

## Course Summary (thus far)

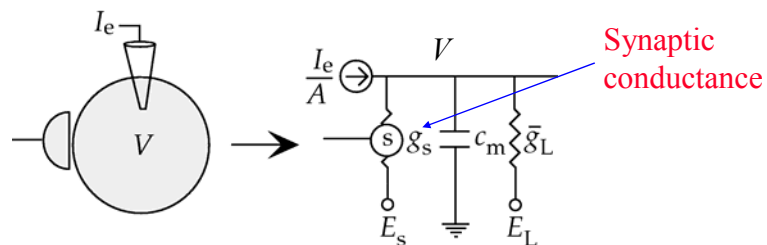
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- ◆ Neural Encoding
  - ⇒ What makes a neuron fire? (STA, covariance analysis)
  - ⇒ Poisson model
- ◆ Neural Decoding
  - ⇒ Stimulus Discrimination based on firing rate
  - ⇒ Spike-train based decoding of stimulus
  - ⇒ Population decoding (Bayesian estimation)
- ◆ Single Neuron Models
  - ⇒ RC circuit model of membrane
  - ⇒ Integrate-and-fire model
  - ⇒ Conductance-based and Compartmental Models

## Today's Agenda

- ◆ Computation in Networks of Neurons
  - ⇒ From spiking to firing-rate based networks
  - ⇒ Feedforward Networks
    - ◆ E.g. Coordinate transformations in the brain
  - ⇒ Linear Recurrent Networks
    - ◆ Can amplify inputs
    - ◆ Can integrate inputs
    - ◆ Can function as short-term memory

## Flashback Modeling Synaptic Inputs from other Neurons



$$\tau_m \frac{dV}{dt} = -(V - E_L) - r_m g_s (V - E_s) + I_e R_m$$

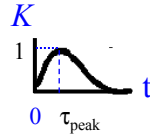
$$g_s = g_{s, \max} P_{rel} P_s$$

← Probability of postsynaptic channel opening  
(= fraction of channels opened)  
← Probability of transmitter release given an input spike

## Flashback 2 Simplified Synapse Model

◆ “Alpha Function” model:

Synaptic kernel  $K(t) = \frac{t}{\tau_{peak}} e^{-\frac{t}{\tau_{peak}}}$



Synaptic current:  $I_s(t) = w_s \int_{-\infty}^t K(t-\tau) \rho_s(\tau) d\tau$

where  $\rho_s(t)$  is the input spike train:

$$\rho_s(\tau) = \sum_i \delta(\tau - t_i) \quad (t_i \text{ are the spike times})$$

## Modeling Networks of Neurons

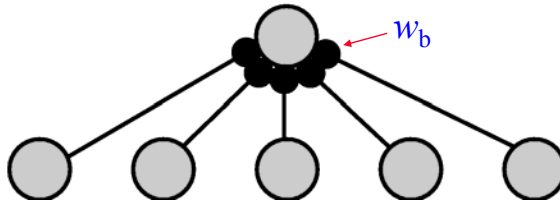
- ◆ **Option 1:** Use *spiking* neurons (e.g. I & F neurons)
  - ⇨ *Advantages:* Allows computation and learning based on:
    - ◆ Spike Timing
    - ◆ Spike Correlations/Synchrony between neurons
  - ⇨ *Disadvantages:* Computationally expensive
- ◆ **Option 2:** Use neurons with *firing-rate outputs*
  - ⇨ *Advantages:* Greater efficiency, scales well to large networks
  - ⇨ *Disadvantages:* Ignores spike timing issues
- ◆ **Question:** How are these two approaches related?

## Network Notation

output  $v$

weights  $\mathbf{w}$

input  $\mathbf{u}$



Current at synapse  $b$   $I_b(t) = w_b \int_{-\infty}^t K(t-\tau) \rho_b(\tau) d\tau$  Spike train  $\rho_b(t)$

$\approx w_b \int_{-\infty}^t K(t-\tau) u_b(\tau) d\tau$  Firing rate  $u_b(t)$

Total synaptic current  $I_s(t) = \sum_b I_b(t)$

## Synaptic Current Dynamics

- ◆ If synaptic kernel  $K$  is an exponential function:  $K(t) = e^{-t/\tau_s} / \tau_s$

Differentiating  $I_s(t) = \sum_b w_b \int_{-\infty}^t K(t-\tau) u_b(\tau) d\tau$

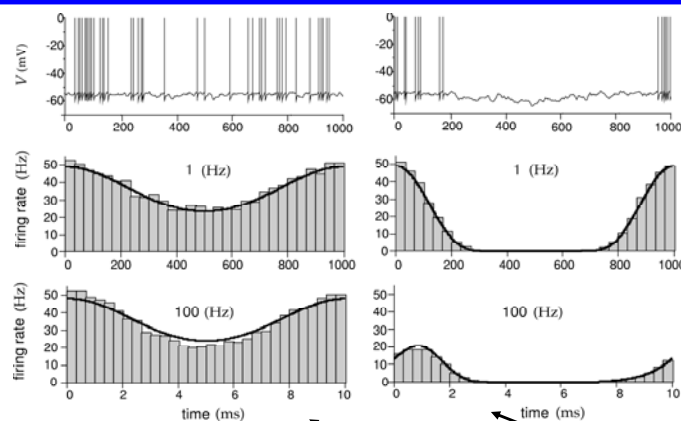
We get  $\tau_s \frac{dI_s}{dt} = -I_s + \sum_b w_b u_b$

$= -I_s + \mathbf{w} \cdot \mathbf{u}$

## Output Firing-Rate Dynamics

- ◆ How is the output firing rate  $\nu$  related to synaptic inputs?
- ◆ On-board derivations...  
(see also pages 234-236 in the text)

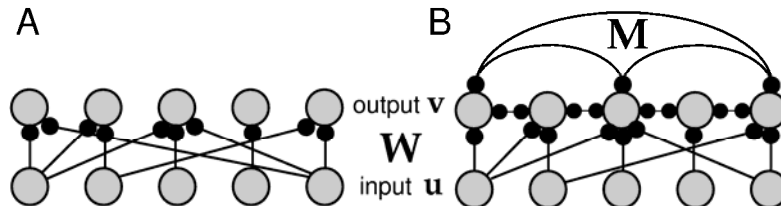
## How good are the Firing Rate Models?



Firing rate  $\nu(t) = F(I(t))$  describes this well but not this case

Input  $I(t) = I_0 + I_1 \cos(\omega t)$

## Feedforward versus Recurrent Networks

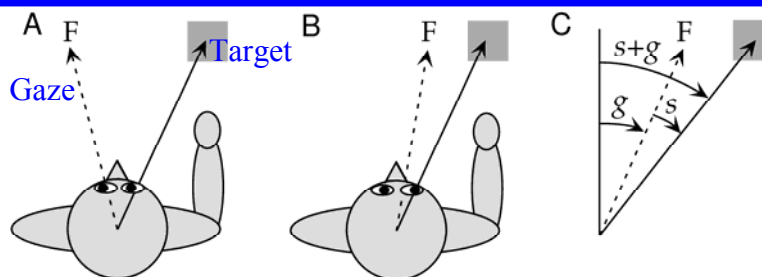


$$\tau \frac{d\mathbf{v}}{dt} = -\mathbf{v} + F(\mathbf{W}\mathbf{u} + \mathbf{M}\mathbf{v})$$

Output    Decay    Input    Feedback

(For feedforward networks, matrix  $\mathbf{M} = 0$ )

## The Problem of Coordinate Transformations



$g$  = gaze angle *relative to body*

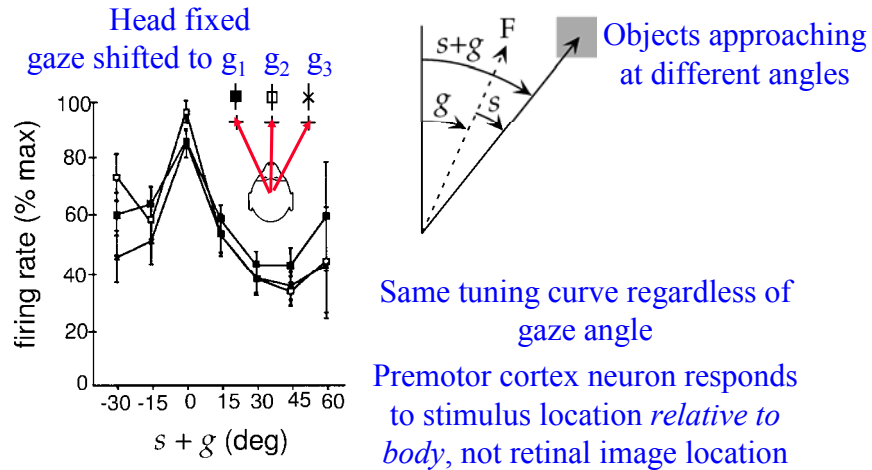
$s$  = stimulus or target angle *relative to gaze (retinal coordinates)*

$s+g$  = stimulus relative to body

Same arm movement required in A and B but  $s$  and  $g$  are different

How does the brain solve this problem?

## Body-Based Representation in the Monkey

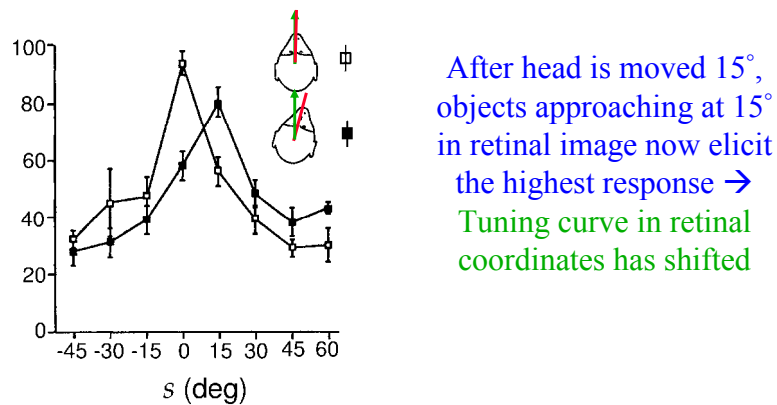


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## Body-Based Representation in the Monkey

When head is moved but  
gaze remains unchanged:



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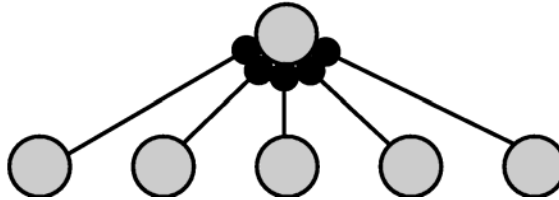
## Suggested Feedforward Network

Output: Premotor Cortex Neuron with Body-Based Tuning Curves

output  $v$

weights  $\mathbf{w}$

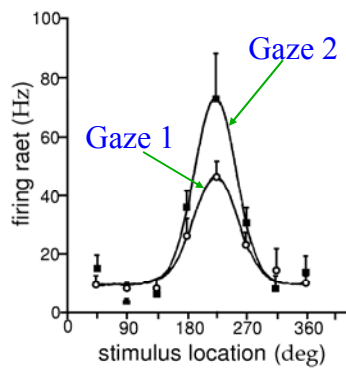
input  $\mathbf{u}$



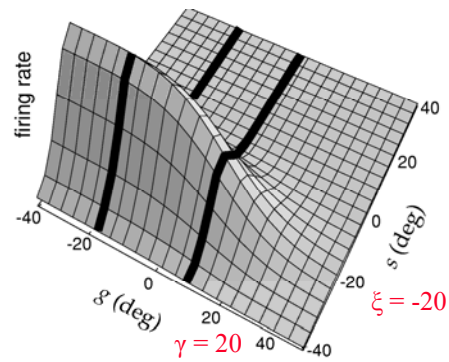
Input: Area 7a Neurons with Gaze-Dependent Tuning Curves

Input neurons exhibit gaze-dependent gain modulation

## Gaze-Dependent Gain Modulation



Responses of Area 7a neuron



Example of a gain-modulated tuning curve



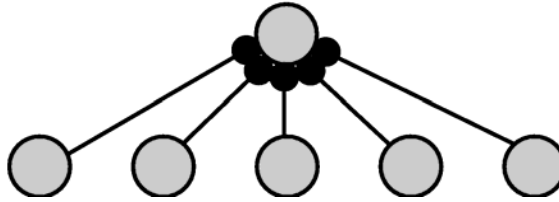
## What should the weights be?

Output: Premotor Cortex Neuron with Body-Based Tuning Curves

output  $v$

weights  $w$

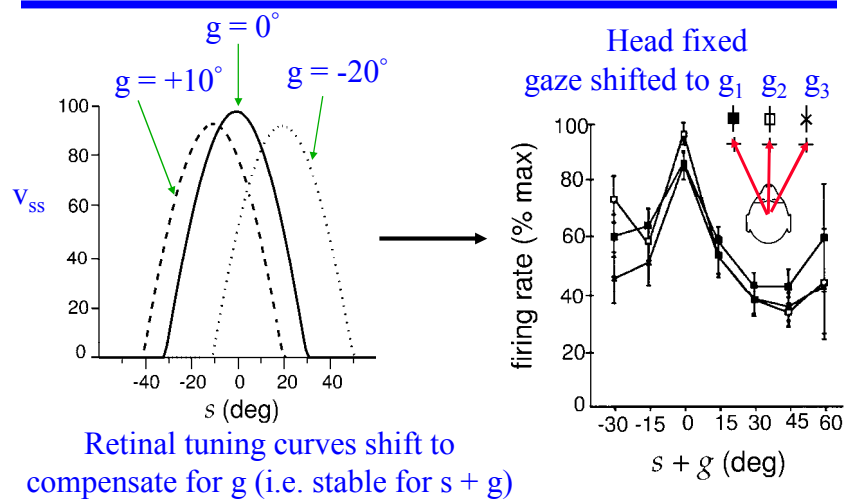
input  $u$



Input: Area 7a Neurons with Gaze-Dependent Tuning Curves

Weights  $w(\xi, \gamma)$  need to be a function of  $\xi + \gamma$

## Output of a Simulated Feedforward Network



## Next Class: More on Networks

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- ◆ Things to do:
  - ⇒ Finish reading Chapter 7
  - ⇒ Homework #3 due next Tuesday
  - ⇒ Start working on mini-project