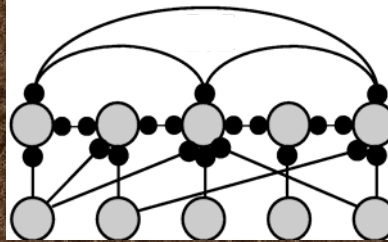


## CSE/NB 528

### Final Lecture: All Good Things Must...



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Lecture figures are from Dayan & Abbott's book  
<http://people.brandeis.edu/~abbott/book/index.html>

### Course Summary (All good things must...come to an end)

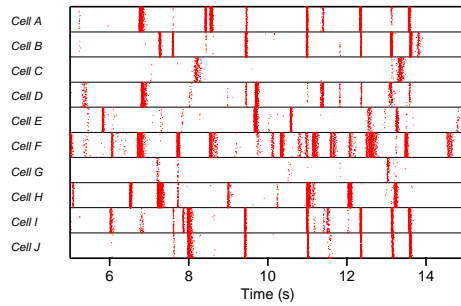
- ◆ **Where have we been?**
  - ⇒ Course Highlights
- ◆ **Where do we go from here?**
  - ⇒ Challenges and Open Problems
- ◆ **Further Reading**

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## What is the neural code?

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*What is the nature of the code?*

How should we represent the spiking output?

single cells vs populations

rates vs spike times vs intervals

What features of the stimulus does the neural system represent?

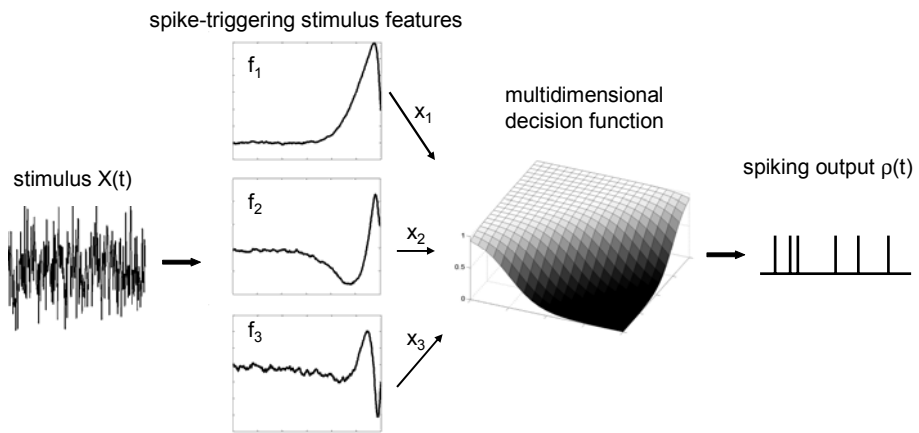
## Encoding and decoding neural information

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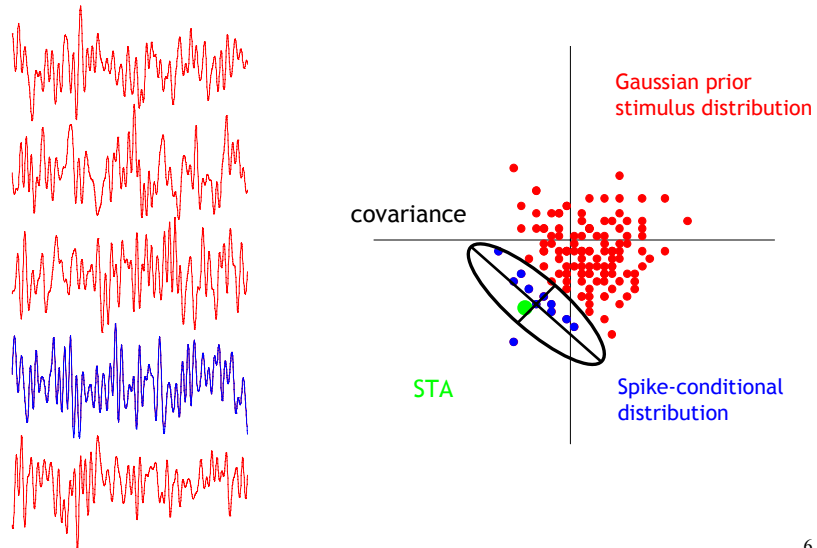
*Encoding:* building functional models of neurons/neural systems and predicting the spiking output given the stimulus

*Decoding:* what can we say about the stimulus given what we observe from the neuron or neural population?

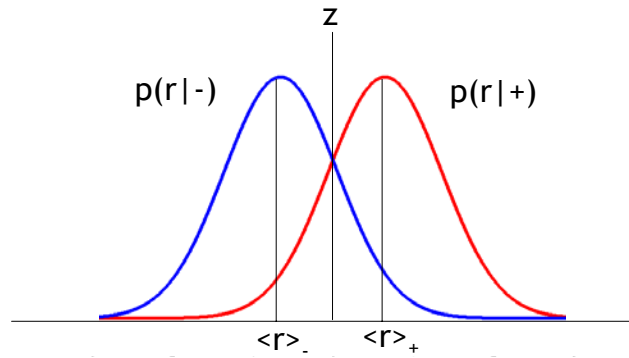
## Highlights: Neural Encoding



## Highlights: Finding the feature space of a neural system



## Decoding: Signal detection theory



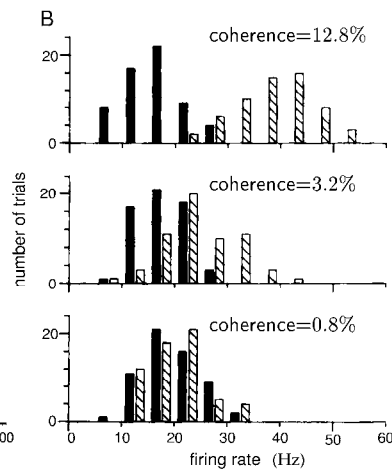
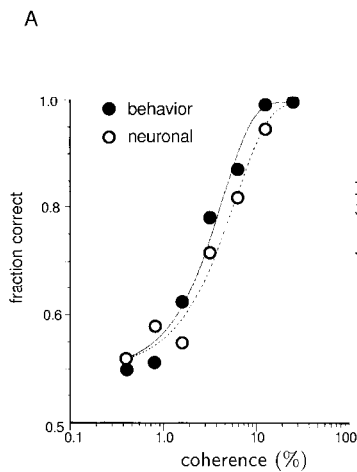
Decoding corresponds to comparing test to threshold.

$$\alpha(z) = P[r \geq z | -]$$

false alarm rate, “size”

$$\beta(z) = P[r \geq z | +]$$

hit rate, “power”

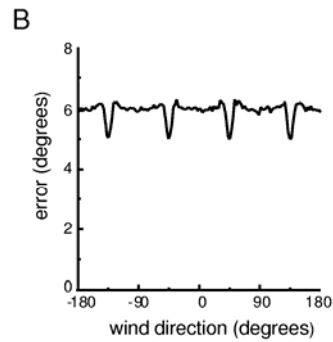
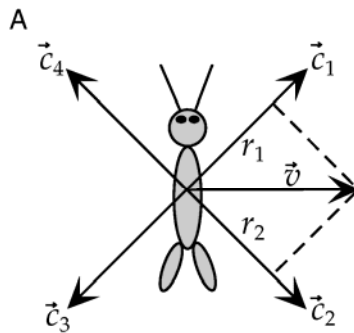


Close correspondence between neural performance and behaviour..

## Decoding from a population

e.g. cosine tuning curves

$$\vec{v}_{\text{pop}} = \sum_{a=1}^4 \left( \frac{r}{r_{\text{max}}} \right)_a \vec{c}_a$$



RMS error in estimate

Theunissen & Miller, 1991

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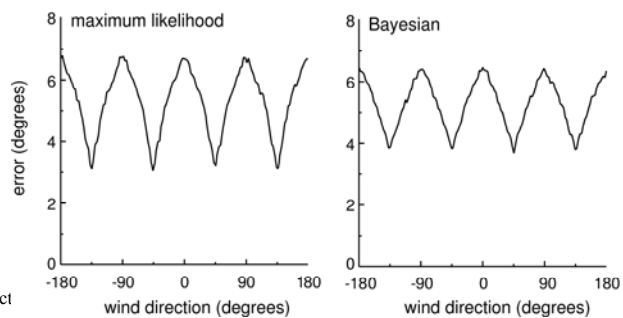
## More general approaches: MAP and ML

MAP:  $s^*$  which maximizes  $p[s|r]$

ML:  $s^*$  which maximizes  $p[r|s]$

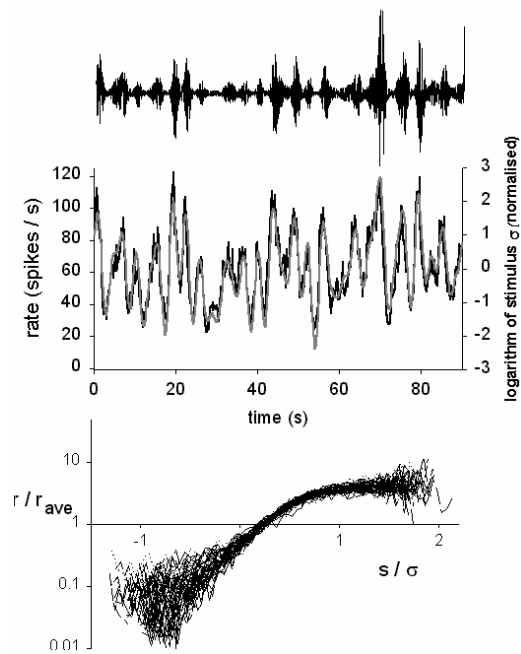
Difference is the role of the prior: differ by factor  $p[s]/p[r]$

For cercal data:



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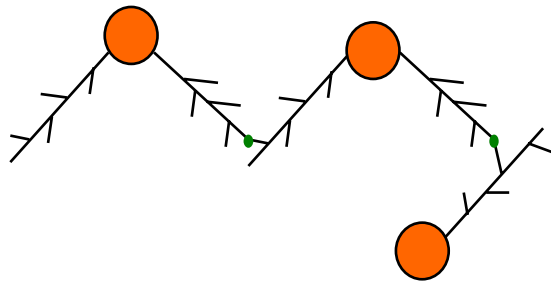
Highlights:  
Information  
maximization  
as a design  
principle of the  
nervous system



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## Biophysical Models

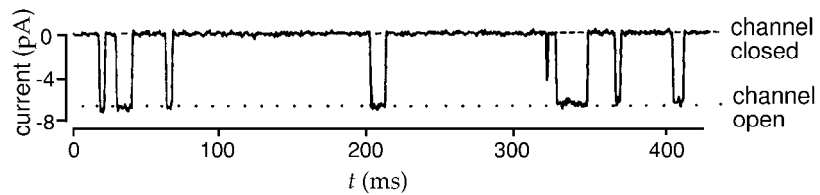
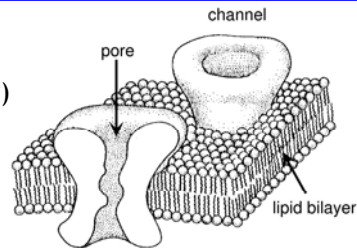
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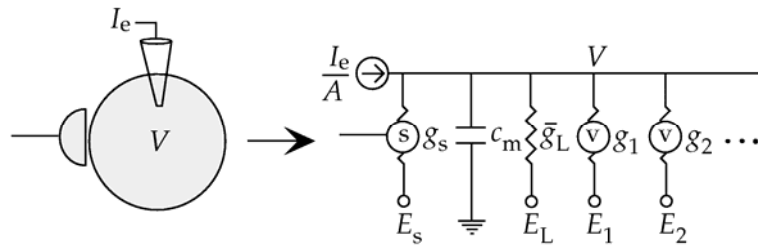
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## Excitability is due to the properties of ion channels

- Voltage dependent
- transmitter dependent (synaptic)
- Ca dependent



## Highlights: The neural equivalent circuit



Ohm's law:  $V = IR$  and Kirchoff's law

$$-C_m \frac{dV}{dt} = \sum_i g_i (V - E_i) + I_e$$

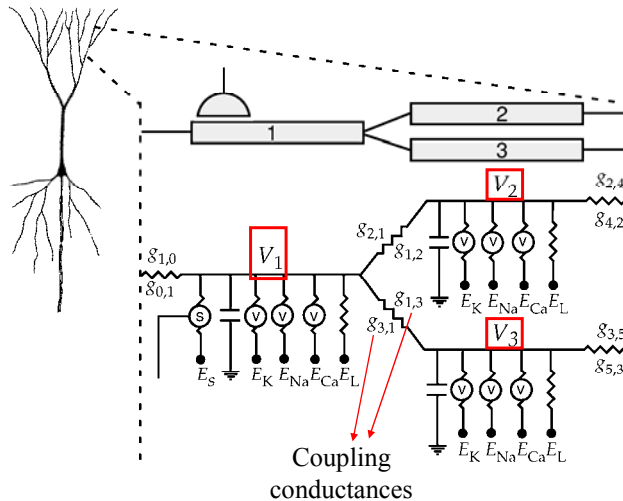
Capacitive  
current

Ionic currents

Externally  
applied current

## Highlights: Compartmental Models

Neuronal structure  
can be modeled  
using electrically  
coupled  
compartments



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## Simplified neural models

A sequence of neural models of increasing complexity  
that approach the behavior of real neurons

*Integrate and fire neuron:*

subthreshold, like a passive membrane  
spiking is due to an imposed threshold at  $V_T$

*Spike response model:*

subthreshold, arbitrary kernel  
spiking is due to an imposed threshold at  $V_T$   
postspike, incorporates afterhyperpolarization

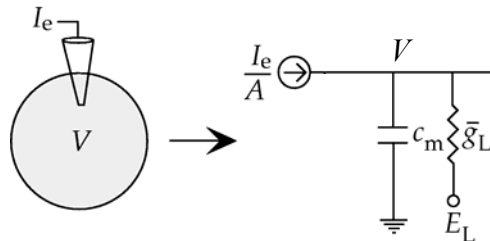
*Simple model:*

complete 2D dynamical system  
spiking threshold is intrinsic  
have to include a reset potential

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## Simplified Models: Integrate-and-Fire

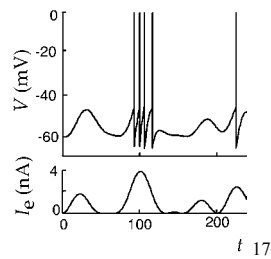


Integrate-and-Fire Model

$$\tau_m \frac{dV}{dt} = -(V - E_L) + I_e R_m$$

If  $V > V_{\text{threshold}} \rightarrow \text{Spike}$

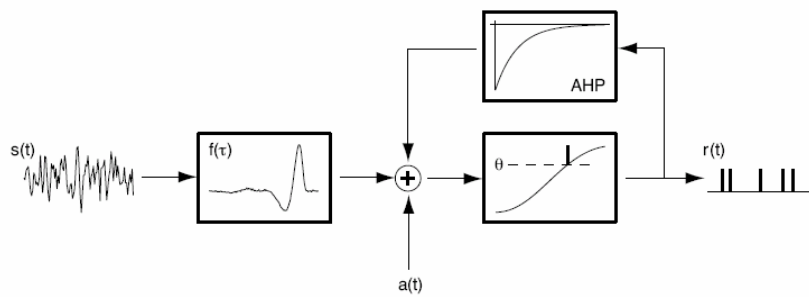
Then reset:  $V = V_{\text{reset}}$



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## Simplified Models: Spike response model

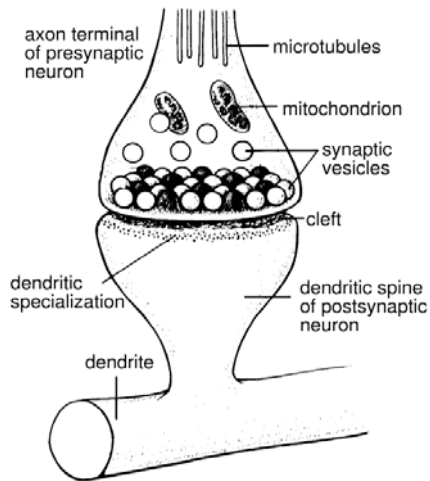


Keat, Reinagel and Meister

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## Connecting neurons: Synapses



Presynaptic voltage spikes cause neurotransmitter to cross the cleft, triggering postsynaptic receptors allowing ions to flow in, changing postsynaptic potential

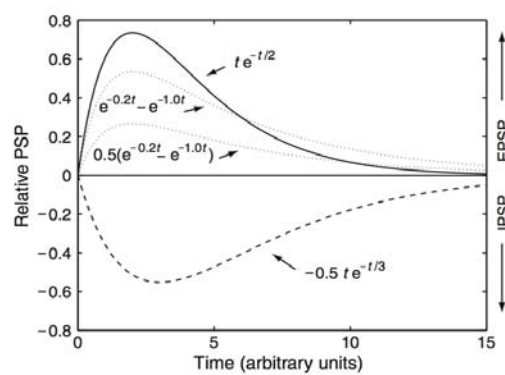
Glutamate: excitatory

GABA<sub>A</sub>: inhibitory

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## Synaptic voltage changes



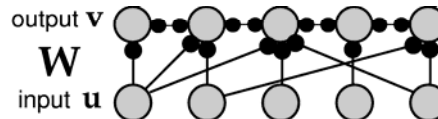
Size of the PSP is a measure of synaptic strength.  
 Can vary on the short term due to input history  
 on the long term due to synaptic plasticity  
 .. one way to build circuits that learn

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## Modeling Networks of Neurons

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$$\tau \frac{d\mathbf{v}}{dt} = -\mathbf{v} + F(\mathbf{W}\mathbf{u})$$

Output      Decay      Input

## Highlights: Unsupervised Learning

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◆ For linear neuron:  $\mathbf{v} = \mathbf{w}^T \mathbf{u} = \mathbf{u}^T \mathbf{w}$

◆ Basic Hebb Rule:  $\tau_w \frac{d\mathbf{w}}{dt} = \mathbf{u}\mathbf{v}$



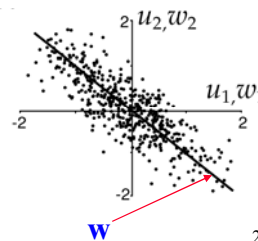
◆ Average effect over many inputs:

$$\tau_w \frac{d\mathbf{w}}{dt} = \langle \mathbf{u}\mathbf{v} \rangle = \mathbf{Q}\mathbf{w}$$

◆  $\mathbf{Q}$  is the input correlation matrix:

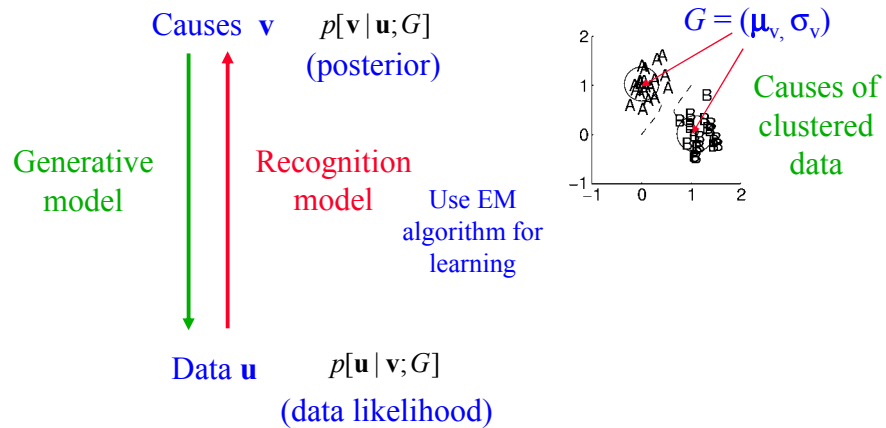
$$\mathbf{Q} = \langle \mathbf{u}\mathbf{u}^T \rangle$$

Hebb rule performs principal component analysis (PCA)



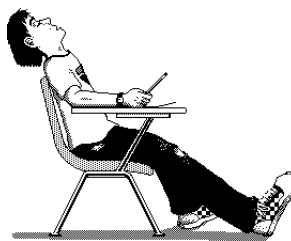
## Highlights: The Connection to Statistics

Unsupervised learning = learning the *hidden causes* of input data



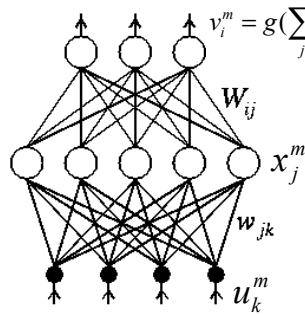
## Highlights: Generative Models

Droning lecture    Lack of sleep    Mathematical derivations



## Highlights: Supervised Learning

### Backpropagation for Multilayered Networks



$$v_i^m = g\left(\sum_j W_{ij} g\left(\sum_k w_{jk} u_k^m\right)\right)$$

Goal: Find  $\mathbf{W}$  and  $\mathbf{w}$  that minimize errors:

$$E(W_{ij}, w_{jk}) = \frac{1}{2} \sum_{m,i} (d_i^m - v_i^m)^2$$

Desired output

Gradient descent learning rules:

$$W_{ij} \rightarrow W_{ij} - \epsilon \frac{\partial E}{\partial W_{ij}} \quad (\text{Delta rule})$$

$$w_{jk} \rightarrow w_{jk} - \epsilon \frac{\partial E}{\partial w_{jk}} = w_{jk} - \epsilon \frac{\partial E}{\partial x_j^m} \cdot \frac{\partial x_j^m}{\partial w_{jk}} \quad (\text{Chain rule})$$

## Highlights: Reinforcement Learning

- Learning to predict rewards:

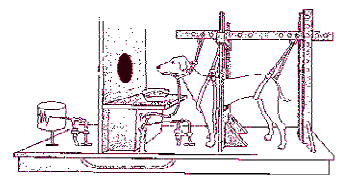
$$w \rightarrow w + \epsilon(r - v)u$$

- Learning to predict **delayed rewards** (TD learning):

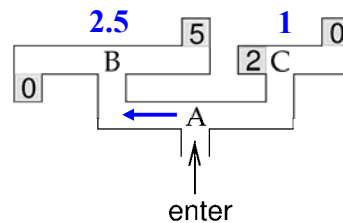
$$w(\tau) \rightarrow w(\tau) + \epsilon [r(t) + v(t+1) - v(t)] u(t - \tau)$$

- Actor-Critic Learning:

- ⇒ Critic learns value of each state using TD learning
- ⇒ Actor learns best actions based on value of next state (using the TD error)



(<http://employees.csbsju.edu/tcreed/pb/pdoganim.html>)



## The Future: Challenges and Open Problems

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- ◆ **How do neurons encode information?** Rate-Based versus Temporal Coding
  - ⇒ **Topics:** Synchrony, Spike-timing based learning, Dynamic synapses
- ◆ **Does a neuron's structure confer computational advantages?**
  - ⇒ **Topics:** Role of dendrites, plasticity in channels and their density
- ◆ How do networks of neurons implement computational principles such as **efficient coding** and **Bayesian inference**?
- ◆ How do networks of neurons learn “**optimal**” representations of their environment and engage in **purposeful behavior**?
  - ⇒ **Topics:** Unsupervised/reinforcement/imitation learning in the brain
- ◆ **Applications:** Machine learning, robotics, brain-machine interfaces

## Further Reading (for the summer and beyond)

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- ◆ *Spikes: Exploring the Neural Code*, F. Rieke et al., MIT Press, 1997
- ◆ *The Biophysics of Computation*, C. Koch, Oxford University Press, 1999
- ◆ *Large-Scale Neuronal Theories of the Brain*, C. Koch and J. L. Davis, MIT Press, 1994
- ◆ *Probabilistic Models of the Brain*, R. Rao et al., MIT Press, 2002
- ◆ *Bayesian Brain*, K. Doya et al., MIT Press, 2007
- ◆ *Reinforcement Learning: An Introduction*, R. Sutton and A. Barto, MIT Press, 1998



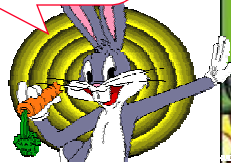
## Next meeting: Project presentations!

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- ◆ Project presentations will be on **Monday, June 4 (10:30am-12:20pm) in the same classroom**
- ◆ Keep your presentation short: **8-10 slides, 15 mins/group**
- ◆ Slides:
  - ⇒ Email Raj your powerpoint (or other) slides **before 8am on Monday, June 4** to use the class laptop
  - OR**
  - ⇒ Bring your own laptop if you want to show videos etc.
- ◆ Projects reports (10-15 pages) also due same day (by email to both Raj and Adrienne before end of the day)
- ◆ No classes next week – work on your projects!

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Have a great summer!



Au revoir!

