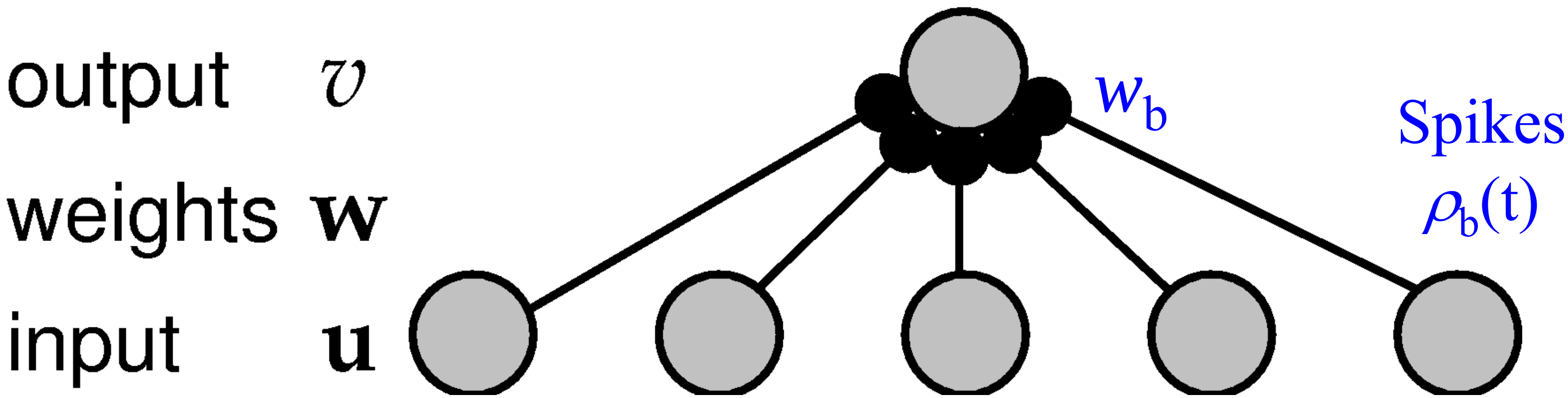
Synaptic kernel: $K(t) = e^{-\frac{t}{\tau_s}} / \tau_s$

Synaptic current: $I_s(t) = w_s \int_{-\infty}^{t} K(t-\tau)\rho_s(\tau)d\tau$

where $\rho_s(t)$ is the input spike train:

$\rho_s(\tau) = \Sigma_i\, \delta(\tau - t_i)$ (t_i are the spike times)

From Spiking to Firing Rate Models



Current at synapse b

$$I_b(t) = w_b \int_{-\infty}^{t} K(t-\tau)\rho_b(\tau)d\tau$$

Spike train $\rho_b(t)$

$$\approx w_b \int_{-\infty}^{t} K(t-\tau)u_b(\tau)d\tau$$

Firing rate $u_b(t)$

Total synaptic current

$$I_s(t) = \sum_b I_b(t)$$

Synaptic Current Dynamics

- If synaptic kernel K is an exponential function: $K(t) = e^{-\frac{t}{\tau_s}} / \tau_s$

Differentiating $I_s(t) = \sum_b w_b \int_{-\infty}^{t} K(t-\tau) u_b(\tau) d\tau$

We get $$\tau_s \frac{dI_s}{dt} = -I_s + \sum_b w_b u_b$$
$$= -I_s + \mathbf{w} \cdot \mathbf{u}$$

Output Firing-Rate Dynamics

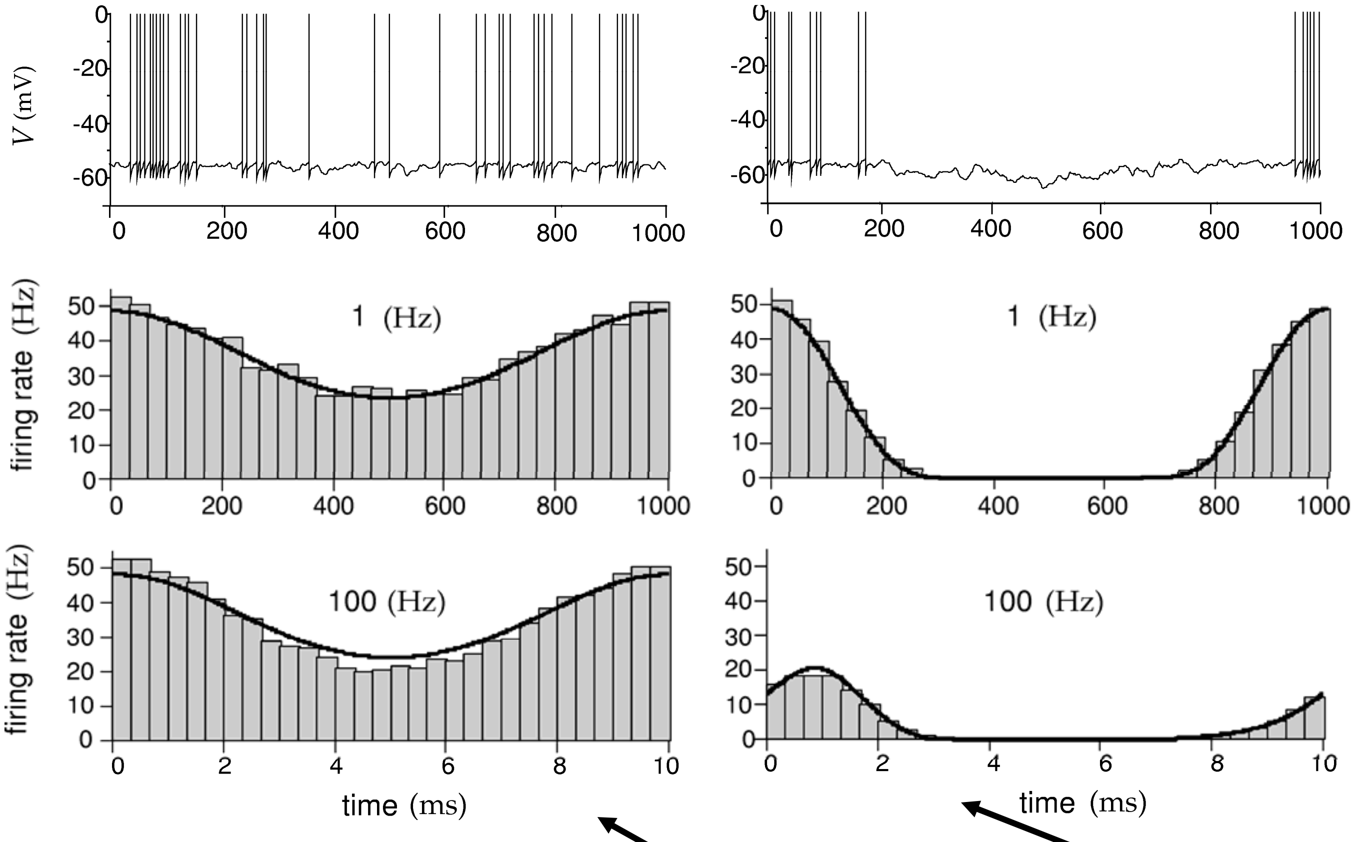
- How is the output firing rate v related to synaptic inputs?

$$\tau_r \frac{dv}{dt} = -v + F(I_s(t))$$

- On-board derivations of special cases obtained from comparing τ_r and τ_s …

  (see also pages 234-236 in the text)

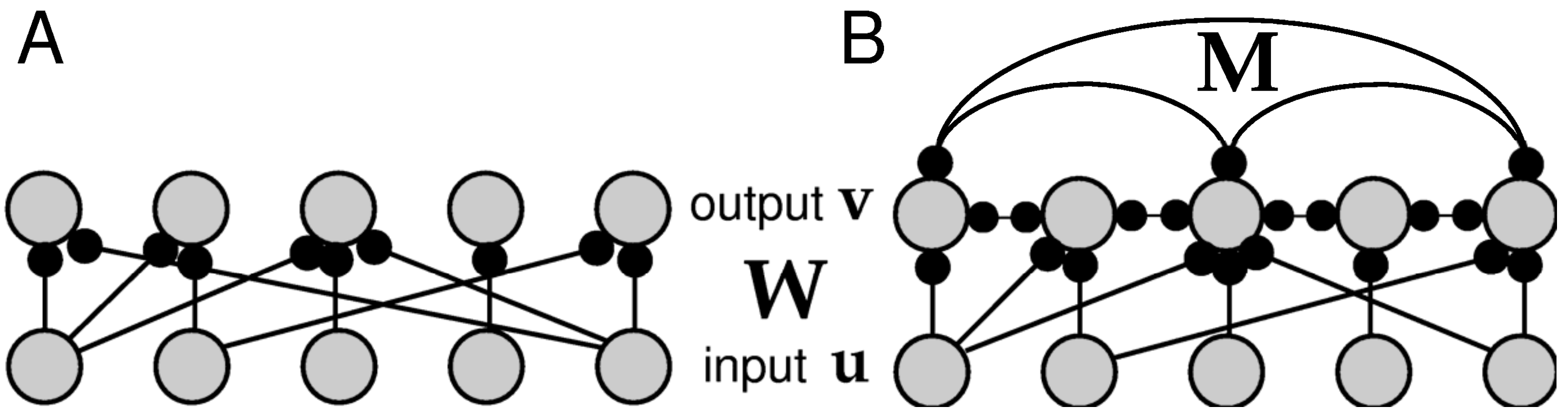
How good are the Firing Rate Models?



Firing rate v(t) = F(I(t)) describes this well but not this case

Input $I(t) = I_0 + I_1\cos(\omega t)$

Feedforward versus Recurrent Networks

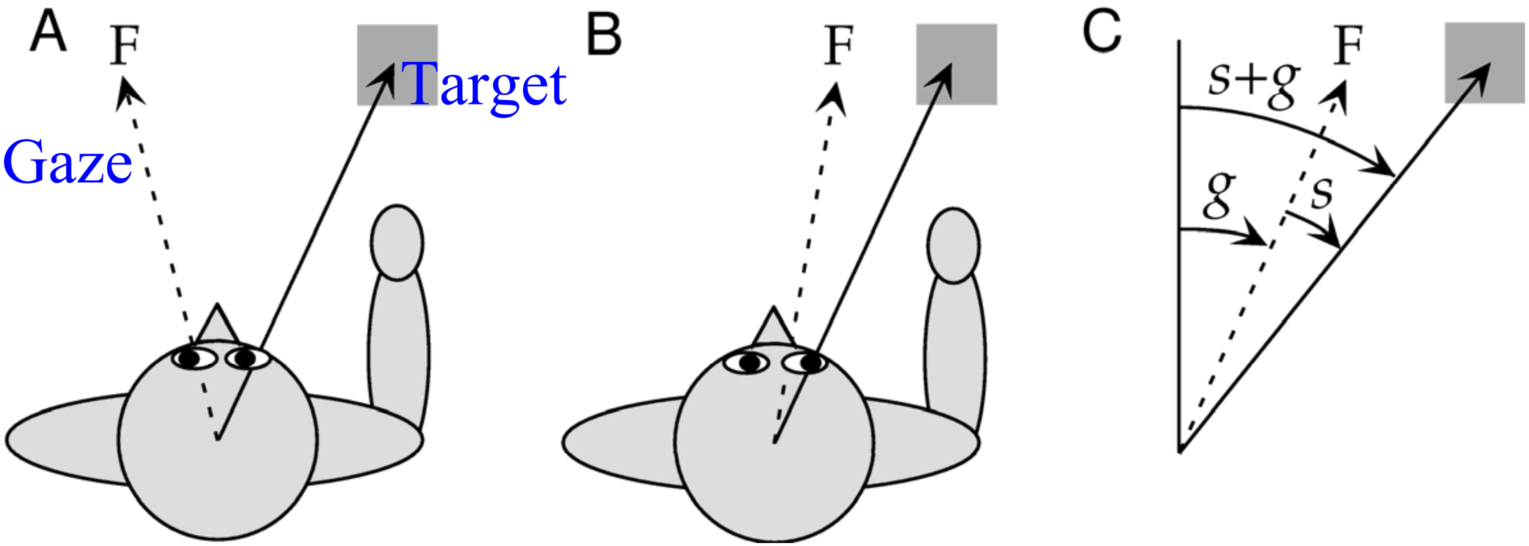


$$\tau \frac{d\mathbf{v}}{dt} = -\mathbf{v} + F(\mathbf{W}\mathbf{u} + \mathbf{M}\mathbf{v})$$

Output Decay Input Feedback

(For feedforward networks, matrix M = 0)

The Problem of Coordinate Transformations



g = gaze angle *relative to body*

s = stimulus or target angle *relative to gaze (retinal coordinates)*

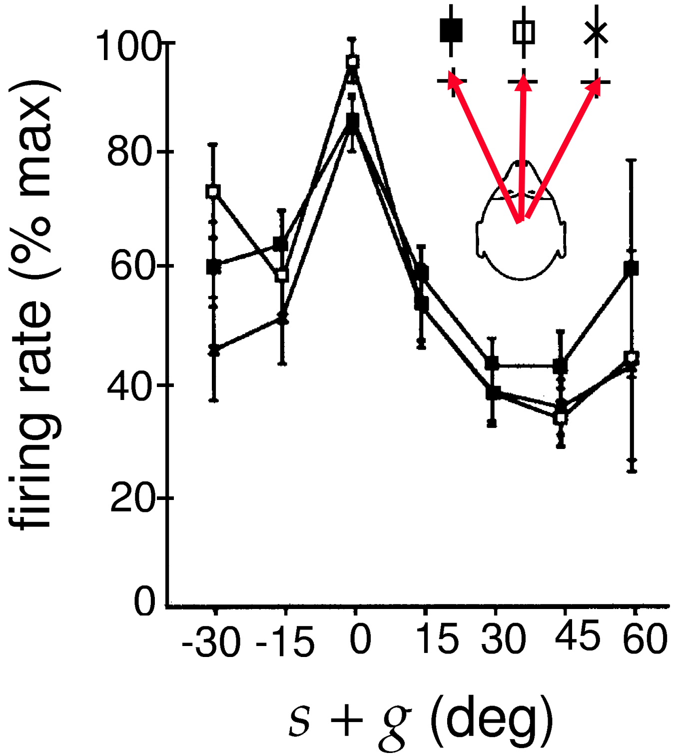
s+g = stimulus relative to body

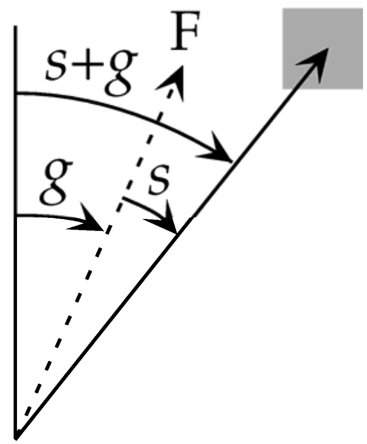
Same arm movement required in A and B but s and g are different

How does the brain solve this problem?

Body-Based Representation in the Monkey

Head fixed
gaze shifted to g_1 g_2 g_3





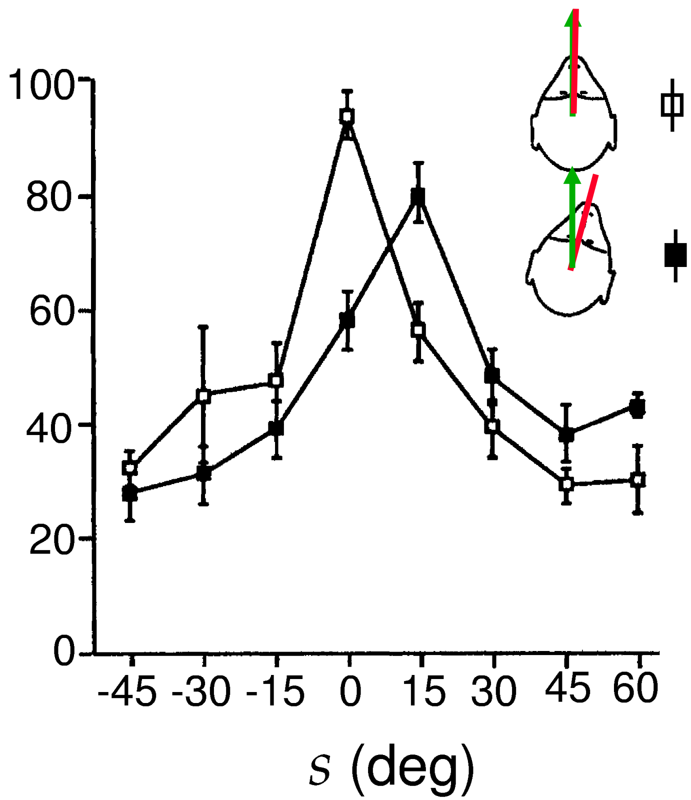
Objects approaching at different angles

Same tuning curve regardless of gaze angle

Premotor cortex neuron responds to stimulus location *relative to body*, not retinal image location

Body-Based Representation in the Monkey

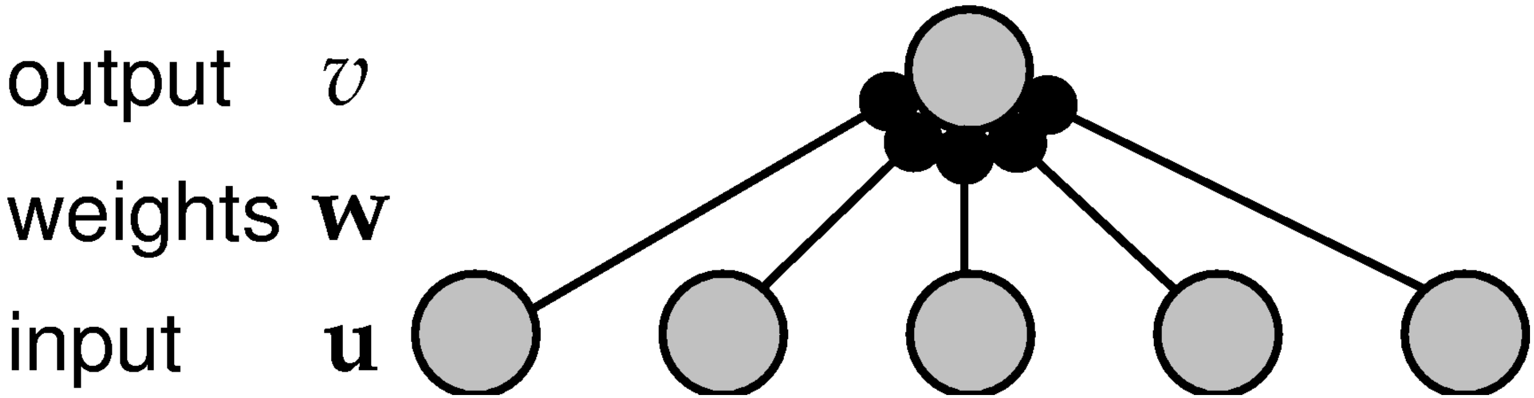
When head is moved but gaze remains unchanged:



After head is moved 15°, objects approaching at 15° in retinal image now elicit the highest response → Tuning curve in retinal coordinates has shifted

Suggested Feedforward Network

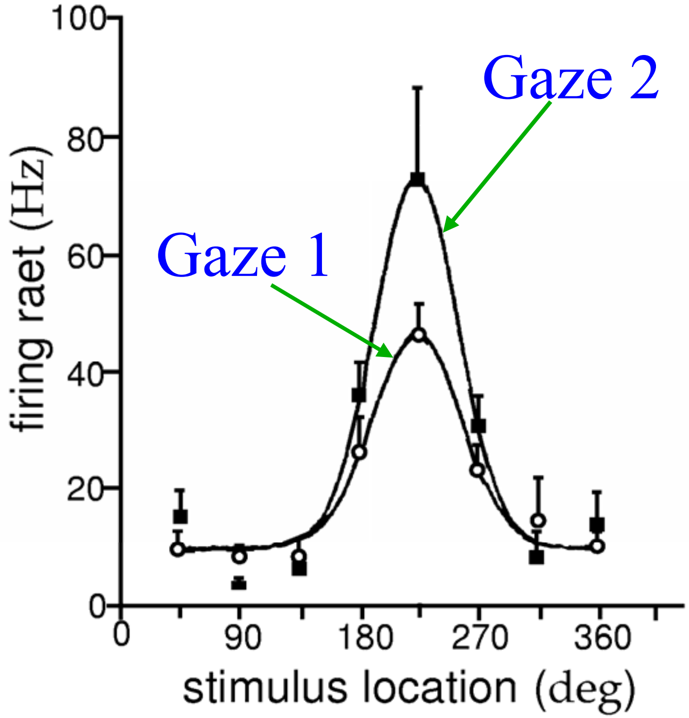
Output: Premotor Cortex Neuron with Body-Based Tuning Curves

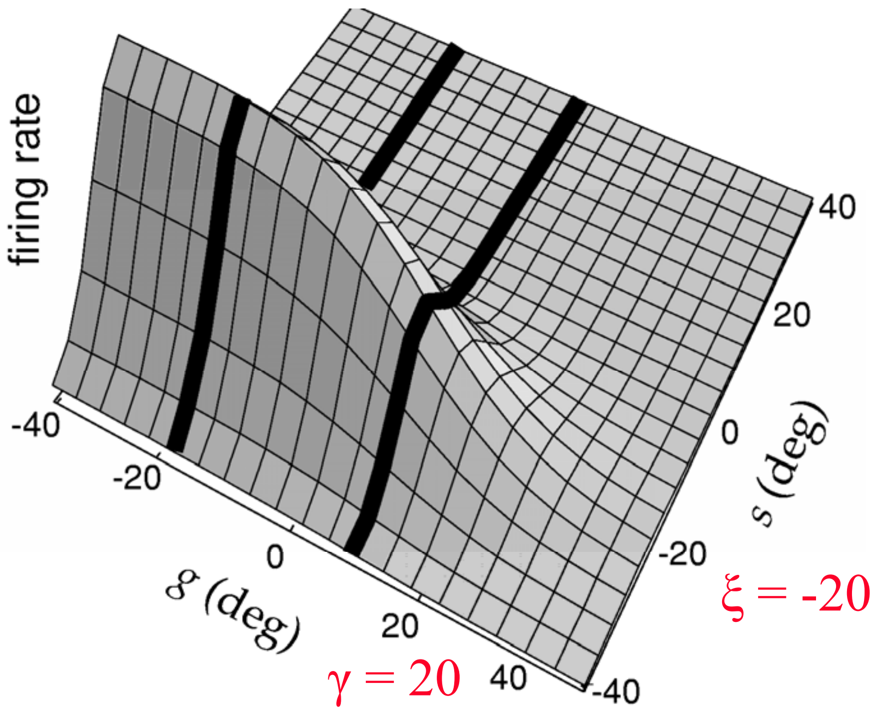


Input: Area 7a Neurons with Gaze-Dependent Tuning Curves

Input neurons exhibit *gaze-dependent* *gain modulation*

Gaze-Dependent Gain Modulation



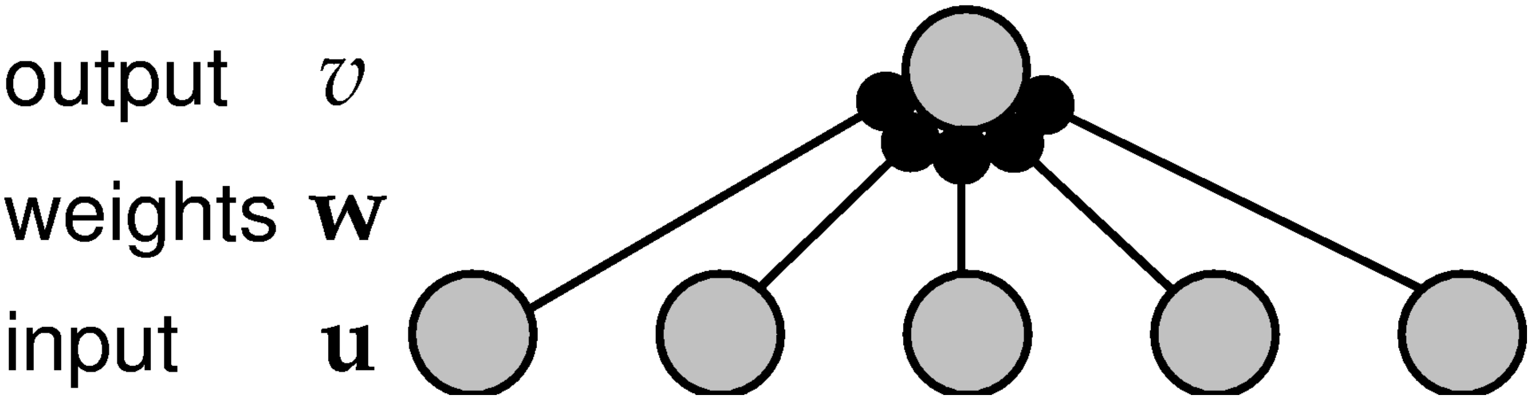


Responses of Area 7a neuron

Example of a gain-modulated tuning curve

What should the weights be?

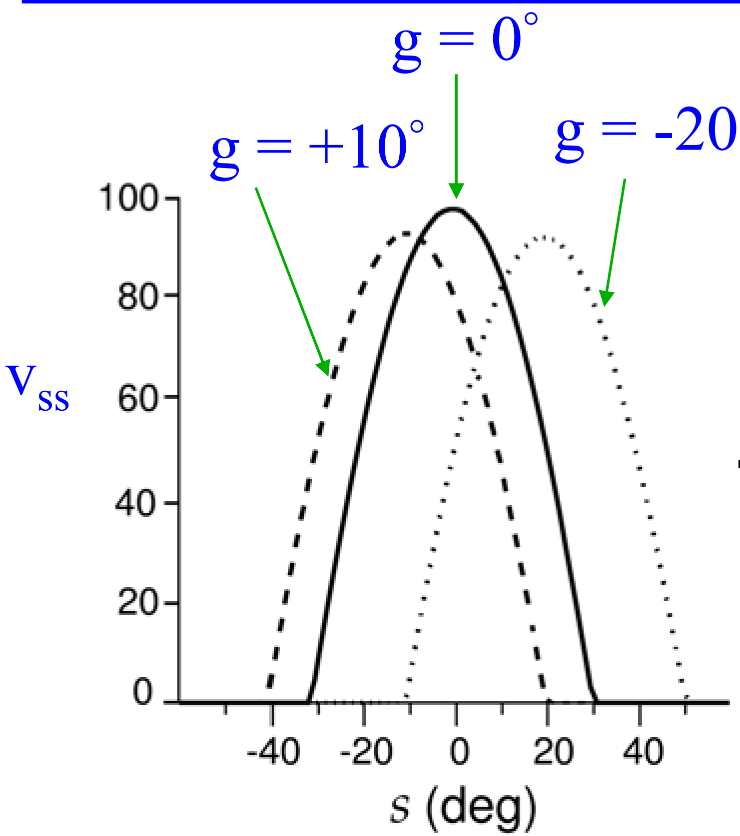
Output: Premotor Cortex Neuron with Body-Based Tuning Curves

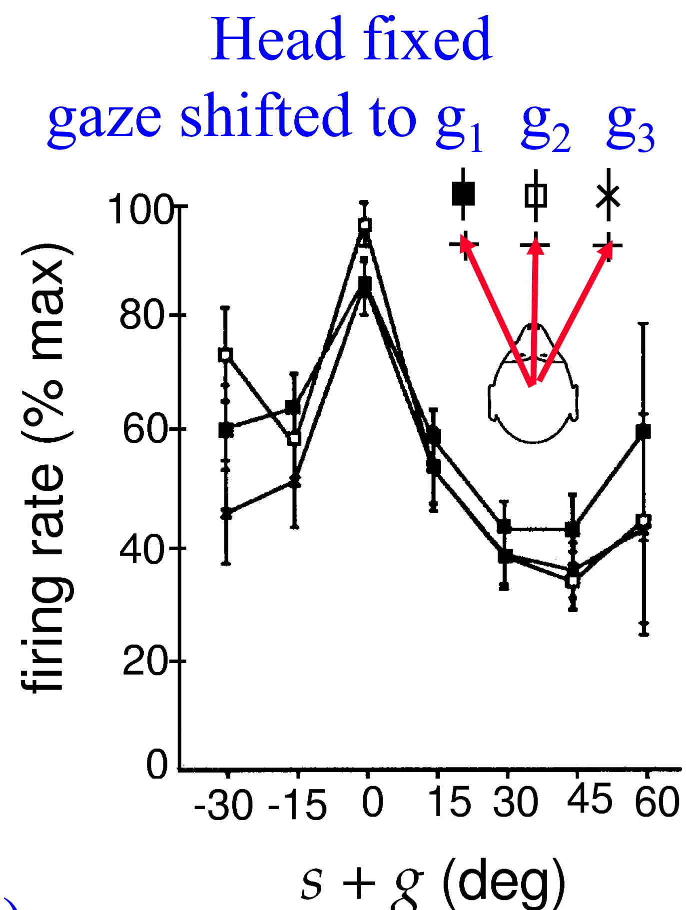


Input: Area 7a Neurons with Gaze-Dependent Tuning Curves

Weights $w(\xi,\gamma)$ need to be a function of $\xi+\gamma$

Output of a Simulated Feedforward Network





Retinal tuning curves shift to compensate for g (i.e. stable for s + g)

Next Class: Recurrent Networks

- Things to do:
  - Finish reading Chapter 7
  - Homework #3 due next Thursday May 14
  - Start working on mini-project